Train of Diverse Multifrequency Radar Pulses

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Abstract— A multifrequency pulse can be generated in many different permutations that are nearly orthogonal to each other. This property allows generating a coherent train of diverse pulses whose ambiguity function is devoid from most of the recurrent lobes found in a coherent train of identical pulses. The ambiguity function volume, removed from the recurrent lobes, fills and raises the sidelobe pedestal.

I. Introduction

Multifrequency complementary phase-coded (MCPC) radar pulse signal was introduced in 2000 [1,2]. A single pulse employs simultaneously N subcarriers, spaced $1/t_b$ apart. Each subcarrier is phase modulated by a complex sequence of M bits, each of duration t_b. A schematic description of an NxM MCPC pulse appears in Fig. 1, where N=M=5. The spectrum of such a pulse is relatively flat with bandwidth of N/t_b. While each subcarrier exhibits fixed real amplitude (due to phase modulation), their sum exhibits variable real amplitude. This drawback of the MCPC signal can be somewhat mitigated by using a limiter.

The $1/t_b$ spacing between subcarriers creates orthogonal frequency division multiplexing (OFDM). Due to this orthogonality the values of the ambiguity function on the grid points $|\chi(mt_b, n/t_b)|$ are equal to the corresponding values of the 2-D autocorrelation of the NxM complex array describing the signal. When the N sequences constitute a complementary set, the elements of the middle row of the 2-D autocorrelation are all zero. Hence the autocorrelation function of the MCPC pulse exhibits absolute nulls at multiples of t_b, which also says that $|\chi(mt_b, 0)| = 0$ for $m \neq 0$. An example based on a simple 4x4 MCPC pulse is shown in Tables 1 and 2 and Figs. 2 and 3. Table 1 presents the complementary set, which in this example is real and binary. The 2-D autocorrelation of the set is presented in Table 2. Note the zero sidelobes in the middle row, which corresponds to the delay axis (zero Doppler). Fig. 2 is a graphical rendering of the absolute values of Table 2. The values of the peaks (after division by MN=16) should match the values of the ambiguity function at the corresponding grid points. Fig. 3 presents quadrants 1 and 2 of the ambiguity function. Indeed we note the nulls at the grid points of the zero-Doppler cut. We can visually match peaks between Figs. 2 and 3, but Fig. 3 contains additional peaks and nulls off the grid points. Note in Fig. 3 that the delay mainlobe width is t_b/N implying a pulse compression ratio of NM. Practical Doppler values are not likely to reach the first Doppler grid line, namely $|\nu| << 1/t_b$. Our interest in the spread of the ambiguity function sidelobes to higher Doppler values is related to the constant-volume property. The further the sidelobe pedestal extends in Doppler, the lower its average height becomes.

