COHERENT PULSE TRAIN

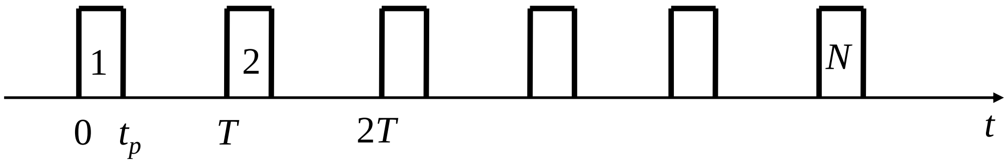
- Extends the coherent signal duration to provide Doppler resolution

- Spreads the constant-volume of the ambiguity function into a “bed-of nails”

- Can use unmodulated as well as compressed pulses

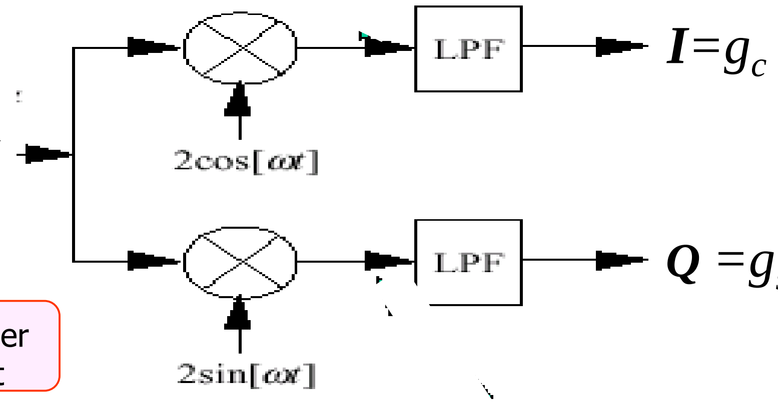
- Delay of target returns are usually shorter than the interval between pulses (but not always)

Doppler resolution is determined by the length of the Coherent Processing Interval (CPI).
Coherent pulse train ➔ long CPI ➔ better (narrower) Doppler resolution.



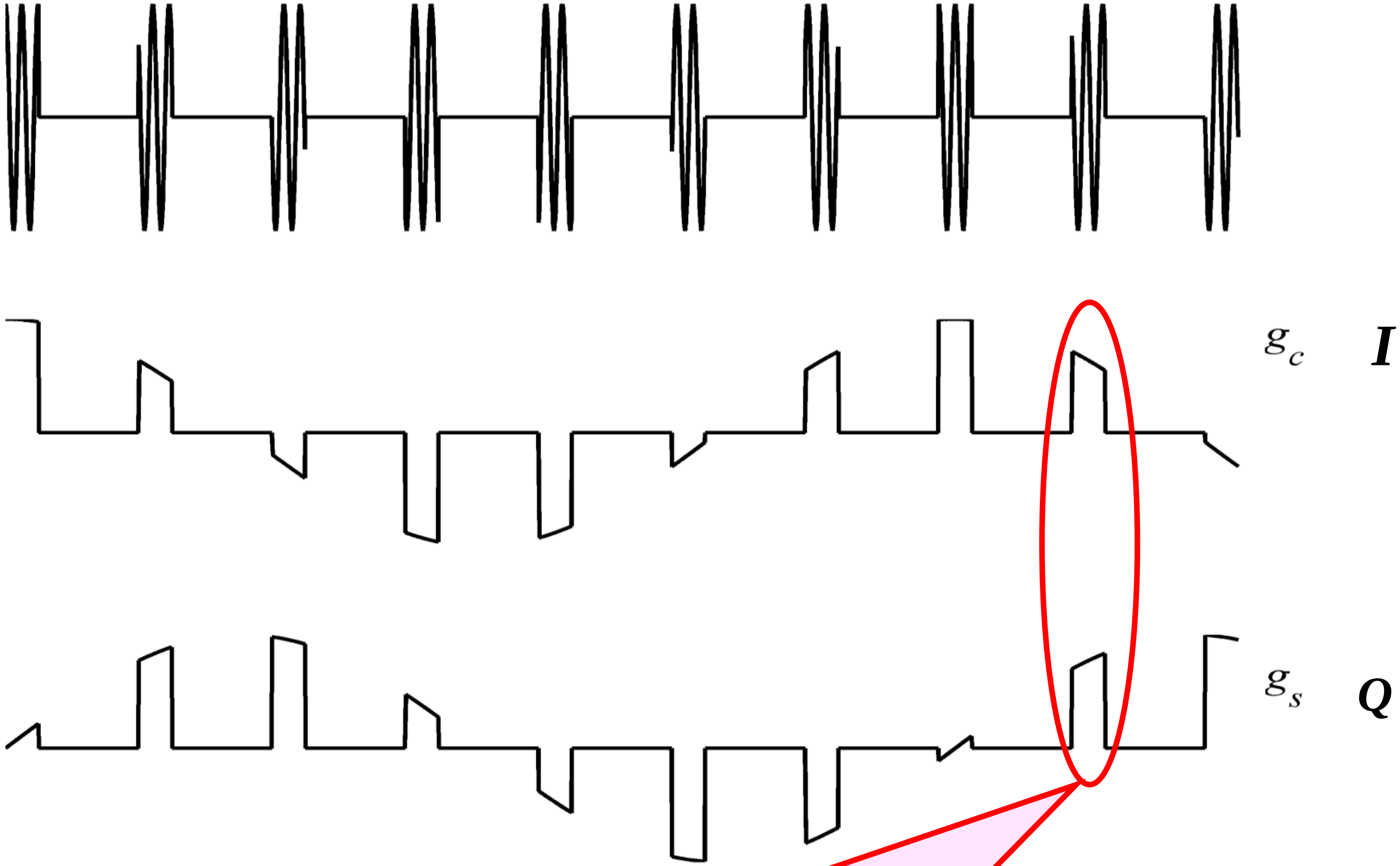
$$s_r(t) = p(t)\cos\left[\left(\omega + 2\pi f_D\right)t\right]$$

Doppler shift



iq_rec

In-phase and quadrature components of synchronously detected Doppler shifted pulses



The slopes indicate phase change,
due to Doppler, during the pulse.
May pose a problem in pulse
compression with low Doppler tolerance.
May require **F**ast-**T**ime **D**oppler **C**ompensation.

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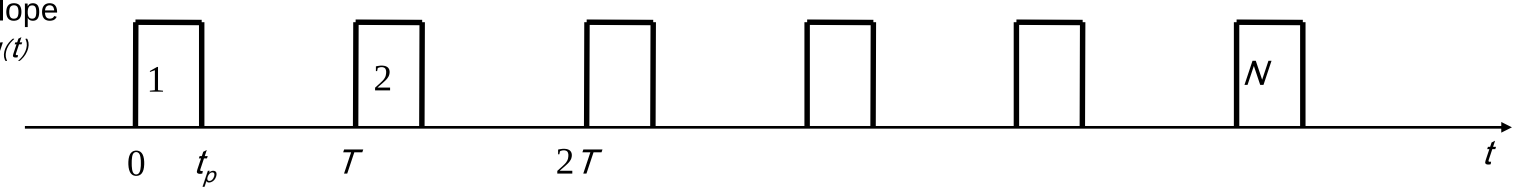
COHERENT PULSE TRAIN

Complex envelope of the signal *u(t)*



$$s(t) = \mathrm{Re}\left[u_N(t)\underbrace{\exp\left(j2\pi f_c t\right)}_{\text{Implies coherency}}\right]$$

$$u_N(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} u_n\left[t - (n-1)T\right]$$

Identical pulses $\Rightarrow$ $u_n(t) = u_1(t)$

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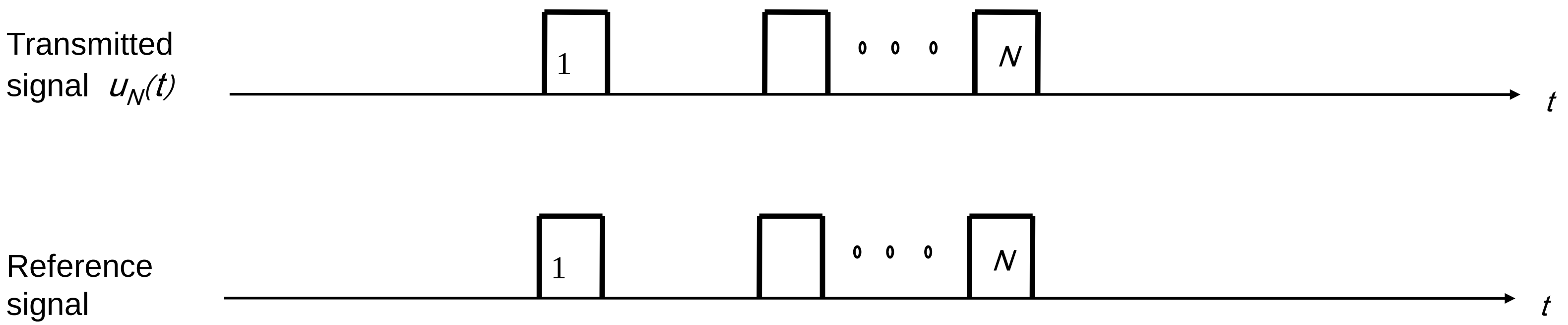
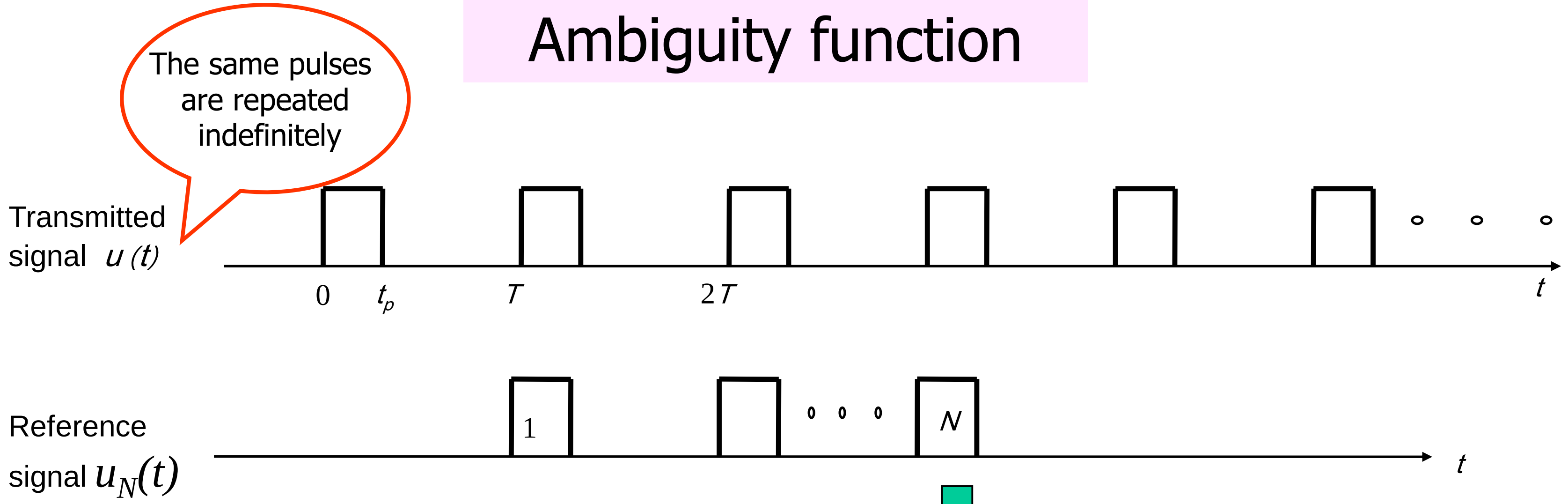


Ambiguity function



$$u(t) = \frac{1}{\sqrt{N}} \sum_{n=-\infty}^{\infty} u_n \left[t - \left(n - 1 \right) T \right], \; u_n = u_1$$

Periodic ambiguity function

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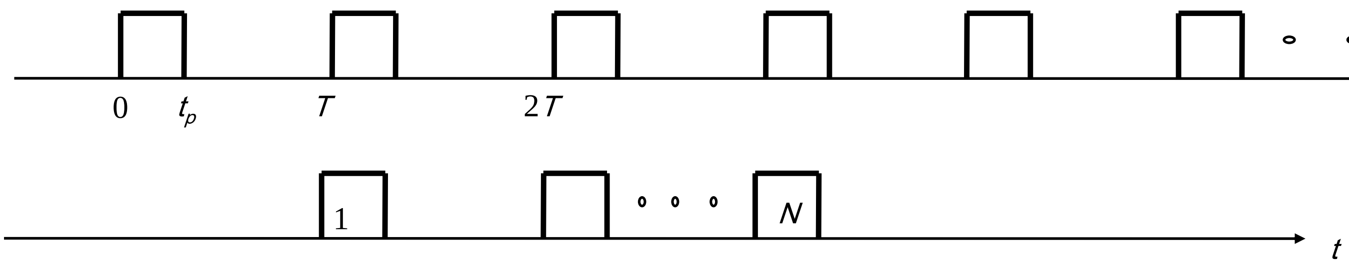
PERIODIC AMBIGUITY FUNCTION

Transmitted signal $u(t)$

Reference signal $u_N(t)$



$$\left|\chi_{NT}(\tau,\nu)\right| = \left|\frac{1}{NT}\int_{-\infty}^{\infty} u_N(t)u^*(t+\tau)\exp(j2\pi\nu t)dt\right|$$

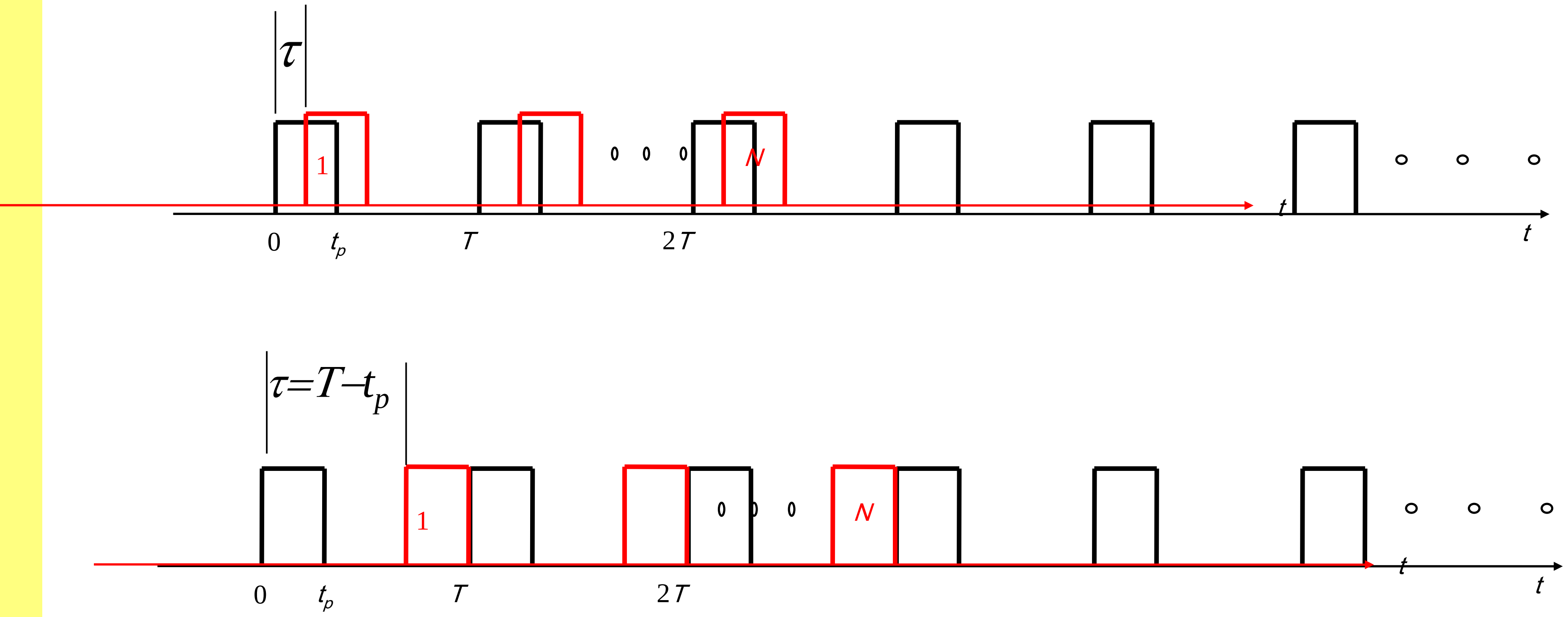
$$\left|\chi_{NT}(\tau,\nu)\right| = \left|\frac{1}{NT}\int_{0}^{NT} u(t)u^*(t+\tau)\exp(j2\pi\nu t)dt\right|$$

$$\left|\chi_{T}(\tau,\nu)\right| = \left|\frac{1}{T}\int_{0}^{T} u(t)u^*(t+\tau)\exp(j2\pi\nu t)dt\right|$$

$$\boxed{\left|\chi_{NT}(\tau,\nu)\right| = \left|\chi_{T}(\tau,\nu)\right|\left|\frac{\sin(N\pi\nu T)}{N\sin(\pi\nu T)}\right|}$$

For $|\tau| < T - t_p$ and any N

the ambiguity function = periodic ambiguity function



For $|\tau|<T-t_p$ and any N
the ambiguity function = periodic ambiguity function

PERIODIC AMBIGUITY FUNCTION

$$\left|\chi_{NT_r}(\tau,\nu)\right| = \left|\chi_{T_r}(\tau,\nu)\right| \underbrace{\left|\frac{\sin(N\pi\nu T_r)}{N\sin(\pi\nu T_r)}\right|}$$

$$\left|\chi_{T_r}(\tau,\nu)\right| = \frac{1}{T_r}\left|\begin{array}{l}\int_0^{\tau} u(t+T_r-\tau)u^*(t)\exp(j2\pi\nu t)\,dt \\ +\int_{\tau}^{T_r} u(t-\tau)u^*(t)\exp(j2\pi\nu t)\,dt\end{array}\right|$$

$$\left|\chi_{T_r}(\tau,\nu)\right| = \left|\chi(\tau,\nu)\right|, \quad t_p < T_r/2$$

Y = diric(X,N) returns a matrix the same size as X whose elements are the **Dirichlet** function of the elements of X. Positive integer N is the number of equally spaced extrema of the function in the interval 0 to 2*pi.

$x = 2\pi\nu T_r$

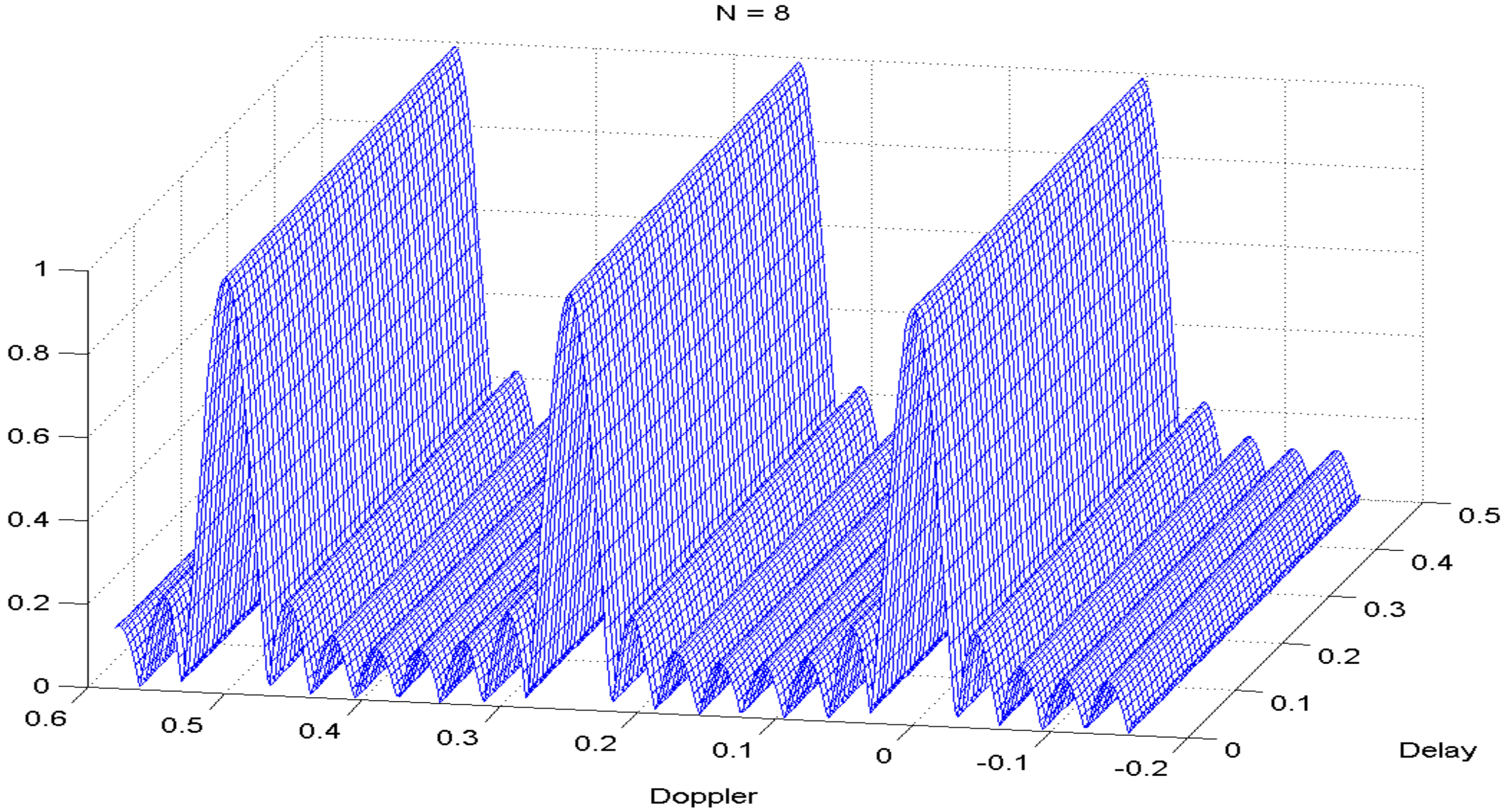
The Dirichlet function is defined as

d(x) = sin(N*x/2)./(N*sin(x/2)) for x not a multiple of 2*pi

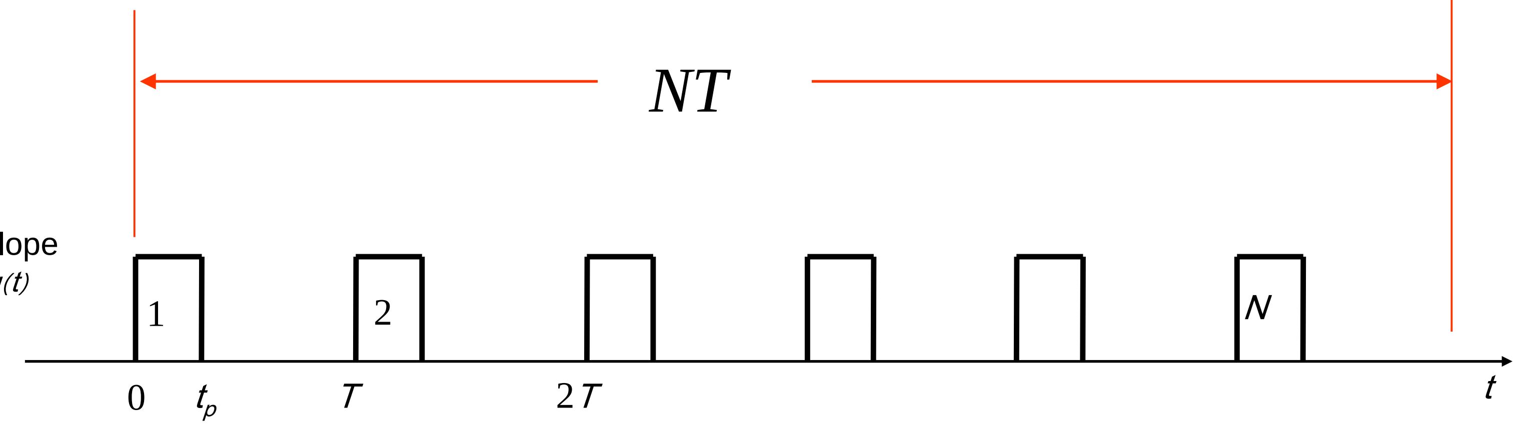
+1 or -1 for x a multiple of 2*pi. (depending on limit)

Also known as "periodic sinc" function or "aliased sinc" function.

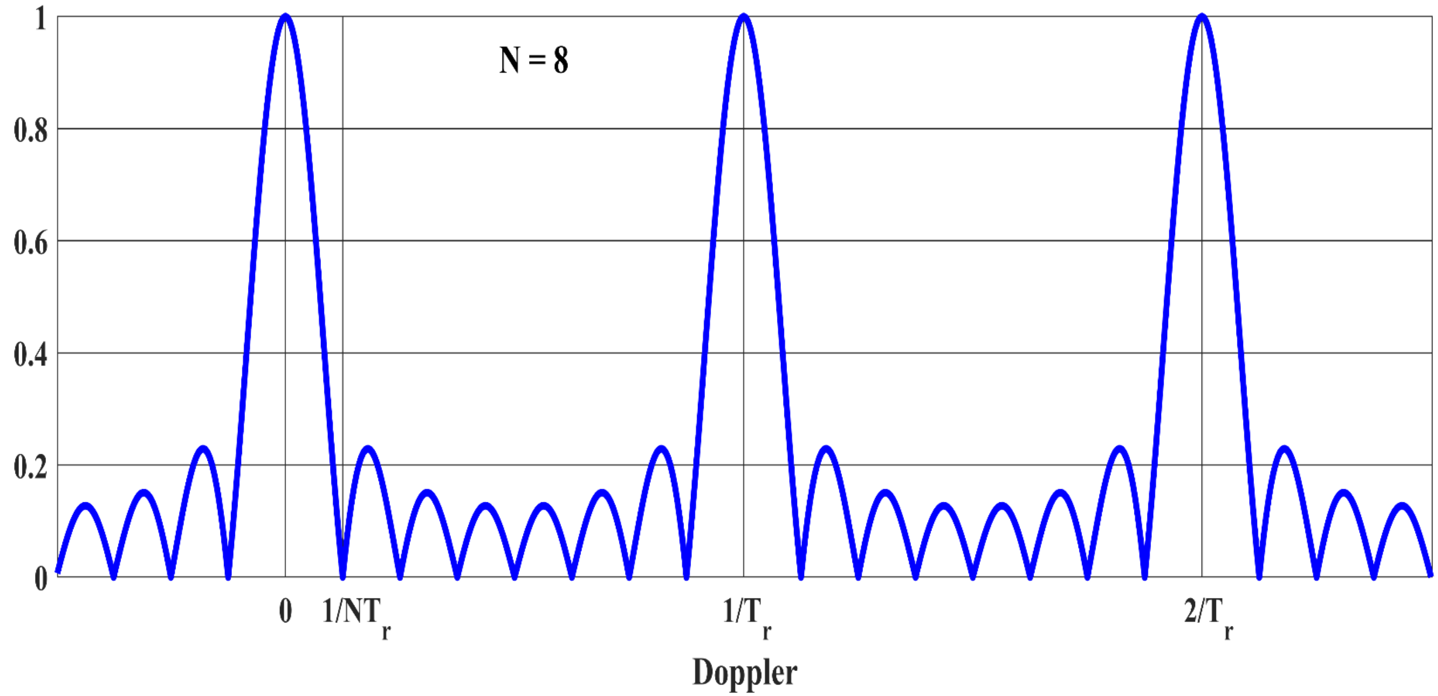
$$\left|\frac{\sin(N\pi\nu T)}{N\sin(\pi\nu T)}\right|$$



