Staggered Costas Signals

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A radar signal, based on coherent processing of a train of staggered Costas bursts, is suggested and investigated. The selection of sequences of each burst is based on a minimum number of collocation of their individual ambiguity function sidelobe peaks. The resulting ambiguity function combines qualities of both "thumbtack" and "bed of nails" signals. Comparison with linear-FM, V-FM, and complementary phase coded (CPC) signals is given, as well as comparison with hybrid signals consisting of both phase and frequency coding.

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I. INTRODUCTION

John P. Costas [1] has recently suggested a new class of radar and sonar signals whose ambiguity function approaches the ideal "thumbtack" configuration. The low-level pedestal is obtained by careful selection of a frequency-hopping pattern within the transmission burst.

A Costas signal of order N is a burst of N contiguous pulses, each of duration T, and each at a different frequency, selected out of N adjacent frequencies spaced $1/T$ apart. The order in which the frequencies are selected is the key feature of the Costas signals.

An easy way to comprehend the Costas signals is by using a binary $N \times N$ matrix as shown in Fig. 1, where a Costas signal is compared with a discrete linear-FM signal (chirp). Both signals are characterized by the fact that no frequency is repeated twice and only one frequency is transmitted at any time slot, i.e., the matrix displays only one dot in each row or column. The signals differ in the order of frequency hopping, and that order affects the ambiguity function. If a thumbtack shape is considered ideal and if frequency hopping is the only kind of modulation applied to the signal, then we will immediately show that the linear hopping sequence, typical of chirp, is the worst possible choice, while a Costas sequence is the best.

A rough idea of the shape of the ambiguity function of frequency-hopping signals can be obtained from the following exercise. Envision (or actually draw) an overlay of the binary matrix of each of the two signals in Fig. 1. First lay the overlay on the original matrix, without any shifts. Obviously, the number of coincidences between dots on the original matrix and on the overlay will be N (= 6) in both signals. This is the (unnormalized) value of the mainlobe of the ambiguity function, and it will always be equal to N, independent of the hopping sequence. Next, shift the overlay right or left, up or down, or any combination thereof. For each combination of shifts count the number of coincidences. The

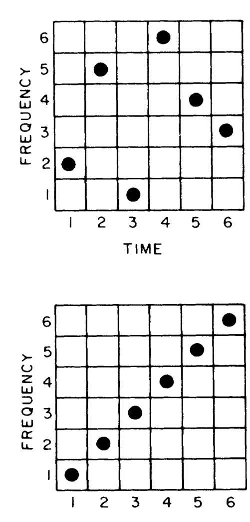


Fig. 1. Binary matrix representation of Costas signal (top) and chirp signal (bottom).

horizontal shift determines the delay coordinate of the ambiguity function, and the vertical shift determines the Doppler coordinate. The number of coincidences is (approximately) the value of the function at those coordinates.

In the case of the chirp signal, one shift to the right and one shift up will yield $N-1$ $(=5)$ coincidences. Two shifts to the right and two up will result in $N-2$ coincidences, etc. Thus, the well-known ridge, typical of the ambiguity function of chirp signals, is created. Similar shifts applied to the Costas signal will never yield more than one coincidence. This is actually the criteria of Costas sequences, namely, those sequences of frequency hopping that will yield no more than one coincidence.

Construction algorithms for such sequences, and studies of the number of Costas signals as a function of N, are given in [1] and [2]. One interesting observation is the increase in the number of different Costas signals as N is increased, together with a decrease in their density. The density is the number of Costas signals divided by $N!$, which is the total number of signals with one dot per row and per column in an $N \times N$ binary matrix.

The exact value of the ambiguity function cannot be determined by simply counting the number of coincidences. However, the qualitative conclusion that the mainlobe-to-sidelobe ratio is usually equal to N, is valid. This fact leads to the conclusion that a large N is necessary in order to obtain a large mainlobe-to-sidelobe ratio.

This paper suggests and investigates a method of increasing the mainlobe-to-sidelobe ratio without increasing N. The idea is to process coherently a train of M bursts, each one being a Costas signal of order N. A key point is that each Costas burst in the train will differ from the others, as much as possible.