A simple way to generate a complementary set of size MxM

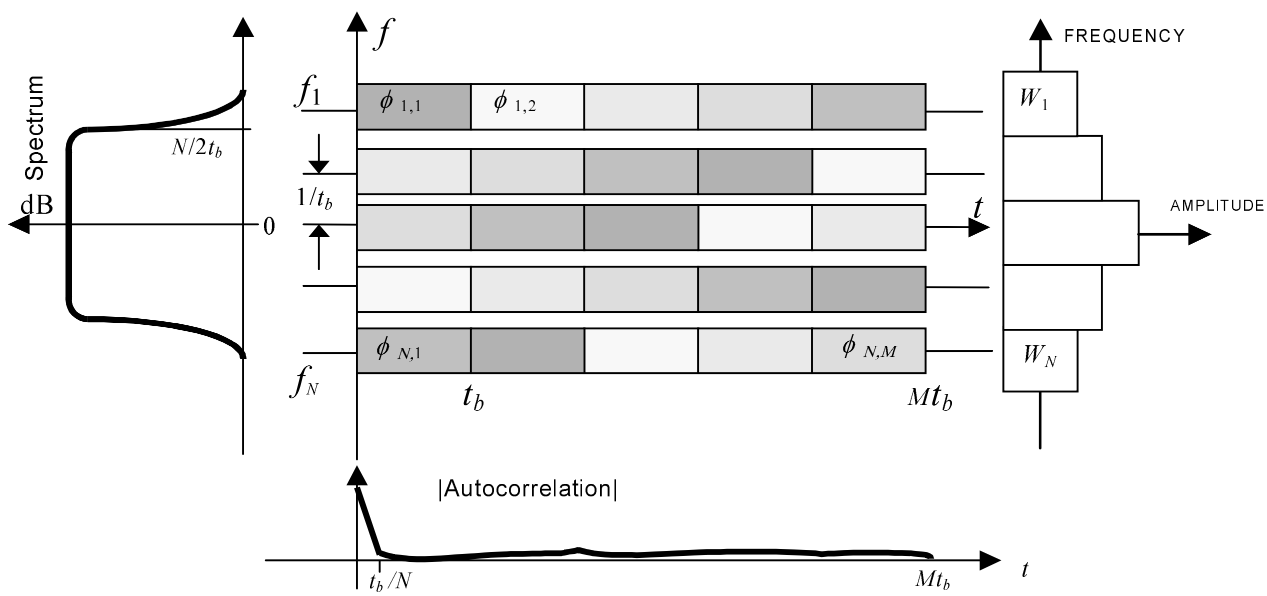

Fig. 1. Structure of an NxM MCPC pulse

TABLE 1
4x4 COMPLEMENTARY SET

1	1	-1	1
-1	1	1	1
1	-1	1	1
1	1	1	-1

TABLE 2
2-D AUTOCORRELATION OF THE SET ABOVE

-1	0	3	0	1	0	1
2	0	-2	0	6	0	2
-1	4	3	0	1	4	1
0	0	0	16	0	0	0
1	4	1	0	3	4	-1
2	0	6	0	-2	0	2
1	0	1	0	3	0	-1

