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Feature Article:

Waveforms Search for Noncoherent Pulse Compression

Nadav Levanon, Tel Aviv University, Tel Aviv 6997801, Israel
Itzik Cohen, Weizmann Institute of Science, Rehovot 7610001, Israel

INTRODUCTION

Sensing, using electromagnetics (RADAR), acoustics (SONAR), or optics (LIDAR, especially direct-detection ones [1]), may have noncoherent waveforms. The most basic such waveform is ON–OFF, namely, signals utilizing only two real values: 0 and 1.

As typical in radar, the propagating signal can be amplified, attenuated, delayed, phase shifted, or Doppler shifted, but an envelope detector at the receiver is sensitive only to amplitude. The envelope detector output dependence on the input amplitude can follow different laws (linear, square, log, etc.), but the envelope detector ignores the phase of the input signal.

It is important to note that the output of the envelope detector will be an analog real waveform that can have any positive value. The digital processor that follows handles the detected signal with all the capabilities of a digital processor.

We will split the discussion of noncoherent pulse compression (NCPC) into two cases: aperiodic and periodic. The *aperiodic* case will resemble processing a single long radar pulse, which is frequency modulated or phase coded. On receive, that pulse is processed by a matched filter (MF) [2] or mismatched filter (MMF) [3], to produce a delay response that exhibits narrow high peak and low sidelobes. We will show that NCPC must use an MMF. The *periodic* case corresponds to periodic continuous wave (CW) radar. The MMF must be an integer number of periodic reference waveform, with the same period duration as the emitted waveform. Fortunately, periodic references can be found that exhibit perfect periodic cross-correlation with periodic ON–OFF waveforms.

Aperiodic NCPC was proposed in [4] and [5]. In these papers, the transmitted pulse burst was a Manchester coded conversion of good binary sequences like Barker or minimum peak sidelobe (MPSL) [6]. Through the ensuing years, various other approaches were tried [7], [8], but the approach of transmitting an ON–OFF version of a binary sequence, and procesing it with minISL MMF (see the Appendix), established itself as the best [9]. In summary: choose a good binary code, convert to ON–OFF, and process it with as long as possible minISL MMF.

In *coherent* pulse compression radar, the correlation delay sidelobes are the most Doppler intolerant parameter. They are likely to increase dramatically if intrapulse Doppler compensation is not provided. However, since the NCPC signal's phase is not considered, Doppler shift by itself has no effect (e.g., direct detection LIDAR). However, in cases where time stretching over the width of the unipolar element is not negligible (e.g., blood flow velocity in a flow cytometer application [9]), the issue has to be considered.

APERIODIC NCPC

The aperiodic equivalent to a coherent phase-coded pulse is a short burst containing equally spaced subpulses, some of which are missing. An example of ON–OFF Barker 5 {1,1,1,0,1} appears in Figure 1.

In the top subplot of Figure 1, the subpulse width equals the element width (considered the default case). As expected, the fourth subpulse is missing. In the bottom subplot, the subpulse width equals one-third of the element's width. In this case, the transmitter emits one-third of the attainable energy and the signal occupies wider bandwidth, but will yield narrower delay response. From the point of view of signal-to-noise ratio (SNR), further processing should include an MF. However, an MMF can produce much lower delay sidelobes. This is demonstrated in Figures 2 and 3 using ON–OFF Barker 5.

The responses shown in Figure 2 were obtained with Barker 5 in which the sub-pulse occupies the entire element width (Figure 1, top). The red dotted line is the MF

Authors' current addresses: Nadav Levanon is with the Faculty of Engineering, Tel Aviv University, Tel Aviv 6997801, Israel (e-mail: levanon@tauex.tau.ac.il). Itzik Cohen is with the Faculty of Mathematics and Computer Science, Weizmann Institute of Science, Rehovot 7610001, Israel (e-mail: bitzikc@gmail.com).
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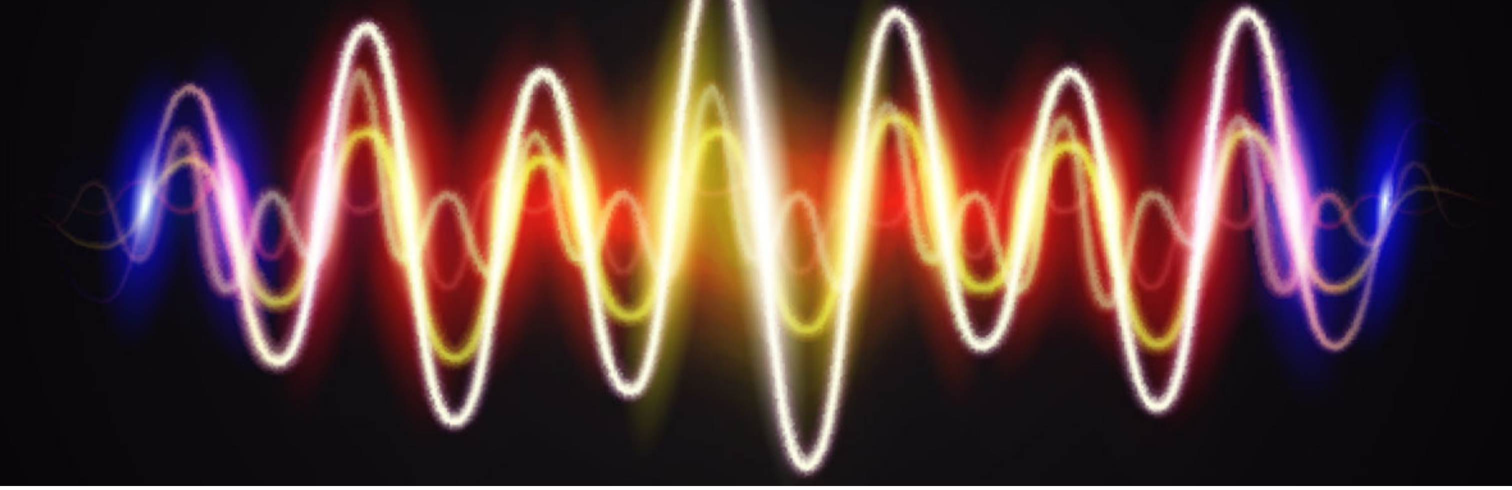