A train of Costas bursts will create additional strips of the ambiguity function at delays which are multiples of the burst repetition period. These strips are similar to the strips found in the "bed of nails" ambiguity function which results from a train of single-frequency pulses. They serve the useful role of moving some of the fixed volume of the ambiguity function further away from the vicinity of the mainlobe. In our case, since the additional strips are produced by cross correlation between differently coded bursts, their typical shape will not include additional "nails" but rather low surfaces. Of more interest is the main strip, around the mainlobe. The sidelobes in the main strip are the result of adding the sidelobes of each burst. If Costas bursts can be found, whose ambiguity function sidelobes do not overlap, then the sidelobes will interleave, and the general level of $1/(NM)$ relative to the mainlobe will be maintained.

Regrettably, it was proven in [3] and [4] that there do not exist two Costas signals of order $N>3$, with completely disjointed ambiguity sidelobe patterns. Even at $N=3$, there are only two such signals. However, fairly good ambiguity functions can be found even if there are some coincidences.

In the next section we investigate trains of two bursts each of order 3, of various codings, and present their exact ambiguity function. The following sections compare staggered Costas trains versus signals with complementary phase modulated coding [5] which bear some similarity in concept. Composite signals with both frequency and phase coding are also investigated.

II. AMBIGUITY FUNCTION OF A TRAIN OF COSTAS BURSTS

The complex envelope of burst number m in a train of M bursts is described by the sum of N contiguous pulses each of duration T:

$$U_m(t) = \sum_{n=1}^{N} P_{nm}(t-nT) \tag{1}$$

where the complex envelope of each pulse is described by

$$P_{nm}(t) = \begin{cases} \exp(j2\pi f_{nm} t), & 0<t<T \\ 0, & \text{elsewhere} \end{cases} \tag{2}$$

which means that the magnitude of the signal is a constant $(=1)$ and the frequency of each pulse is shifted from the carrier frequency by

$$f_{nm} = (\theta_{nm} - 1)/T. \tag{3}$$

The set

$$\{\theta_{nm}\}, \qquad n = 1,2, \ldots, N; \qquad m = 1,2, \ldots, M$$

consists of M Costas sequences, each of order N, and each defines the frequency-hopping pattern of one of the bursts in the train. Each sequence is called the "coding sequence" of that burst. For example, the coding sequence of the single Costas burst which appears in Fig. 1 is $\{\theta_n\} = 2,5,1,6,4,3$.

If the repetition period of bursts within the train is T_r, then the complex envelope of the entire train is given by

$$U(t) = \sum_{m=1}^{M} U_m(t-mT_r). \tag{4}$$

The ambiguity function of a signal with complex envelope $U(t)$ is defined as

$$X(\tau,\nu) = (1/2E) \int_{-\infty}^{\infty} U^*(t)\, U(t-\tau) \exp(j2\pi\nu t)\, dt \tag{5}$$

where E is the total energy of $U(t)$ and $U^*(t)$ is the conjugate of $U(t)$. Costas [1] has given the closed-form expression of $X(\tau,\nu)$ for a single Costas burst. However, we use outputs from a universal computer program [6] which calculates and plots the ambiguity function of any signal.

The following are three important comments with regard to all the following plots. (1) The plots are of the magnitude of the ambiguity function $|X(\tau,\nu)|$, with a linear vertical scale. (2) The delay and Doppler coordinates are based on the normalization $T=1$. (3) Scales and coordinates appear only in the first figure in a group but apply to the remaining figures in that group. However, sometimes the observation viewpoint changes within the group of figures (e.g., in Fig. 3(b) the observation point differs from all the other viewpoints in Fig. 3). In order to easily determine which is the Doppler axis note that the flat strips, corresponding to a zero value of the ambiguity function, are always parallel to the Doppler axis.

The ambiguity function is first calculated for five different signals. Each signal is a train of two bursts, and each burst is constructed from three contiguous pulses. Thus $M=2$ and $N=3$. The binary matrices describing the five signals are shown in Fig. 2. To avoid overlapping of the range sidelobes strips it is necessary to choose $T_r > 2NT$. In our particular case we selected $T_r = 8T$.

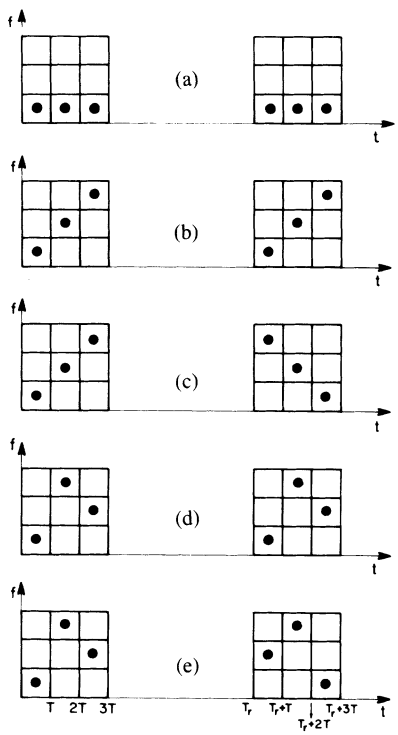


Fig. 2. Binary matrix representation of five signals. (a) Train of two single-frequency pulses. (b) Linear FM. (c) V-FM. (d) Repeated Costas. (e) Staggered Costas.