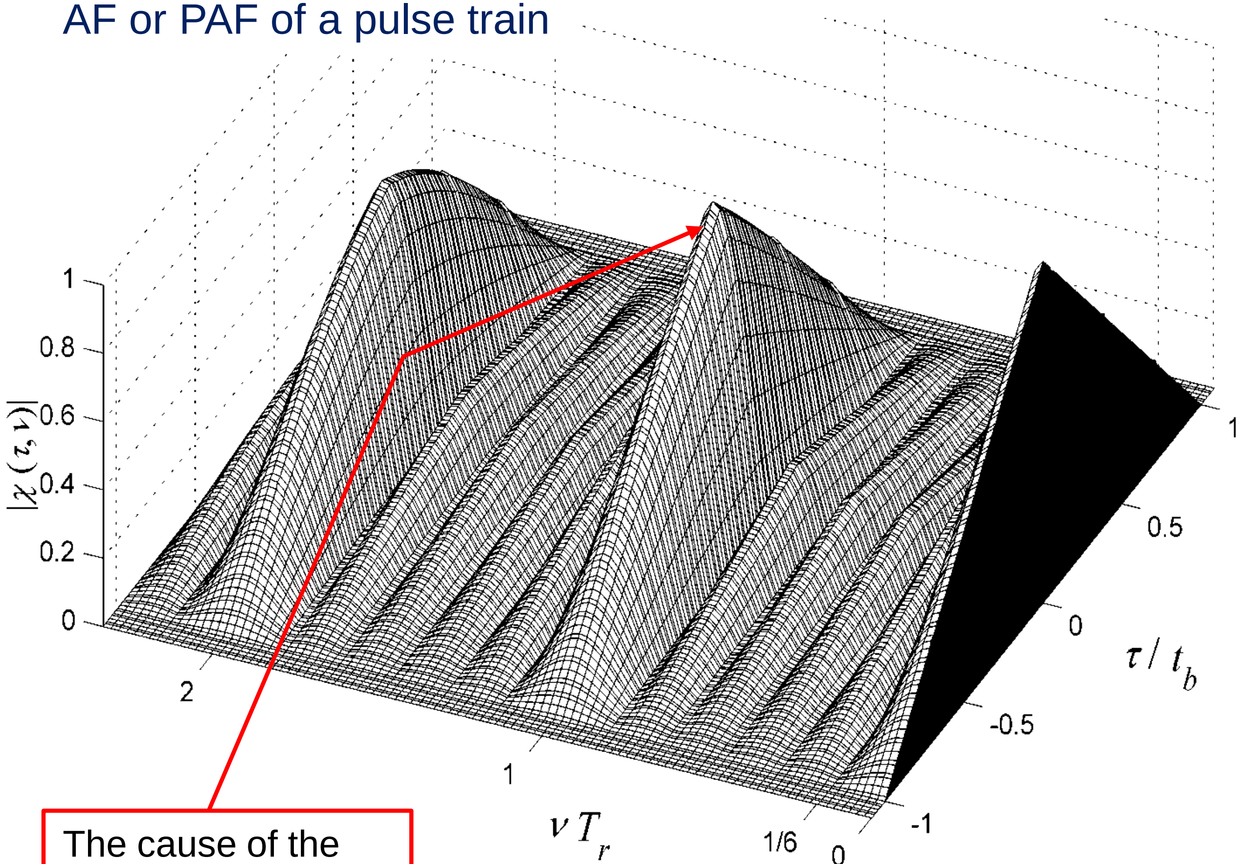
Complex envelope of the signal *u(t)*



$$\left|\frac{\sin(N\pi\nu T)}{N\sin(\pi\nu T)}\right|$$

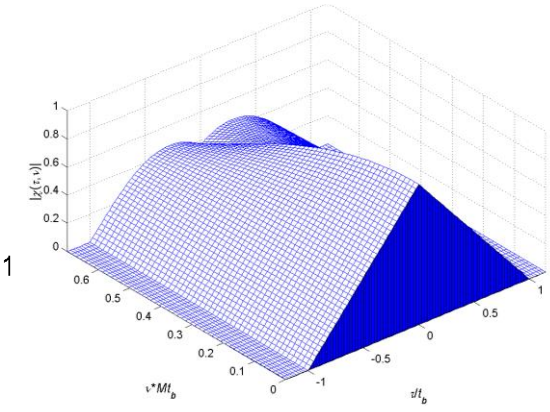


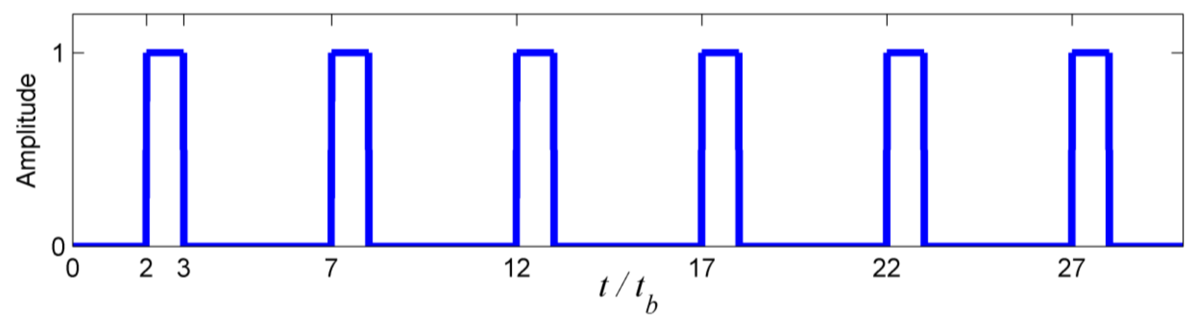
AF or PAF of a pulse train

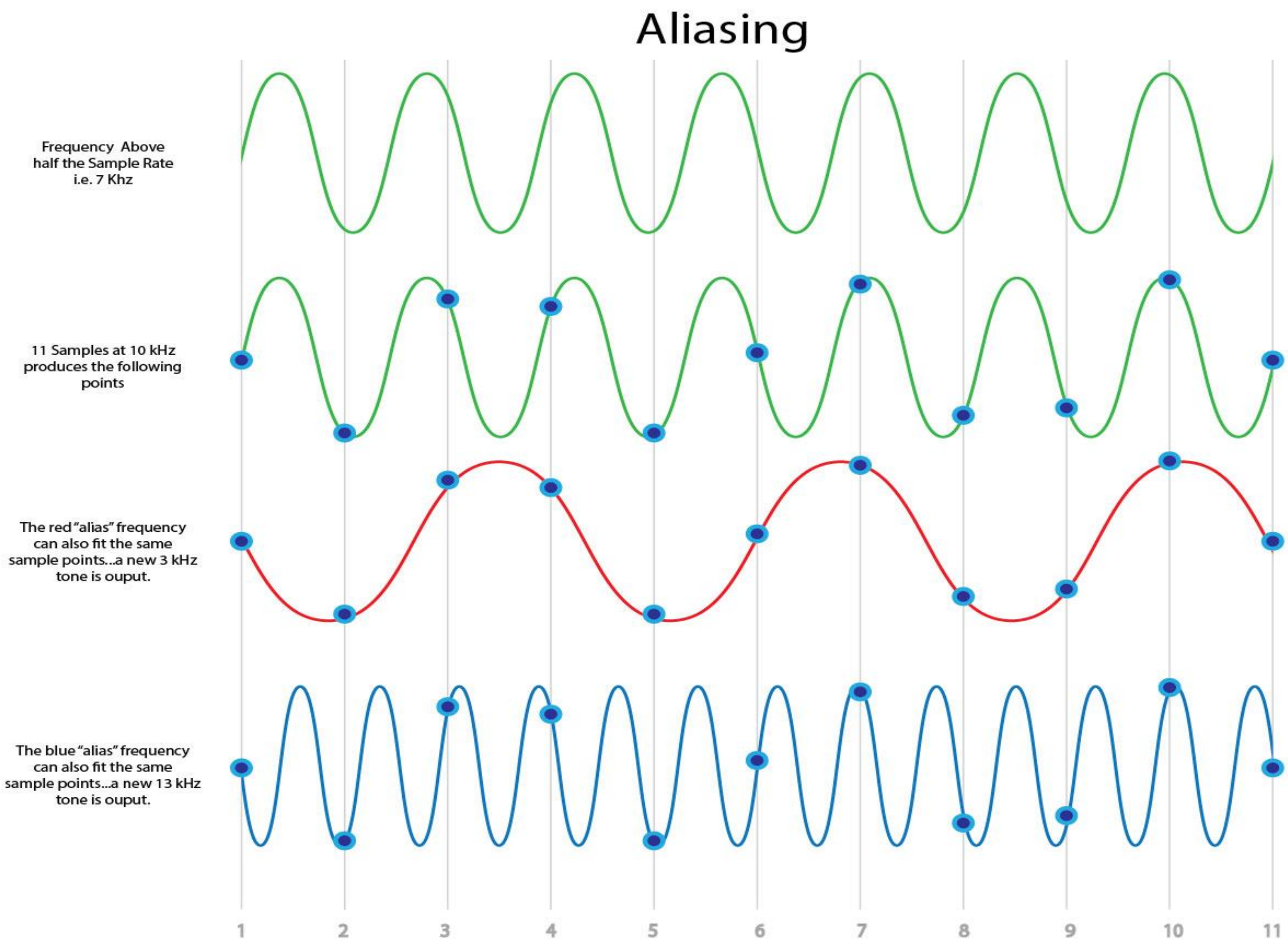


The cause of the ambiguous Doppler peak ? Aliasing !

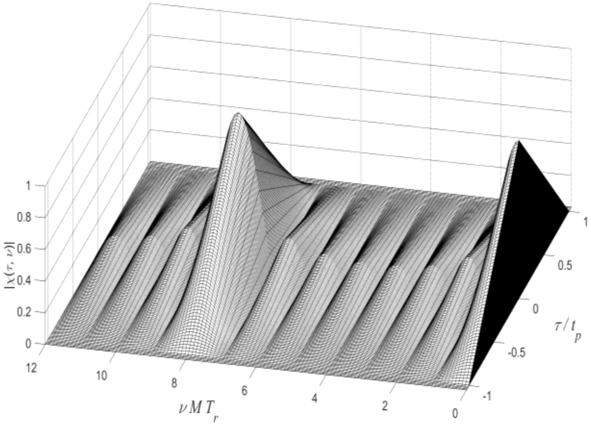
AF of a single pulse



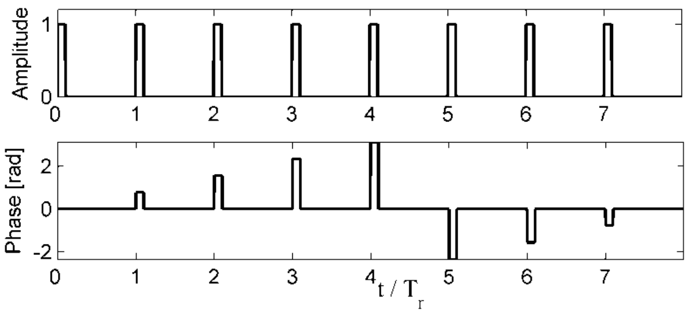



Aliasing
Frequency Above half the Sample Rate i.e. 7 Khz
11 Samples at 10 kHz produces the following points
The red "alias" frequency can also fit the same sample points...a new 3 kHz tone is ouput.
The blue "alias" frequency can also fit the same sample points...a new 13 kHz tone is ouput.
1
2
3
4
5
6
7
8
9
10
11

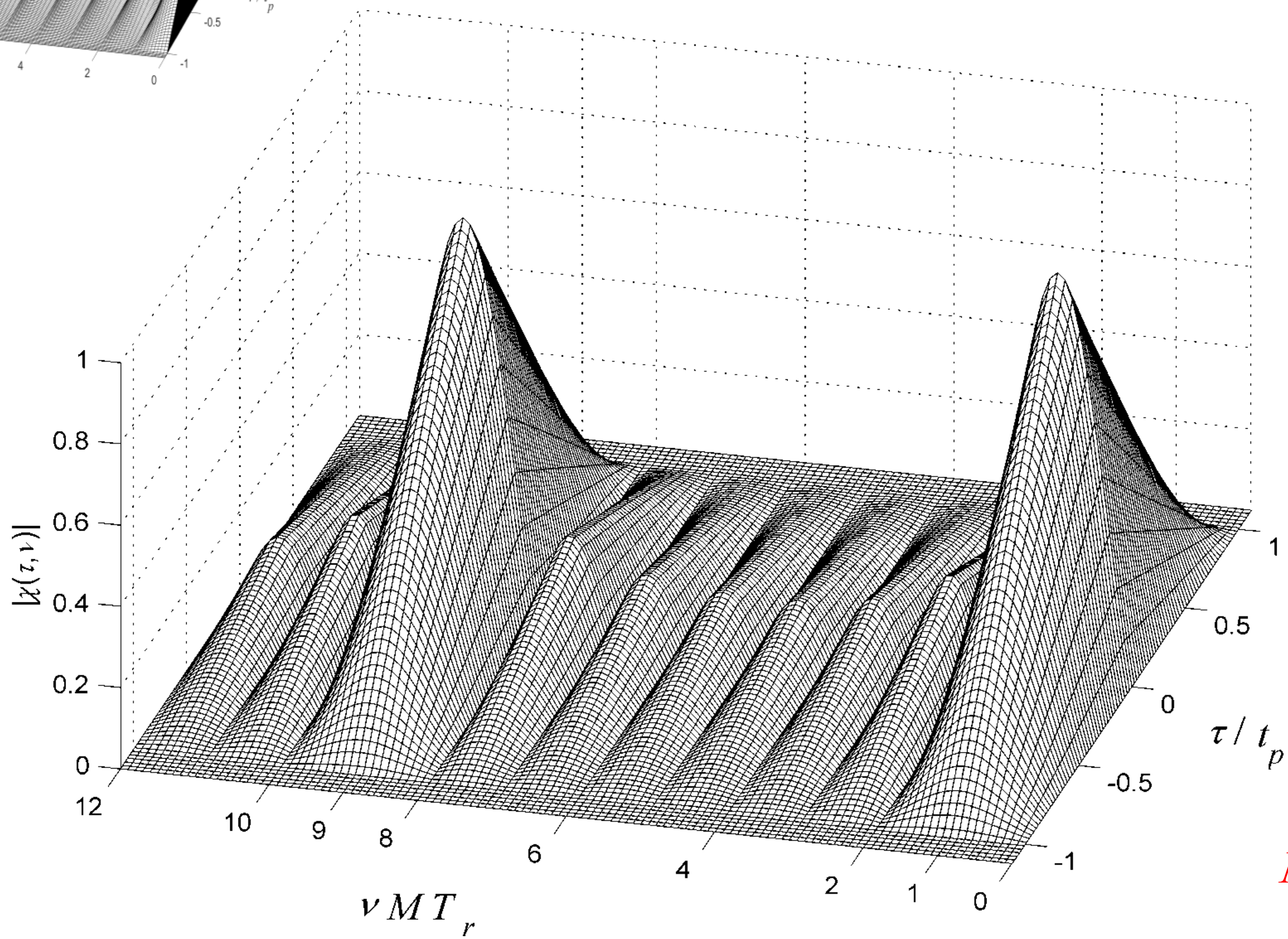
Creating a filter matched to a Doppler-shifted pulse-train return.



N=8



N= 8



N= 8

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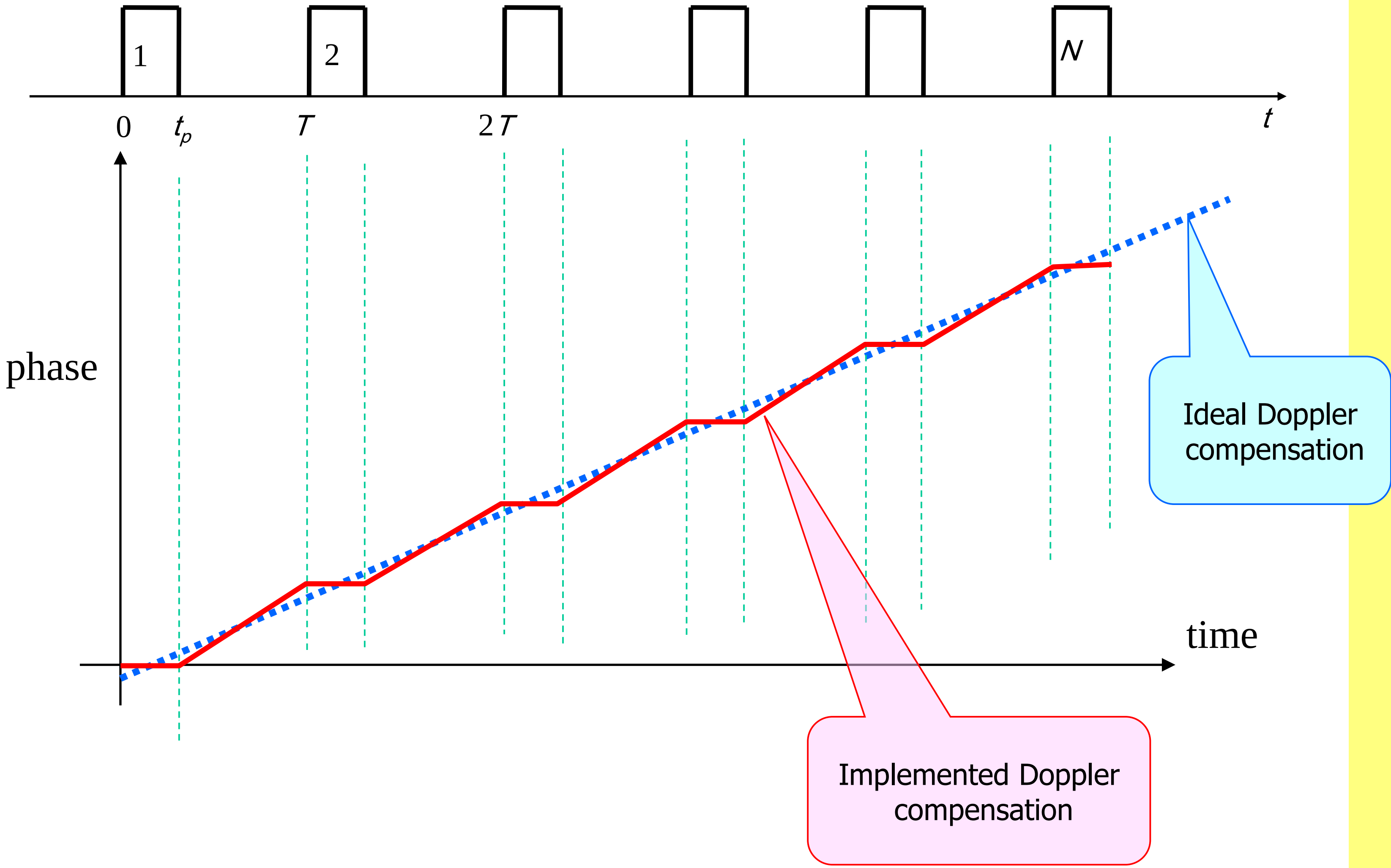
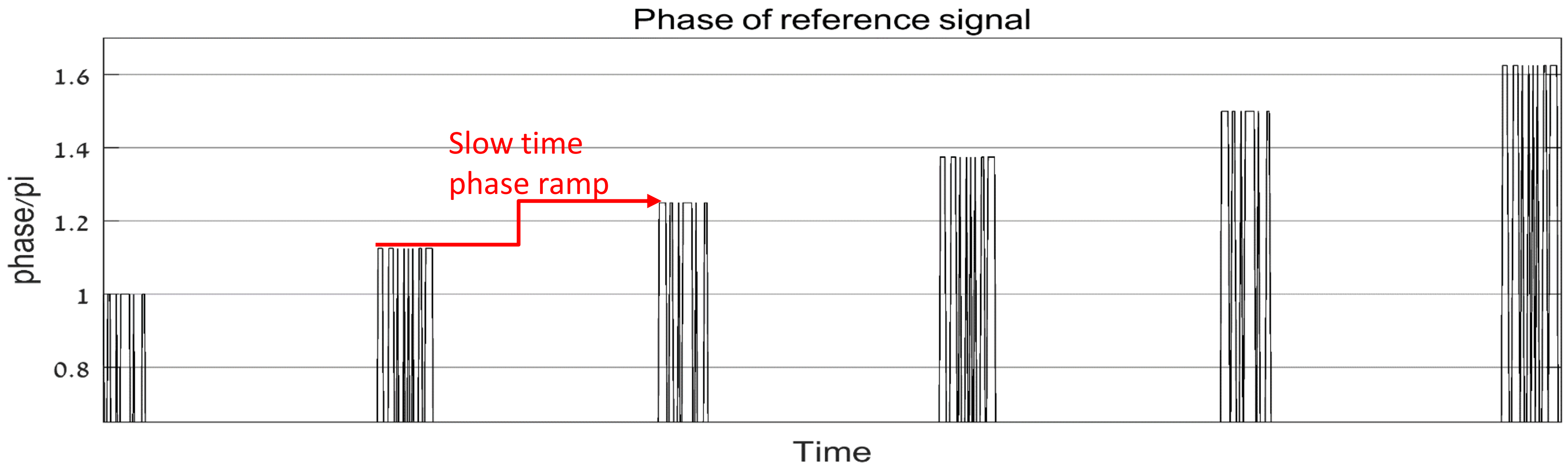
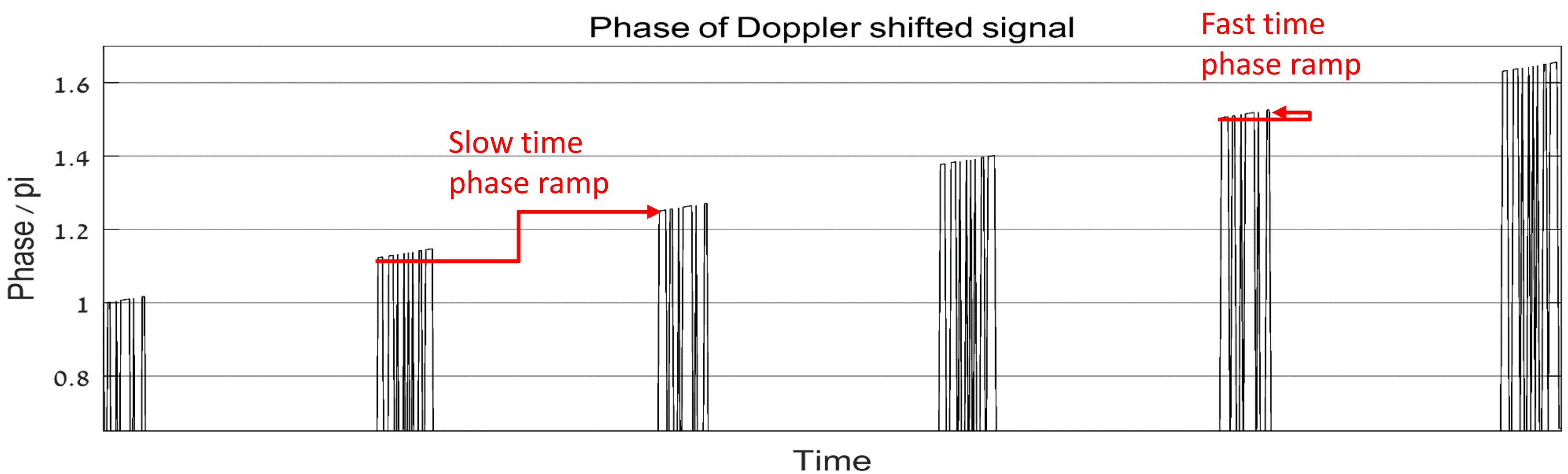


There is no Doppler compensation within each pulse.
Hence, performances degrade with Doppler.

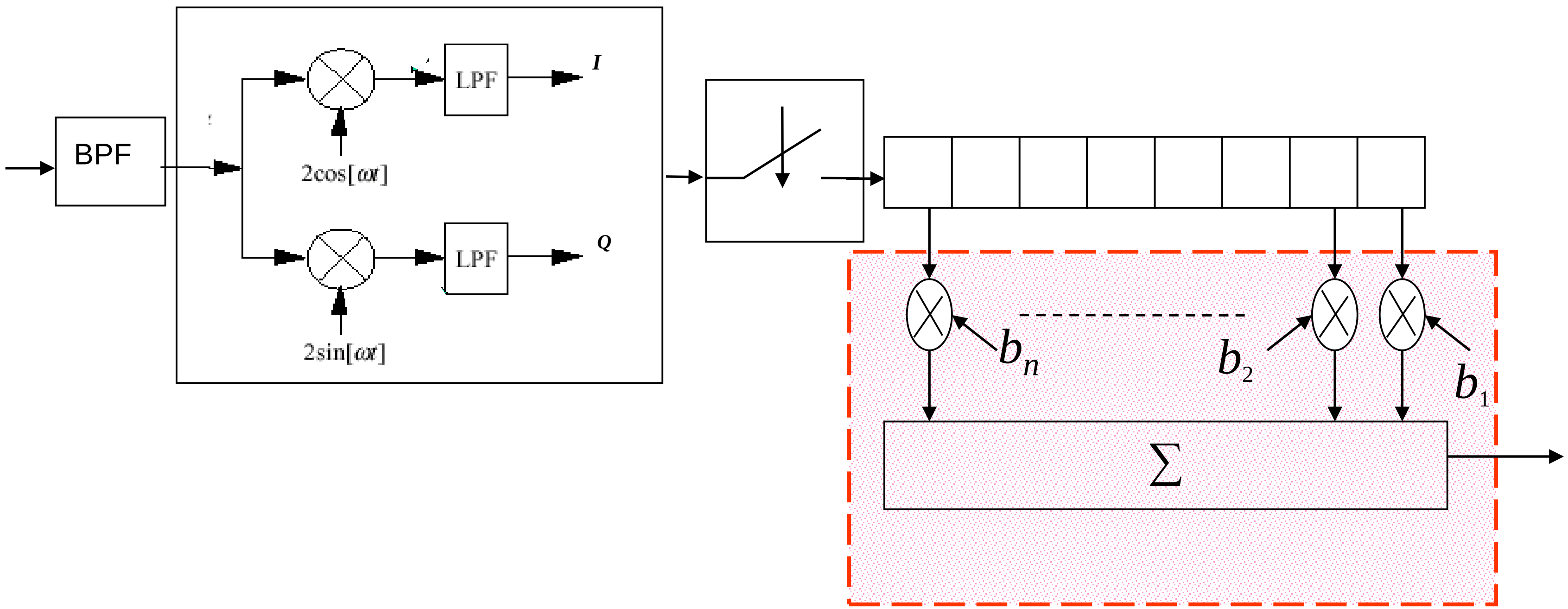
Mitigation: Add **F**ast **T**ime **D**oppler **C**ompensation (**FTDC**)

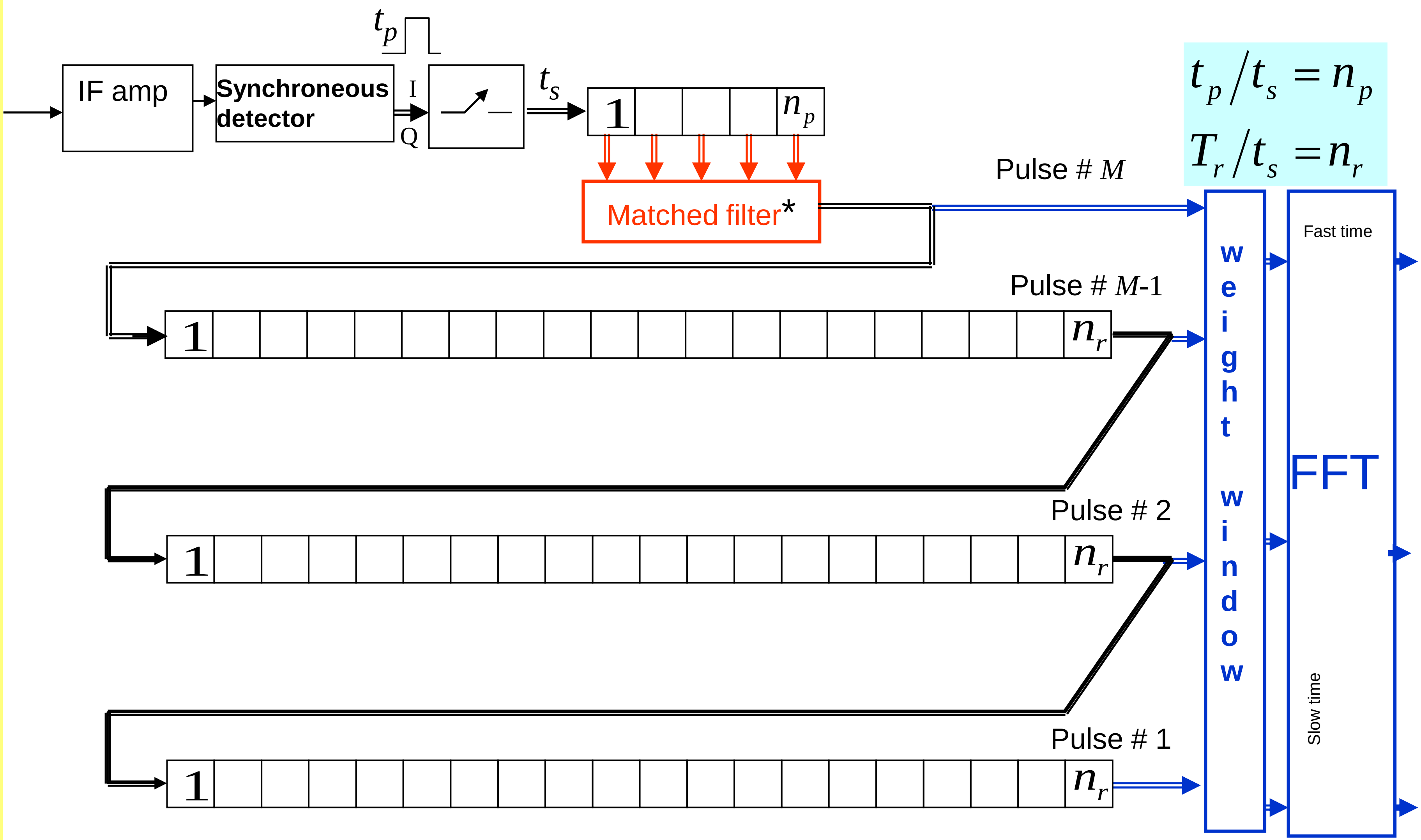
Range sidelobe's sensitivity to Doppler





Synchronous detection and matched filtering

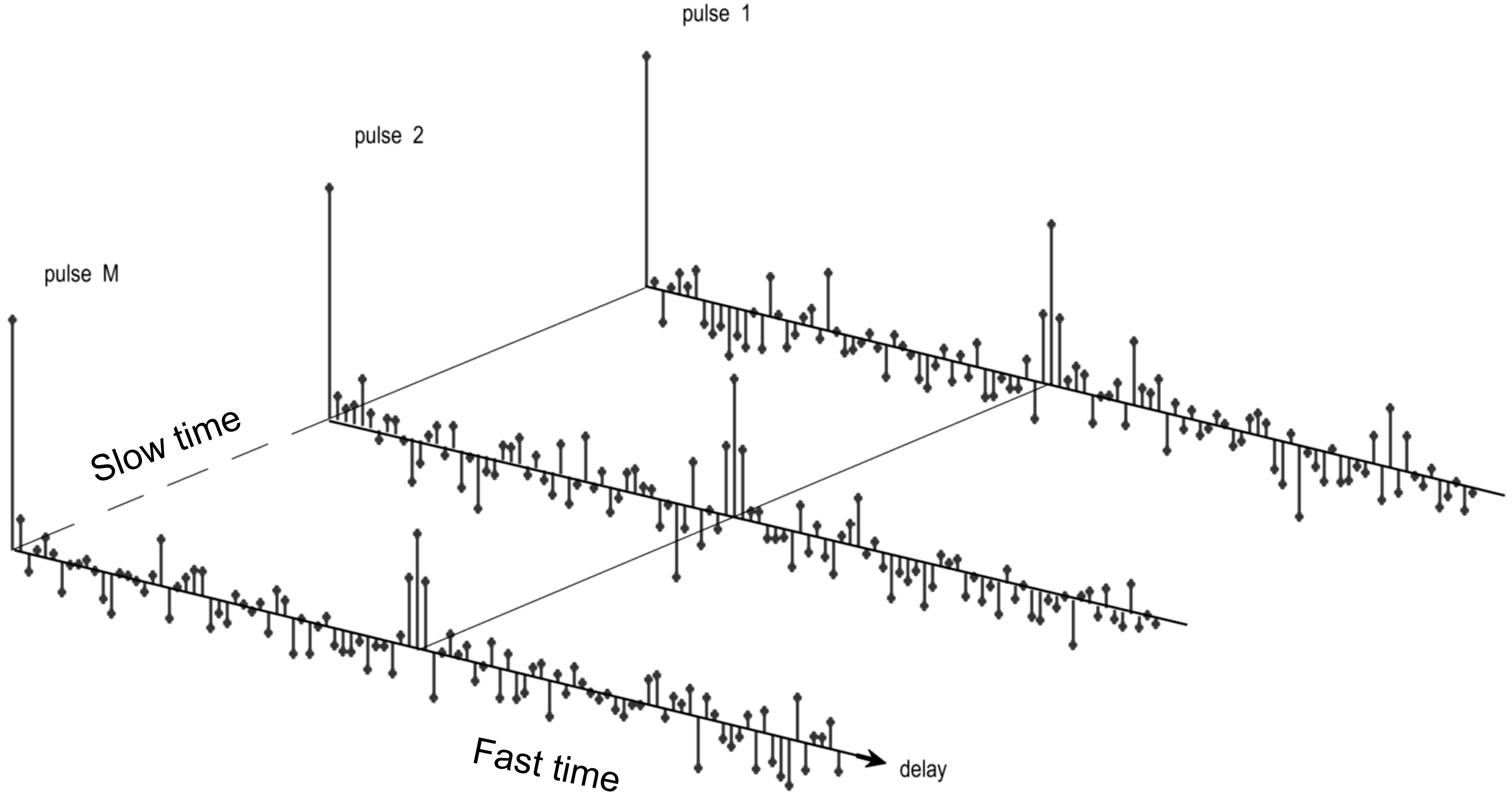




$$t_p / t_s = n_p$$
$$T_r / t_s = n_r$$

Integration of M pulses

Returns from *M* consecutive pulses (*I* component)



שאלה

סעיף זה של השאלה עוסק בעיבוד דופלר ע״י FFT של החזרי רכבת פולסים הנקלטים במכ״ם קוהרנטי.

טבלה 1 מציגה את הדגימות הקומפלקסיות של ההחזרים המתקבלים מ 8 השהיות עוקבות (השורות) עבור 16 פולסים עוקבים (העמודות).

להשגת פשטות נבחר בכל ההשהיות ובכל הפולסים החזר רקע זהה (גם בעצמה וגם בפאסה) שערכו הקומפלקסי = 1.

רק בחלון השהיה מס. 12 נוסף לרקע האחיד גם החזר ממטרה נעה במהירות רדיאלית קבועה. עצמת ההחזר מהמטרה היא קבועה (= 1) ורק הפאסה שלו משתנה מפולס לפולס, כרשום בטבלה 1.