response (also the autocorrelation) and it reaches a peak of 4, which is the sum of the signal's elements.

An 11 element MMF was used, designed to reach minimum integrated sidelobes (minISL) {0.0484 -0.4319 -0.3714 0.4763 0.9365 0.4844 -1.0576 0.9365 -0.2947 -0.3552 0.3552}. Indeed, the MMF response (solid black) exhibits lower peak but much lower sidelobes.

Figure 3 displays the MF and MMF responses to the ON–OFF Barker 5, with narrow subpulses (Figure 1. Bottom). As both drawings show, in an ON–OFF case, only a MMF is an acceptable choice. Note that both MF (= signal) and MMF sequences have identical sum of the elements squares = 4, implying same output variance when the input is noise only. That value is also the peak of the autocorrelation of {1,1,1,0,1}. The lower peak of the MMF response (= 2.83) expresses the typical, relatively high signal to noise ratio (SNR) loss of an MMF to ON–OFF coded pulse **(**$(2.83/4)^2 = 0.5 \Rightarrow -3$ dB**).**

In coherent pulse compression utilizing binary phase coding, the length of the sequence determines the compression (the ratio between the pulse width and the correlation mainlobe width). Once that ratio is set, the design challenge is the shape of the delay response, especially the sidelobes. That issue was studied for years and some good families are given in [6, Table 6.3] and in [10]. In those families the merit of the sequence is determined by a single principal measure—the peak sidelobe level of the aperiodic autocorrelation, hence their name, minimum peaks sidelobe level (MPSL). There is yet no figure of merit for ON–OFF sequences. In the ON–OFF case exhaustive search is doubly more complex because both the sequence and its MMF are involved.

SEARCH FOR VALUABLE ON–OFF APERIODIC SEQUENCES

An aperiodic ON–OFF sequence of length N must start and end with a "1," otherwise its length will be $<N$. Other qualities that add value to a sequence are: (a) Large

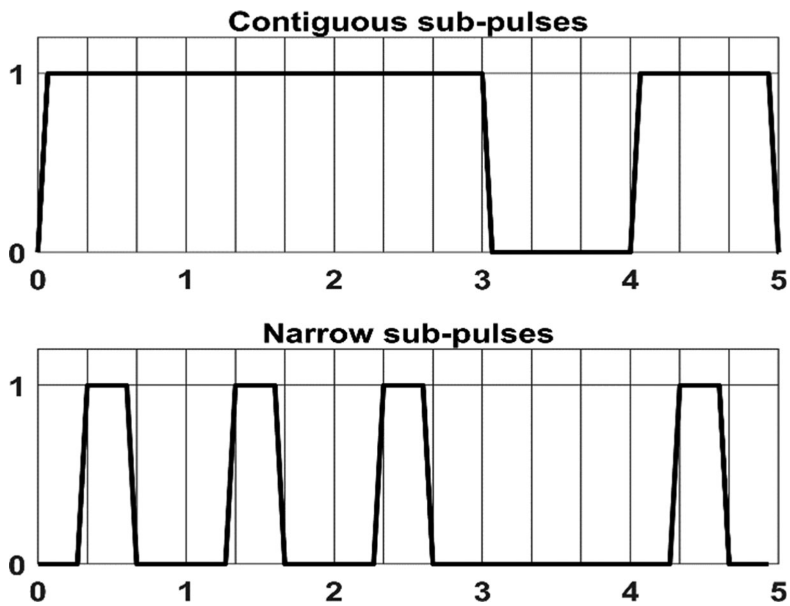

Figure 1.
ON–OFF Barker 5.

Table 1.