A. Single Frequency Pulse Train (Fig. 2(a))

The first signal is the simple case of two coherent single-frequency bursts. Using the terminology defined in (1)–(4) the coding sequences are

$$\theta_{nm} = 1, \qquad n = 1,2,3; \qquad m = 1,2. \tag{6}$$

The ambiguity function, shown in Fig. 3(a) exhibits the wide central peak typical of a single pulse, plus two additional peaks at $\pm T_r$ $(= \pm 8)$, which are similar in shape but of half the magnitude.

B. Linear FM (Fig. 2(b))

This signal is a discrete version of the well-known chirp signal. The coding sequence of the two bursts is

$$\theta_{nm} = n, \qquad n = 1,2,3; \qquad m = 1,2. \tag{7}$$

The ambiguity function (Fig. 3(b)) reflects the ridge typical of chirp signals. However, because of the discrete nature of the signal the ridge is not smooth but is broken into many small peaks.

C. *V*-shaped FM (Fig. 2(c))

Here

$$\theta_{n1} = n, \qquad n = 1,2,3$$

$$\theta_{n2} = 4 - n, \qquad n = 1,2,3. \tag{8}$$

This signal, while not Costas, has the desired quality that the coding is not repeated from burst to burst. This results in lower sidelobes in the main strip and shallow side strips around $\tau = \pm T_r$ (Fig. 3(c)).

D. Repeated Costas Coding (Fig. 2(d))

Here

$$\{\theta_{nm}\} = 1,3,2, \qquad m = 1,2. \tag{9}$$

Each burst is Costas, but the bursts are identical. This causes peaks in the side strips, which are half the height of those in the main strip (Fig. 3(d)).

E. Staggered Costas Bursts (Fig. 2(e))

Here

$$\{\theta_{n1}\} = 1,3,2$$

$$\{\theta_{n2}\} = 2,3,1. \tag{10}$$

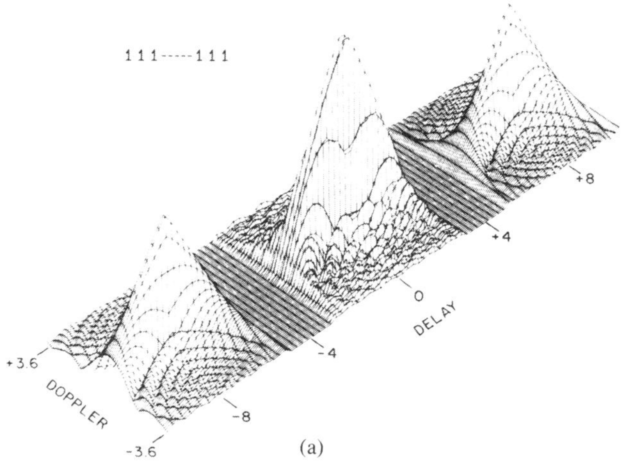
The fact that the ambiguity sidelobes of the first burst do not coincide with those of the second burst creates a lower level of peaks in the main strip of sidelobes (around $\tau = 0$), as can be clearly seen by comparing Fig. 3(e) with Fig. 3(d). Furthermore, it eliminates the sharp peaks (nails) at the side strips ($\tau = \pm T_r$).

Comparing Figs. 3(a)–(e) it is easy to see that the ambiguity function of the staggered Costas signal is superior to all the others shown. In order to further demonstrate this fact we present an expanded three-dimensional view of the main ambiguity strip for all five signals, in Figs. 4(a)–(e).