Slow time

Fast time

TABLE 1	Complex input data from 16 consecutive pulses															
Pulse #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Delay/dt																
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	$1+e^{j0}$	$1+e^{j\pi/8}$	$1+e^{j2\pi/8}$	$1+e^{j3\pi/8}$	$1+e^{j4\pi/8}$	$1+e^{j5\pi/8}$	$1+e^{j6\pi/8}$	$1+e^{j7\pi/8}$	$1+e^{j8\pi/8}$	$1+e^{j9\pi/8}$	$1+e^{j10\pi/8}$	$1+e^{j11\pi/8}$	$1+e^{j12\pi/8}$	$1+e^{j13\pi/8}$	$1+e^{j14\pi/8}$	$1+e^{j15\pi/8}$
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	16 inputs FFT															

ההחזר מהמטרה הנעה, הנמצאת בהשהיה 12, משלים תוספת מופע של 2π בדיוק, במשך 16 הפולסים.

עבור כל השהיה (שורה בטבלה 1) מוזנים (ישירות ללא פונקציית משקול) הערכים המתאימים מ 16 הפולסים למעבד FFT בעל 16 כניסות ו 16 יציאות. מוצא ה FFT מתעדכן אחרי כל השהיה ומתקבלת טבלה 2, המציגה את הערך המוחלט של מוצאי ה FFT . התוצאה היא מפת השהייה - דופלר .

לא בהכרח

בטבלה 2 חסרים הערכים המתאימים לחלון ההשהיה מס. 12.

יש לחשב את כל הערכים החסרים בשורה. יש להסביר את החישוב, ואת משמעות התוצאה שנתקבלה.

<u>עזרה ותזכורת</u>: במוצא עיבוד ע״י FFT המקבל N כניסות מN פולסים עוקבים, היציאה מספר 0 איננה מקזזת שום הסט דופלר (רמפת פאסה) אלא פשוט מסכמת את N הכניסות. יציאה מספר 1 מקזזת הסט דופלר המשלים בדיוק מחזור דופלר אחד במשךN הפולסים (בלומר רמפת פאסה המשלימה 2π במשך N הפולסים). יציאה מספר 2 מקזזת הסט דופלר המשלים בדיוק 2 מחזורי דופלר במשך N הפולסים, וכן הלאה.

TABLE 2	Doppler cell				**Output data after 16 input FFT: Absolute intensities**											
Doppler filter # / **Delay/dt**	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
18	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16 Delay	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

תרומת הרקע הנייח.
זהה בכל ההשהיות.

תרומת המטרה

<u>תשובה מילולית</u> :

ה FFT מבצע פעולה לינארית ולכן אפשר להפריד הפעולה על הרקע הקבוע בחלון השהיה 12, שהיא זהה למה שקורה בשאר חלונות ההשהיה. כלומר סיכום 16 ערכים של 1. לבין הפעולה על ההחזר מהמטרה. רמפת הפאסה עקב המטרה מתאימה <u>בדיוק</u> לקיזוז שמבצע ה FFT כשהוא יוצר את מוצא ה FFT מס. 1. אחרי הקיזוז מתבצע גם כאן סיכום 16 ערכים של 1 ולכן המוצא המתאים לדופלר 1 יהיה גם הוא שווה ל 16.

<u>תשובה מבוססת על נוסחאת ה FFT</u> :

ב FFT בעל N כניסות $x[n],\ n=0,\,1,\,2,\,\ldots,\,N-1$ ו N יציאות $X[k],\ k=0,\,1,\,2,\,\ldots,\,N-1$

היציאה ה k נתונה ע"י:

$$\boxed{X[k]=\sum_{n=0}^{N-1}x[n]\exp(-j2\pi k\,n/N)}$$

בכל ההשהיות, **<u>מלבד בהשהיה ה 12</u>**, כל 16 הכניסות ל FFT הן פאזור קבוע בעוצמה ובפאסה הנתון ע"י 1. ובהן מתקיים לכן:

$$X[0]=\sum_{n=0}^{16-1}1\bullet\exp(-j0)=\mathbf{16}\quad,\quad X[k]=\sum_{n=0}^{16-1}1\bullet\exp(-j2\pi k\,n/16)=\mathbf{0},\ k=1,2,\ldots,15$$

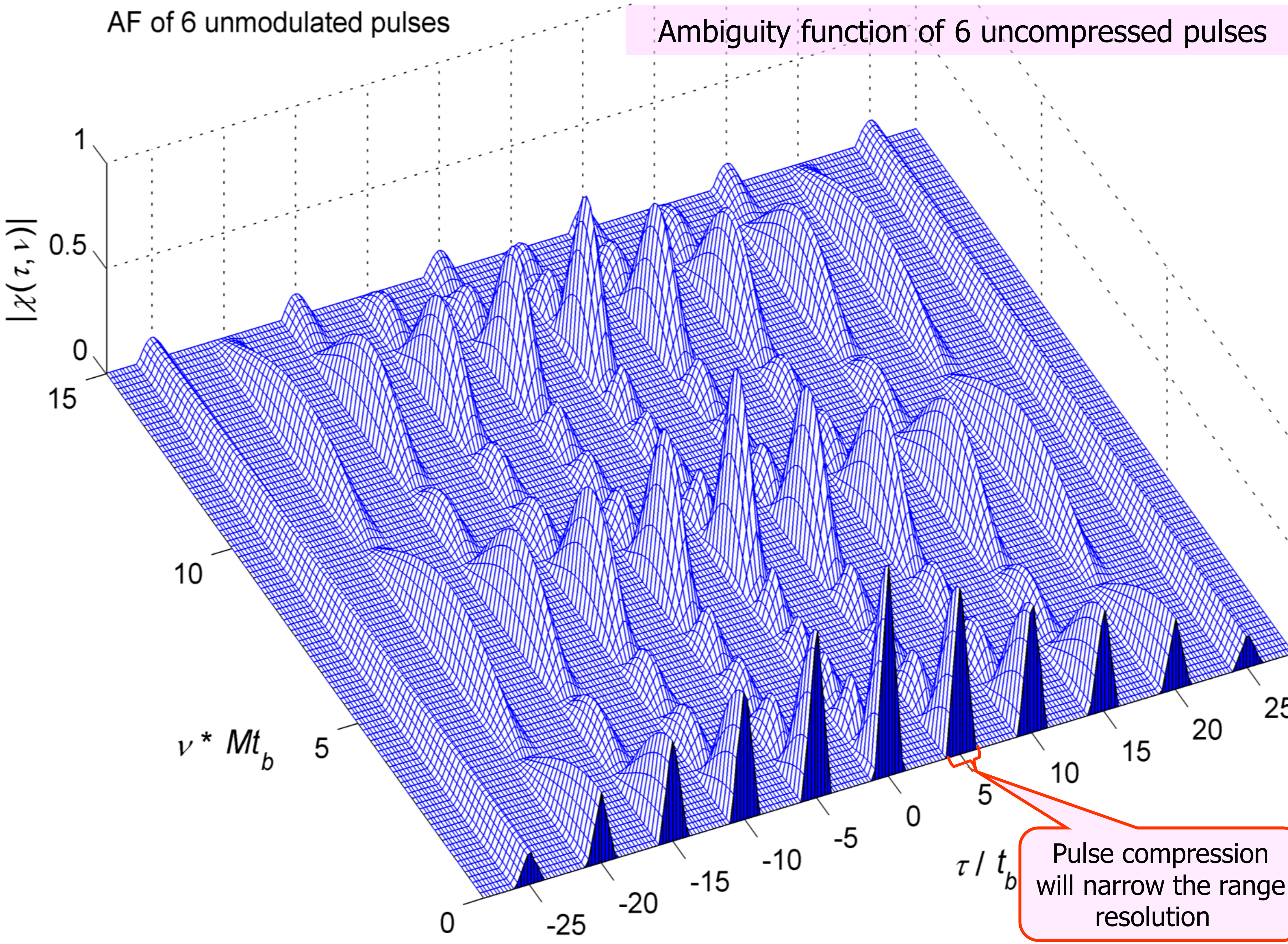
<u>בהשהיה ה 12</u> נתון כי 16 דגימות הכניסה ל FFT נתונות ע"י:

$$x[n]=1+\exp(j2\pi\,n/16),\ n=0,\,1,\,2,\,\ldots,15$$

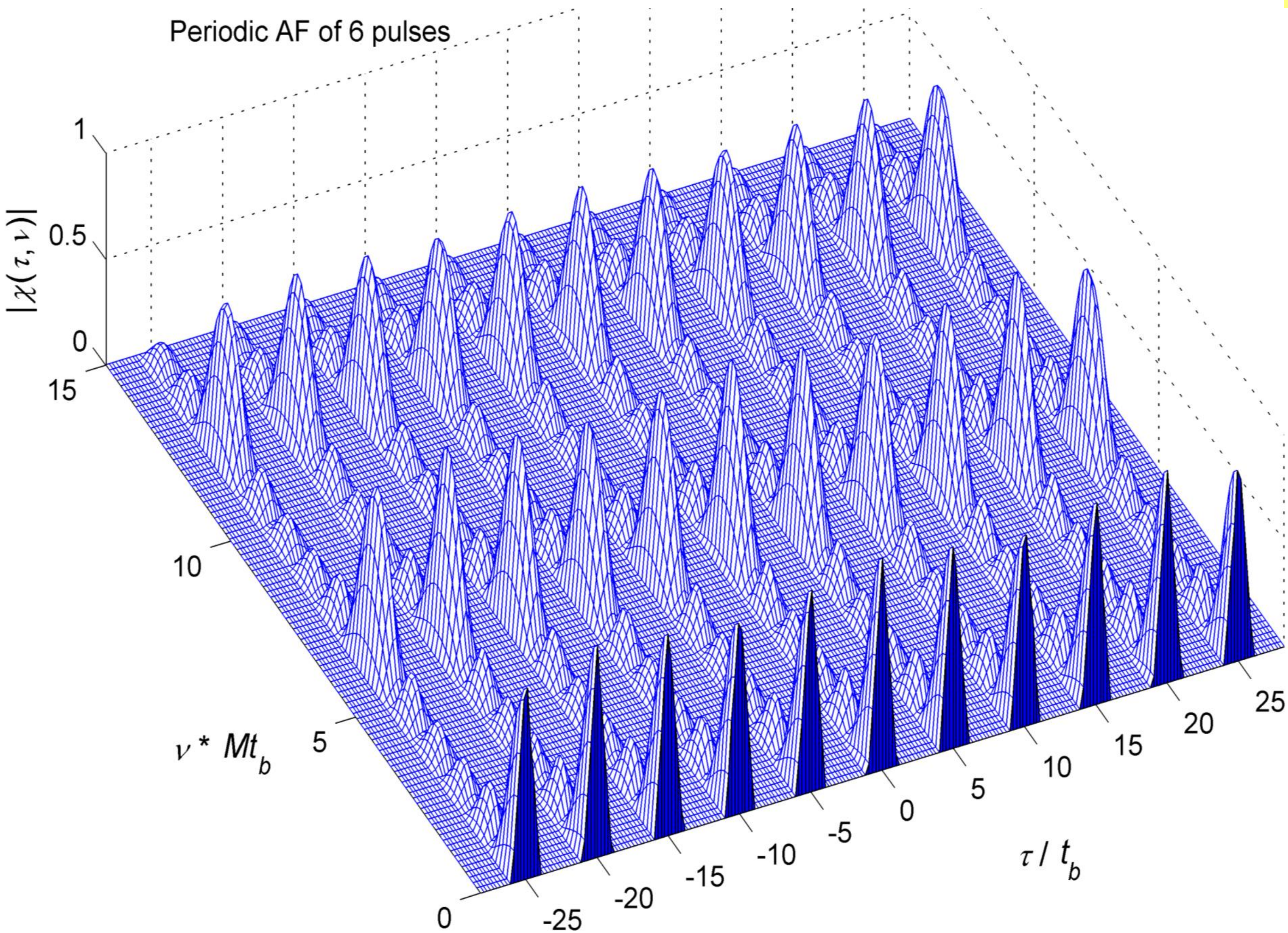
הצבתן במשוואת ה FFT תתן:

$$X[k]=\sum_{n=0}^{15}\left[1+\exp(j2\pi\,n/16)\right]\exp(-j2\pi k\,n/16)=\begin{cases}\mathbf{16} & k=0,1\\ \mathbf{0} & k=2,3,\ldots,15\end{cases}$$

Ambiguity function of 6 uncompressed pulses



Pulse compression will narrow the range resolution


Periodic AF of 6 pulses
$|\chi(\tau,\nu)|$
1
0.5
0
15
10
5
0
$\nu * Mt_b$
-25
-20
-15
-10
-5
0
5
10
15
20
25
τ / t_b

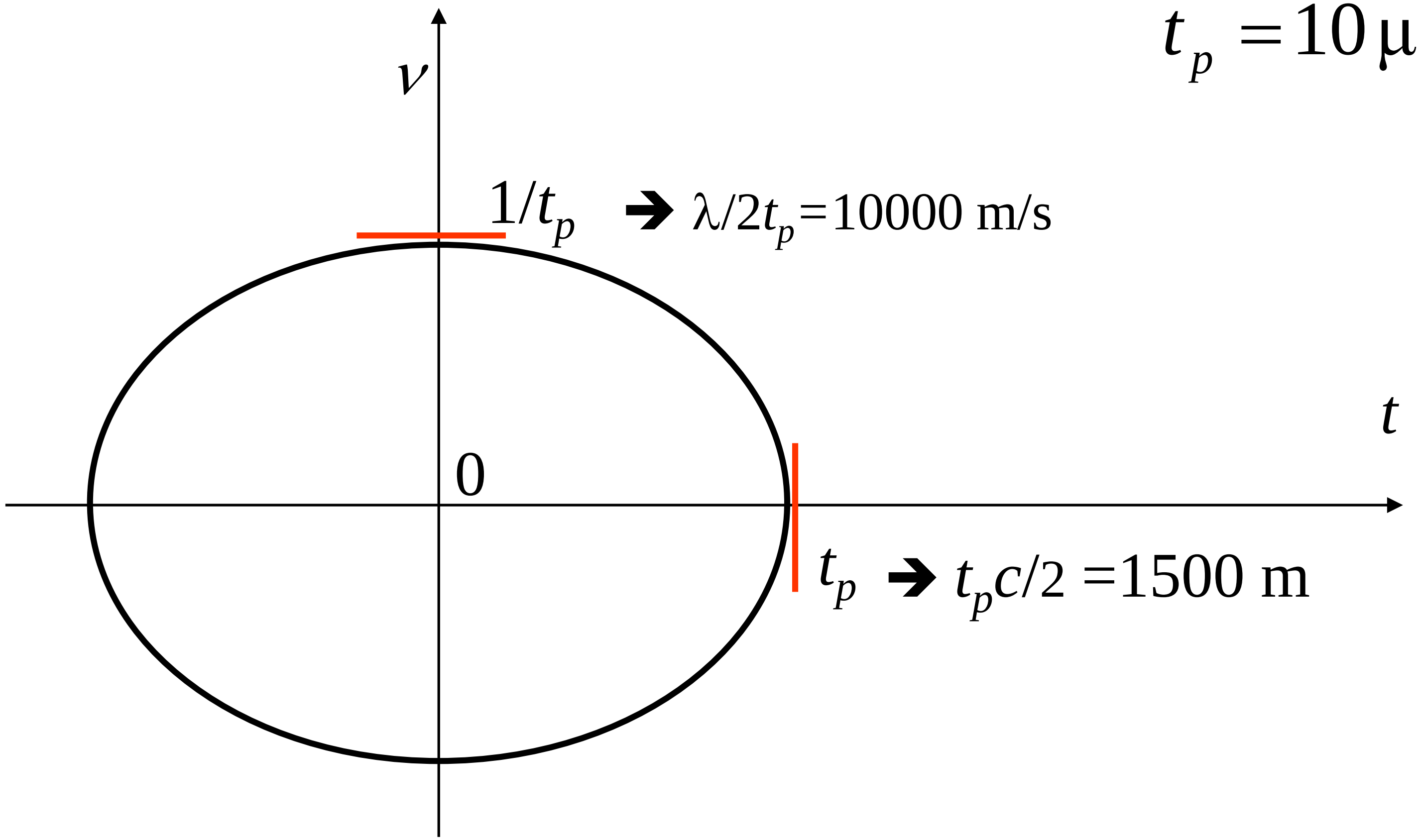
Numerical Example

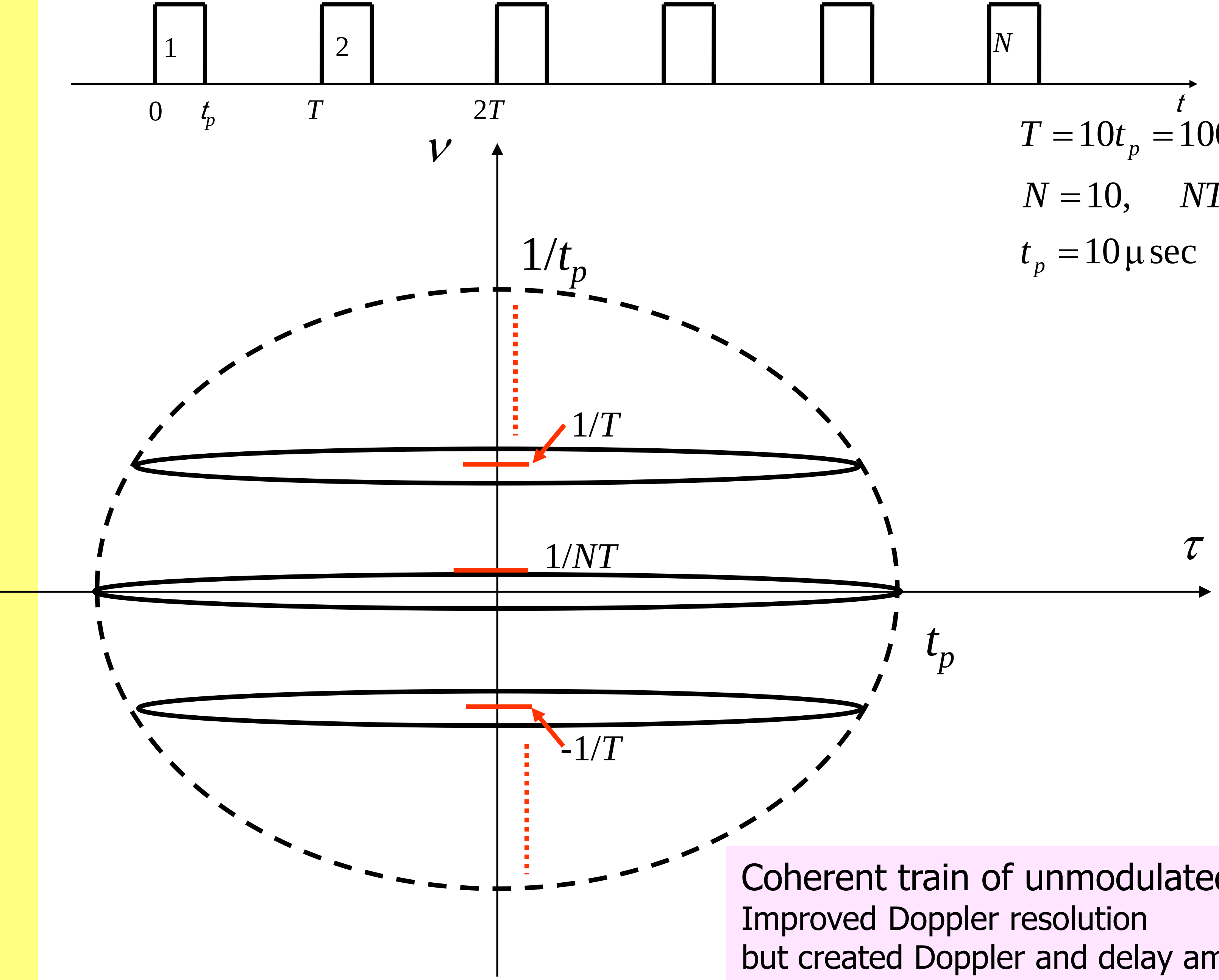
Single unmodulated pulse

$$f_c = 1500\,\mathrm{MHz}$$

$$\lambda = 0.2\,\mathrm{m}$$

$$t_p = 10\,\mu\mathrm{sec}$$





$T = 10t_p = 100\,\mu\text{sec}$

$N = 10, \quad NT = 1000\,\mu\text{sec}$

$t_p = 10\,\mu\text{sec}$

Coherent train of unmodulated pulses:
Improved Doppler resolution
but created Doppler and delay ambiguities

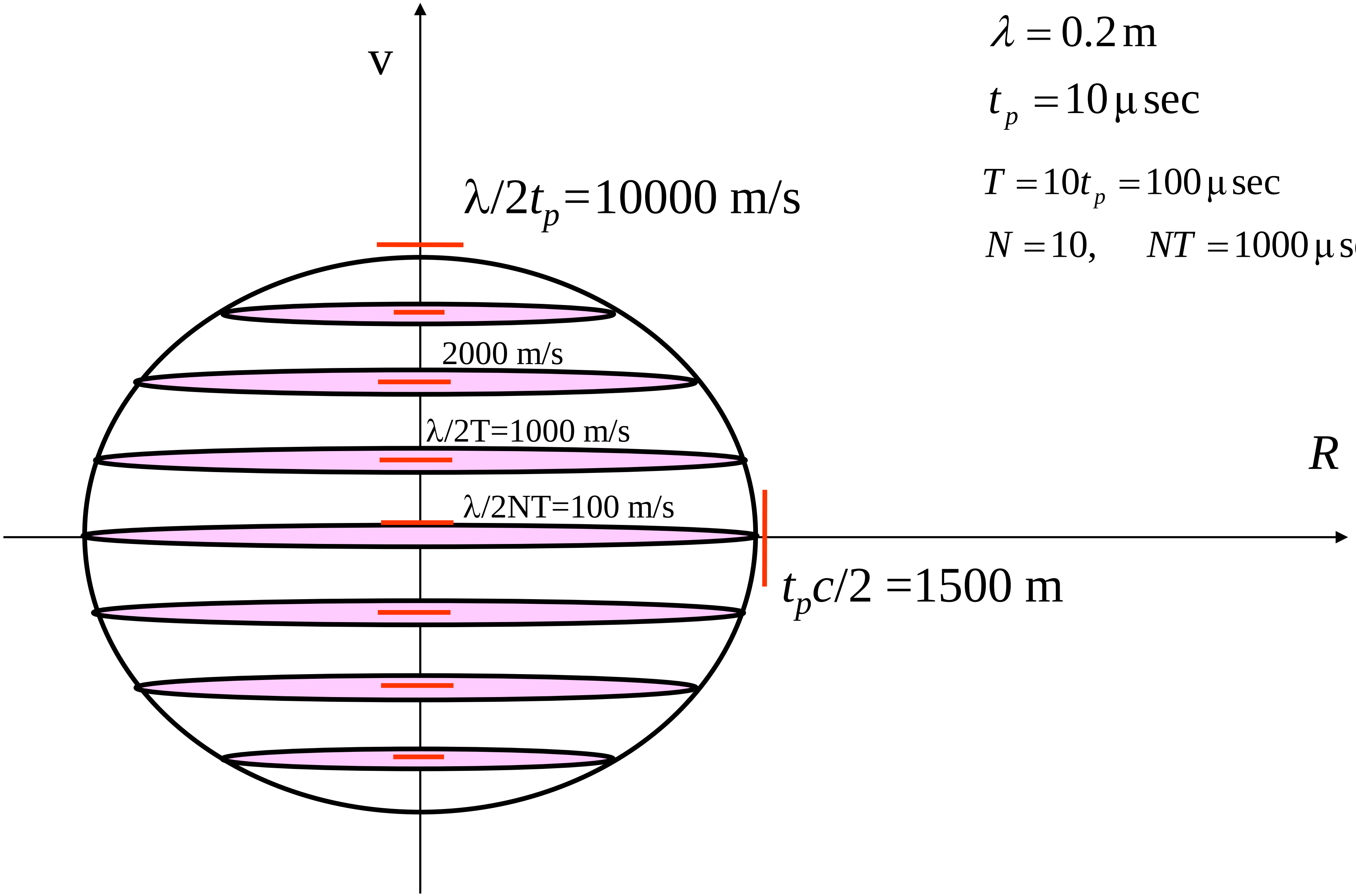
$$f_c = 1500\,\text{MHz}$$

$$\lambda = 0.2\,\text{m}$$

$$t_p = 10\,\mu\text{sec}$$

$$T = 10 t_p = 100\,\mu\text{sec}$$

$$N = 10, \quad NT = 1000\,\mu\text{sec}$$



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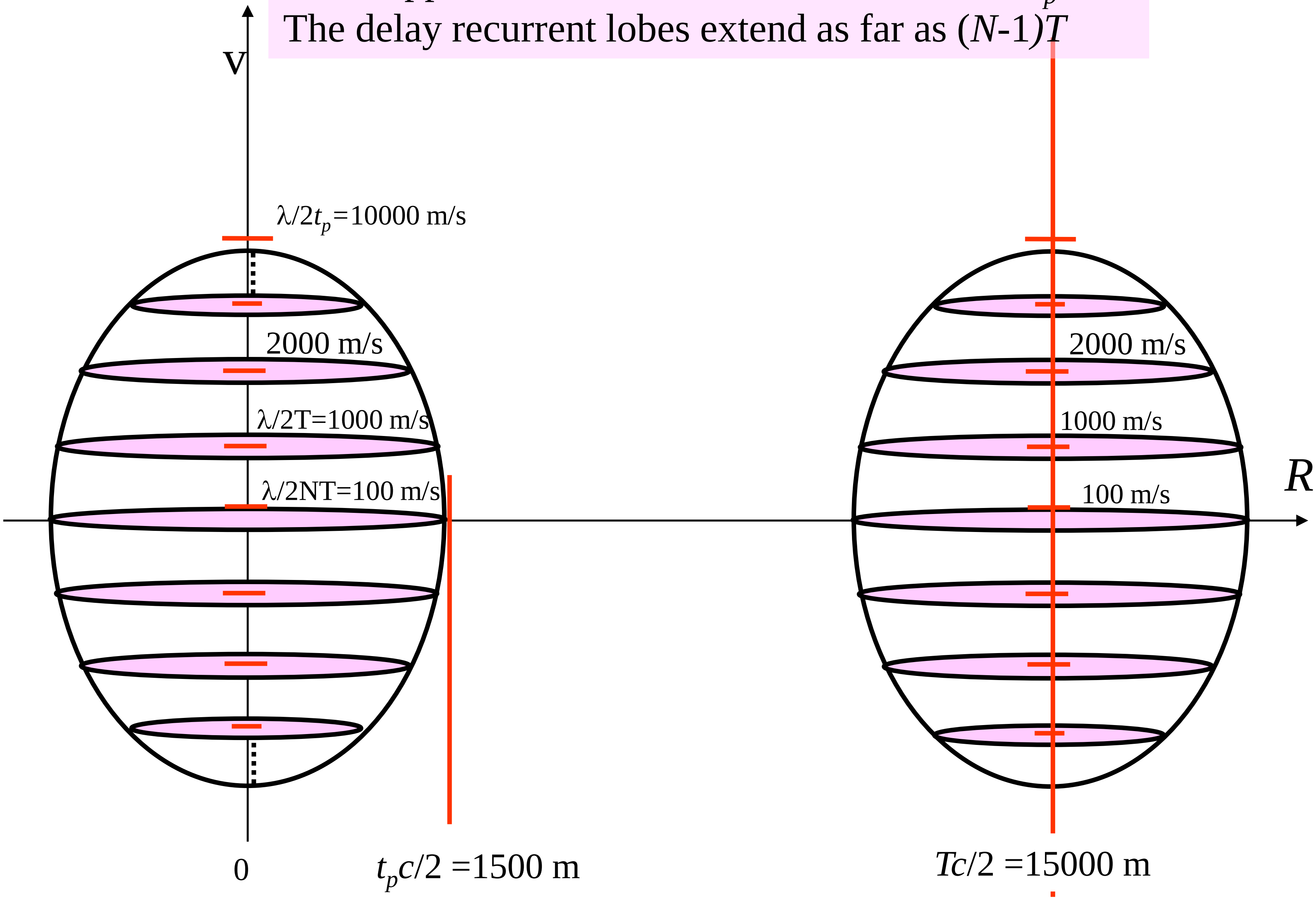
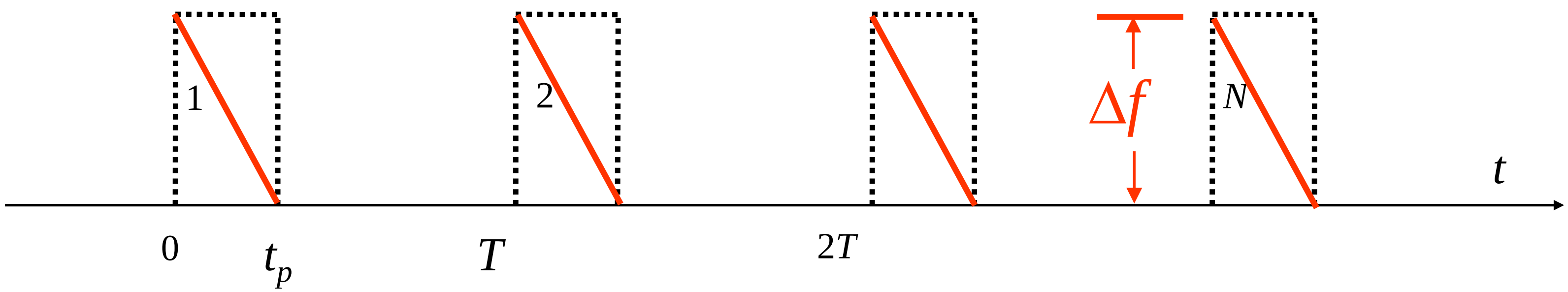
The Doppler recurrent lobes extend as far as $1/t_p$
The delay recurrent lobes extend as far as $(N\text{-}1)T$