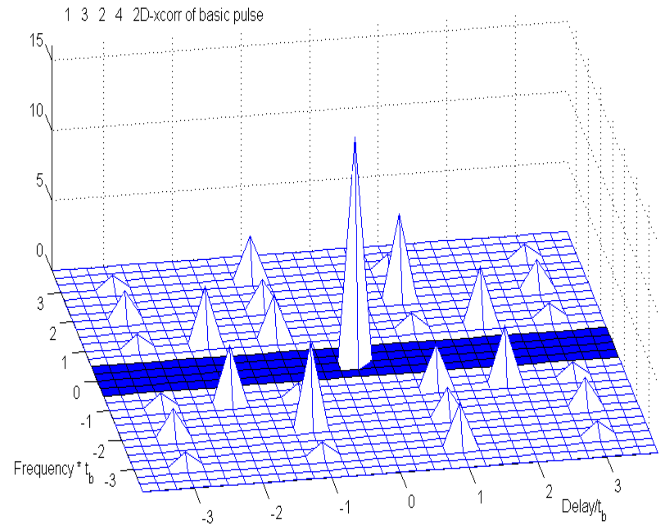

Fig. 2. Graphical rendering of the 2-D autocorrelation in Table 2

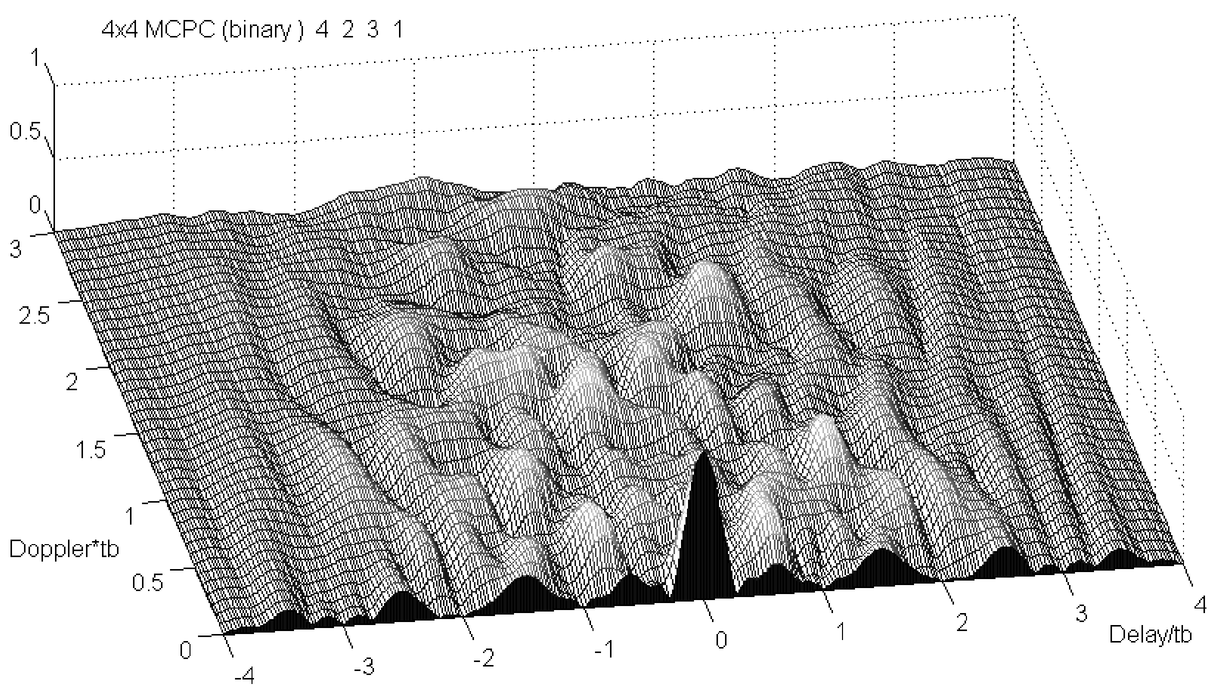

Fig. 3. Ambiguity function (Quadrants 1 and 2) of the 4x4 MCPC pulse defined in Table 1.

is to use a basic sequence of length *M*, that exhibits ideal periodic autocorrelation (e.g., P3, P4, Golomb), and to construct the *M* sequences from all the *M* cyclic shifts of that sequence [3]. Henceforth, we will discuss mainly *M*x*M* MCPC pulses. Note that an *M*x*M* complementary set remains complementary for all the *M*! possible permutations of ordering the rows of the set. Different permutations yield different sidelobe patterns of the ambiguity function and different peak-to-mean envelope power ratios (PMEPR).

II. DIVERSE PULSE TRAIN

The present paper emphasizes the *periodic* ambiguity function [4] of a train of *M* diverse *M*x*M* MCPC pulses with frequency weighting. Diversity is created by generating each pulse as a different cyclic frequency shift of the first pulse, *thus creating a complementary set along each frequency, as well as along each pulse*. Here too there are *M*! different ways to order the *M* pulses. This type of pulse diversity produces two favorable results:

(a) Such a diversity reduces the autocorrelation sidelobes for the delay range $t_b < |\tau| < Mt_b$. When combined with frequency weighting, sidelobe reduction can be extended over the entire sidelobe duration, namely $t_b/M < |\tau| < Mt_b$.

(b) Diversity eliminates recurrent lobes at multiples of the pulse repetition interval T_r. Recurrent lobes are found in the ambiguity function of a coherent train of identical pulses. There are applications where recurrent lobes are unacceptable, and pulse diversity was suggested as a remedy, for example by changing the chirp rate of Linear FM pulses [5]. However, the near-perfect orthogonality between diverse MCPC pulses can produce complete nulls where recurrent lobes were expected.

Frequency weighting can be easily implemented by assigning different amplitude to each subcarrier, as indicated on the r.h.s. of Fig. 1. Since, constant amplitude was given-up already, there is no reason to limit this kind of frequency weighting to the receiver (which will prevent it from being a matched filter). Hence, we will equally split frequency weighting between transmitter and receiver, and each side will implement the square root of the weight window (e.g., Hamming). As a matter of fact, it was noted that a somewhat lower exponent α (< 0.5) yields better sidelobe reduction.

Ambiguity function sidelobes along the Doppler axis can be reduced, by *interpulse time weighting* at the receiver. Such weighting degrades complementarity along each frequency.

The performances of a coherent train of eight diverse 8x8 MCPC pulses will be compared to a train of eight identical P3 pulses with 64 elements (yielding the same pulse compression). Both frequency and time weighting will be used. In the P3 signal, frequency weighting is implemented through *intrapulse* time weighting. The real envelopes of the transmitted and reference signals are presented in Fig. 4.

According to the constant-volume property of the ambiguity function (which applies also to the *periodic* ambiguity function) the volume lost by eliminating the recurrent lobes must reappear somewhere else. Indeed, this is demonstrated in Fig. 5 by comparing a partial plot of the periodic autocorrelation function of a train of the eight diverse 8x8 MCPC pulses (top) with a similar plot of the train of eight identical P3 pulses (bottom). The corresponding ambiguity functions appear in Figs. 6 and 7. The comparison shows that for MCPC, the volume removed from the missing recurrent lobes, reappears in the form of higher sidelobe pedestal strips around each multiple of the pulse repetition interval, namely over ambiguity function strips defined by $t_b/M < |\tau - nT_r| < Mt_b$, $n = 1,2,\ldots$. Note from Fig. 5 that the autocorrelation around the mainlobe (n=0) is lower than around recurrent lobes. It would have been even lower without interpulse weighting. In Figs. 5-7 we set $T_r = 4Mt_b$.