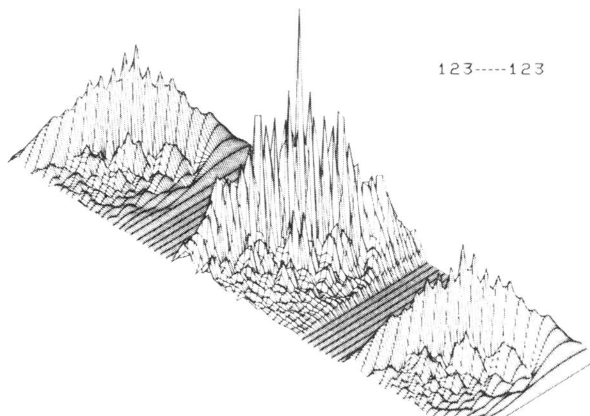
III. COMPARISON WITH COMPLEMENTARY SEQUENCES

So far we have ignored the phase issue. A more general form of expressing the complex envelope of each pulse is to modify (2) by including phase terms. Thus the complex envelope of pulse number n in burst number m is given by

$$P_{nm}(t) = \begin{cases} \exp(j2\pi f_{nm} t + j\phi_{nm}), & 0<t<T \\ 0, & \text{elsewhere}. \end{cases} \tag{11}$$



(a)



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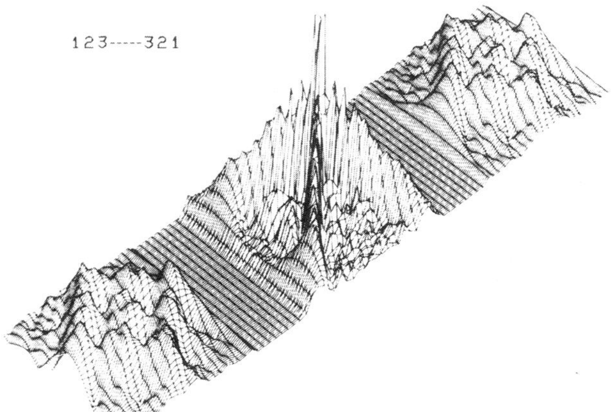

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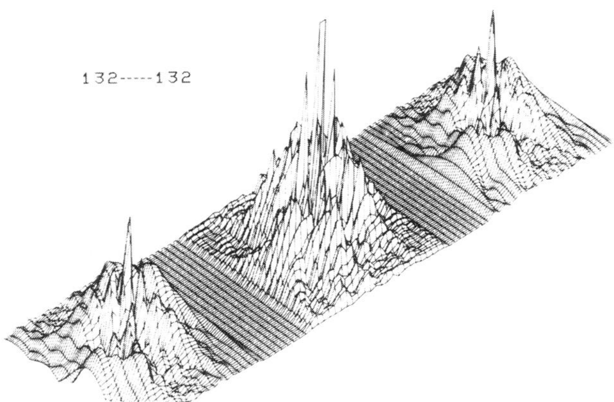
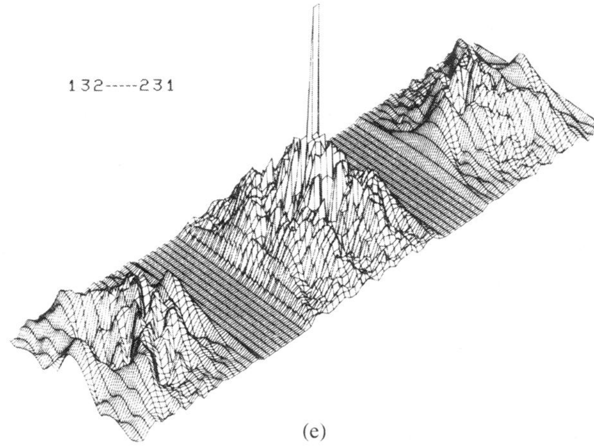


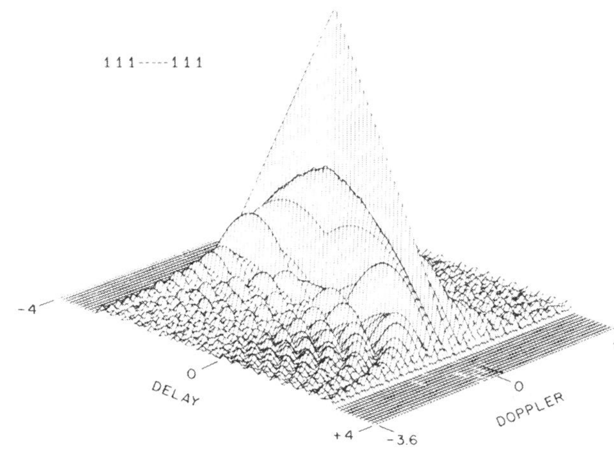
(d)