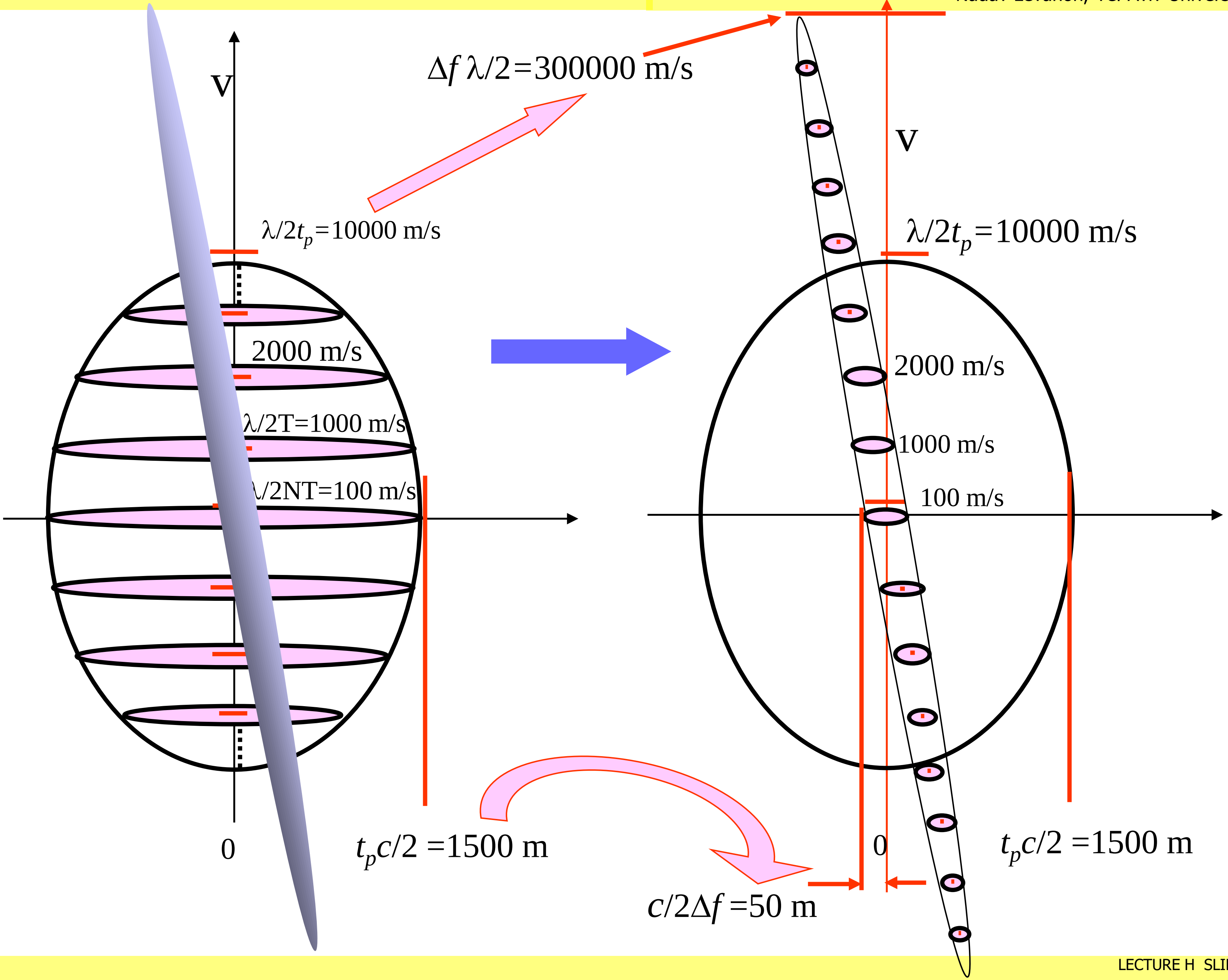
L F M

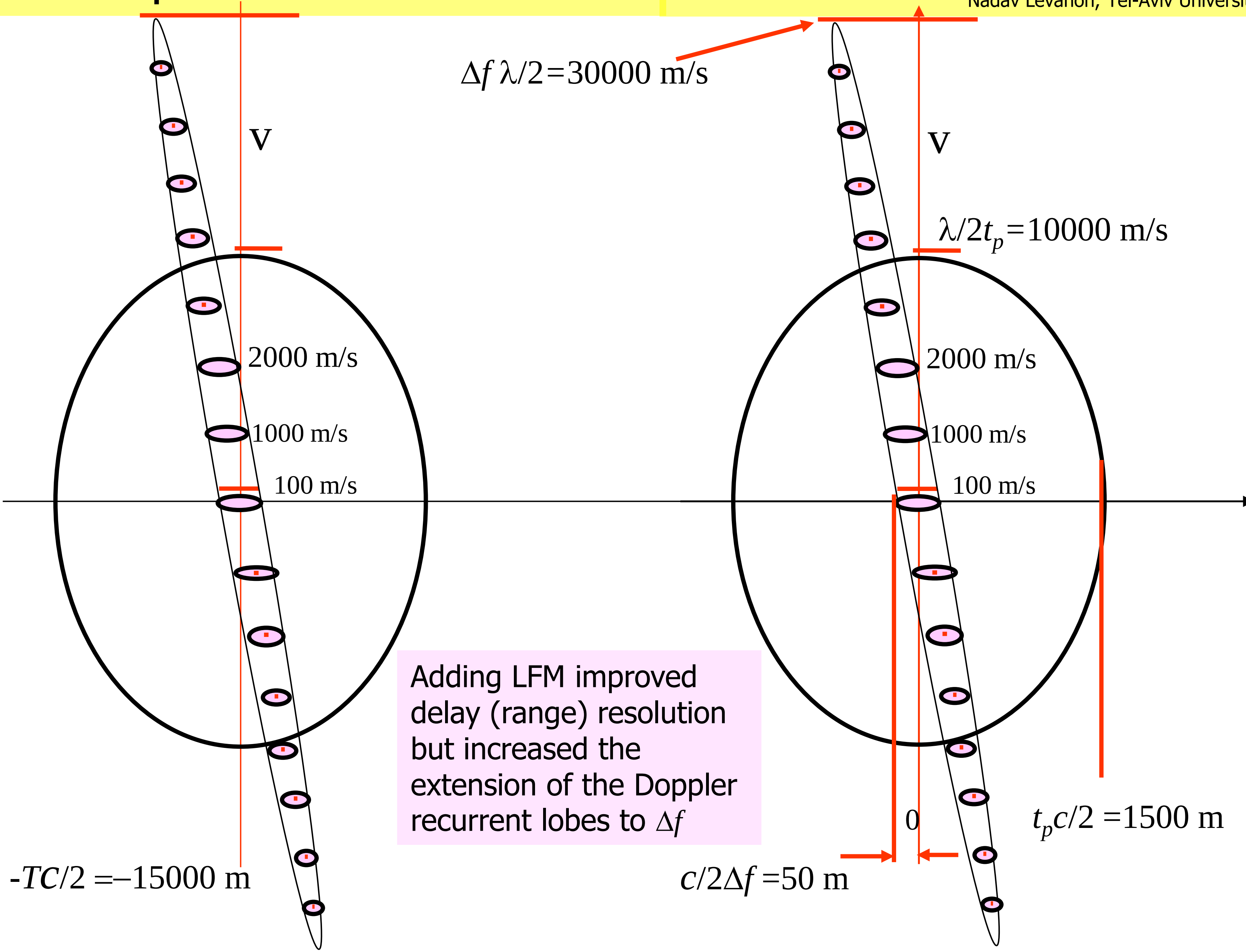
is added



$$\text{Compression ratio} = t_p \Delta f = 30$$

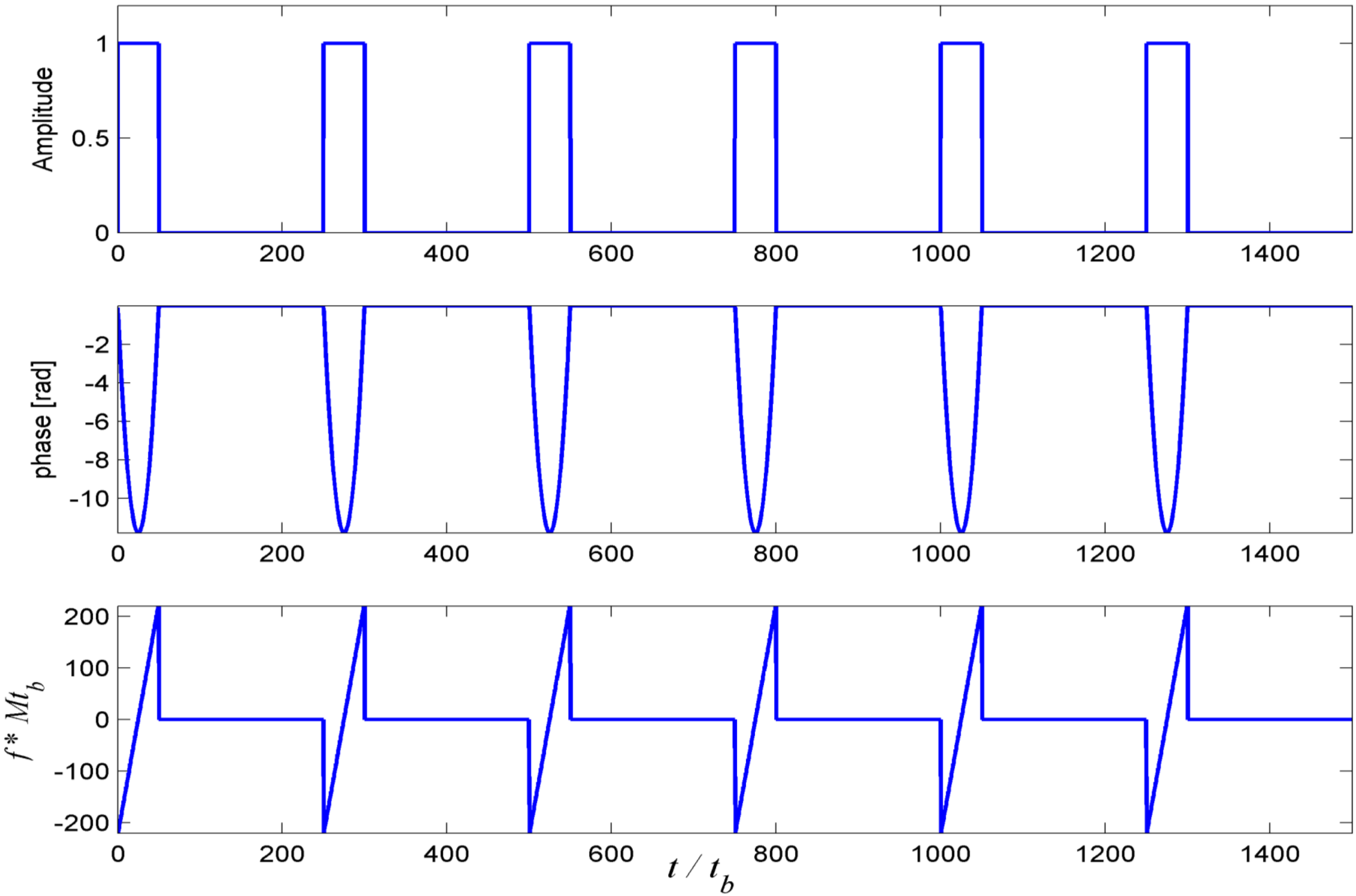
$$\Delta f = 30 / t_p = 30 / 10^{-5} = 3\,\text{MHz}$$


V
$\lambda/2t_p$=10000 m/s
2000 m/s
λ/2T=1000 m/s
λ/2NT=100 m/s
0
$t_pc/2$ =1500 m
$\Delta f\ \lambda/2$=300000 m/s
V
$\lambda/2t_p$=10000 m/s
2000 m/s
1000 m/s
100 m/s
0
$t_pc/2$ =1500 m
$c/2\Delta f$ =50 m

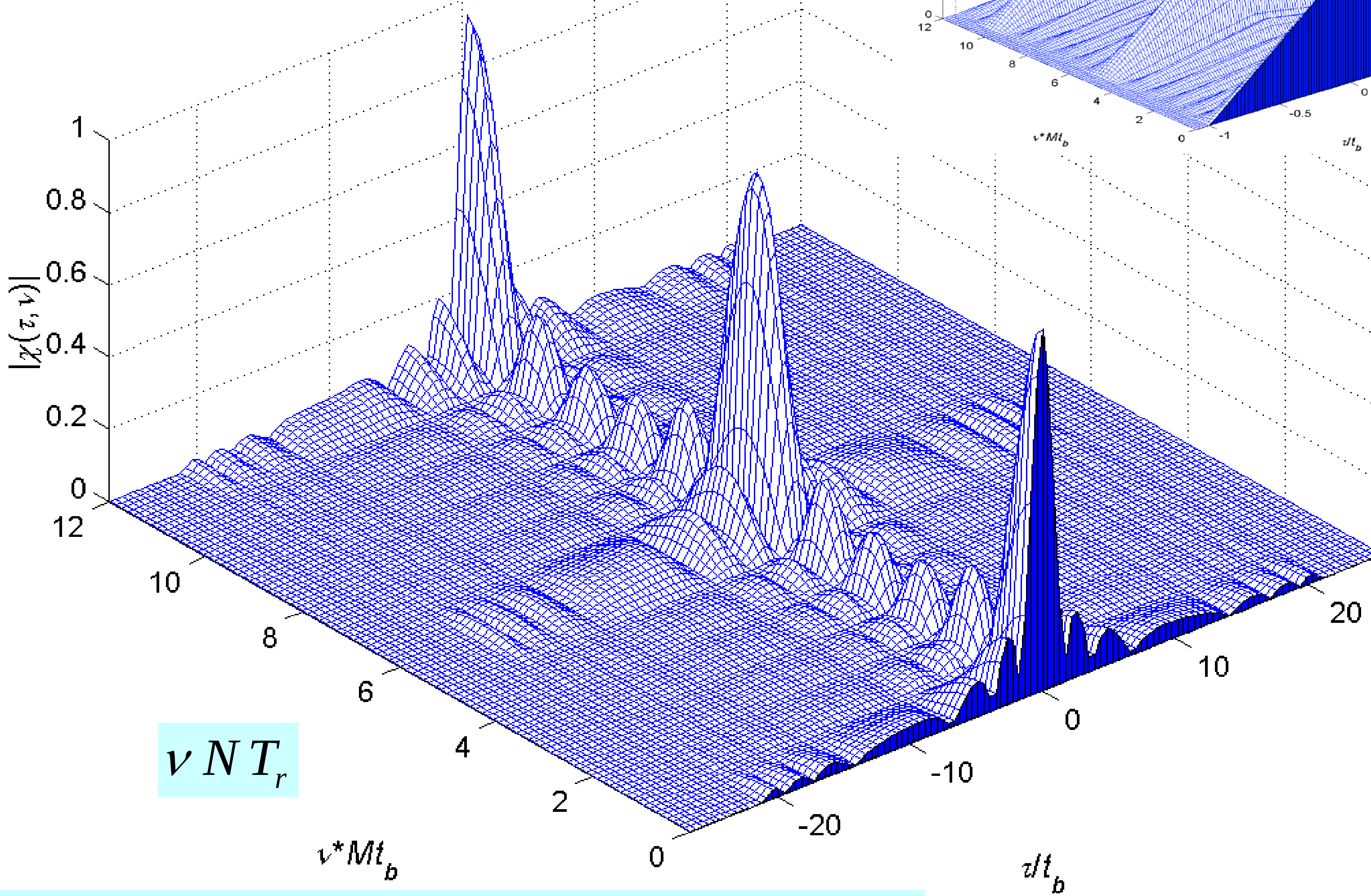


Adding LFM improved delay (range) resolution but increased the extension of the Doppler recurrent lobes to Δf

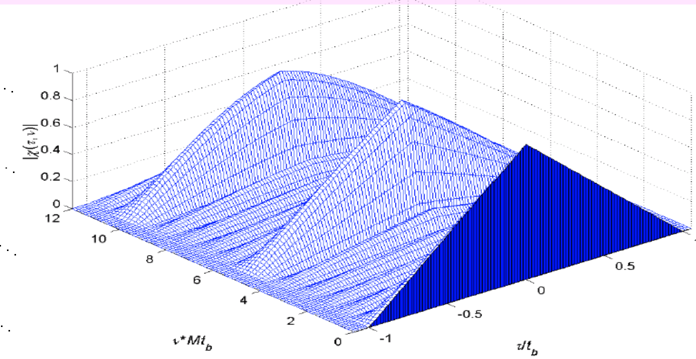
TRAIN OF 6 LFM PULSE



Ambiguity function (zoom) of 6 LFM pulses



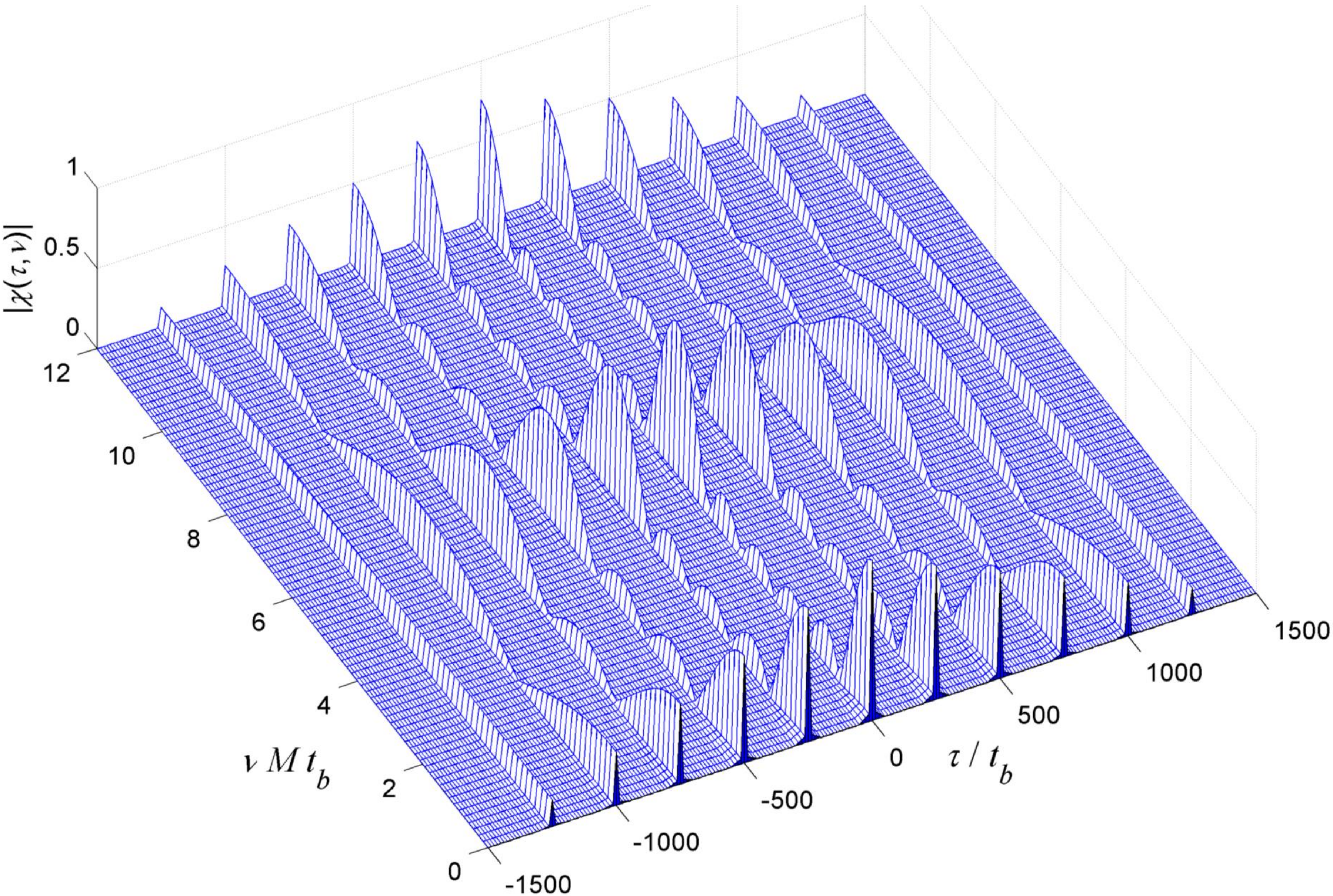
Ambiguity function of 6 unmodulated pulses



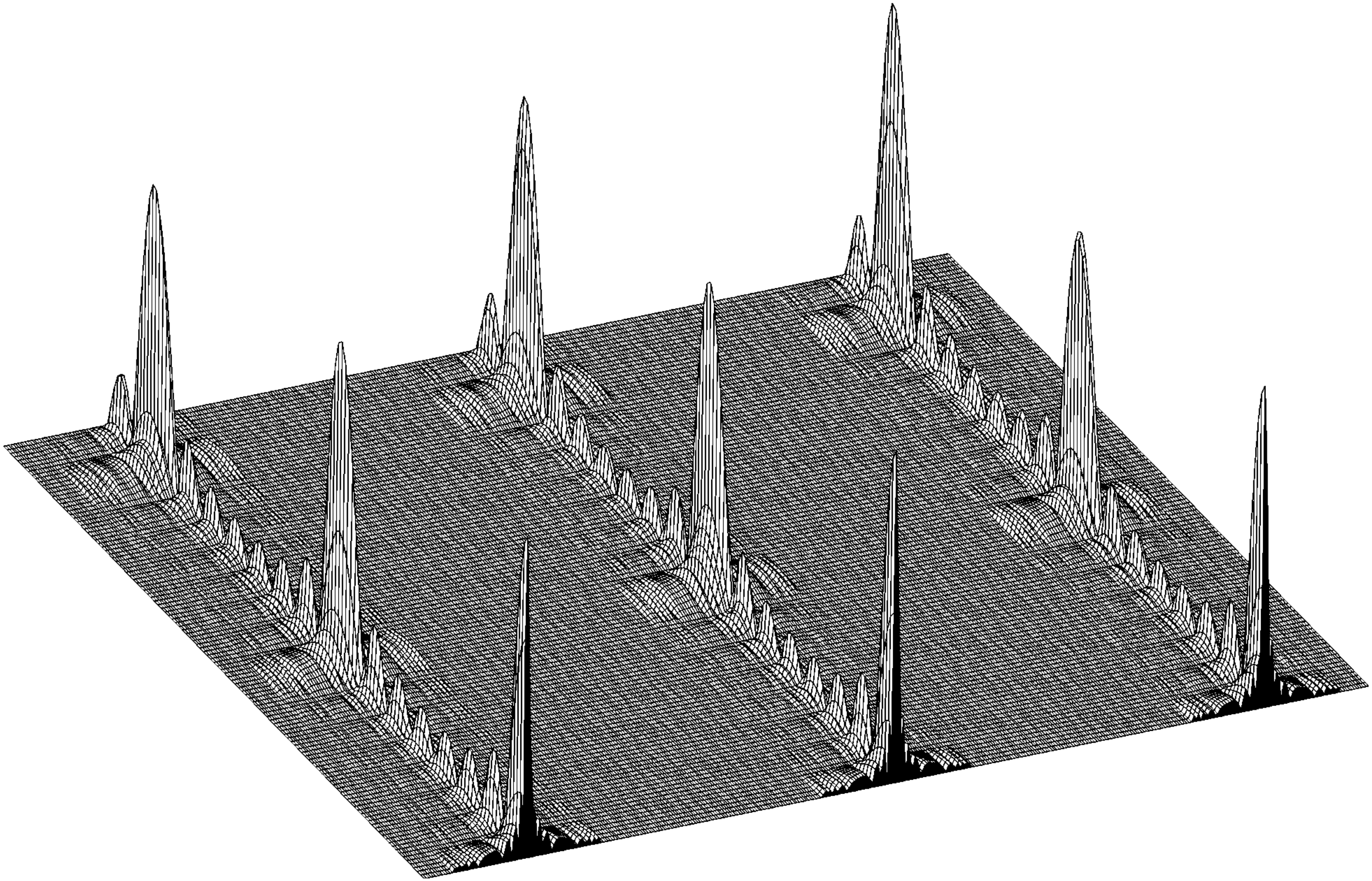
$\nu N T_r$

$$\nu N T_r = 6 \quad , \quad N = 6 \quad \Rightarrow \quad \nu T_r = 1 \quad \Rightarrow \quad \nu = 1/T_r$$

Ambiguity function of 6 LFM pulses



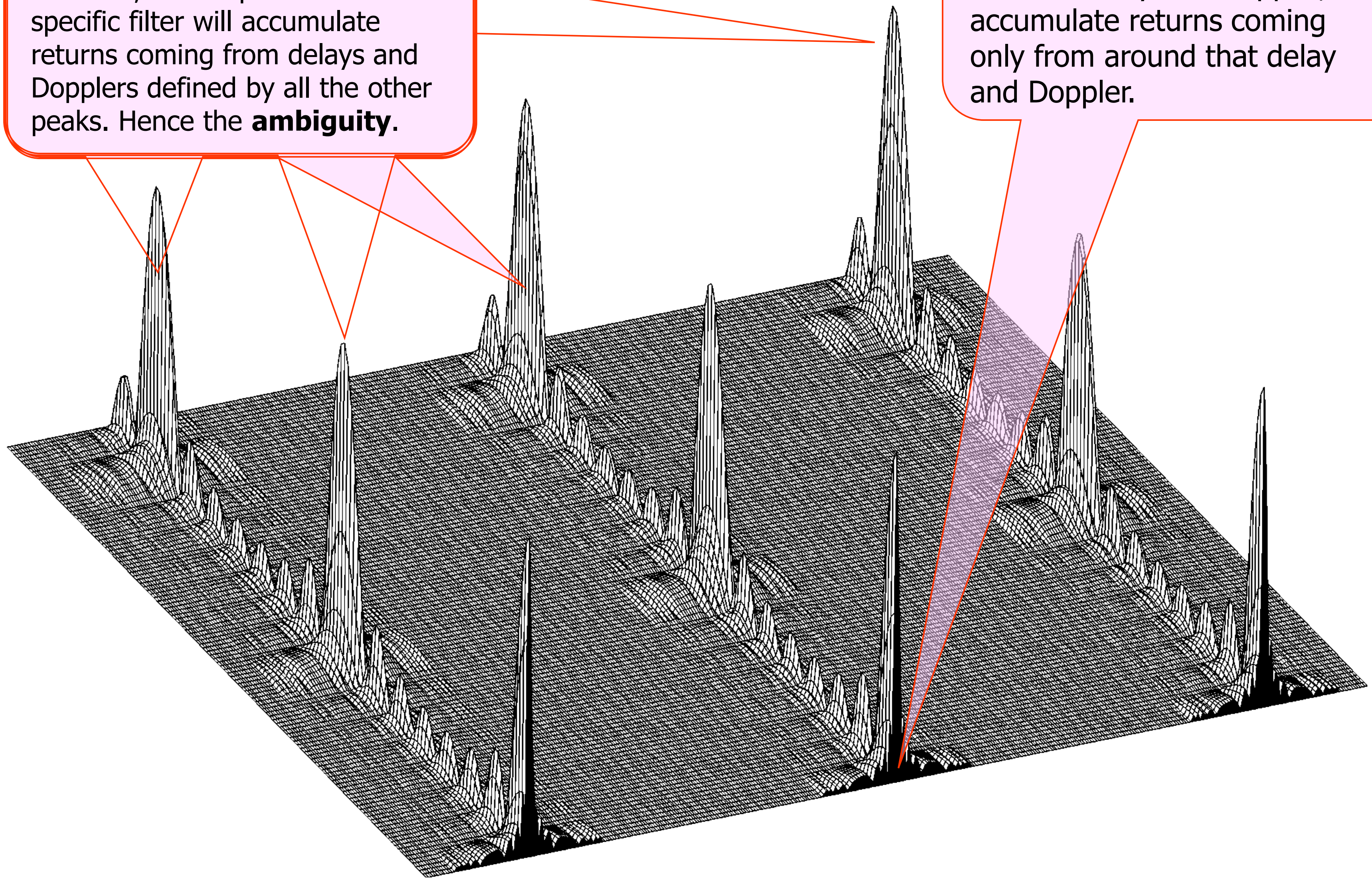
Periodic ambiguity function of a coherent train of LFM pulses

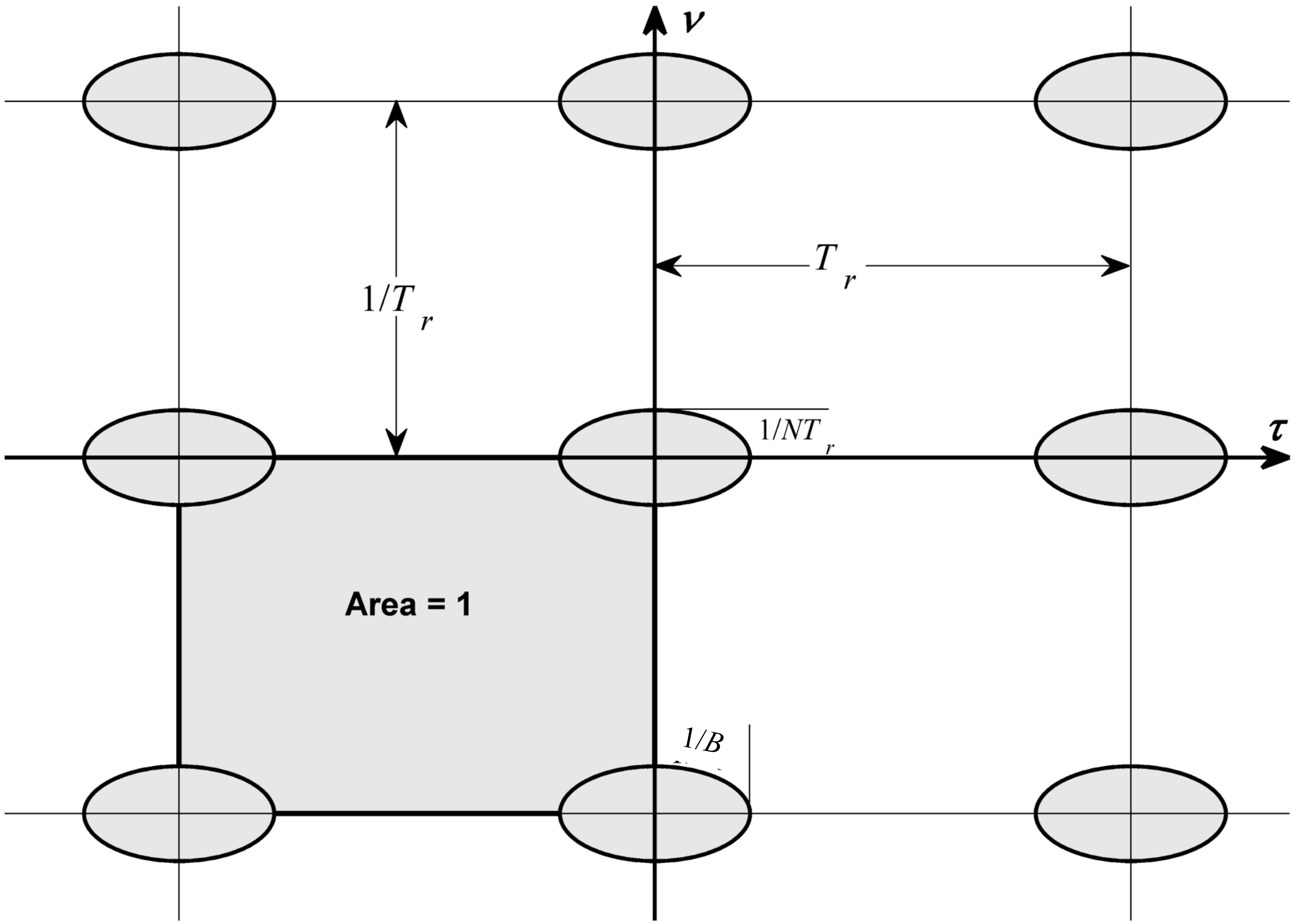


Why the name "**ambiguity** function"?