Preferred ON–OFF Sequences

N	dec	No "1"s	SNR loss [dB]	Peak SL [dB]	Binary	HEX
15	1623	9	-3.90	-29.91	100110010101111	4CAF
	2919	10	-3.39	-28.16	101011011001111	56CF
16	9643	10	-3.14	-25.27	1100101101010111	CB57
	11371	10	-3.21	-24.54	1101100011010111	D8D7
17	12179	11	-3.53	-23.99	10101111100100111	15F27
	26939	11	-3.57	-23.47	11101001001110111	1D277
18	40239	12	-3.82	-28.47	110011101001011111	33A5F
	46451	12	-3.77	-26.53	110110101011100111	36AE7
19	81323	13	-3.93	-24.74	1100111101101010111	67B57
	105303	13	-4.13	-27.83	1110011011010101111	736AF
20	183031	14	-4.05	-30.96	11011001010111101111	D95EF
	232942	14	-4.33	-29.75	11110001101111011101	F1BDD
21	339695	15	-4.78	-29.93	110100101110111011111	1A5DDF
	453455	15	-4.29	-28.16	111011101011010011111	1DD69F
22	765419	16	-4.56	-26.08	1101110101101111010111	375BD7
	842431	16	-4.84	-30.63	1110011011010101111111	39B57F
23	1518575	17	-4.76	-29.15	11011100101011111011111	6E57DF
	1816943	17	-4.76	-29.71	11101110111001011011111	7772DF
24	3008351	18	-4.83	-27.72	110110111100111010111111	DBCEBF
	3010391	18	-5.07	-29.17	110110111101111010101111	DBDEAF
	3369727	18	-5.48	-32.14	111001101101010111111111	E6D5FF
25	6139455	18	-4.97	-32.41	1101110110101110001111111	1BB5C7F
	8215417	18	-4.56	-32.03	1111110101011011011110011	1FAB6F3
	7059383	19	-5.14	-26.36	1110101110110111101101111	1D76F6F
	7273895	19	-5.21	-30.30	1110111011111101101001111	1DDFB4F

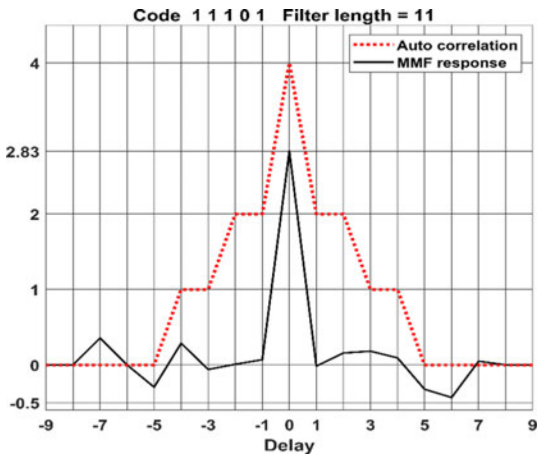

Figure 2.
Matched and mismatched filters responses to ON–OFF Barker 5 with contiguous subpulses.

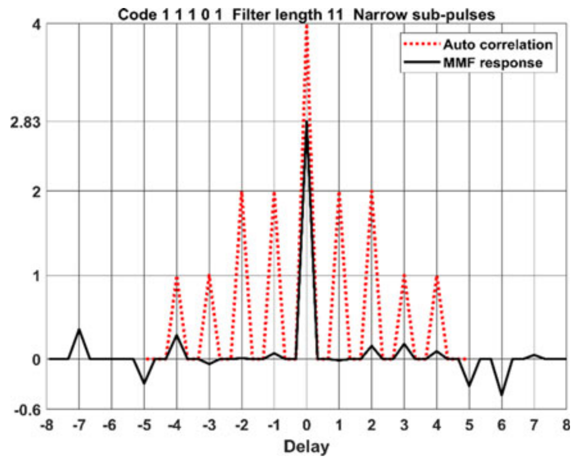

Figure 3.
Matched and mismatched filters responses to ON–OFF Barker 5 with narrow subpulses.

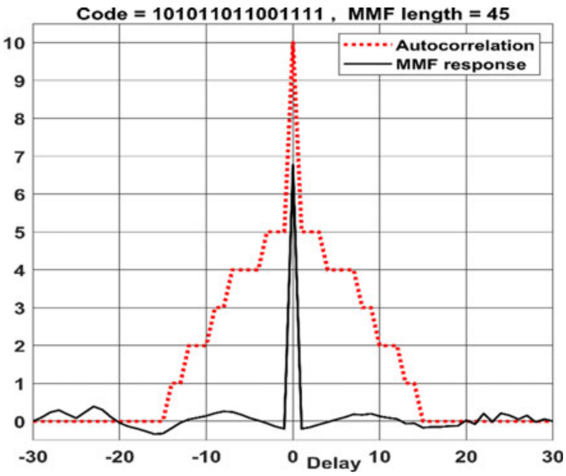

Figure 4.
MF and minISL MMF responses to 15 element ON–OFF sequence 2919 ''56CF.''