Two-valued signals with ideal periodic autocorrelation, suggested by Golomb [6], can also serve as a basis for MCPC pulse train. An example of length 15 is based on the sequence

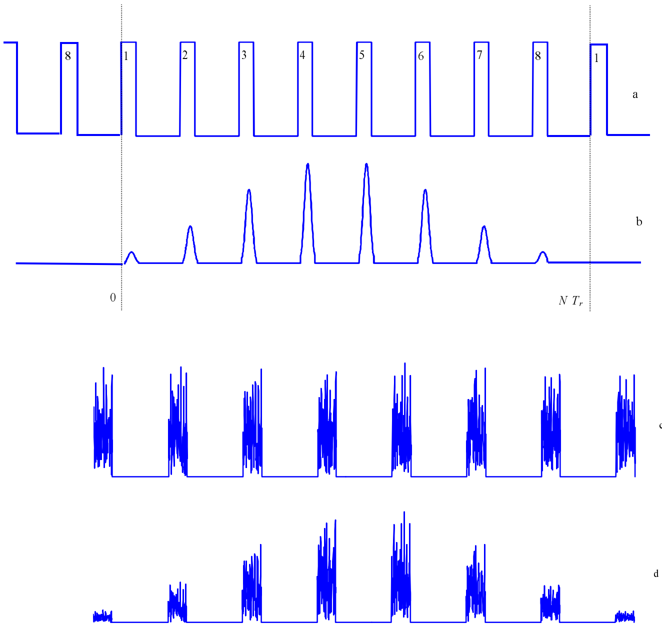

Fig. 4. Real envelope of: (a) Transmitted P3 pulses, (b) Reference P3 pulses with intrapulse (frequency) and interpulse weighting, (c) Trans. diverse 8x8 MCPC pulses, (d) Ref. MCPC pulses with interpulse weighting.

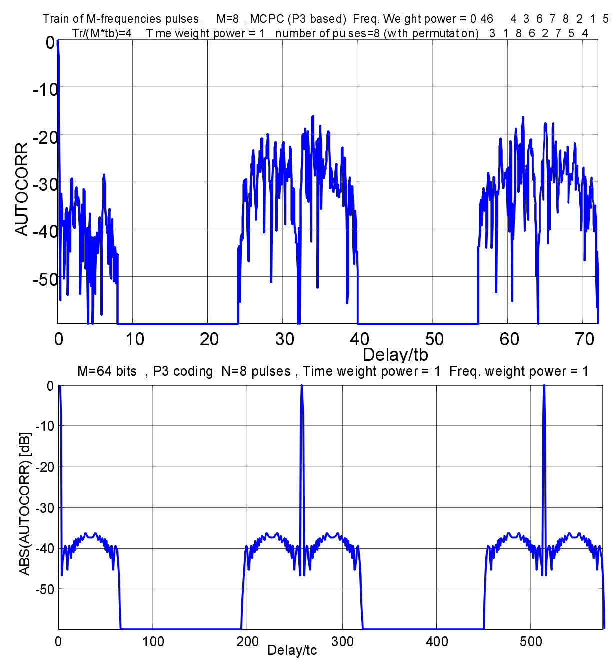

Fig. 5. Partial autocorrelation in dB of a train of 8 MCPC (top) and P3 (bottom) pulses.

$\{1\ 1\ 1\ 1\ h\ 1\ h\ 1\ 1\ h\ h\ 1\ h\ h\ h\}$, $h = \exp\left[j\cos^{-1}(-7/8)\right]$. The partial ambiguity function (the mainlobe and its pedestal) is given in Fig. 8. The autocorrelation sidelobes for $t_b < |\tau| < Mt_b$ are below –35 dB (–52 dB without interpulse weighting). Complementary diversity can also be applied to a single-frequency pulse-train, but the number of pulses must equal the compression ratio, not its square root, as in MCPC.