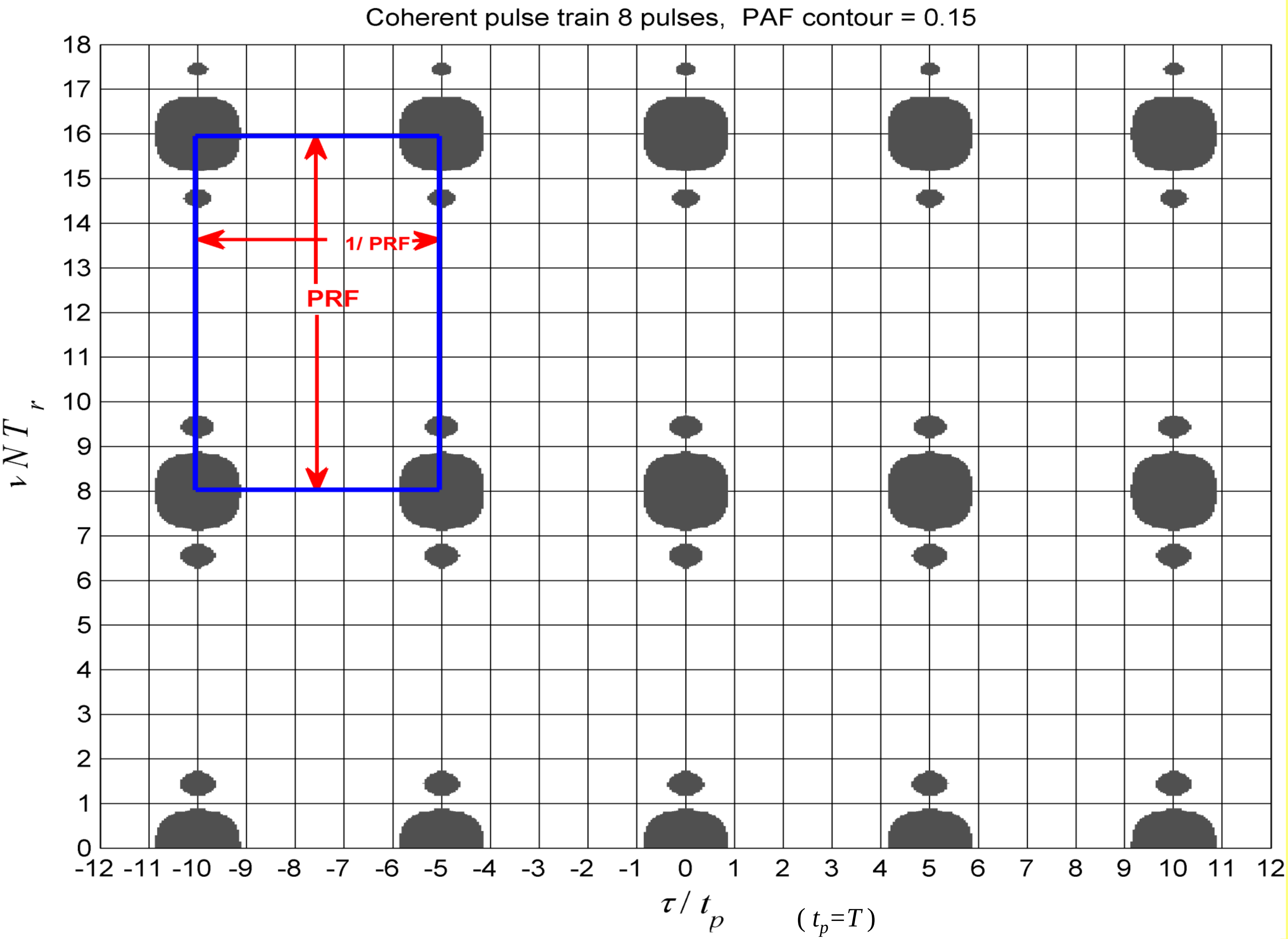
We hoped that the output of the filter, designed for the nominal delay and Doppler, will accumulate returns coming only from around that delay and Doppler.

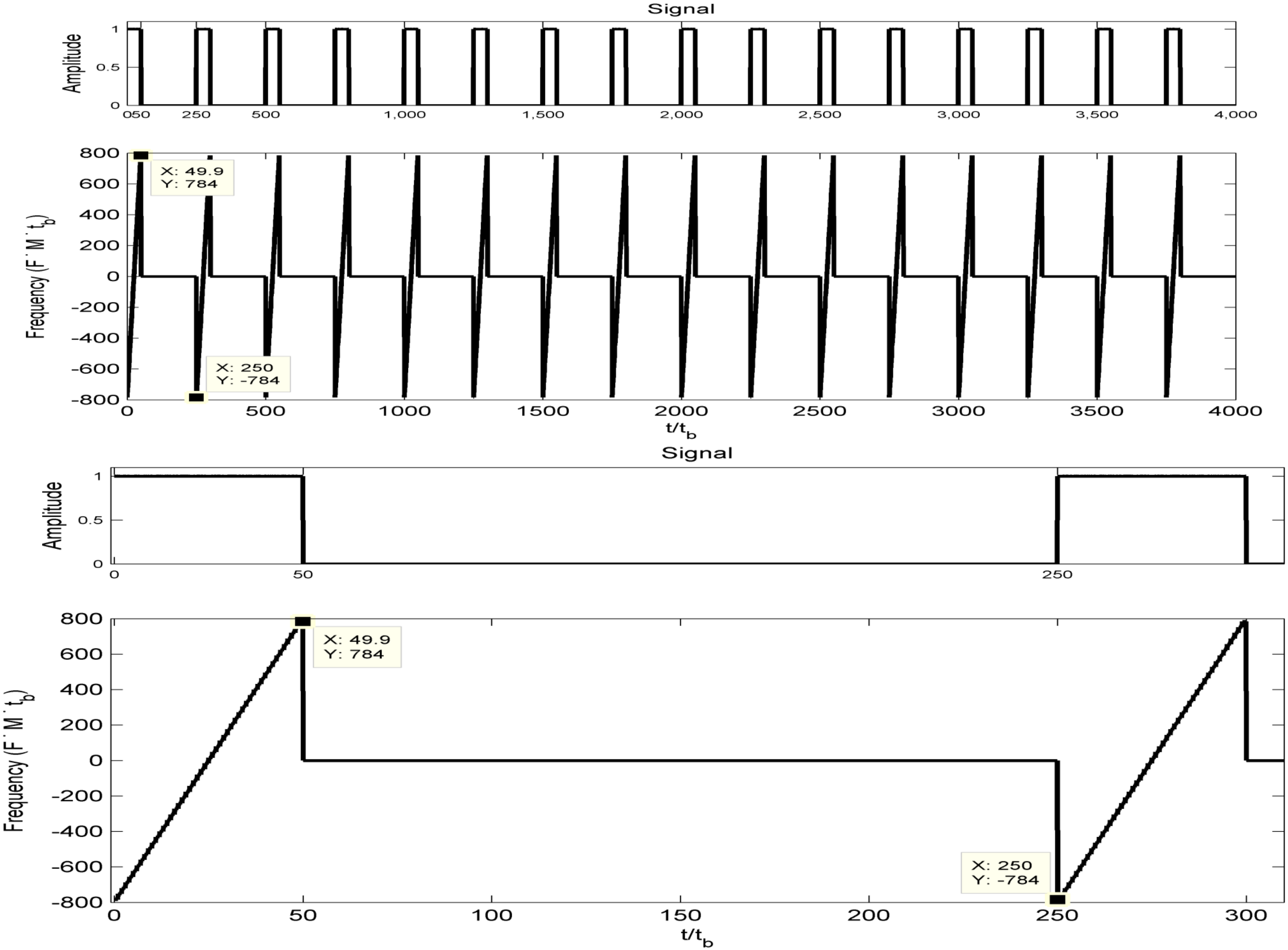
However, the output of that specific filter will accumulate returns coming from delays and Dopplers defined by all the other peaks. Hence the **ambiguity**.

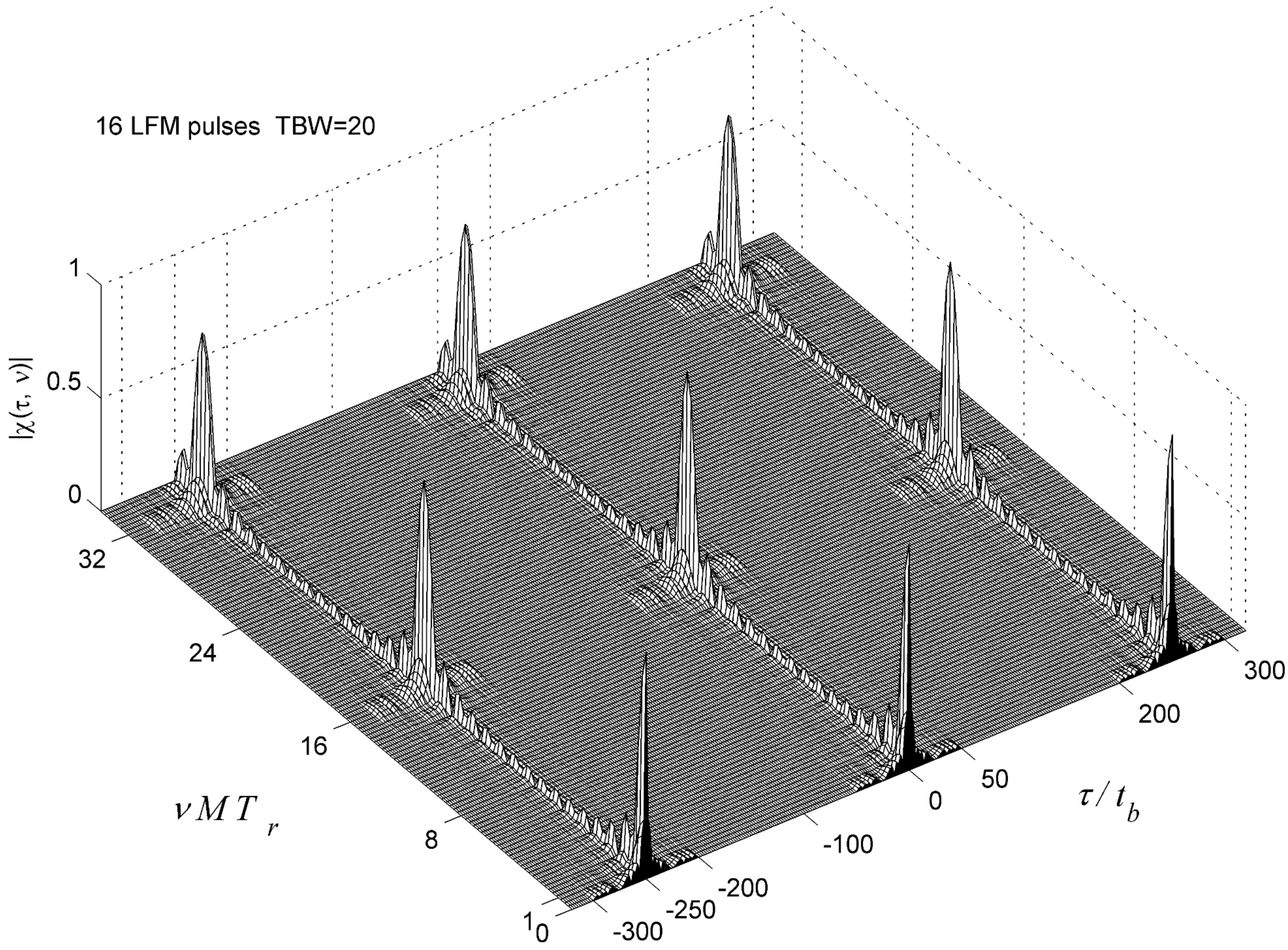




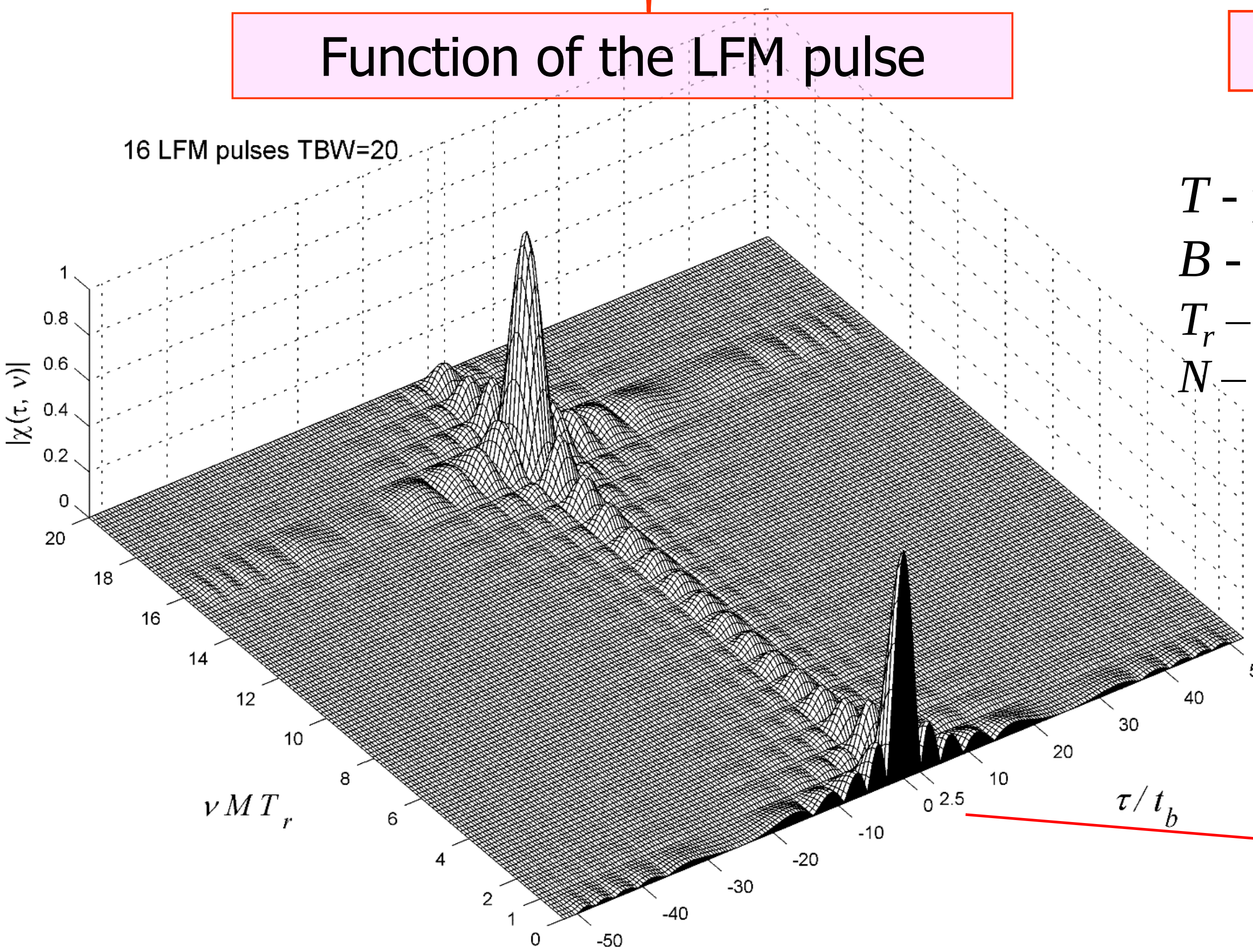
Horizontal cut through a "bed of nails" periodic ambiguity function


Coherent pulse train 8 pulses, PAF contour = 0.15
1/ PRF
PRF
$\nu N T_r$
τ / t_p
$(t_p = T)$


Signal
Amplitude
1
0.5
0
050
250
500
1,000
1,500
2,000
2,500
3,000
3,500
4,000
X: 49.9
Y: 784
X: 250
Y: -784
Frequency (F · M · t_b)
800
600
400
200
0
-200
-400
-600
-800
0
500
1000
1500
2000
2500
3000
3500
4000
t/t_b
Signal
Amplitude
1
0.5
0
0
50
250
Frequency (F · M · t_b)
800
600
400
200
0
-200
-400
-600
-800
X: 49.9
Y: 784
X: 250
Y: -784
0
50
100
150
200
250
300
t/t_b


16 LFM pulses TBW=20
1
0.5
0
$|\chi(\tau, \nu)|$
32
24
16
8
1
0
$\nu M T_r$
-300
-250
-200
-100
0
50
200
300
τ / t_b

$$\left|\chi(\tau,\nu)\right|_{|\tau|\le T}=\left|\underbrace{\left(1-\frac{|\tau|}{T}\right)\operatorname{sinc}\left[T\left(\nu-B\frac{\tau}{T}\right)\left(1-\frac{|\tau|}{T}\right)\right]}_{\text{Function of the LFM pulse}}\underbrace{\frac{\sin\left[N\pi\left(T_r\nu\right)\right]}{N\sin\left[\pi\left(T_r\nu\right)\right]}}_{\text{Function of } N \text{ pulses}}\right|,\quad |\tau|\le T$$



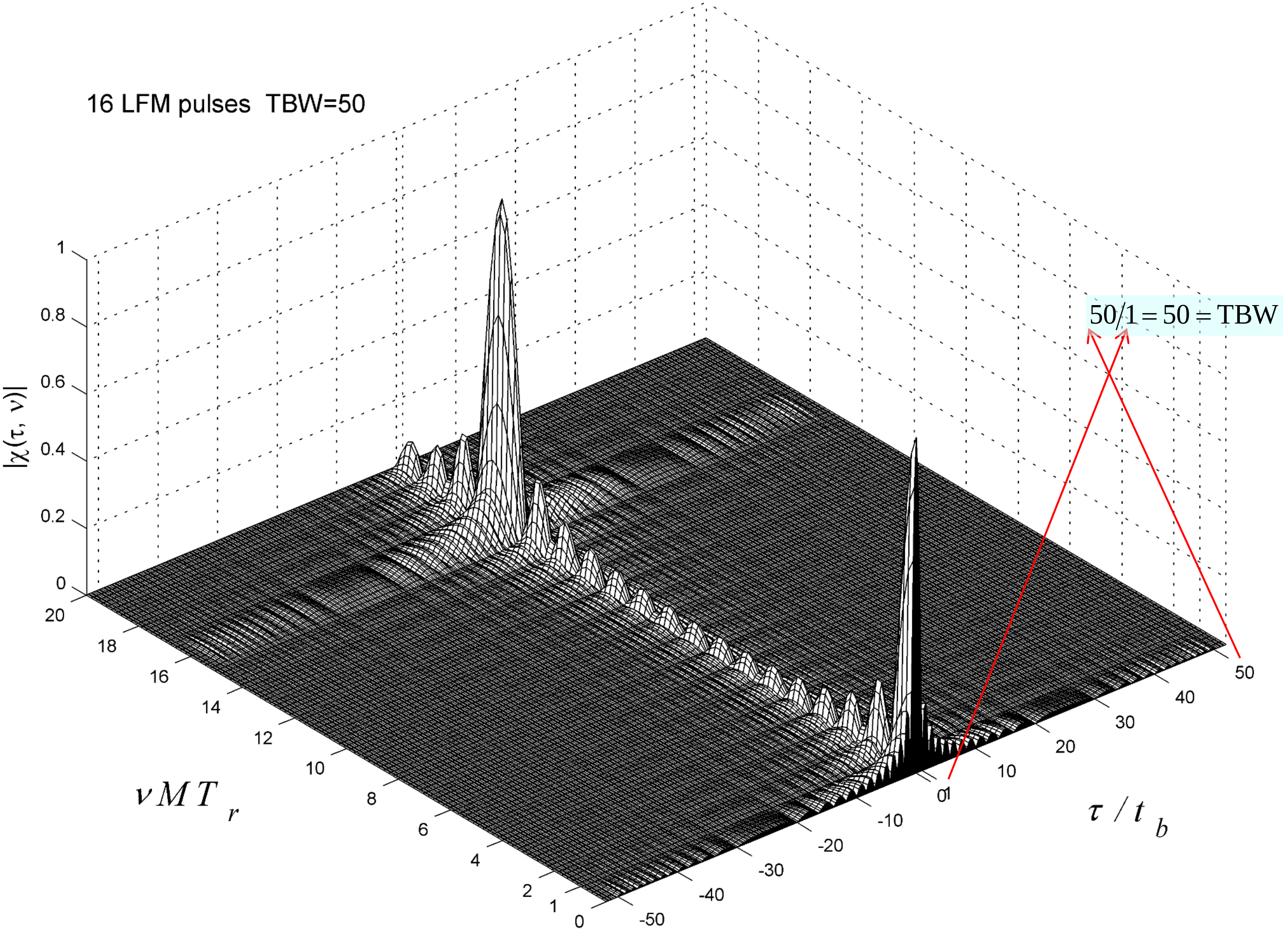
T - pulse duration ($=t_p=50t_b$)

B - pulse bandwidth ($=\Delta f$)

T_r – pulse repetition interval

N – number of pulses ($=M$)

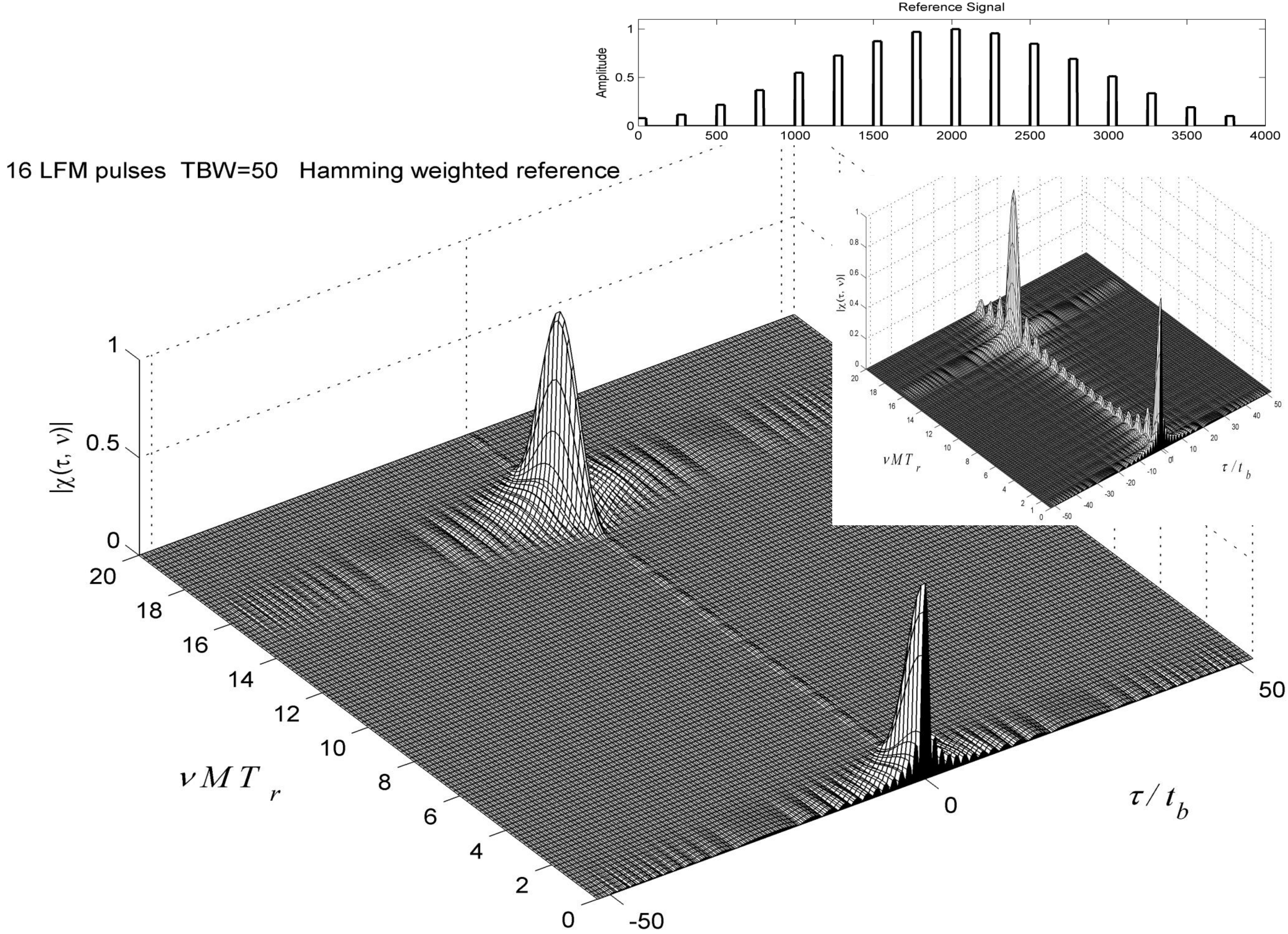
$50/2.5 = 20 = \text{TBW}$

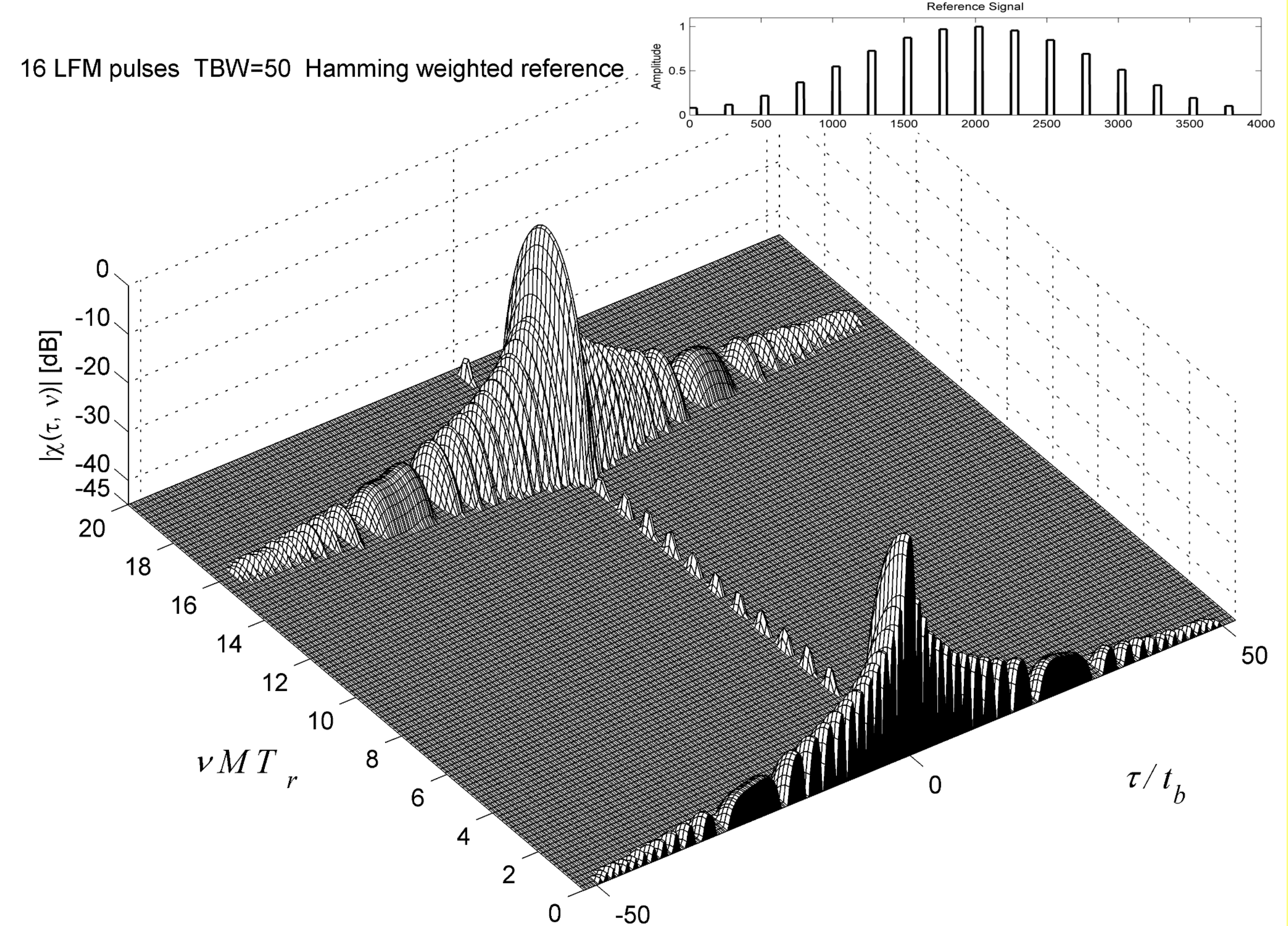

16 LFM pulses TBW=50
50/1 = 50 = TBW
$|\chi(\tau, \nu)|$
1
0.8
0.6
0.4
0.2
0
$\nu M T_r$
20
18
16
14
12
10
8
6
4
2
1
0
τ / t_b
-50
-40
-30
-20
-10
0
1
10
20
30
40
50

Intra-pulse weighting = frequency weighting $\Rightarrow$ reduces delay sidelobes

Inter-pulse weighting = slow-time weighting $\Rightarrow$ reduces Doppler sidelobes

16 LFM pulses TBW=50 Hamming weighted reference

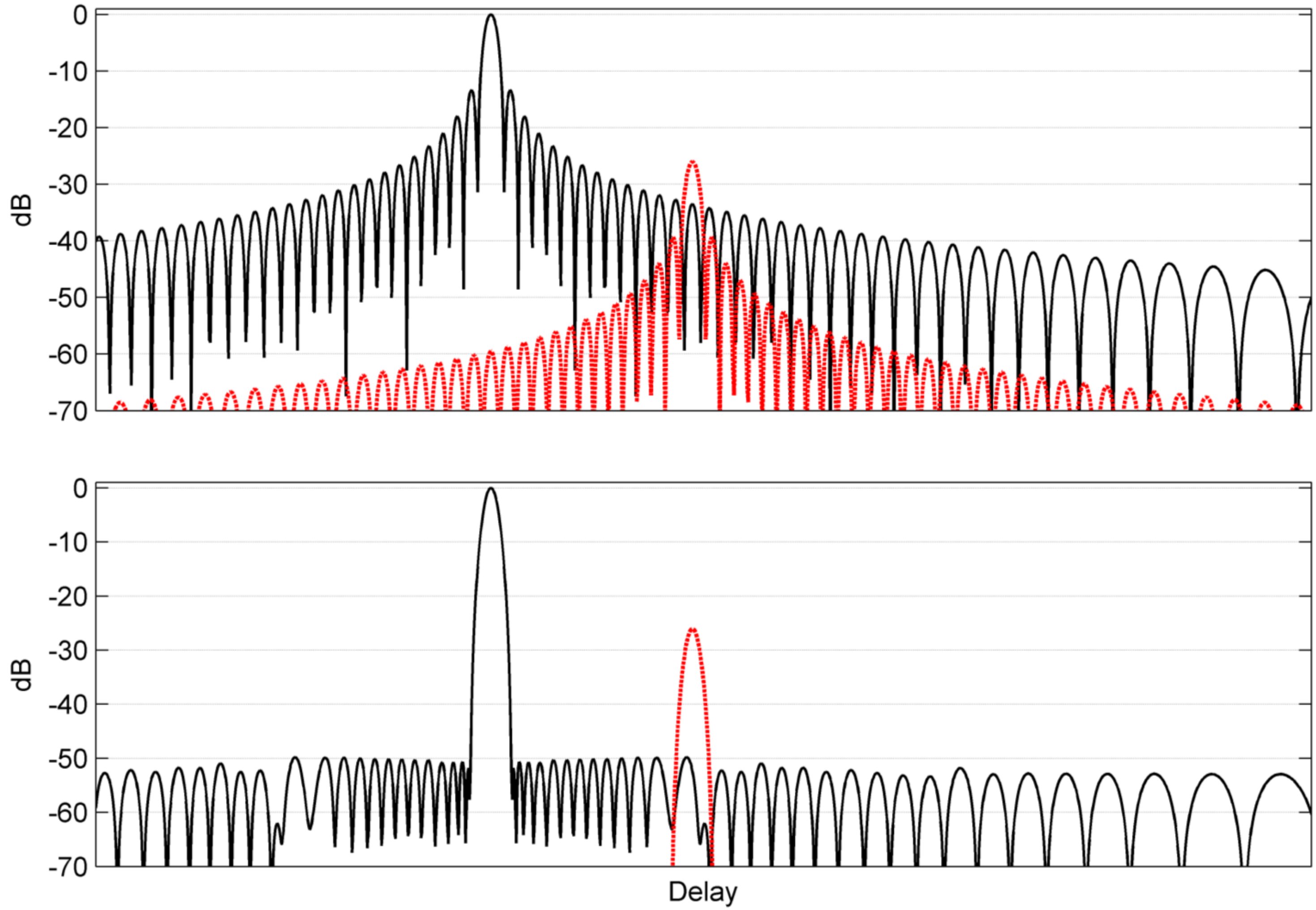



Reference Signal
Amplitude
1
0.5
0
0
500
1000
1500
2000
2500
3000
3500
4000
16 LFM pulses TBW=50 Hamming weighted reference
0
-10
-20
-30
-40
-45
|χ(τ, ν)| [dB]
20
18
16
14
12
10
8
6
4
2
0
ν M T r
-50
0
50
τ / t b

Nadav Levanon, Tel-Aviv University

DELAY SIDELOBE REDUCTION

Sidelobe masking - why is it so important to reduce sidelobes?