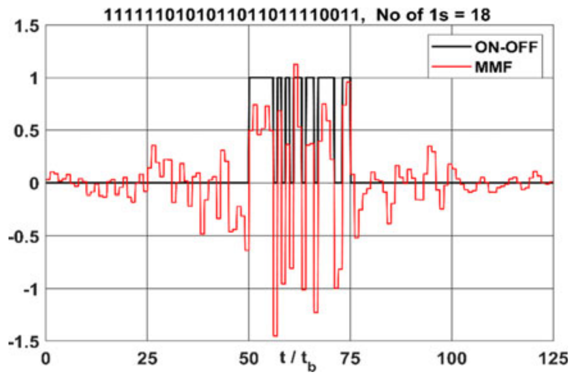

Figure 6.
A total of 125 element minISL MMF response to 25 element ON–OFF sequence found in exhaustive search.

number of "1"s, implying larger emitted energy; (b) Low SNR loss of the MMF delay response; (c) Low sidelobes of the MMF response (e.g., low peak SL). The order hints to the weight given to those three degrees of freedom, but this is a rather subjective issue.

An exhaustive search of all the ON–OFF sequences of length N involves checking all the 2^{N-2} binary combination, after adding "1" at the start and end of all the combinations. Tentative checking involves designing a $3N$ minimum integrated sidelobes (minISL) MMF and judging the response (cross-correlation between the ON–OFF sequence and its MMF), according to the above three qualities. We chose an MMF that minimizes the integrated sidelobes, rather than one that minimizes the peak sidelobe, because the design of the minISL MMF requires a simple matrix operation (see the Appendix), while finding a minimum peak sidelobes (minPSL) MMF requires an algorithm.

Table 1 presents the outcome of an exhaustive search for $N = 15$ to 25 element ON–OFF sequences. The second column displays the decimal value of the middle N-2 element binary code. The third column displays the number of "1"s in the sequence. The fourth column lists the SNR loss caused by using a minISL MMF of length $3N$.

The fifth column displays the peak SL. The sixth and seventh columns display the binary and Hex formats of the sequences. Figure 4 displays the MMF response of the second sequence in Table 1. It will be used in the section "Detection in the Presence of Noise."

The SNR losses quoted in Table 1 represent the loss caused by using an MMF (a must) rather than an MF. But using a code with smaller number of "1"s contributes an additional SNR loss. For example, with regard to the $N =$ 25 sequences in Table 1, the additional loss of using a code with 18 instead of 19 "1"s should have been $20\log_{10}(18/19) = -0.47\,\text{dB}$. However, the two sequences with 19 "1"s show larger overall SNR losses than the two codes with 18 "1"s, indicating that the codes with 19 "1"s do not lend themselves so well to MMF processing. The two $N = 25$ codes in the comparison will be code 1111110101011011011110011 from Table 1 and the MPSL sequence of length 25 whose binary presentation is 1001001010100000011100111. Note the large difference

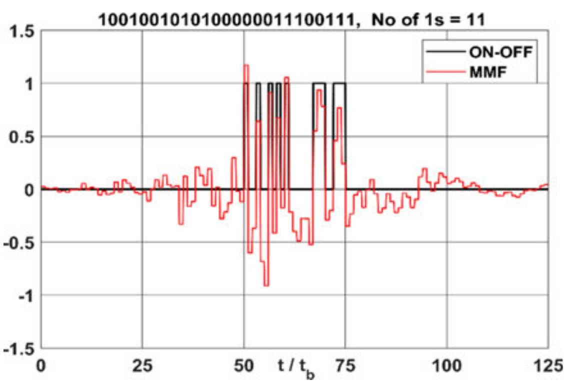

Figure 5.
A total of 125 element minISL MMF response to 25 element ON–OFF sequence based on MPSL 25.

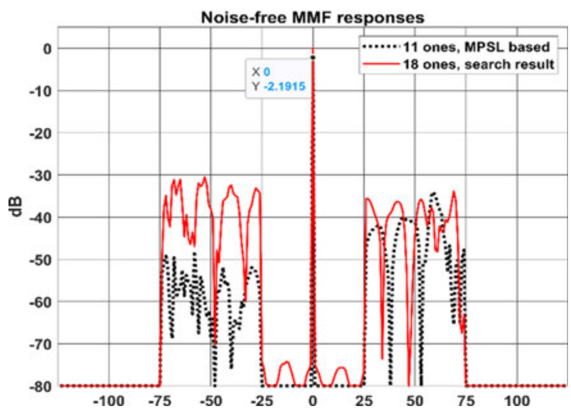

Figure 7.
A total of 125 element MMF responses of noise-free two 25 element ON–OFF signals.