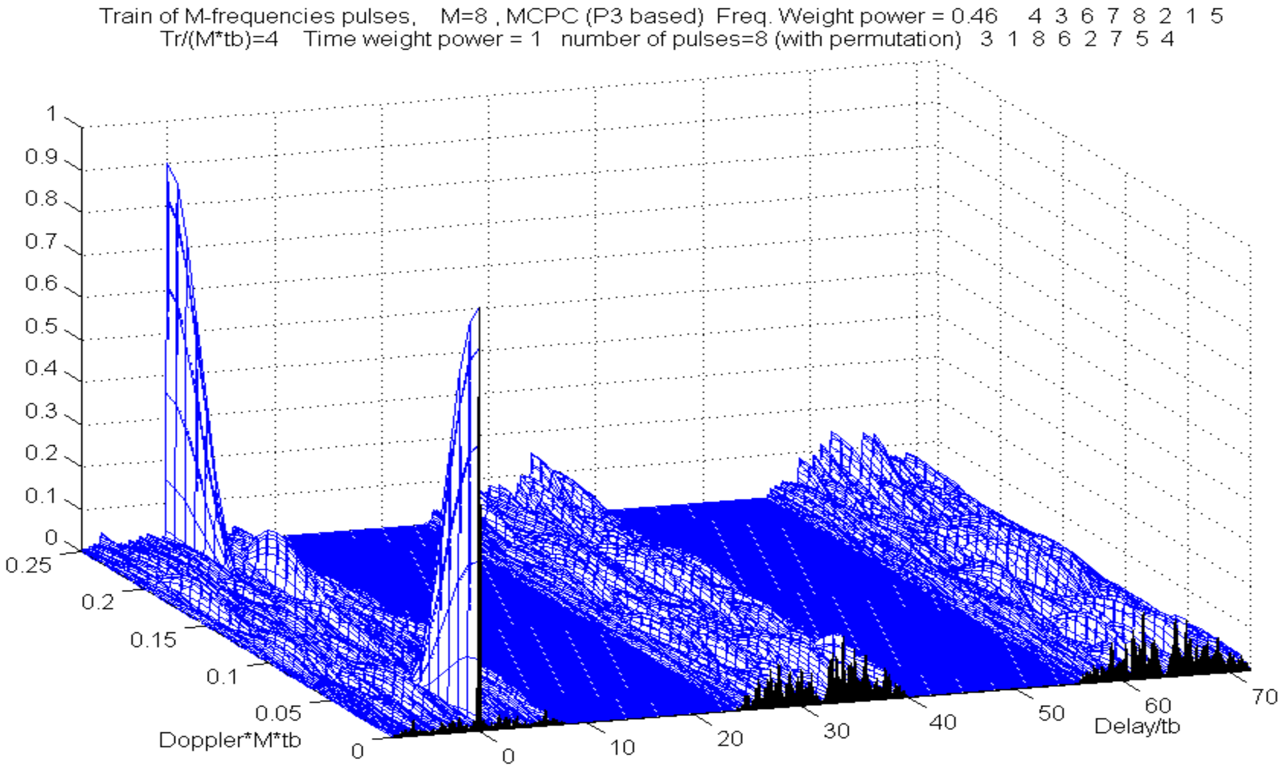

Fig. 6. Partial periodic ambiguity function of eight diverse 8x8 MCPC pulses (P3 based)