Delay responses of a compressed pulse: (top) without and (bottom) with sidelobe reduction

$$\left|\chi(\tau,0)\right| = \left|\int_{-\infty}^{\infty} u(t)u^*(t+\tau)dt\right| = \left|R(\tau)\right|$$

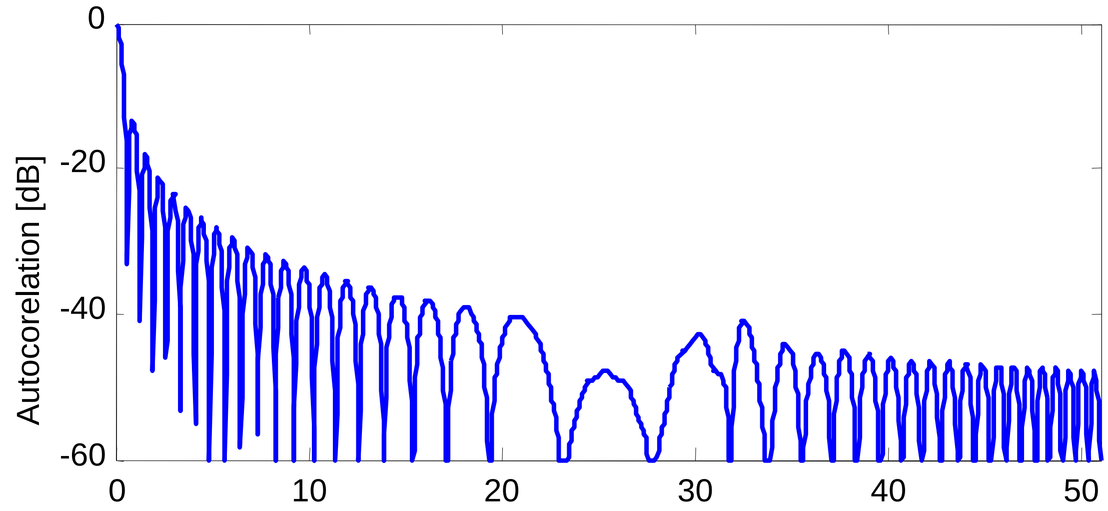
Range window

$\Downarrow\Uparrow$

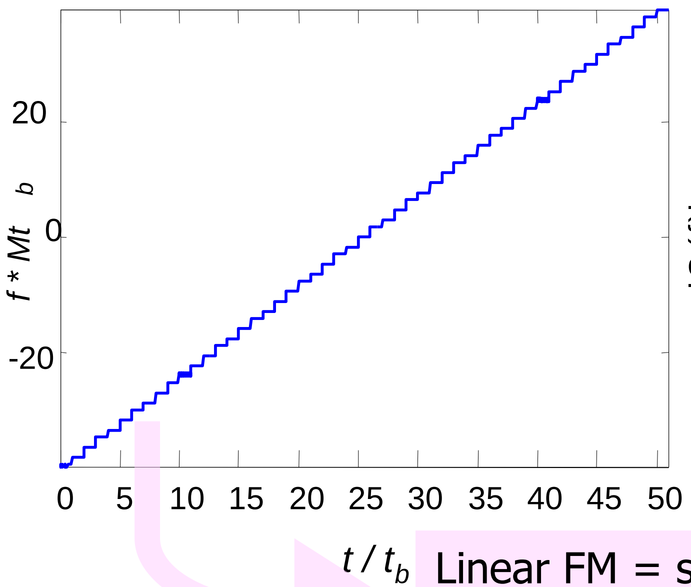
Autcorrelation

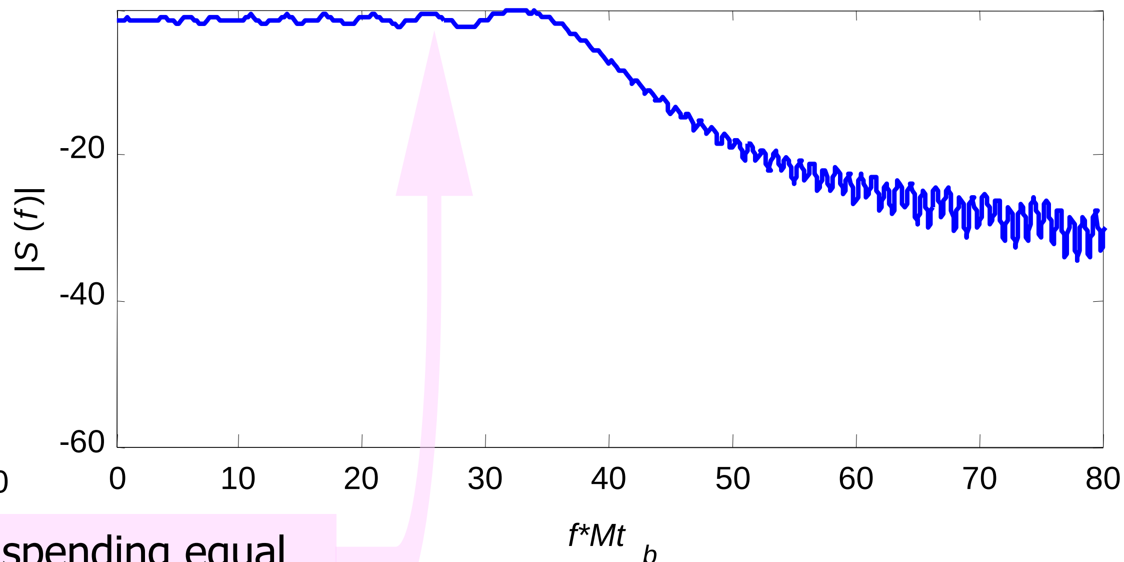
$\Downarrow\Uparrow$

$\mathbf{F}^{-1}$ **{** Power spectral density **}**



Rectangular spectrum $\Rightarrow$
sinc autocorrelation





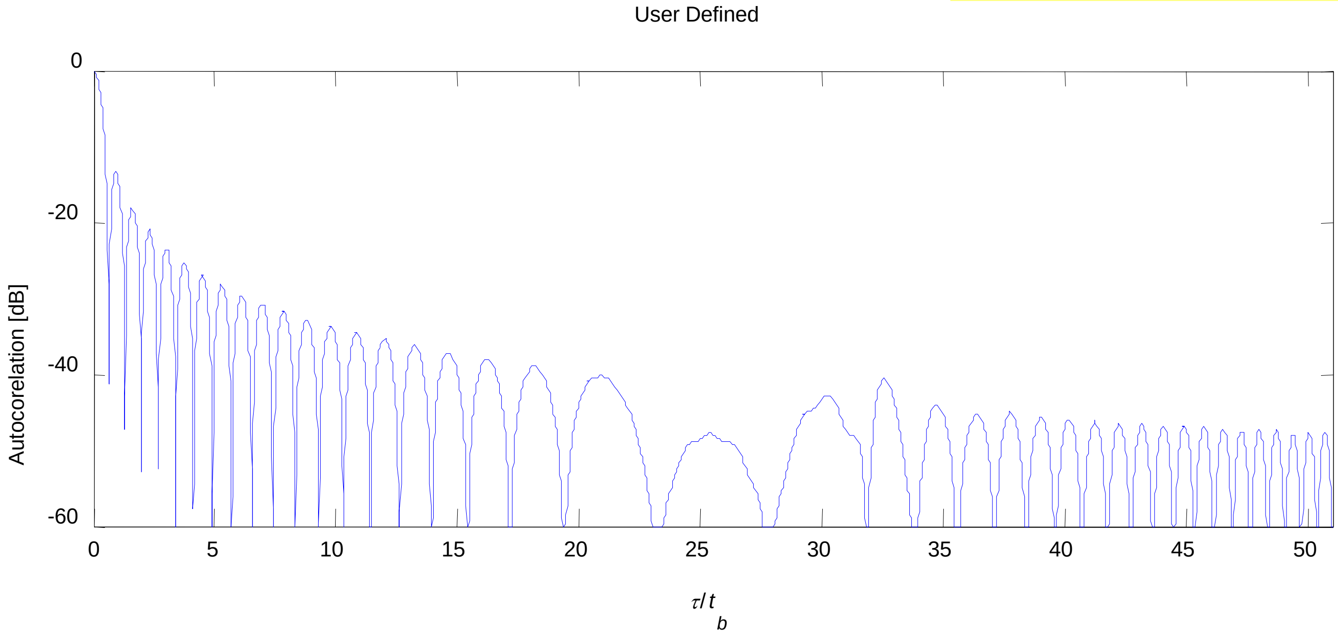
Linear FM = spending equal time at all frequencies within the band $\approx$ flat spectrum

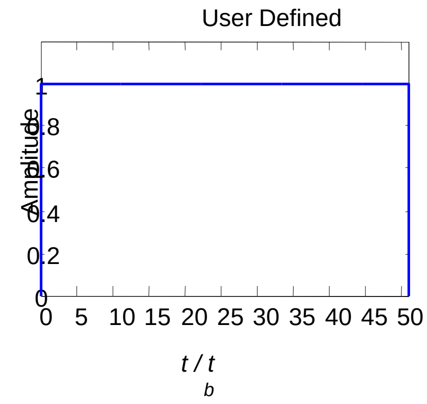
Rectangular spectrum $\Rightarrow$ sinc autocorrelation $\Rightarrow$ -13 dB peak sidelobe

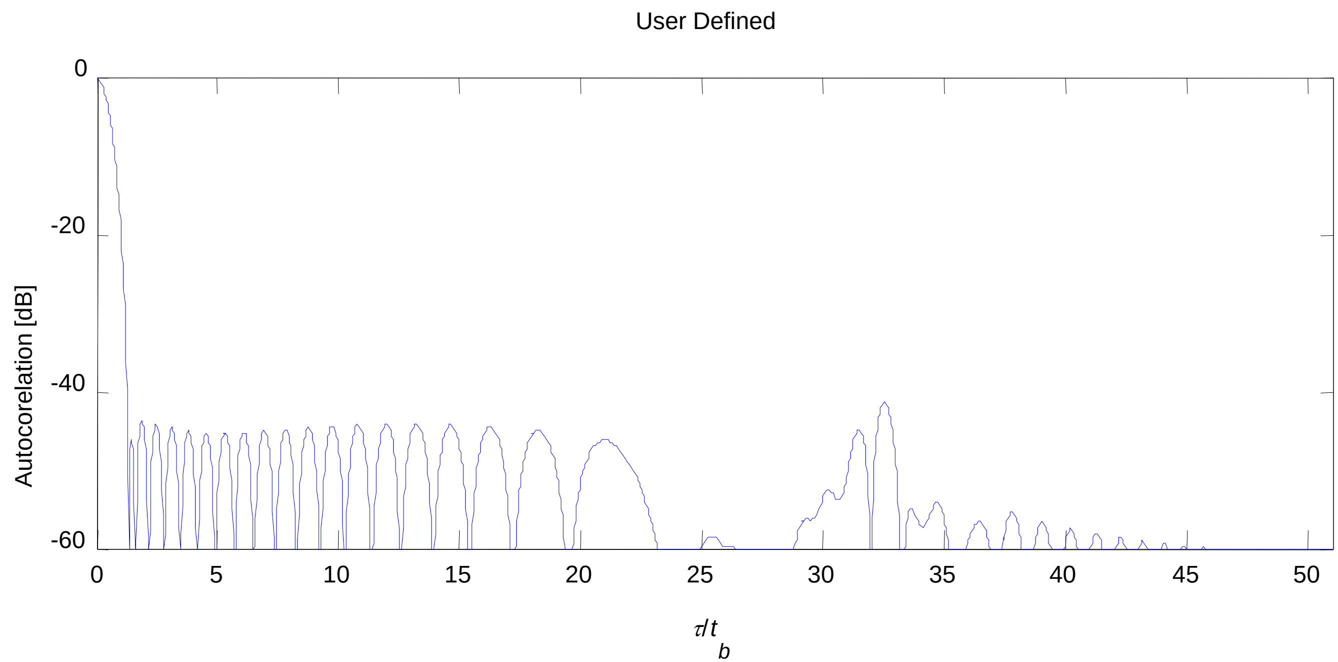
Reshaping the spectrum will lower the autocorrelation sidelobes.
Desired spectral shapes (windows):
Hamming, Hann, Chebyshev, ...

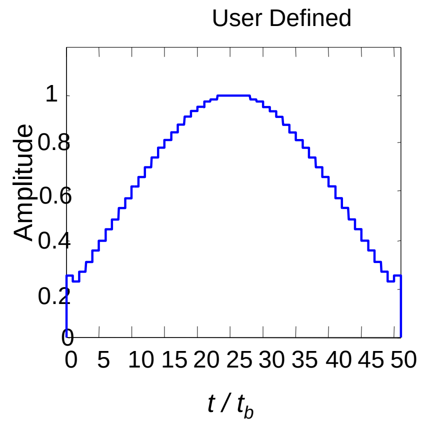
Two spectral shaping methods:

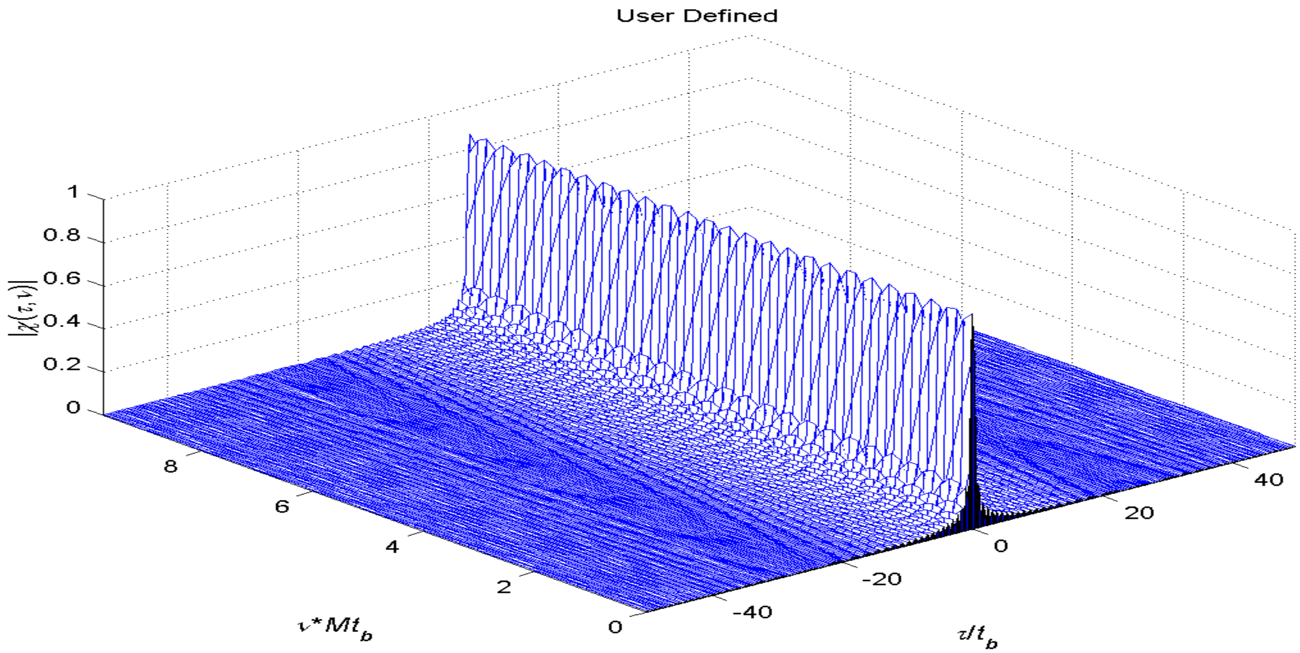
- Amplitude modulating the pulse (using the linear relationship between time and frequency, to increase the weight of mid-band frequencies)
- Frequency modulation (Non-linear FM) (spend more time at mid-band frequencies)

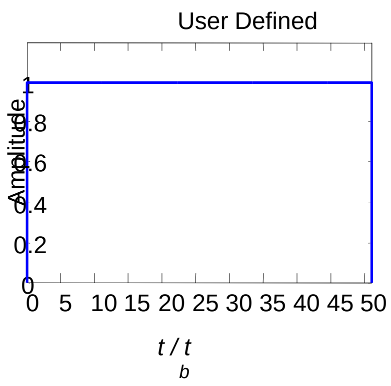

User Defined
Autocorelation [dB]
0
-20
-40
-60
0 5 10 15 20 25 30 35 40 45 50
τ/t_b

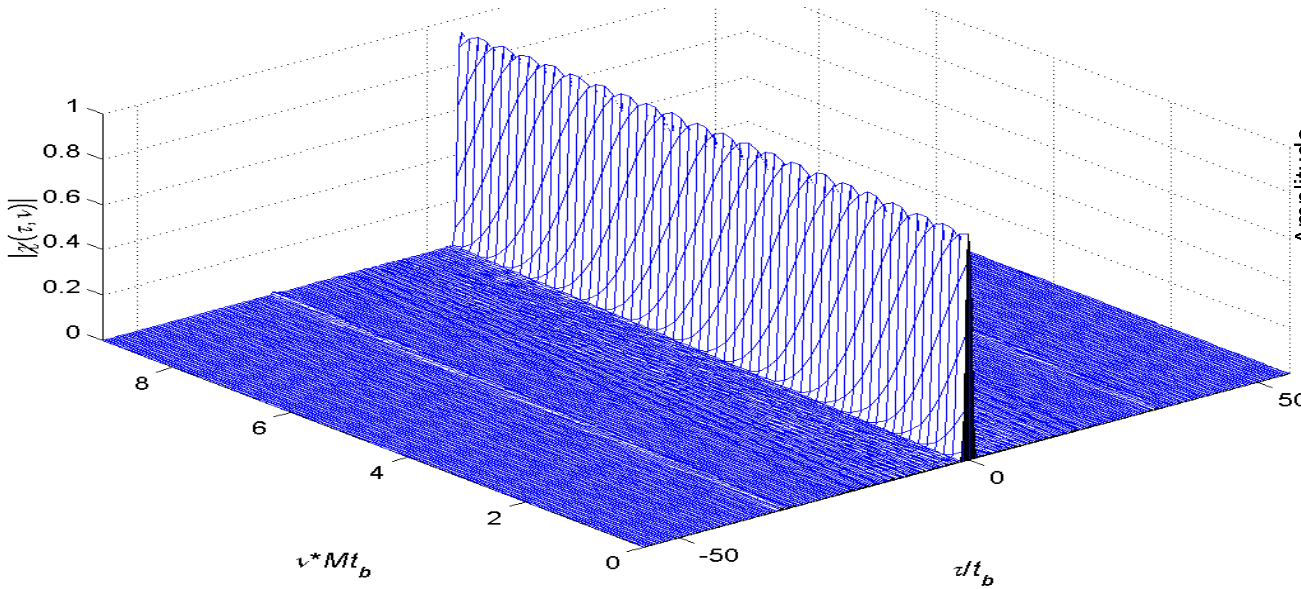

User Defined
Amplitude
0 0.2 0.4 0.6 0.8 1
0 5 10 15 20 25 30 35 40 45 50
t / t_b

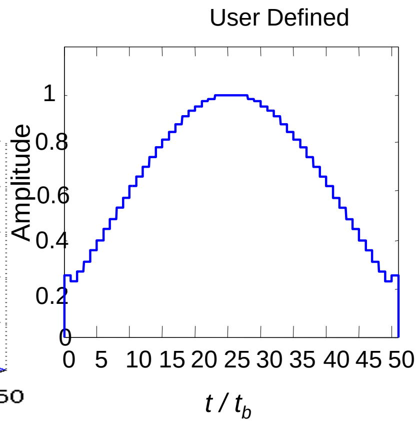

User Defined
Autocorelation [dB]
0
-20
-40
-60
0 5 10 15 20 25 30 35 40 45 50
τ/t_b


User Defined
Amplitude
0 0.2 0.4 0.6 0.8 1
0 5 10 15 20 25 30 35 40 45 50
t / t_b


User Defined
1
0.8
0.6
0.4
0.2
0
$|\chi(\tau,\nu)|$
8
6
4
2
0
$\nu*Mt_b$
-40
-20
0
20
40
τ/t_b


User Defined
Amplitude
1
0.8
0.6
0.4
0.2
0
0 5 10 15 20 25 30 35 40 45 50
t / t_b

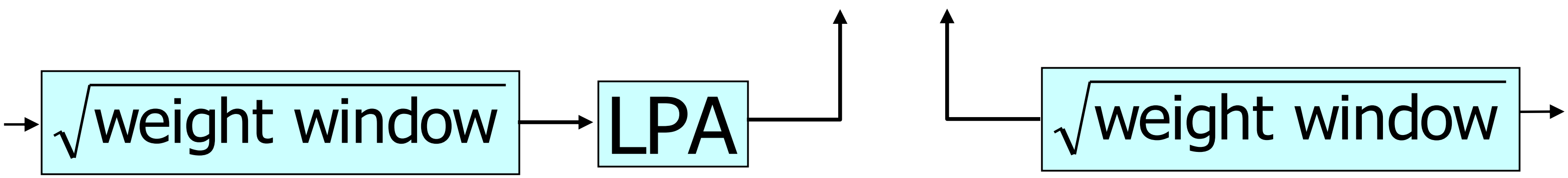

1
0.8
0.6
0.4
0.2
0
$|\chi(\tau,\nu)|$
8
6
4
2
0
$\nu*Mt_b$
-50
0
50
τ/t_b


User Defined
Amplitude
1
0.8
0.6
0.4
0.2
0
0 5 10 15 20 25 30 35 40 45 50
t / t_b

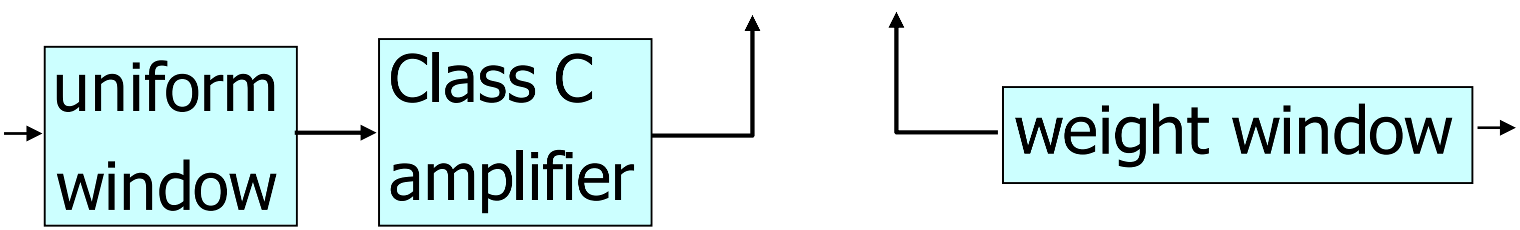
Spectrum shaping:

- Lowers the autocorrelation sidelobes.
- Widens autocorrelation mainlobe.

Spectrum shaping by amplitude modulating the pulse requires a transmitter with linear power amplifier (LPA).



Concentrating the weighting in the receiver causes mismatch (= SNR loss).



Concentrating the weighting in the receiver causes mismatch (= SNR loss).

uniform window → Class C amplifier

weight window →

$$\text{weight window} = \{C_1, C_2, ..., C_N\}$$

Uniform signal samples with amplitude A, noise RMS value is σ

$$SNR = \frac{A^2\left(\sum_{n=1}^{N} C_n\right)^2}{\sigma^2 \sum_{n=1}^{N} C_n^2}$$

Signal samples add up coherently (power of sum).
Noise samples add up noncoherently (sum of powers).

$$\text{Uniform weight}: C_n = 1$$

$$SNR_{\text{uni}} = \frac{A^2 N^2}{\sigma^2 N} = \frac{A^2}{\sigma^2} N$$

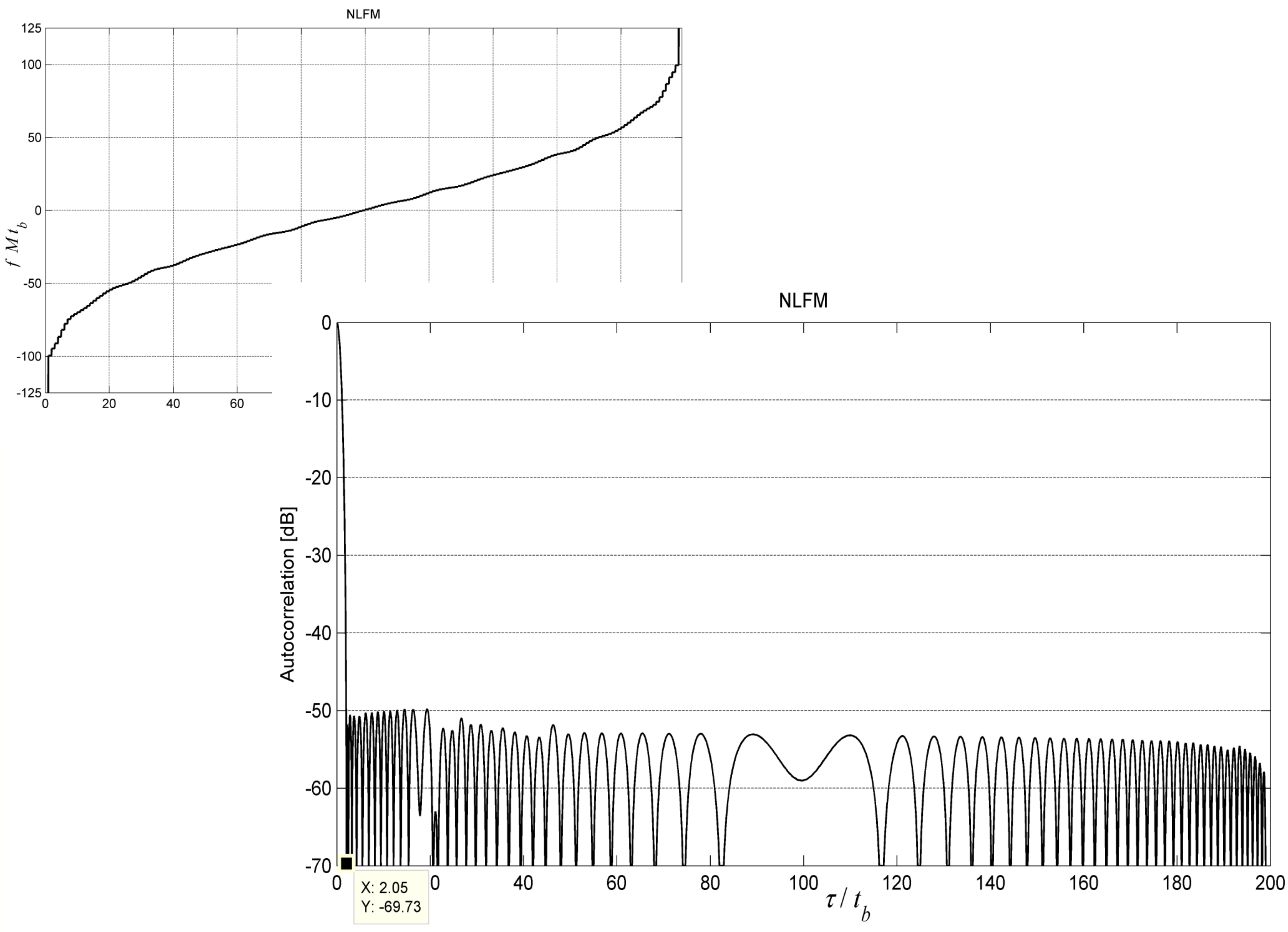
$$SNR_{\text{loss}} = \frac{SNR}{SNR_{\text{uni}}}$$

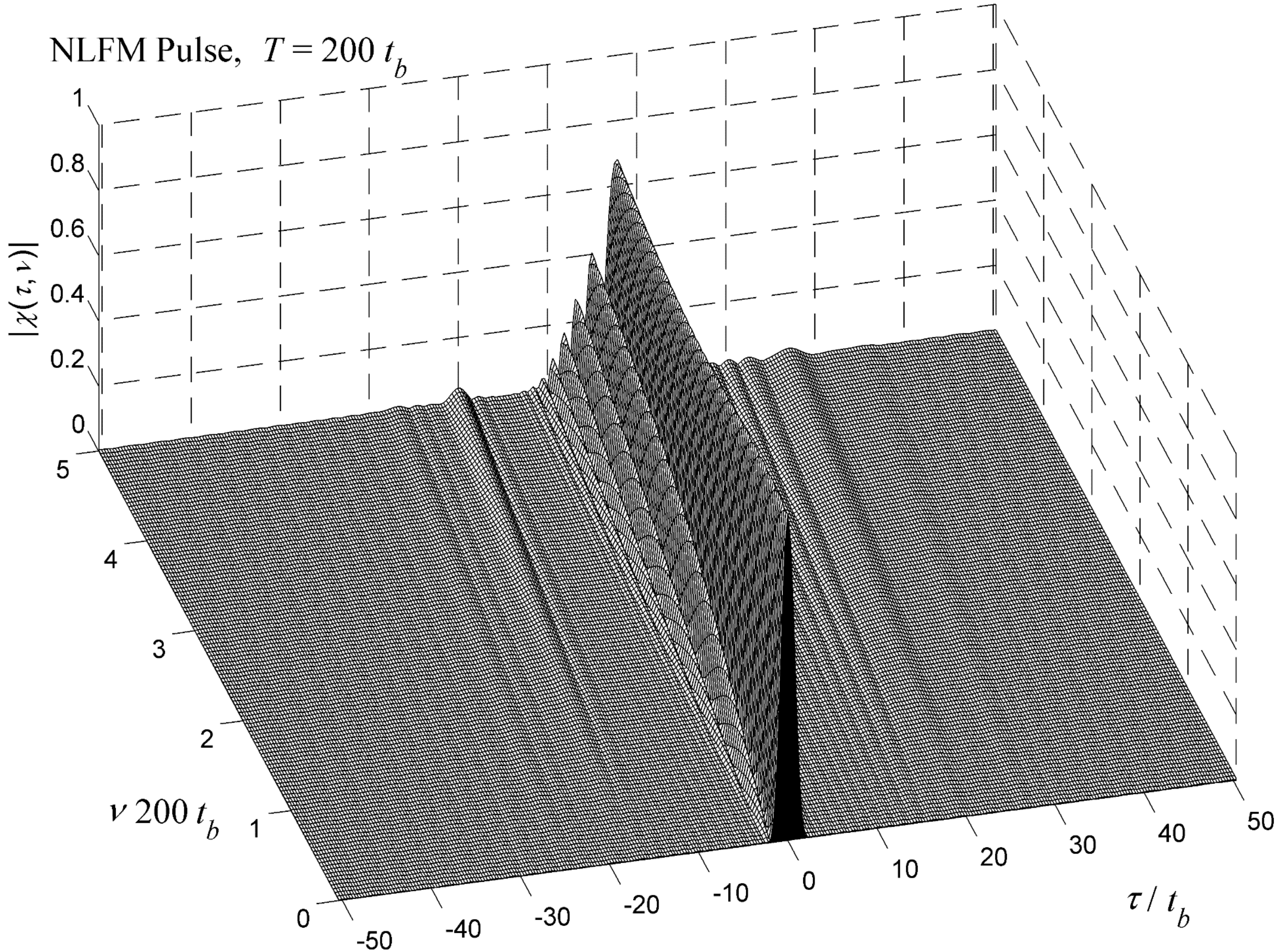
$$SNR_{\text{loss}} = \frac{\left(\sum_{n=1}^{N} C_n\right)^2}{N \sum_{n=1}^{N} C_n^2}$$