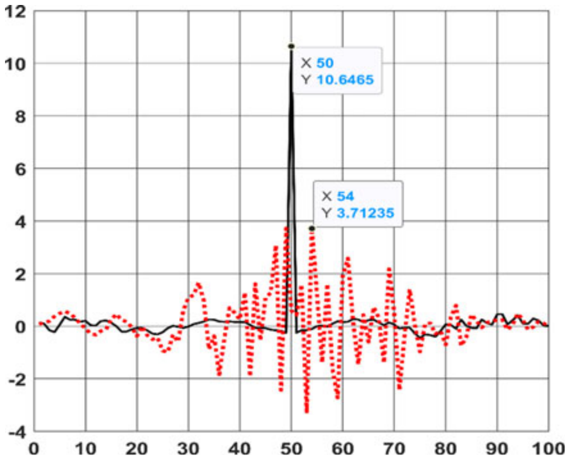

Figure 8.
Cross-correlation between ON–OFF code and (black) it is MMF, (red) MMF designed for another ON–OFF code.

between the number of "1"s in the two sequences (18 versus 11). The MMFs used in the comparison were 125 element long and the diagonal of the weight matrix **F** (see the Appendix) was modified to lower the cross-correlation near sidelobes. The codes and MMF are shown in Figures 5 and 6. The noise-free response is given in Figure 7, which shows how well the increase of the weight assigned to lower the near sidelobes (see the Appendix) worked, and shows also the height of the remaining sidelobes.

In the noise-free case displayed in Figure 7, the 0 dB level was set at the level of the peak reached by the code with 18 "1"s (red line). The MPSL-based code (dotted, black) exhibits lower sidelobes, but its peak is also lower, by about 2.2 dB. That difference is smaller than the ratio $20\log_{10}(18/11) = 4.28\ dB$, which suggests that the code with 18 "1"s, did not lend itself so well to processing by an MMF, bringing its performances closer to those of the MPSL based code, having only 11 "1"s.

A likely application of NCPC is automotive LIDAR. NCPC offers two benfits for that application. The obvious one is the ability to transmit a longer pulse at lower power and then process it to effectively perform like a narrow pulse of high power. The other property is that the particular coding sequence and its unique MMF serve as a *stamp*. The automotive application expects many users and when that happens the probability increases of one LIDAR receiving reflections or illuminations triggered by another LIDAR [11]. The mutual interference will be reduced if different codings will be used and each processor will favor its own coded pulse. An example is given in Figure 8. In that example, the received noise-free code was the 25 element ON–OFF code 1FAB6F3. The cross-correlation (black) was between it and the MMF designed for it. A clear strong peak is evident. The noise-like cross-correlation (red) was between it and an MMF designed for the wrong ON–OFF code 1BB5C7F.

A nonradar application [9] reported a selection of an ON–OFF sequence of length 42. A "1" in the sequence represents a slot in a strip, illuminated by a moving fluorescent blood cell.

```
1 1 1 0 1 1 1 0 1 1 1 0 1 1 1 0 0 0 0 1 0 0 0 1 1 1 0 0
0 1 0 1 1 0 1 0 0 1 0 1 1
```

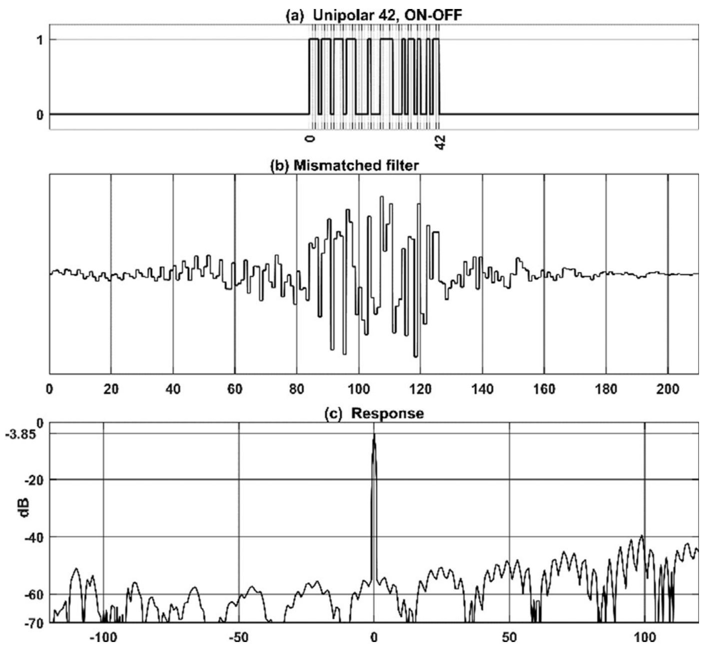

Figure 9.
Flow cytometer. (a) Slotted mask. (b) minISL MMF. (c) Response.

The sequence is a negated MPSL 42 ([6, Table 6.3]). Figure 9 displays the sequence, a 210 (= 42x5) element minISL MMF and the resulted response. The SNR loss, caused by the MMF, is 3.85 dB.