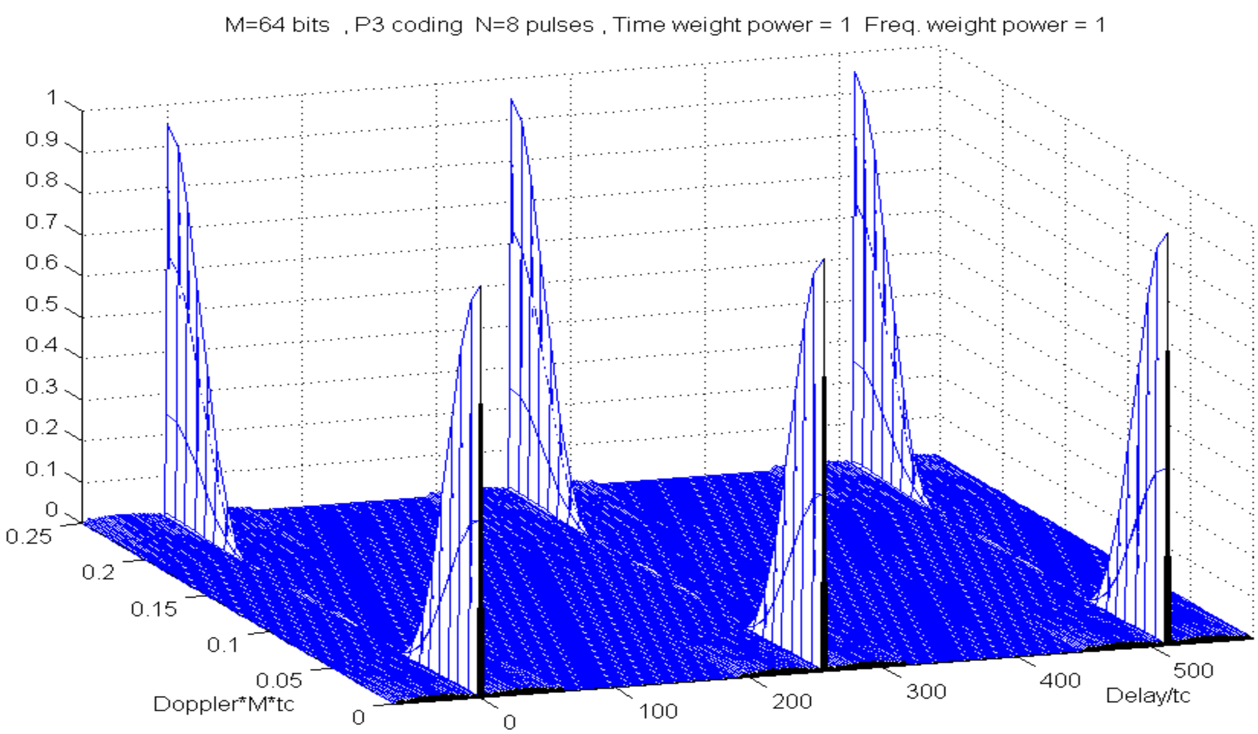

Fig. 7 Partial periodic ambiguity function of eight P3 pulses (64 elements).