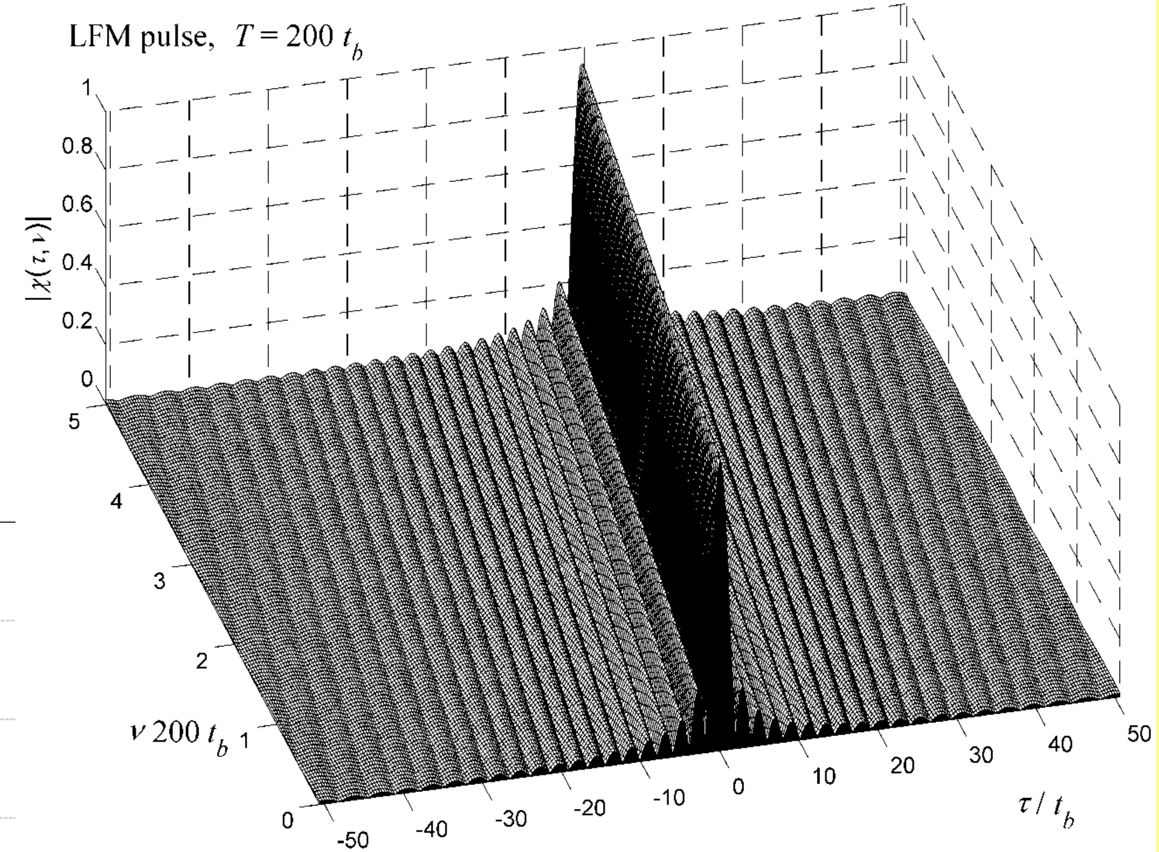
Hamming, $N \rightarrow \infty$ SNR loss $\rightarrow$ -1.345 dB

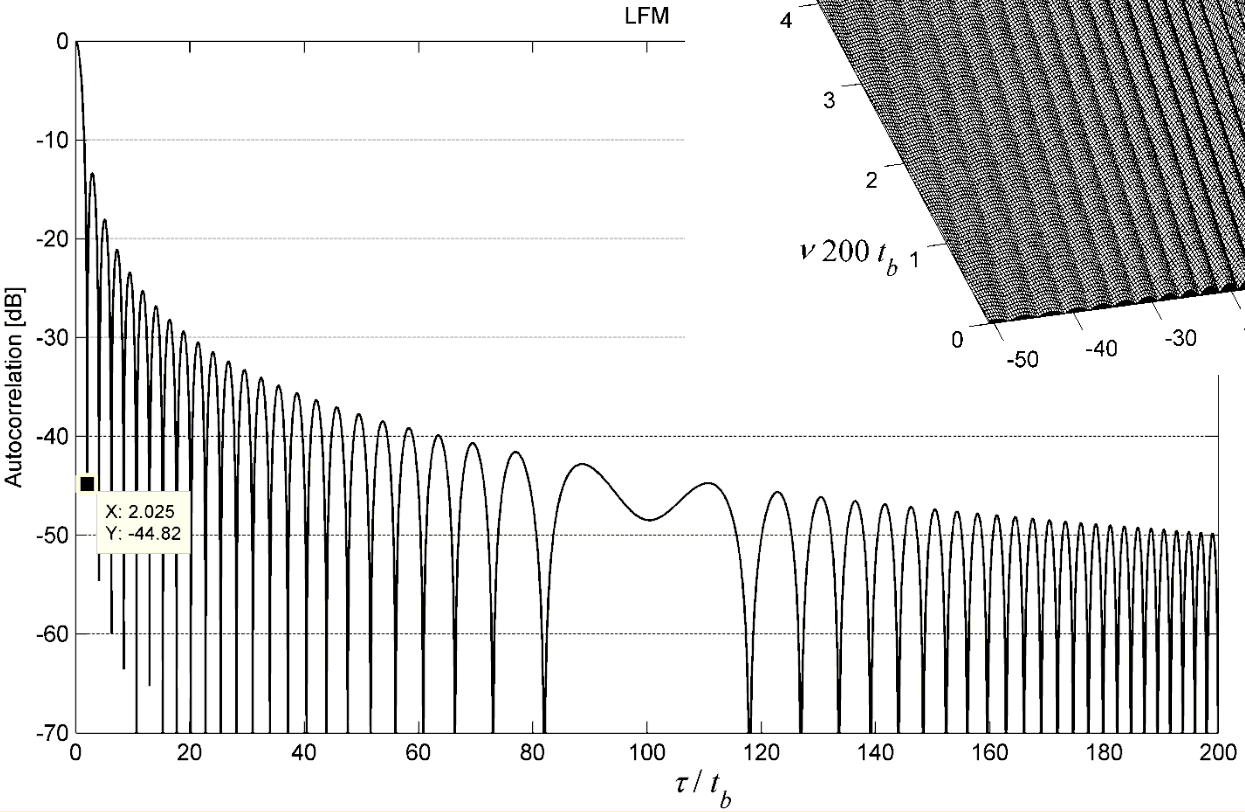
NON-LINEAR FM

Shapes the spectrum to a desired window, not through a non-uniform amplitude, but through a non-linear frequency modulation
(spends more time at the central frequencies)


NLFM
125
100
50
0
-50
-100
-125
$f\ M\ t_b$
0
20
40
60
NLFM
0
-10
-20
-30
-40
-50
-60
-70
Autocorrelation [dB]
X: 2.05
Y: -69.73
0
40
60
80
100
120
140
160
180
200
τ / t_b

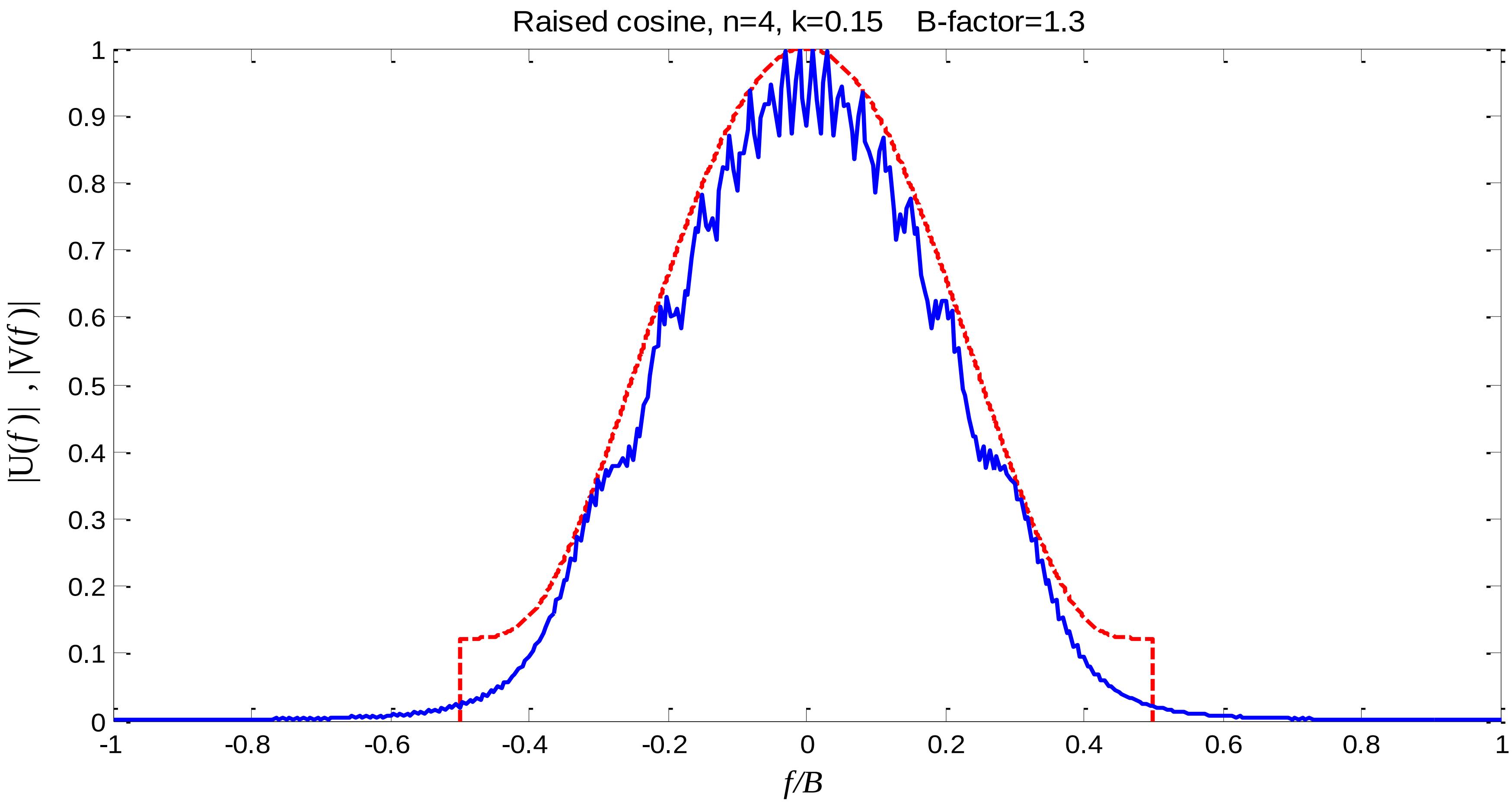

NLFM Pulse, $T = 200\ t_b$
$|\chi(\tau,\nu)|$
1
0.8
0.6
0.4
0.2
0
5
4
3
2
$\nu\ 200\ t_b$
1
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
$\tau\ /\ t_b$


LFM pulse, $T = 200\ t_b$
$|\chi(\tau,\nu)|$
1
0.8
0.6
0.4
0.2
0
5
4
3
2
$\nu\ 200\ t_b$ 1
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
τ / t_b

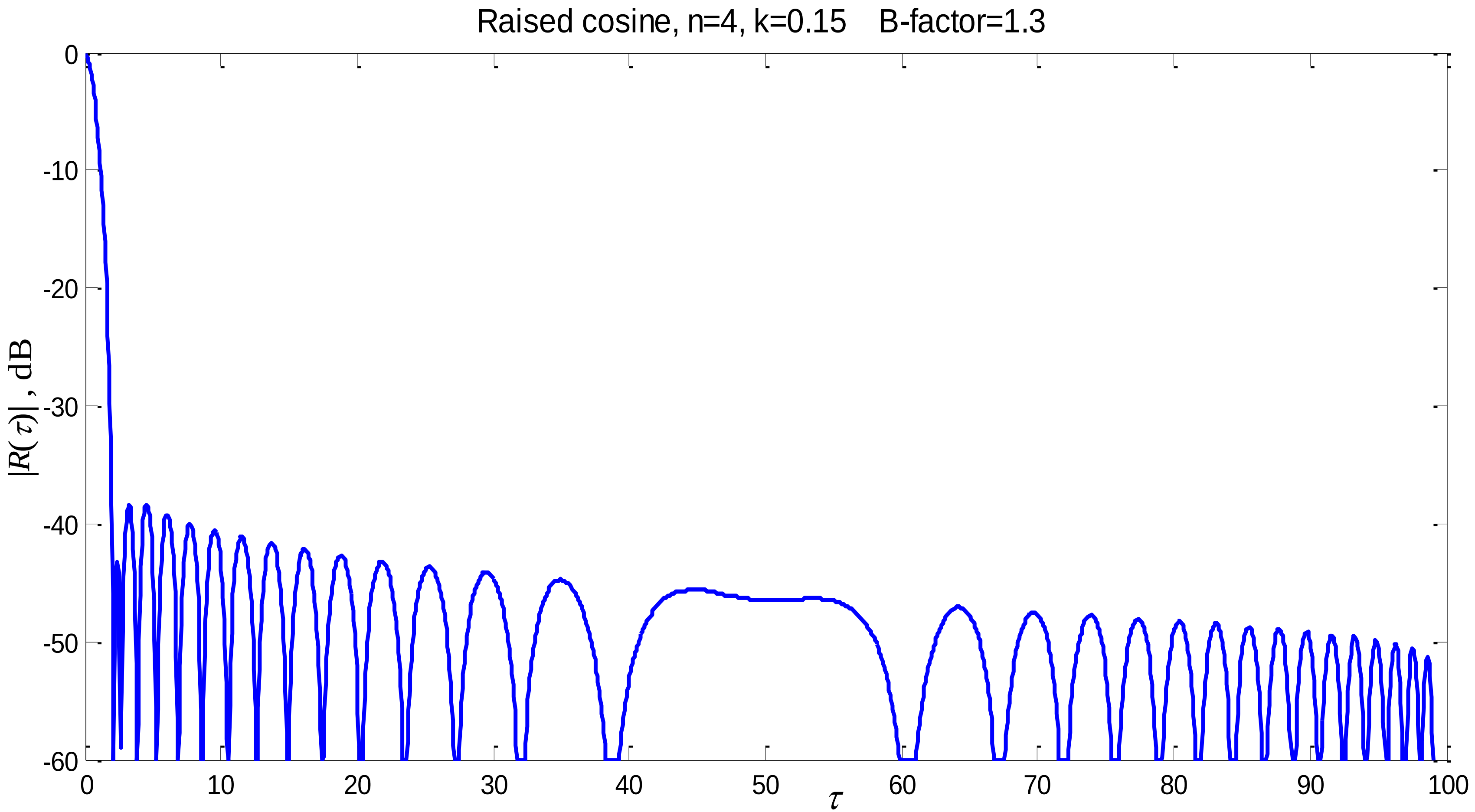

LFM
Autocorrelation [dB]
0
-10
-20
-30
-40
-50
-60
-70
X: 2.025
Y: -44.82
0
20
40
60
80
100
120
140
160
180
200
τ / t_b

Example: $V(f)$ is raised cosine

$$V(f)=\left[k+(1-k)\cos^{n}\left(\pi\frac{f}{B}\right)\right]^{1/2}$$



The designed (dotted) and achieved (solid) spectrum of NLFM signal allowing spectrum spreading of 30%

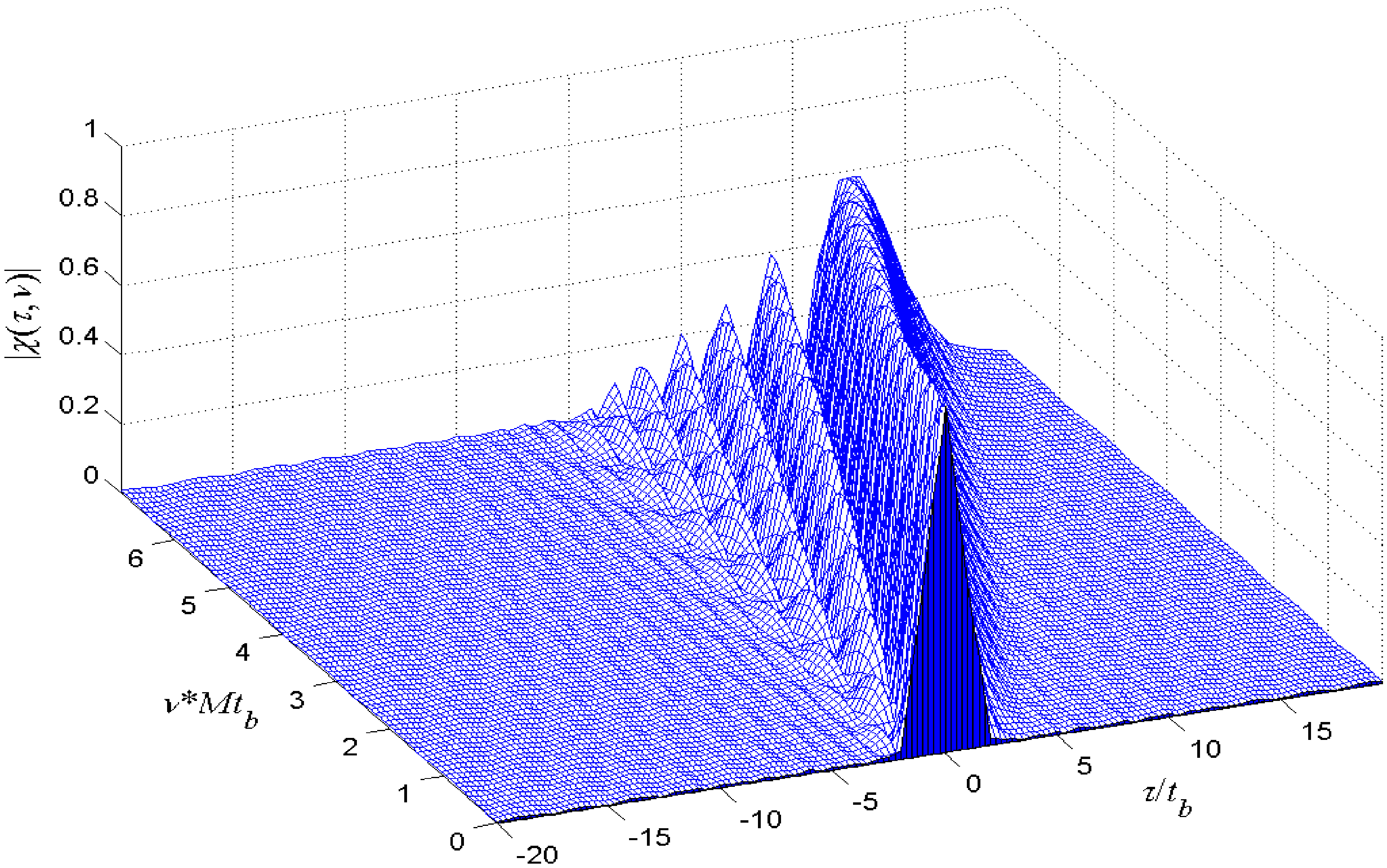


The autocorrelation function of the NLFM signal whose spectrum appears in the previous slide

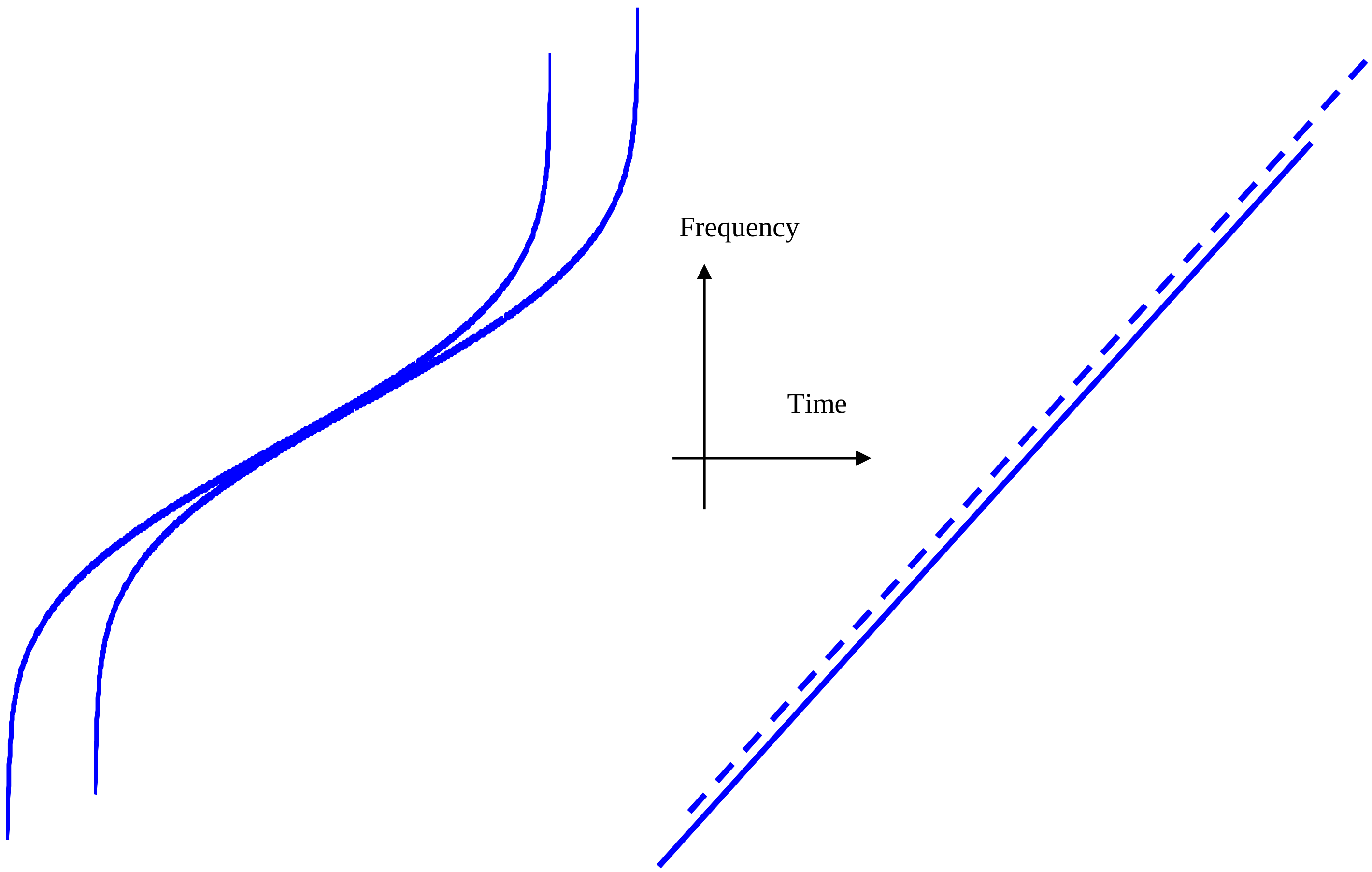
$$\text{Peak SL}_{[\text{dB}]} = -20\log_{10}(BT) - 3$$

$BT = 130$ =>

$$-20\log_{10}(130) - 3 = 39.3\text{dB}$$



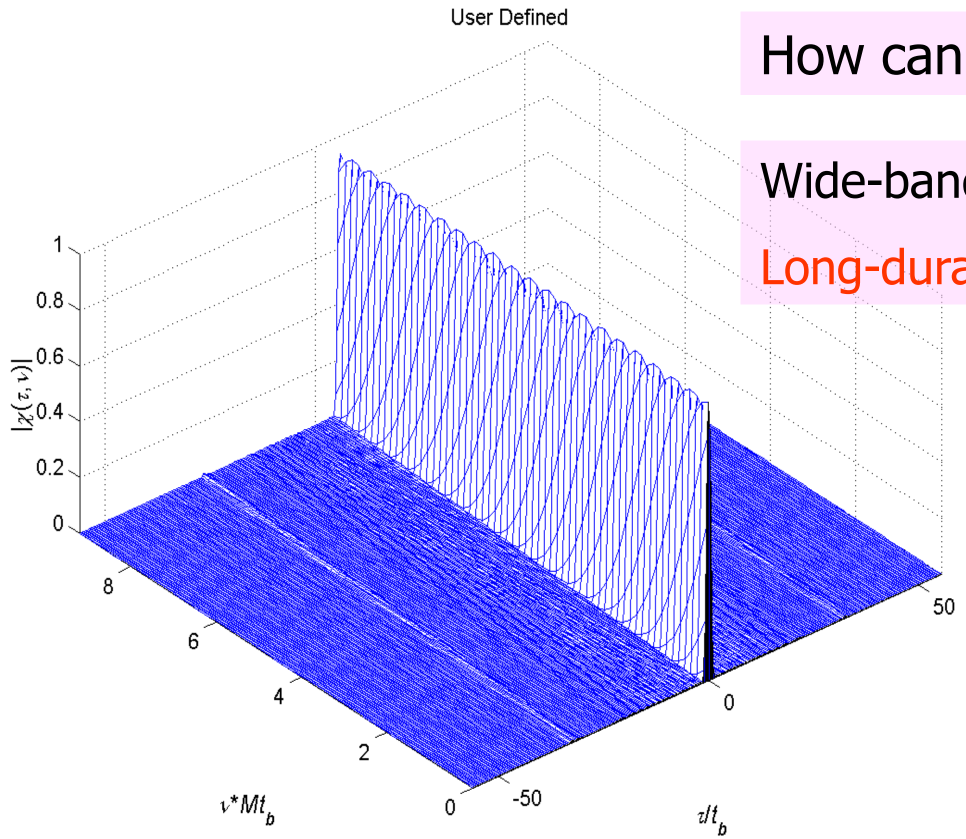
Partial ambiguity function (zoom in delay and Doppler) of the NLFM signal



Alignment of frequency characteristics of two NLFM signals (left) and two LFM signals (right)

Weighted LFM (or NLFM) provides near-ideal Doppler-tolerant ambiguity function:

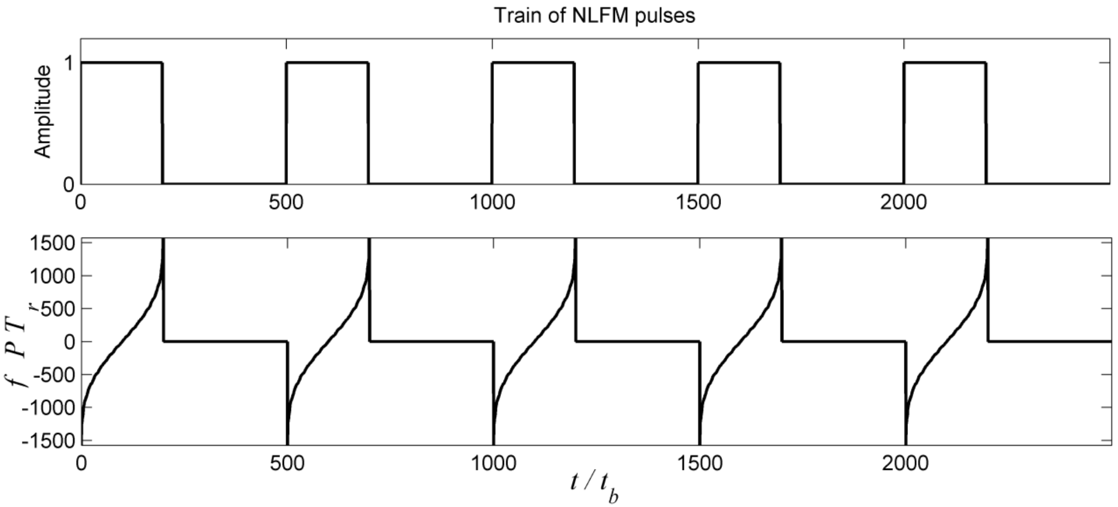
- Narrow ridge = high range resolution.
- Extending very far in Doppler.
- Almost parallel to the Doppler axis (can't be along the Doppler axis - because the Doppler cut is not affected by frequency or phase modulation).

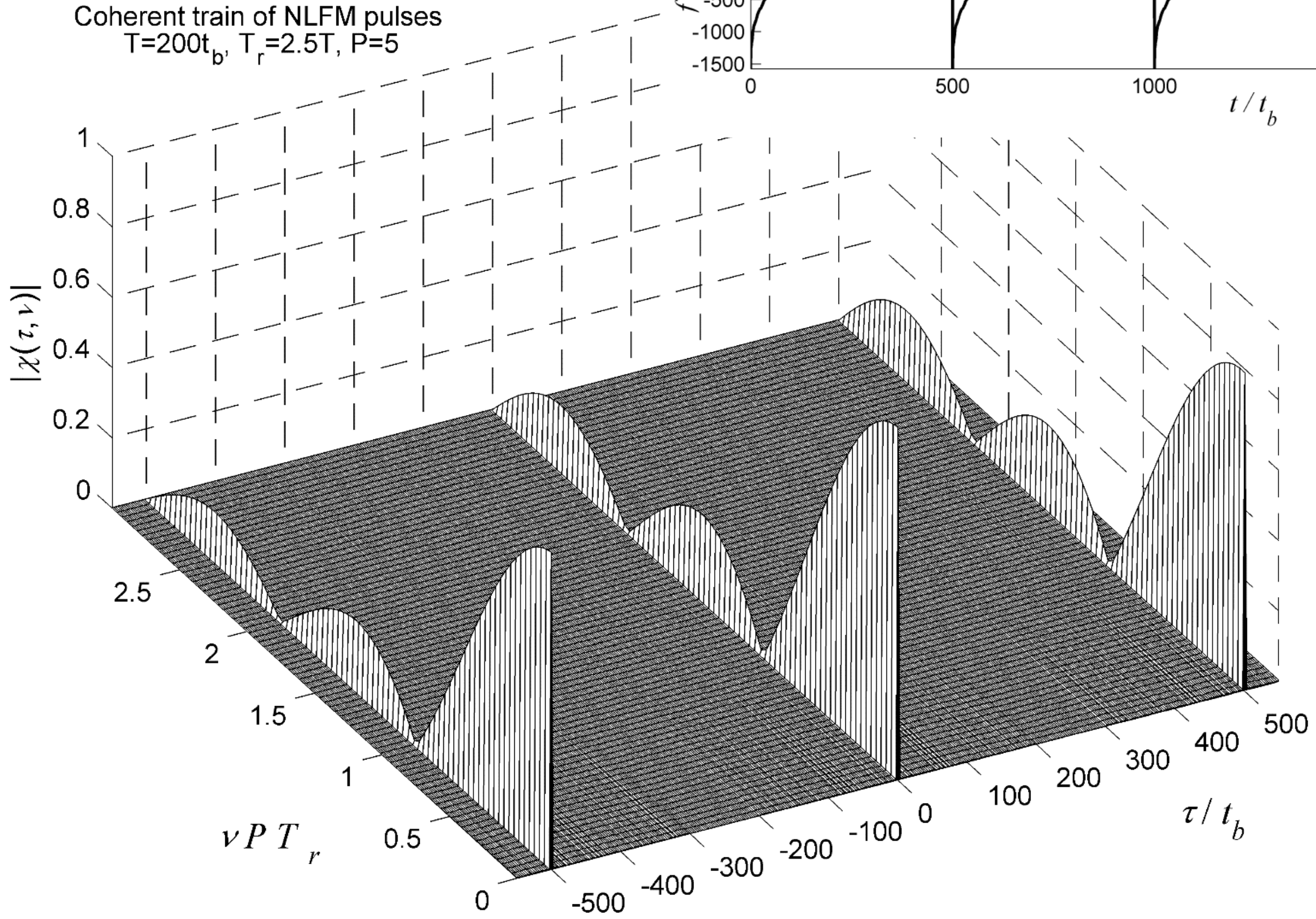


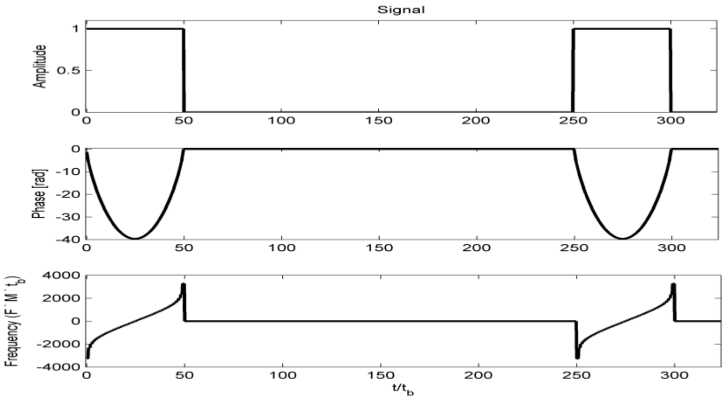
How can we add Doppler resolution ?