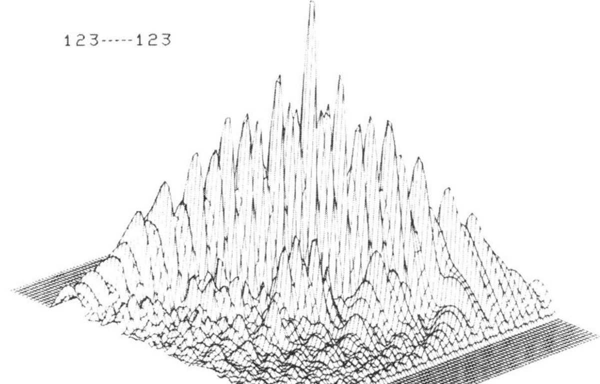
(e)

Fig. 3. Complete ambiguity function of five signals shown in Fig. 2.

This is a more general signal than (2), since it includes an additional degree of freedom, phase hopping. Before getting further into the phase issue, it should be noted that an advantage of Costas signals is their limited sensitivity to phase coherence. Costas's original application was sonar, where phase coherence is hard to maintain. He was also using noncoherent processors, i.e., filters and detectors for each frequency, before adding the contribution from each frequency at the proper delay. By



(a)



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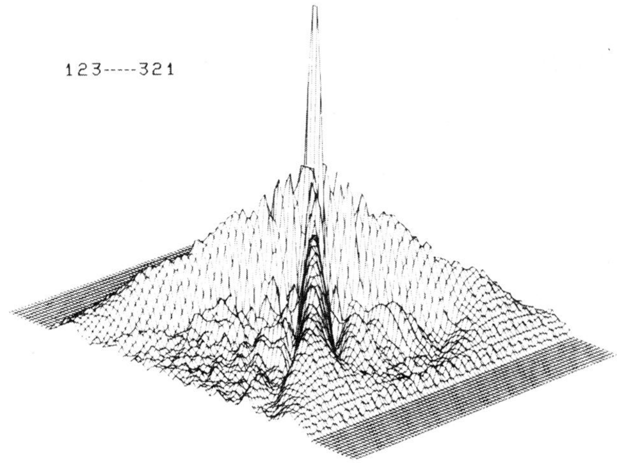

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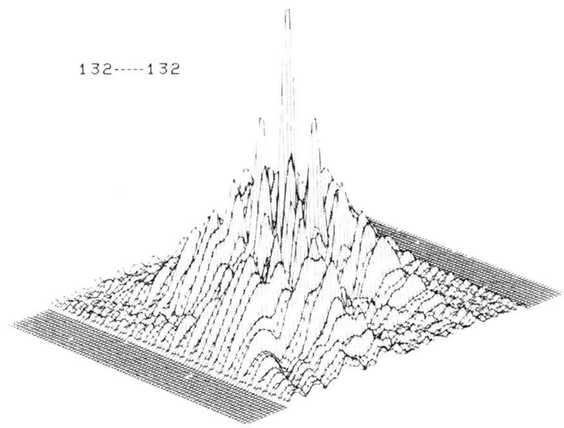

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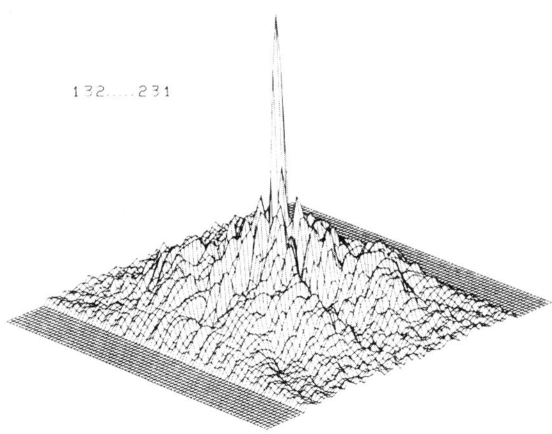


(e)

Fig. 4. Main strip of five ambiguity functions shown in Fig. 3. (continued)

adding phase hopping we are likely to lose this quality of Costas signals, but other qualities may be gained. It is well known that signals at different frequencies exhibit zero correlation while signals at different phases can exhibit negative correlation.