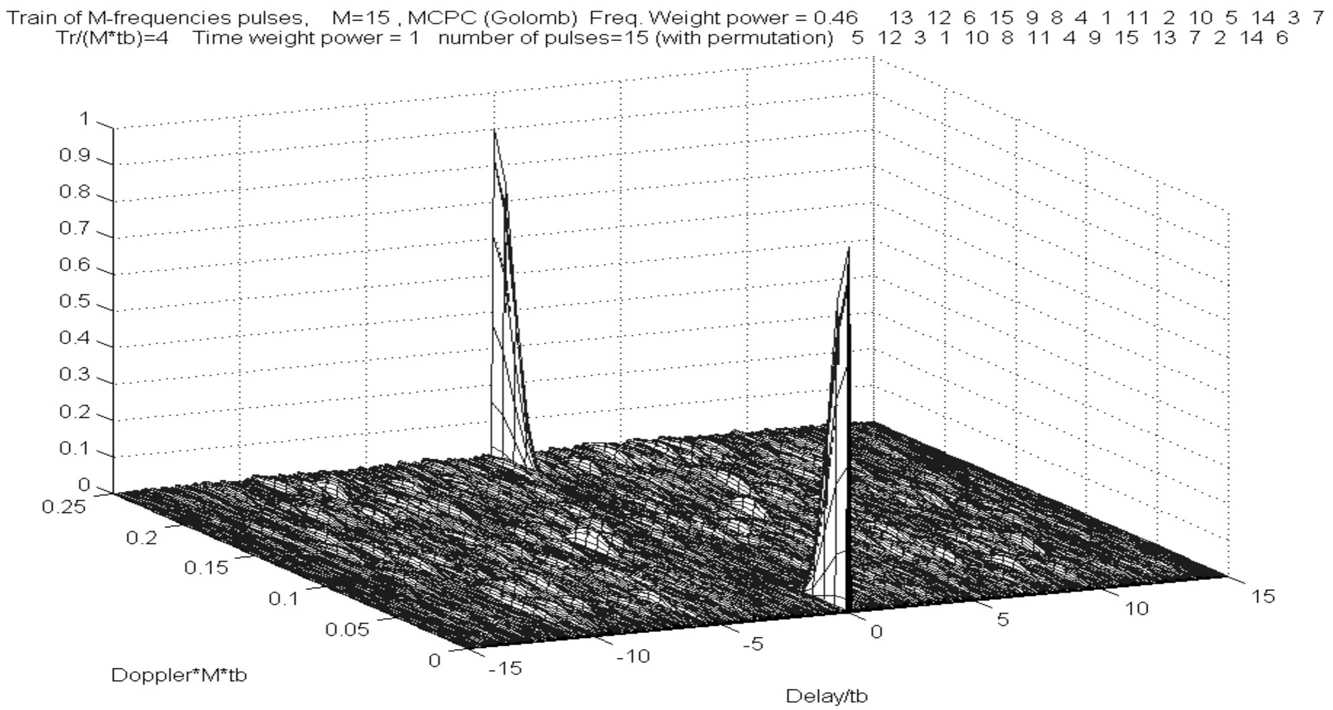

Fig. 8. Partial ambiguity function of a diverse train of fifteen 15x15 MCPC Golomb pulses with frequency and time weighting.

III. CW MODE

In continuous-wave mode, MCPC signals compete against single-frequency signals with ideal periodic autocorrelation (e.g. P3, P4 or Golomb). Namely, against signals that, by definition, exhibit zero periodic autocorrelation sidelobes. MCPC signals do not have this property; hence their periodic autocorrelation cannot be completely free of sidelobes. However, we found out that a modified MCPC signal can be almost free of periodic autocorrelation sidelobes. The modified signal is based on N *consecutive* cyclic shifts of an M element sequence with ideal periodic autocorrelation (e.g. P3, P4 or Golomb). However, three restrictions exist: (a) If M is odd, N must be odd. (b) $N < M/2$. (c) Ordering the cyclically shifted sequences along the N carriers must be consecutive (e.g., 1,2,…,N; or 2,3,…,N+1; or in reverse).

When these restrictions are met, the periodic autocorrelation function exhibits zero sidelobe level for $Nt_b < \tau < (M-N)t_b$. The pulse compression becomes MN. Frequency weighting can still be used in CW. It will lower the remaining delay sidelobes but will widen the mainlobe.

For higher Doppler resolution, coherent processing can extend over several *identical* periods (no diversity in CW mode). If several periods are used, interperiod time weighting (continuous is better than stepwise) can be added in the receiver. Fig. 9 presents the periodic autocorrelation of a 9x63 MCPC signal based on the first 9 cyclic shifts of a length M=63 Golomb signal. The pulse compression ratio is 284 (not 567, because of frequency weighting). Note the complete elimination of sidelobes beyond Nt_b where N=9. Fig. 10 is a plot of the periodic ambiguity function of the same signal (the delay span is one period, Mt_b). Two additional parameters affect the ambiguity function and should be mentioned: (a) The reference signal contained P=8 periods, processed coherently, and (b) A smooth continuous Hamming weight function multiplied the reference signal (rather than a P-steps function). With uniform time weighting the Doppler resolution (first null) would have been $1/(PMt_b)$, but with prominent sidelobes along the Doppler axis at $\tau = 0, Mt_b, 2Mt_b, \ldots$. Because of the Hamming time

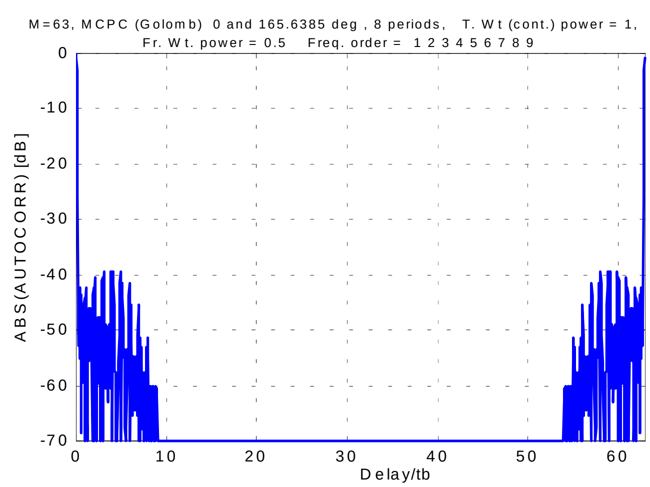

Fig. 9. Periodic autocorrelation of a 9x63 MCPC CW signal (Golomb)