The MPSL 42 has 24 "1"s. A nonexhaustive search found a 42 element ON–OFF sequences, with relatively good MMF response, which contains 32 "1"s, but its minISL MMF caused SNR loss of 7.34 dB.

```
1 0 1 1 1 0 1 1 0 1 0 1 1 1 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0
1 1 1 1 1 0 1 1 1 1 1 1 1
```

Here, again, we see a conflict between more "1"s, but larger MMF induced SNR loss.

PERIODIC NCPC

Periodic continuous wave (CW) sensors may also prefer ON–OFF emitted waveforms. Converting a two-valued pseudonoise periodic phase code, like shift register [12] or Legendre [13], to ON–OFF, is the preferred way to create a periodic transmitted waveform. As in aperiodic NCPC, a MMF is needed to obtain acceptable periodic cross correlation. In aperiodic NCPC, the MMF length was the key to low correlation sidelobes (SL). But in periodic CW waveform, the periods are contiguous hence a period of the MMF must have the same duration as the transmitted period. Fortunately, ON–OFF versions of shift register, Legendre and few more sequence families exhibit perfect periodic cross correlation with a same-period MMF.

A very simple example is Barker 3. Its ON–OFF version is {1,1, 0}, and its same period MMF is {1,1, −1}. Recall that the periodic cross correlation with an MMF is performed in the digital processor of the receiver, after envelope detection, hence, the MMF can use any real or complex value.

In MATLAB, *aperiodic* cross correlation between sequences x and y (not necessarily of the same length) is given by

$$\mathbf{z} = \mathbf{xcorr}\,(\mathbf{x}, \mathbf{y})$$

In contrast, the *periodic* cross correlation between two sequences of the same length is given by

$$\mathbf{z} = \mathbf{ifft}(\mathbf{fft}(\mathbf{x}).^{*}\mathbf{conj}(\mathbf{fft}(\mathbf{y})))$$

Using the Barker 3 example, we will get

$$\begin{array}{llll} \mathbf{x} = & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{y} = & \mathbf{1} & \mathbf{1} & \mathbf{-1} \\ \mathbf{z} = & \mathbf{2.000} & \mathbf{0.000} & \mathbf{0.000} \end{array}$$

That example assumed that the reference contained only one period. Using a reference containing two periods will produce

$$\begin{array}{lllllll} \mathbf{x} = & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{y} = & \mathbf{1} & \mathbf{1} & \mathbf{-1} & \mathbf{1} & \mathbf{1} & \mathbf{-1} \\ \mathbf{z} = & \mathbf{4.000} & \mathbf{0.000} & \mathbf{0.000} & \mathbf{4.000} & \mathbf{0.000} & \mathbf{0.0000} \end{array}$$

Table 2.

MMF *b* Values Replacing 0 in Barker Codes of Length *N*

N	3	4	5	7	11	13
b	−1	−2	−3	−1	−2/3	−2

Note that in both examples the sidelobes of z are zero, which implies perfect periodic cross-correlation (PPCC). Note also that the peak values, 2 and 4, were the sum of the elements of the ON–OFF sequence x.

One more member of the Barker family exhibits the same property. It is Barker 7 whose ON–OFF version is {1,1,1,0,0,1,0}. The other members of the Barker family will exhibit PPCC if the MMF value replaces the value 0 by the variable *b* according to Table 2.

Note that a negative value of *b* that is larger than −1 will cause higher SNR loss of the MMF processing. We will present longer families of ON–OFF sequences, producing PPCC, in which the MMF replaces the value of 0 by −1.

LEGENDRE ON–OFF SEQUENCES

Legendre two-valued phase-coded periodic sequences were described in [14, Sec. 15.3]. Their ON–OFF (unipolar) version turns out to be a very important family in periodic NCPC. The Legendre family is split into two groups: (a) All unipolar Legendre sequences of a *prime* length *P* that obey

$$P = 4k - 1, \quad k = 1, 2, 3, 4, \ldots; \quad P \text{ is a prime.} \tag{1}$$

(b) All unipolar Legendre sequences of a *prime* length *P* that obey

$$P = 4k - 3, \quad k = 2, 3, 4, \ldots; \quad P \text{ is a prime.} \tag{2}$$

Group (b) obeys Fermat's "sum of two squares" theorem. For example, using $k = 4$ in (2) yields $P = 13 = 3^2 + 2^2$.

The following MATLAB function generates the: transmitted Legendre sequence, its MMF, and the resulted periodic cross-correlation. The function applies to both Legendre groups.

```
function[code,mmf,crosscorr]=GenerateOnOffLege
ndre(Length)
mm=Length;
if isprime(mm)==0
    disp('Not a prime')
    return
end
code=ones(1,mm);
code(unique(mod((1:mm-1).^2,mm))+1)=0;
mmf=code*2-1;
if rem((mm+3)/4,1)==0
    code(1)=0;
end
crosscorr=ifft(fft(mmf).*conj(fft(code)));
```

Table 3.