A. Phase-Hopping Signals

A special case of (11) occurs when $f_{nm} = 0$. This case is by itself a large family of signals of which a very interesting one is when ϕ_{nm} are multiphase complementary codes [5]. An example of a simple multiphase complementary sequence is the pair:

$$\{\phi_{n1}\} = 0,0,\pi; \quad \{\phi_{n2}\} = 0,\pi/2,0. \tag{12}$$

A major quality of such a pair of bursts can be observed when the spacing between the bursts is longer than the length of each burst. In that case the ambiguity function at zero Doppler (which is the autocorrelation function) for $|\tau| < NT$, is the sum of the autocorrelations of each burst, yielding zero everywhere except for a $2N$ peak at zero delay. For the particular pair represented by (12) the autocorrelations (positive delays) are $3,0,-1$ and $3,0,1$ and their sum is $6,0,0$. Cross-correlation terms of the two signals appear only around $\tau = \pm T_r$, i.e., when the delay is equal to the burst repetition period.

The cancellation of sidelobes on the zero Doppler axis does not persist at Dopplers other than zero. The results of calculating the ambiguity function are given in Figs. 5–8. Fig. 5 is the three-dimensional view of the entire function including the two side strips. Fig. 6 is a blow-up of the main strip. These two figures should be compared with Figs. 3(e) and 4(e) which are the corresponding ones for the staggered Costas signals. Fig. 7 is a cut along zero Doppler, which indeed indicates zero sidelobes when $\nu=0$ and $1<|\tau|<5$ $(5=T_r-NT)$. Fig. 8 is a cut parallel

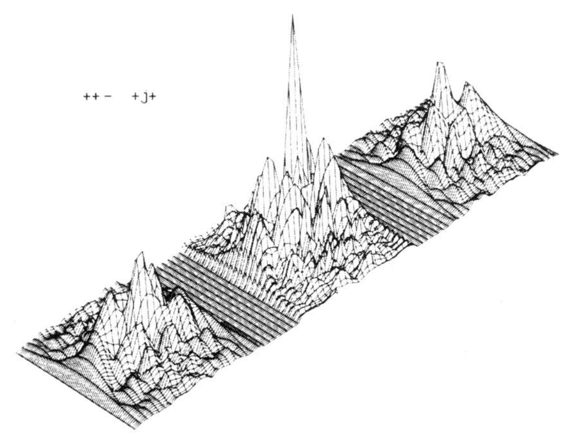


Fig. 5. Complete ambiguity function of 2×3 CPC signal given in (12).

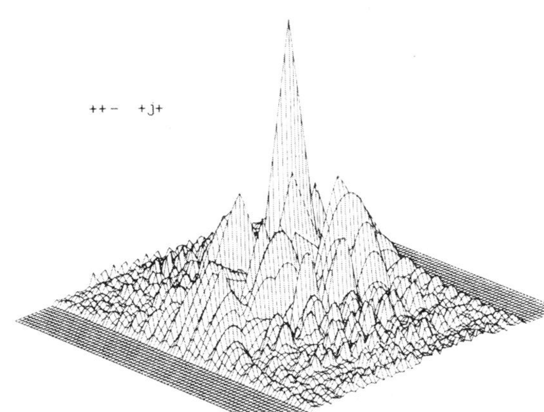


Fig. 6. Main strip of ambiguity function shown in Fig. 5.

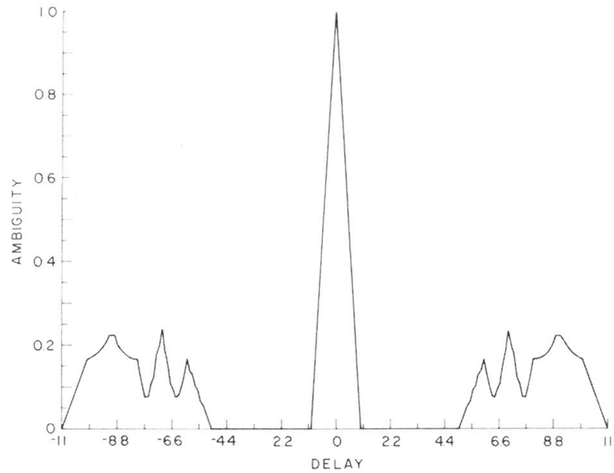


Fig. 7. Cut along zero Doppler ($\nu = 0$) of the ambiguity function shown in Fig. 5.