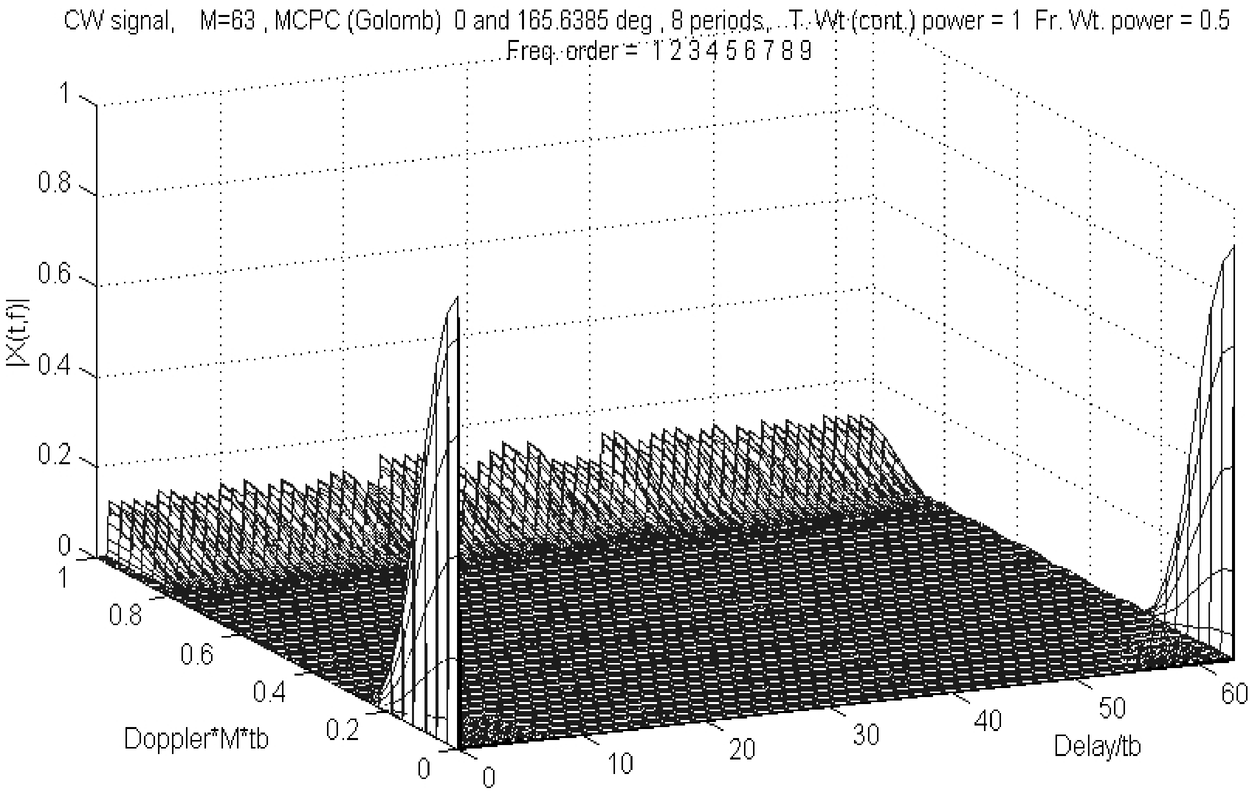

Fig. 10. Periodic ambiguity function of eight periods of a 9x63 MCPC CW signal

weighting, those Doppler sidelobes were drastically reduced. However, the main Doppler lobe (first null) now extends to $2/(PMt_b)=1/(4Mt_b)$. Note how the bulk of the sidelobes volume was pushed far in Doppler, to $\nu = 1/(Mt_b)$.

IV. Conclusions

Most pulse compression waveforms are defined by a vector. Our multicarrier signal is defined by an array. The added dimension provides more control over the ambiguity function. Using many carriers increases the bandwidth and reduces the delay resolution to a fraction of a bit-width. In order to reduce ambiguity sidelobes around zero Doppler, the array should constitute a complementary set. Frequency (i.e. row) weighting reduces delay sidelobes but violates the complementarity of the set. Using diverse coherent train of pulses corrects the problem by creating a complimentary set at each frequency. Furthermore, the frequency-cycled pulses are orthogonal enough to practically eliminate recurrent delay lobes. A single MxM MCPC pulse can be created in $M!$ permutations, most nearly-orthogonal. A train of M pulses can be created in $(M!)^2$ permutations. The orthogonality reduces susceptibility to mutual or intentional interference.

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