ON–OFF Legendre 13		
Code	**MMF**	**PCC**
0	1	6
0	-1	0
1	1	0
0	-1	0
0	1	0
1	1	0
1	1	0
1	1	0
1	1	0
0	-1	0
0	-1	0
1	1	0
0	-1	0

Table 3 presents the code and MMF sequences as well as the periodic cross correlation between the two. Because 13 belongs to group (b) of the primes, the first element of the transmitted ON–OFF sequence had to be switched from 1 to 0 in order to produce PPCC [15]. Since we are dealing with periodic sequences, which are repeated contiguously, cyclic shifts make no difference. Any cyclic shift of the first column, and the same cyclic shift in the second column, will produce PPCC.

An example of Legendre 13 (Table 3) with a cyclic shift of 2 elements appears in Figure 10. The top subplot contains six periods of the cyclic shifted ON–OFF transmission. The middle subplot contains two periods of the cyclic shifted binary reference. The bottom subplot displays the resulted perfect periodic cross correlation. The PPCC peak value of 12 equals the sum of the number of "1" in two transmitted periods.

SHIFT REGISTER ON–OFF SEQUENCES

All unipolar shift register sequences (or *m*-sequences) exhibit PPCC, with their corresponding bipolar reference ($b = -1$).

m-sequences [12] are available at lengths

$$N = 2^m - 1, \quad m = 2, 3, 4, \ldots \tag{3}$$

which are more sparse than the prime numbers. However, in contrast to Legendre sequences, m-sequences provide several different sequences at each length, according to the number of primitive polynomials over the Galois field GF(2^m).

The use of periodic NCPC in Laser range-finder was reported in [16]. Its use in fiber sensing was reported in [17].

DETECTION IN THE PRESENCE OF NOISE

Detection performances will be described by performing Monte-Carlo simulations using ad-hoc specifications. A 15-element ON–OFF signal, detailed in the second row of Table 1 and possessing unit amplitude,

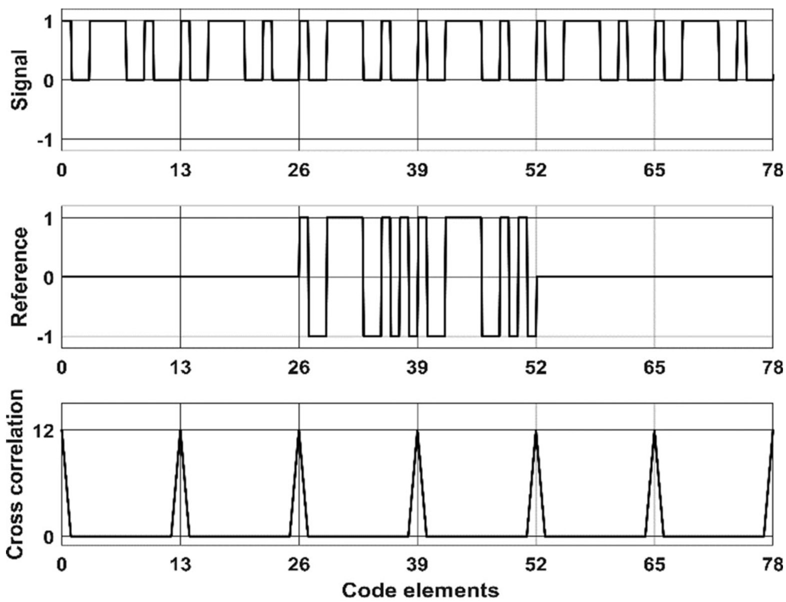

Figure 10.
Cyclic shifted Legendre 13.

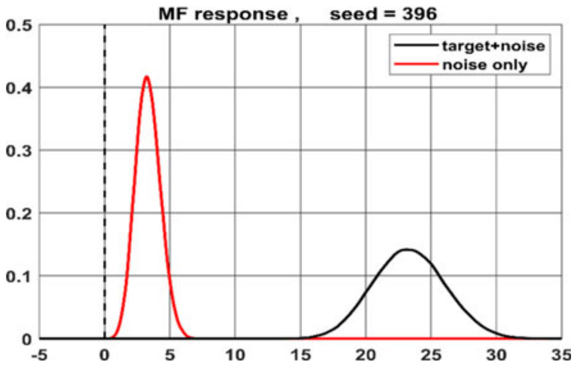

Figure 11.
Simulation obtained PDFs of noise only and noise+signal, obtained from cross correlation with MF.

served as the test signal. The noise was *uniformly* distributed between 0 and 1. With these choices the signal plus noise have only positive values. It then passed through a square-law envelope detector whose output was cross-correlated with two filters: (a) Matched filter, (b) minISL MMF of length 45. A noise-free example of the two responses appears in Figure 4. Note the many negative values in the MMF and the only positive values of the MF. The number of trials was 100,000. Probability density functions (PDFs) derived from the simulations were generated using MATLAB's ksdensity instruction, employing the Epanechnikov kernel. Note that using *Rayleigh* distributed noise, with scale parameter$\sigma = 0.4$, produced very similar PDFs.