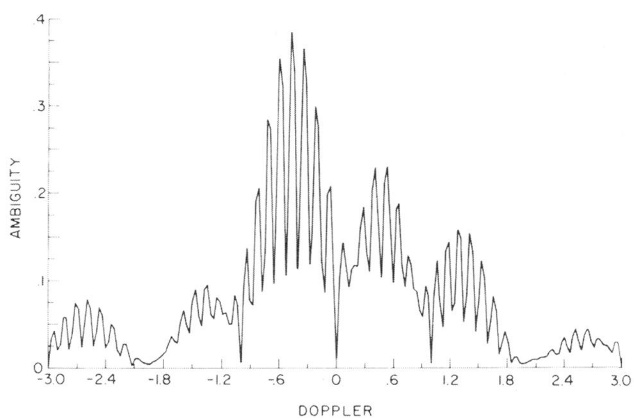


Fig. 8. Cut parallel to Doppler axis at ($\tau = 1.5$) of the ambiguity function shown in Fig. 6.

to the Doppler axis at $\tau = 1.5$. It was plotted in order to demonstrate how narrow the canyon is around $\nu = 0$ in which the sidelobes level is zero. The cut reveals also zero level of the ambiguity function at $\nu = \pm 1$.

IV. PHASE AND FREQUENCY HOPPING

A more general case is obtained when both phase and frequency are coded. Clearly the number of different signals is increased considerably. Here we present preliminary results only.

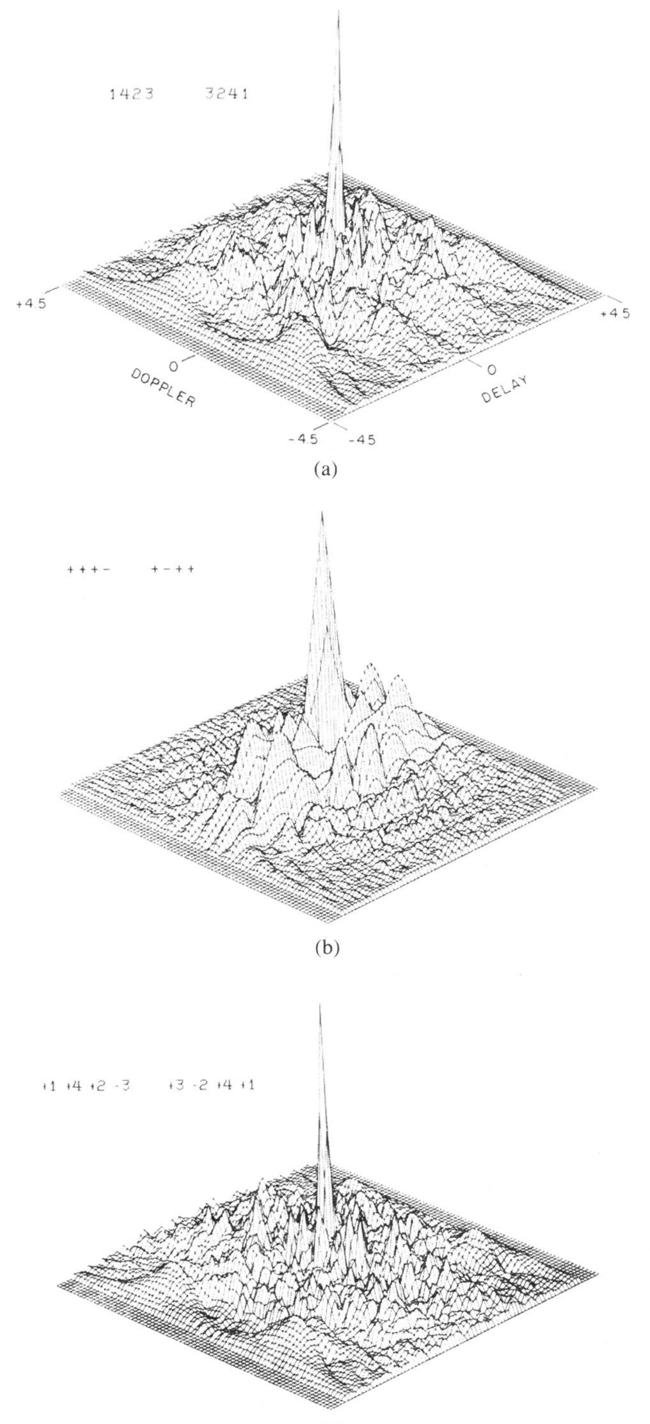
As a basis for comparison we choose, this time, a train of two bursts, each consisting of four contiguous pulses. The minimum number of coincidences among pairs of Costas signals of length $N = 4$ is four. The pair that yields the coincidences furthest from the origin results in lower peaks in the ambiguity function and was selected as the basis for the following comparison. The corresponding coding sequence is

$$\{\theta_{n1}\} = 1,4,2,3; \qquad \{\theta_{n2}\} = 3,2,4,1. \tag{13}$$

A complementary phase-coding (CPC) pair of the same dimension could be a binary signal, and the one selected has the phase sequence:

$$\{\phi_{n1}\} = 0,0,0,\pi; \qquad \{\phi_{n2}\} = 0,\pi,0,0. \tag{14}$$

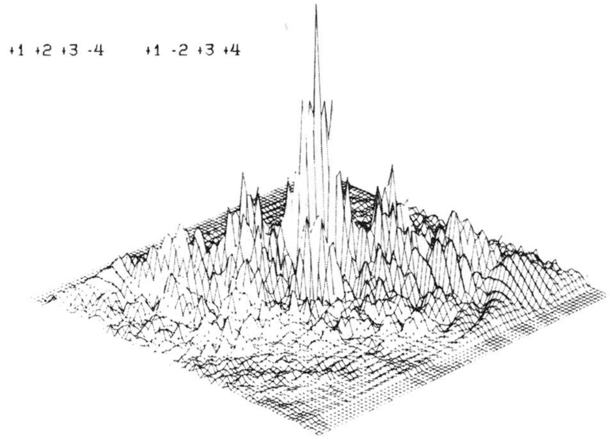
We begin the comparison by first presenting the ambiguity function of each coding by itself. Figs. 9(a) and 9(b) are the three-dimensional plots of the ambiguity function of the Costas and of the CPC signals, respectively ($T_r = 9T$ was selected.) If the thumbtack shape is the ideal, then clearly the Costas signal is superior to the CPC signal. Fig. 9(c) is the three-dimensional plot of the ambiguity function of a hybrid signal (11), in which the frequency is coded according to



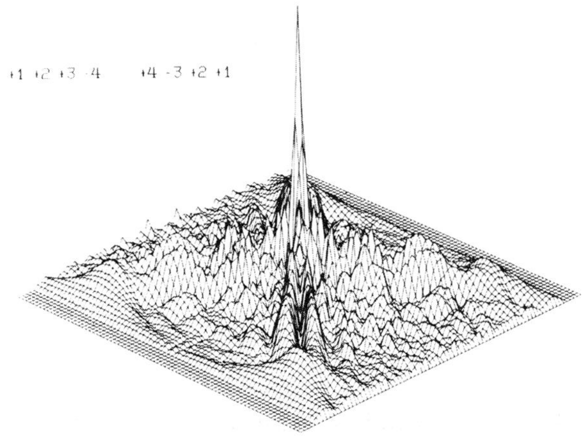
(a)

(b)

(c)



(d)



(e)

Fig. 9. Main ambiguity strip of five 2×4 signals. (a) Staggered Costas. (b) CPC. (c) Staggered Costas and CPC. (d) Linear FM and CPC. (e) V-FM and CPC.

(13) and the phase according to (14). The three-dimensional view reveals only minor improvements of the Costas + CPC hybrid signal over the Costas signal. There is a pronounced improvement over the CPC signal.

Other hybrid signals of interest are CPC combined with linear-FM and CPC with V-FM. Their ambiguity functions are given in Figs. 9(d) and 9(e), respectively.

V. CONCLUSIONS

Staggered Costas signals, introduced in this paper, can yield favorable ambiguity function shapes. They combine the qualities of both thumbtack and bed of nails signals. Some of the volume is removed to side strips, thus allowing a narrow mainlobe surrounded by a relatively low pedestal. At the same time, the side strips are shallow, missing the undesired narrow peaks (the nails). Plots of the ambiguity function of staggered Costas signals reveal their superiority over other frequency modulated signals, such as linear FM and V-FM, and over phase-coded signals such as CPC. A limited number of hybrid signals such as CPC and Costas coding simultaneously were also investigated.

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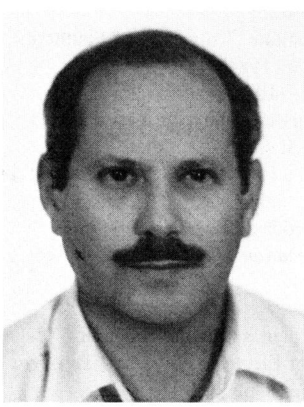

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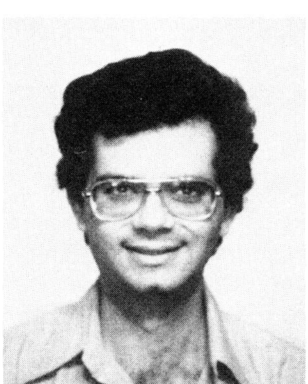

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