Significant distinctions are evident when comparing the PDFs illustrated in Figures 11 and 12: (a) While the noise-only PDF in the MF case extends only over positive values, in the MMF case it extends also to negative values. This is caused by the negative valued elements of the MMF. (b) The separation between the means of the signal+noise and noise-only, dropped considerably in the MMF case. This reflects the relatively high SNR loss of MMFs designed for ON–OFF signals. (c) The two noise-only PDF shapes are almost identical, indicating similar noise variance.

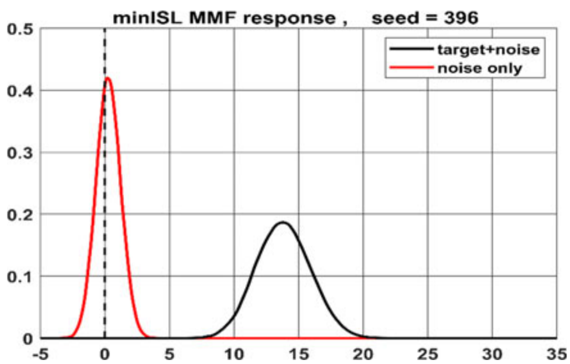

Figure 12.
Simulation obtained PDFs of noise only and noise+signal, obtained from cross-correlation with minISL MMF.

CONCLUSION

ON–OFF sequences were proposed for noncoherent sensors emitting periodic or aperiodic waveforms and using envelope detection. It was shown that in order to obtain acceptable pulse-compressed delay response, the receiver must employ a mismatched filter. Merit measures were suggested and used in a search for good mid-length (15 to 25) sequences. Detection performances in the presence of noise were demonstrated using Monte-Carlo simulation. Already reported potential applications of NCPC are: Noncoherent RADAR, direct detection LIDAR, range-finders, fiber sensing, and a medical flow cytometer.

APPENDIX

minISL MMF Design

This Appendix is included because an MMF is an essential part of the aperiodic noncoherent pulse compression (NCPC). MMF also plays a role in the search for good ON–OFF sequences. The minISL type was selected because it is well defined, while minPSL is an iterative process with many variations. The outline of minISL MMF design follows [18].

In order to be effective, the length P of the MMF x must be equal or longer than the length N of the ON–OFF sequence c. Let the code sequence be

$$\mathbf{c}' = [c_0\ c_1\ \ldots..\ c_{N-1}]. \tag{A.1}$$

The filter sequence of length P will be obtained from $\mathbf{c}$ by nearly symmetrical zero padding

$$\mathbf{x}' = [0\ \ldots..0\ c_0\ c_1\ \ldots..\ c_{N-1}\ 0\ \ldots..0]. \tag{A.2}$$

$\boldsymbol{\psi}$ is a $Px(2P-1)$ Hankel matrix of $\mathbf{x}$

$$\boldsymbol{\psi} = \begin{bmatrix} 0 & 0 & \ldots.. & x_{p-2} & x_{p-1} \\ 0 & 0 & \ldots.. & x_{p-1} & 0 \\ 0 & x_0 & \ldots.. & 0 & 0 \\ x_0 & x_1 & \ldots.. & 0 & 0 \end{bmatrix}. \tag{A.3}$$

Let the filter sequence be

$$\mathbf{h}' = [h_0\ h_1\ \ldots..\ h_{P-1}]. \tag{A.4}$$

The response of the MMF is

$$y_m = \sum_{n=0}^{P-1} x_n h^*_{n-m}\,,\ m = -(P-1),\ \ldots..,\ (P-1) \tag{A.5}$$

and in a vector form

$$\mathbf{y} = \mathbf{h}'\boldsymbol{\psi}\ \ ,\ \ ()'\ \text{implies conjugate transpose} \tag{A.6}$$

$\mathbf{F}$ is a $(2P-1)x(2P-1)$identity matrix in which the (p, p) element is zero. An example for $P = 3$ is given in

(A.7). Let the total energy in the sidelobes be represented by $\mathbf{E}$, given in (A.8), which also defines the matrix B

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.7}$$

$$\mathbf{E} = \mathbf{yFy}' = (\mathbf{h}\boldsymbol{\psi})\mathbf{F}(\mathbf{h}\boldsymbol{\psi})' = h'(\boldsymbol{\psi}\mathbf{F}\boldsymbol{\psi}') = \mathbf{h}'\mathbf{Bh}. \tag{A.8}$$

The unnormalized MMF that minimizes the total sidelobes' energy $\mathbf{E}$ is given by

$$\mathbf{h}_0 = \mathbf{B}^{-1}\mathbf{x}\ldots \tag{A.9}$$

In order to get an MMF $\mathbf{h}$ whose energy $\mathbf{h}'\mathbf{h}$ is equal to the energy $\mathbf{x}'\mathbf{x}$ of the matched filter, $\mathbf{h}_0$ has to be normalized as follows:

$$\mathbf{h} = \mathbf{h}_0\sqrt{\frac{\mathbf{x}'\mathbf{x}}{\mathbf{h}_0'\mathbf{h}_0}}. \tag{A.10}$$

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