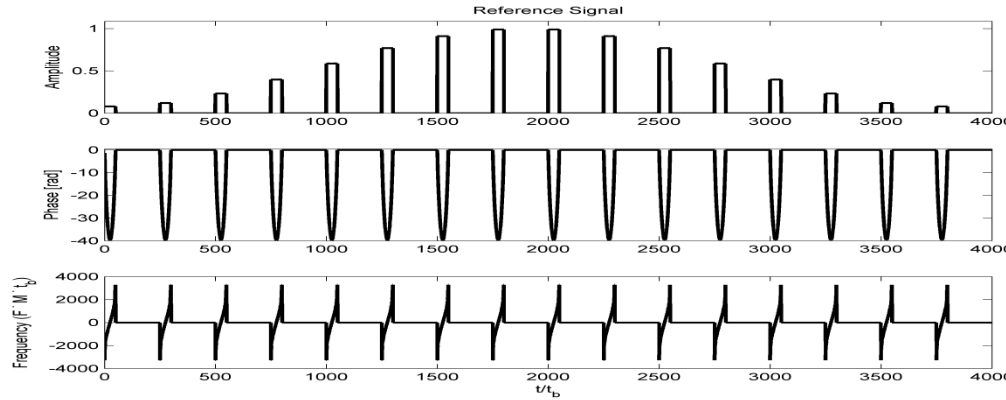
Wide-bandwidth signal $\Rightarrow$ range resolution

Long-duration signal $\Rightarrow$ Doppler resolution

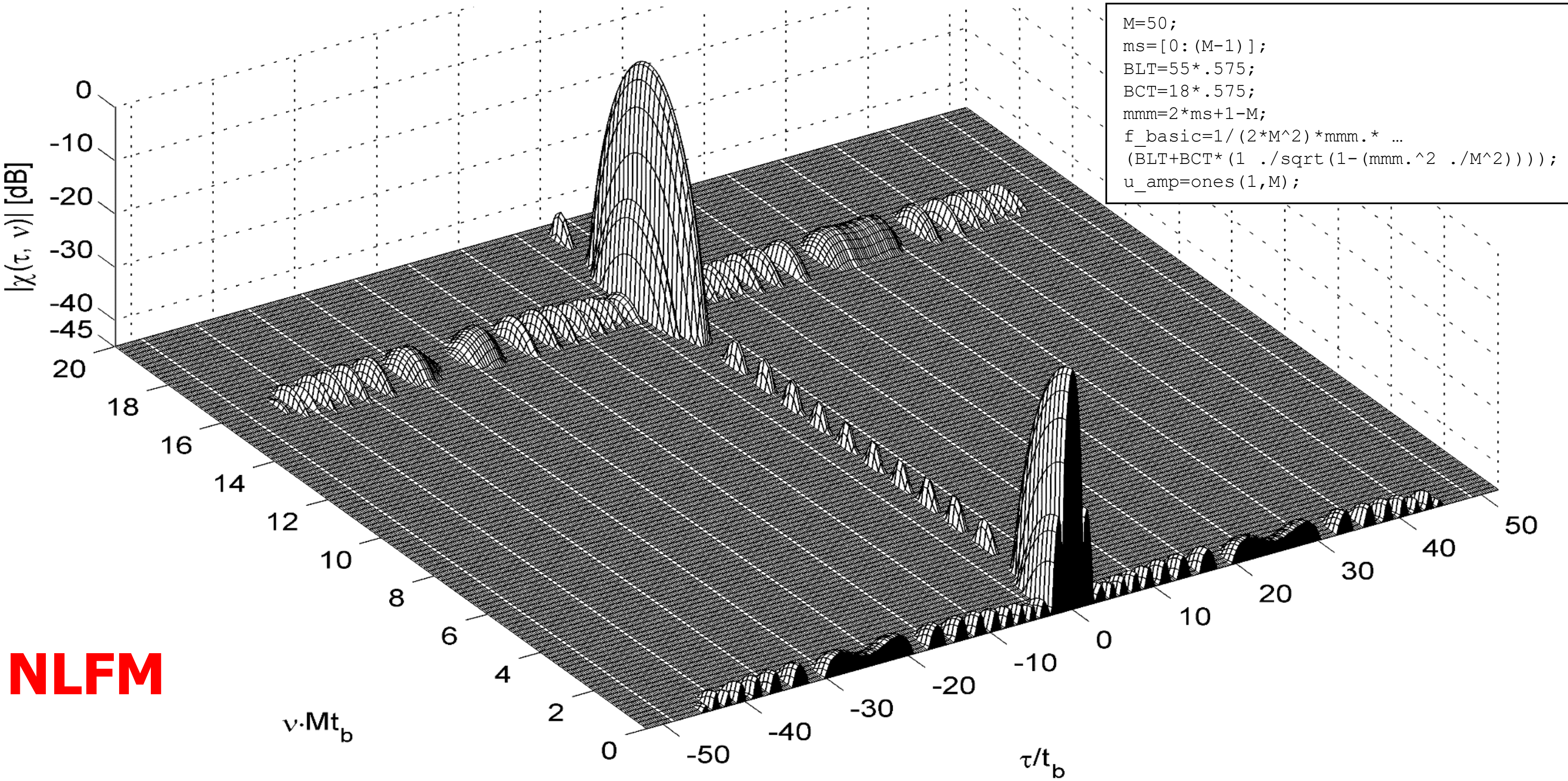

Train of NLFM pulses
Amplitude
1
0
0
500
1000
1500
2000
$f\ P\ T_r$
1500
1000
500
0
-500
-1000
-1500
$t\,/\,t_b$


Coherent train of NLFM pulses
$T=200t_b$, $T_r=2.5T$, $P=5$
$|\chi(\tau,\nu)|$
1
0.8
0.6
0.4
0.2
0
2.5
2
1.5
1
0.5
0
$\nu\,P\,T_r$
-500
-400
-300
-200
-100
0
100
200
300
400
500
$\tau\,/\,t_b$



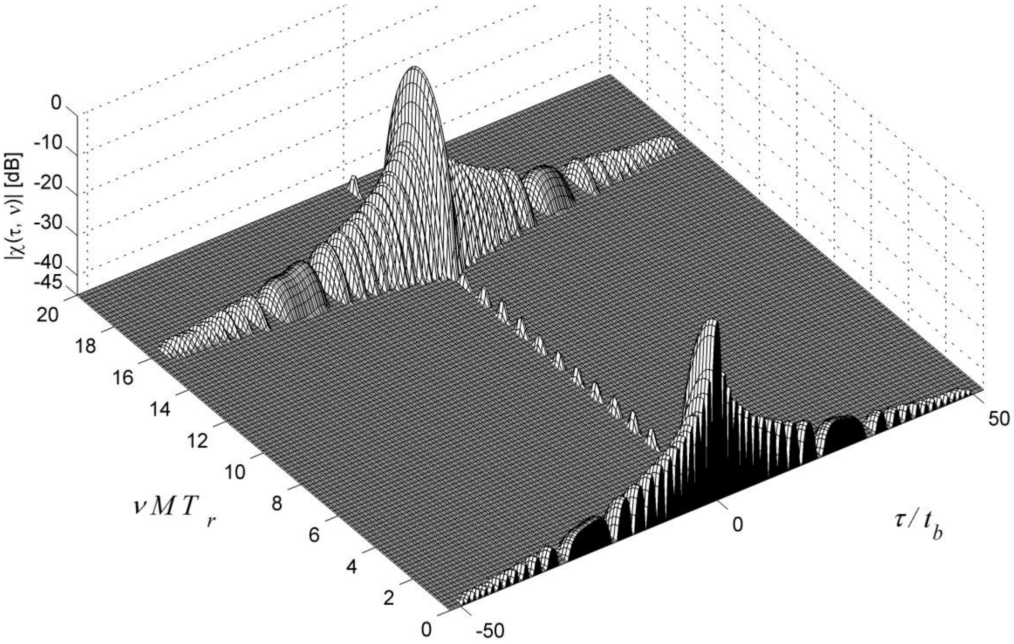


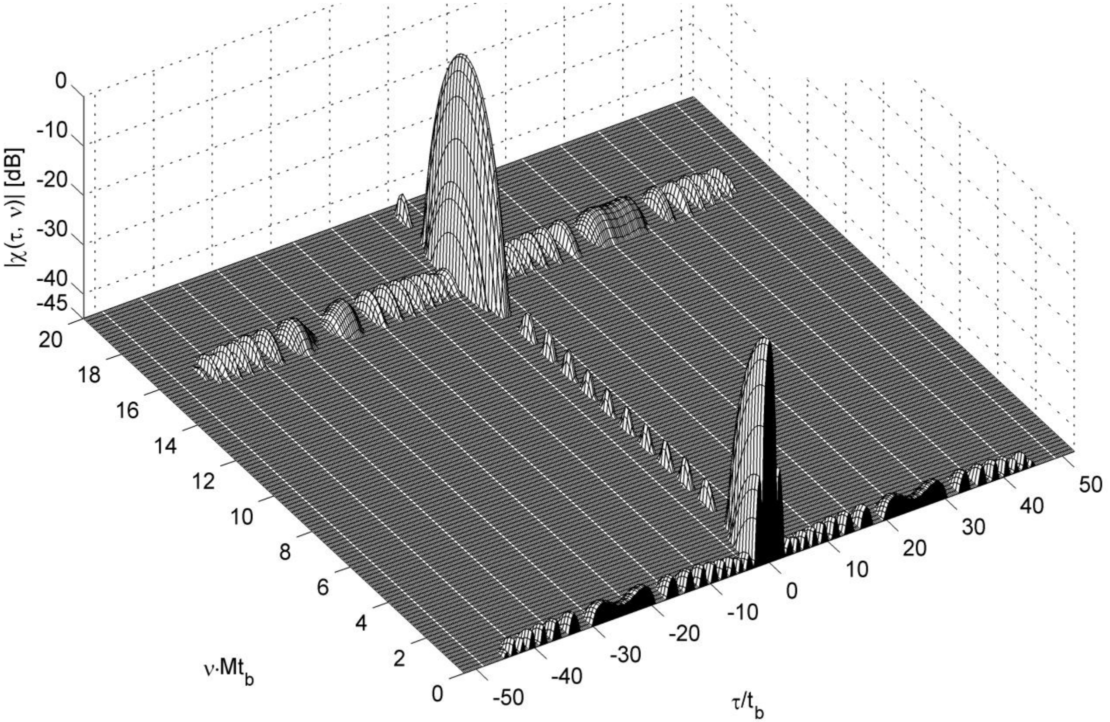

```
M=50;
ms=[0:(M-1)];
BLT=55*.575;
BCT=18*.575;
mmm=2*ms+1-M;
f_basic=1/(2*M^2)*mmm.* …
(BLT+BCT*(1 ./sqrt(1-(mmm.^2 ./M^2))));
u_amp=ones(1,M);
```



NLFM

Coherent train of LFM pulses,
Hamming inter-pulse weighting.





Coherent train of NLFM pulses,
Hamming inter-pulse weighting.

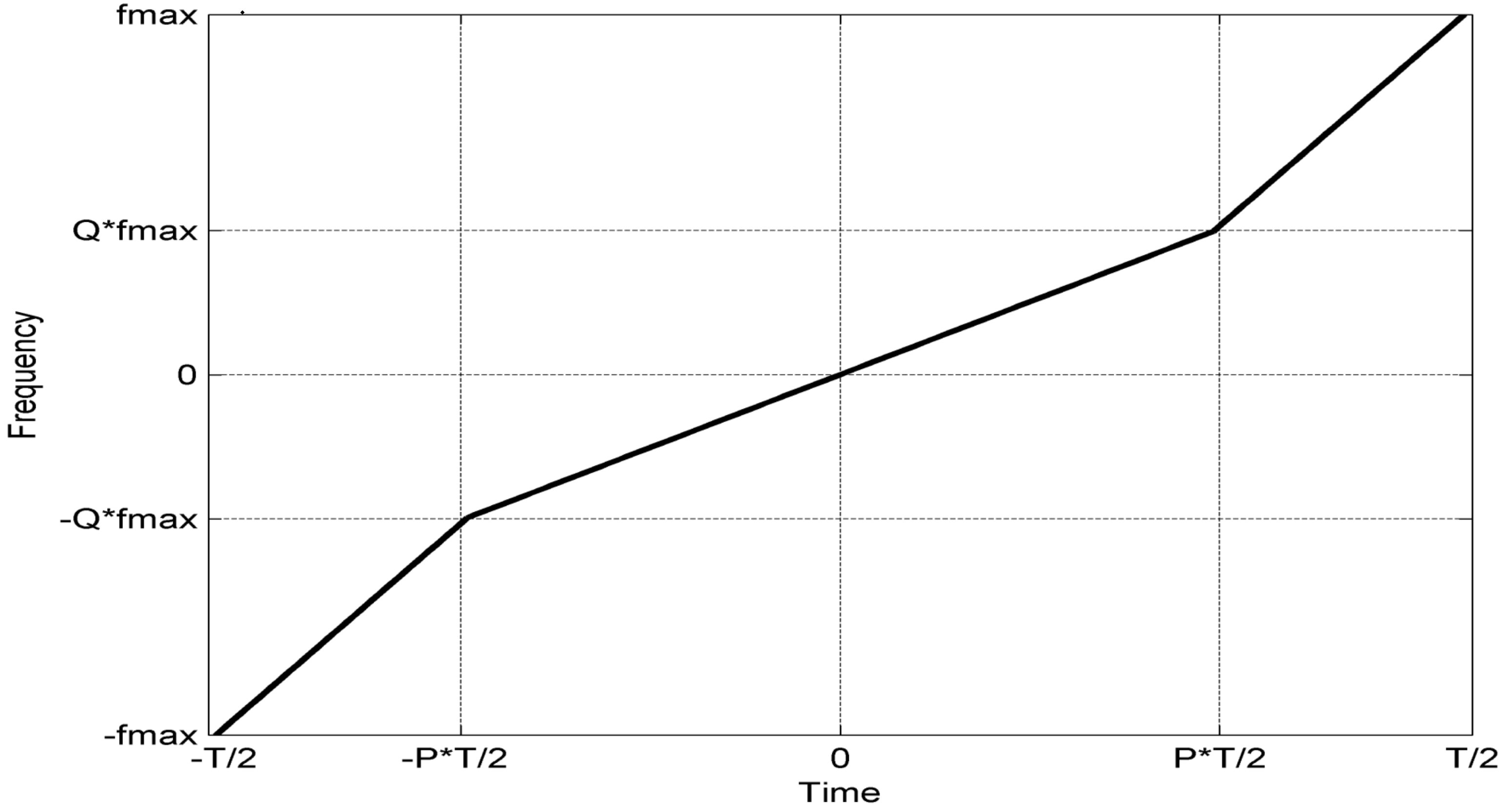
Piecewise Non-linear FM תכנון אות

פונקצית האוטוקורלציה (ACF) של אות Linear-FM (LFM) סובלת מאונות צד גבוהות יחסית. אונת הצד הגבוהה ביותר (PSL) מגיעה ל -13.5 dB . את אונות הצד ניתן להנמיך ע"י אות Non-linear FM (NLFM). אחת הדרכים לממש אות NLFM היא ע"י הרכבתו מקטעים ישרים.

בשאלה נעסוק באות פשוט יחסית שמהלך התדר שלו (משני צידי התדר הנושא) מתואר ע"י שלושה קטעים ישרים, כמוצג בציור.

שתי נקודות הברך הן סימטריות לגבי הראשית, ומוגדרות ע"י שני הפרמטרים: $0<P<1\,,\ 0<Q<1$

מכפלת רוחב הפס במשך האות תוגדר כ: $TBW = 2 f_{max} T$



1. עבור $TBW = 40$

יש למצוא את P ו- Q שיתנו לאות פונקציית אוטוקורלציה עם PSL מינימלי.

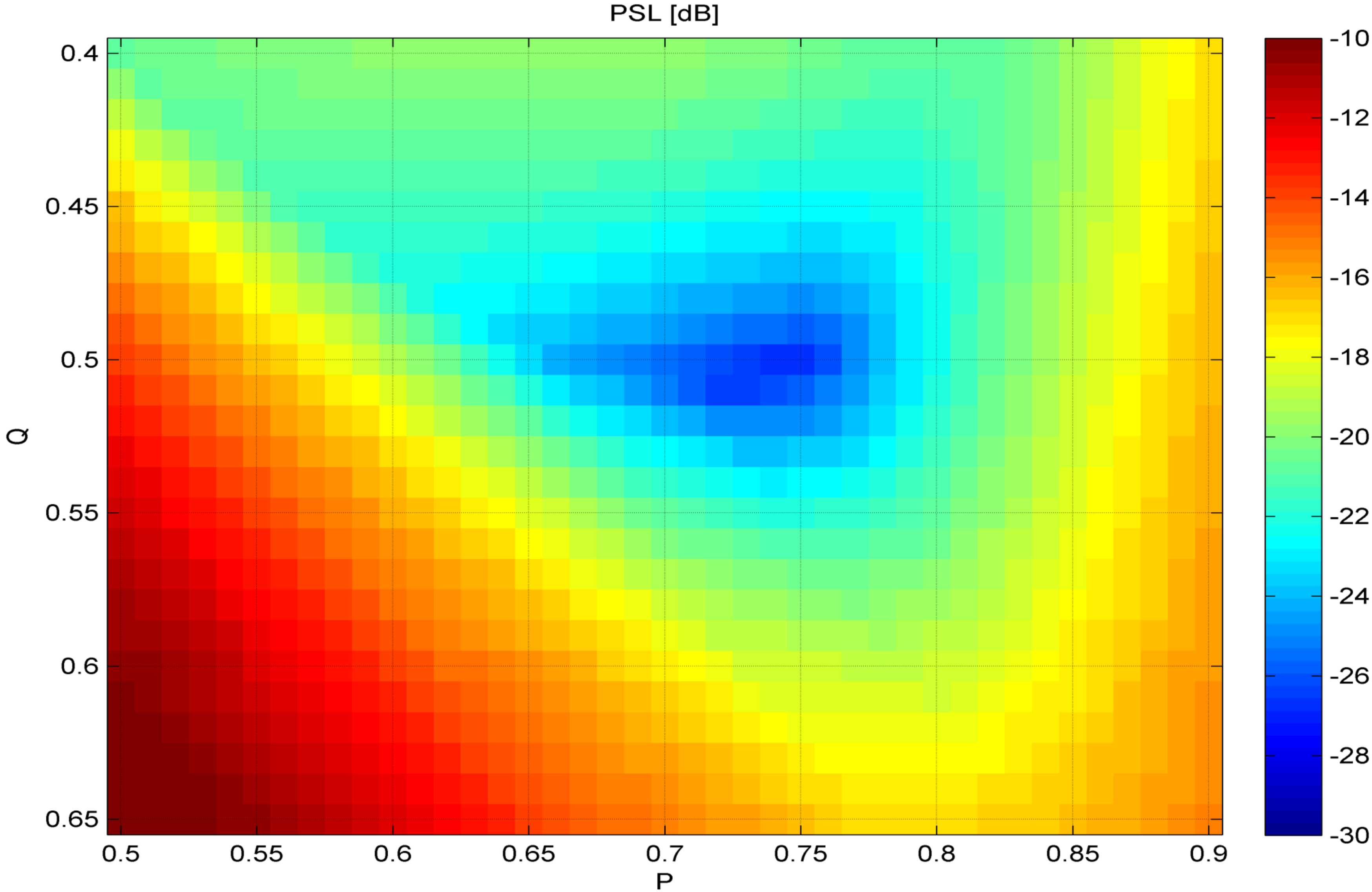
2. עבור האות שמתקבל עם אותם P ו- Q יש לצייר את פונקציית האוטוקורלציה (מספיק לציירה עבור השהיות חיוביות), ולהשוותה לפונקציית האוטוקורלציה של אות LFM עם אותו TBW. המהדרים מוזמנים להוסיף את פונקציות האמביגיוטי המתאימות.

רמז: מוצע להגדיר את האות על ידי $M = 200$ מדרגות תדר שמשכן t_b קבוע, כך שמתקיים $T = Mt_b$
גובה קפיצת התדר ישתנה כמובן לפי השיפוע של מהלך התדר בזמן.

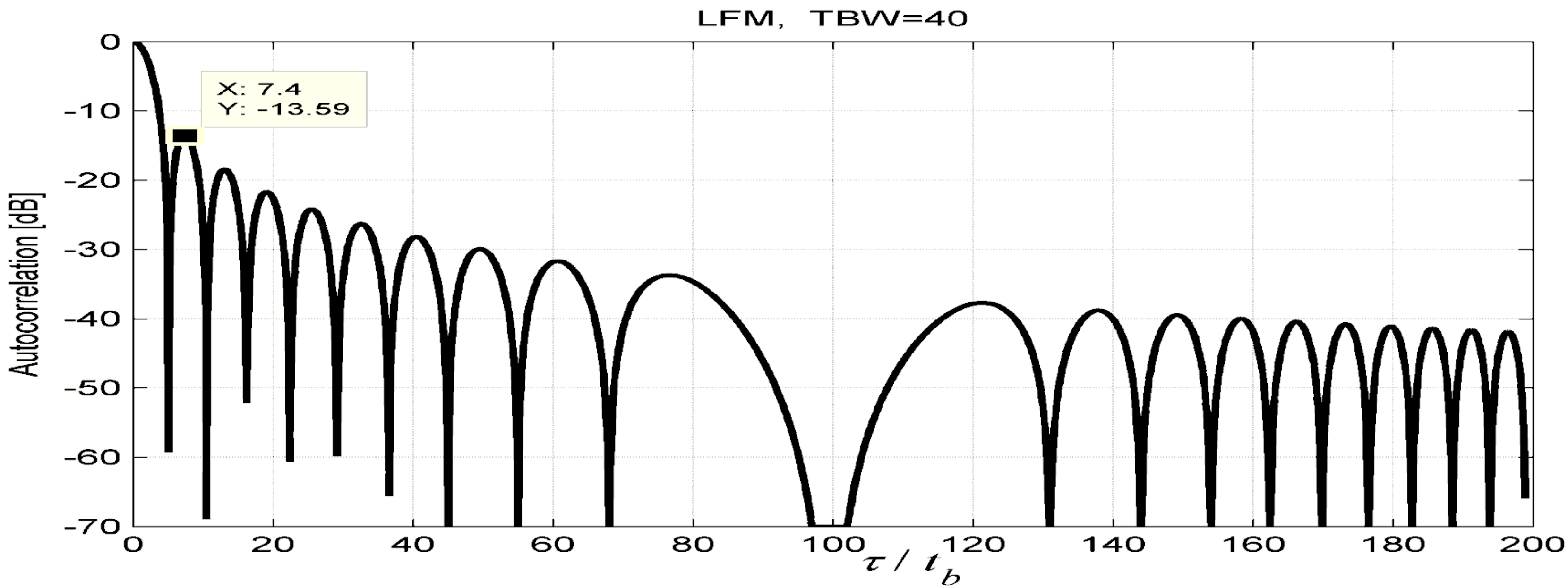
פתרון:

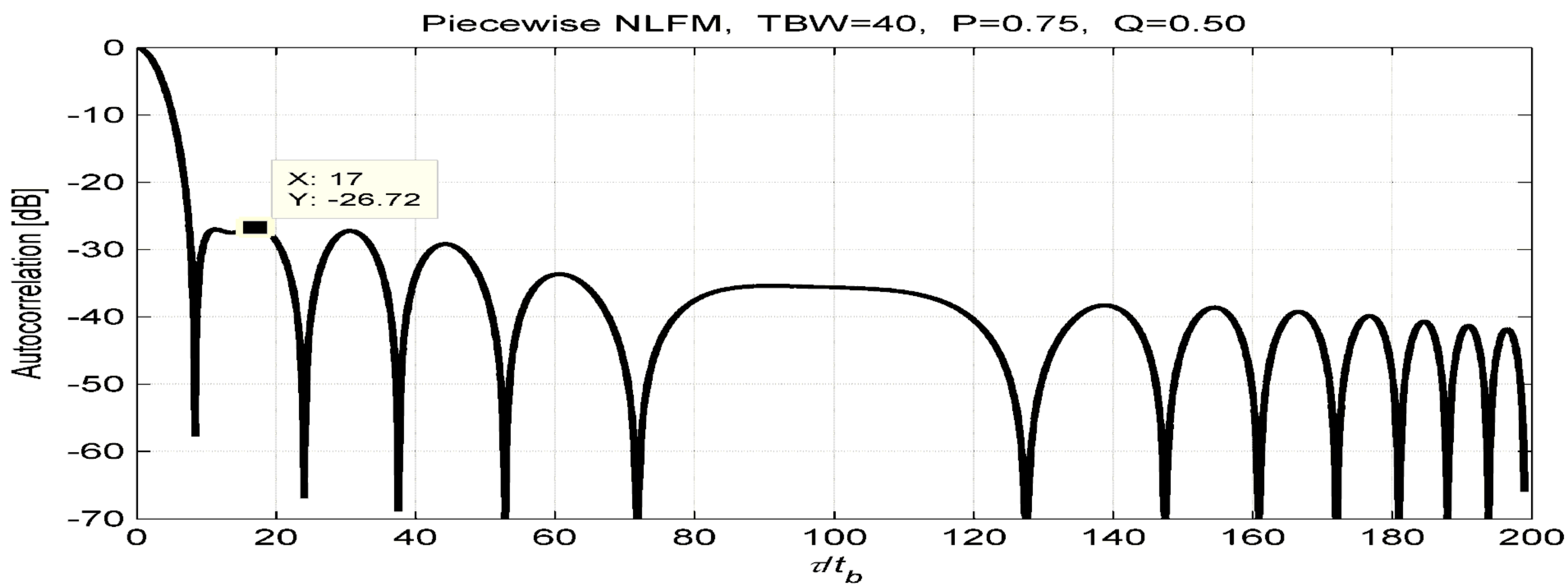
נכתב קוד MATLAB היוצר את האות, מחשב את ה ACF ומוצא את אונת הצד הגבוהה ביותר. בעזרתו בוצע חיפוש דו-ממדי במרחב P ו- Q. תוצאות החיפוש מופיעות בציור (1).

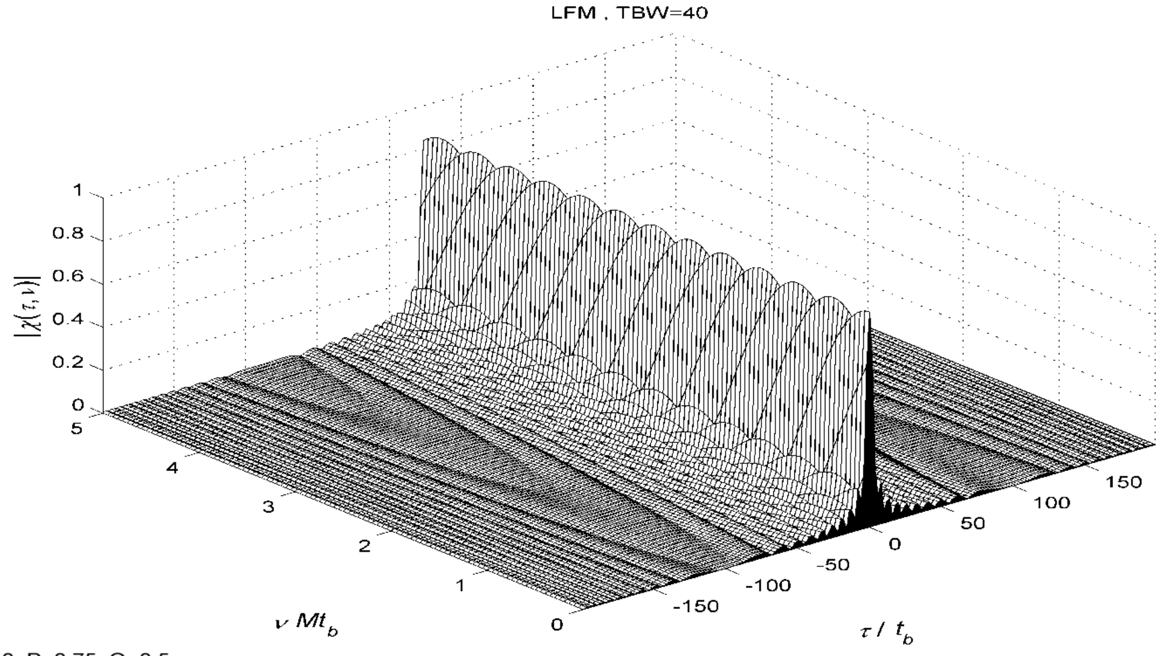
פונקציות ה ACF וה AF עבור אות ה Piecewise NLFM עם הברך המוגדרת ע"י P ו Q הנ"ל, ועבור אות LFM, שניהם עם $TBW = 2f_{max}T = 40$, מופיעות בציורים (2) (3).

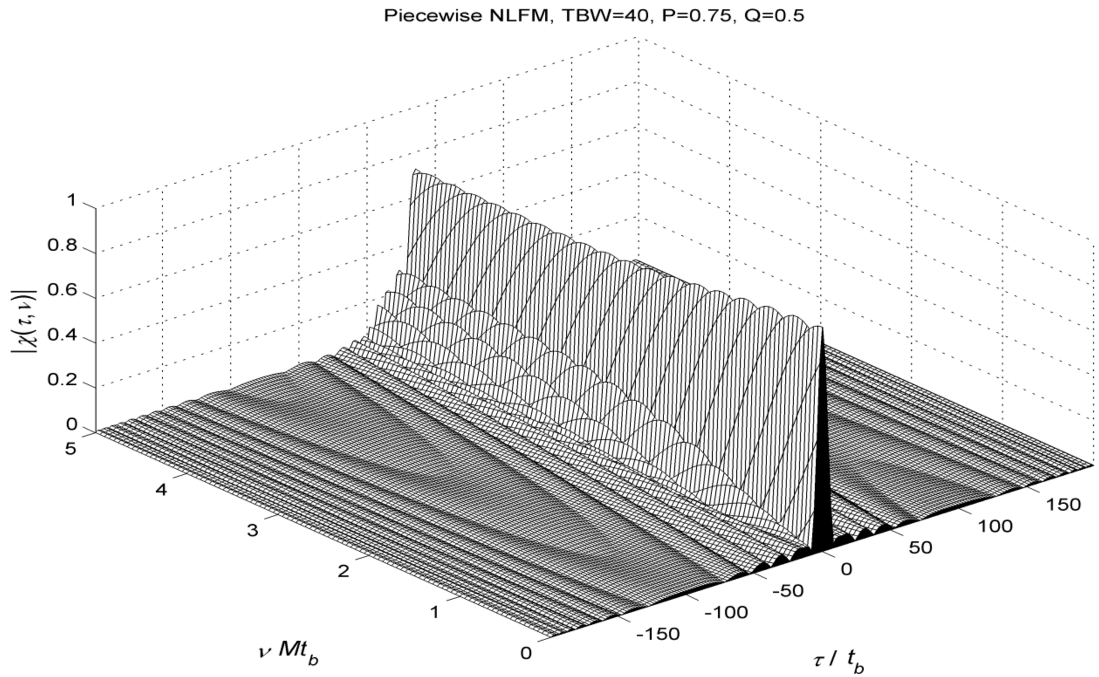


ה PSL הנמוך ביותר נמצא בערכים $P = 0.75\ ,\ Q = 0.50$


LFM, TBW=40
X: 7.4
Y: -13.59
Autocorrelation [dB]
0
-10
-20
-30
-40
-50
-60
-70
0
20
40
60
80
100
120
140
160
180
200
τ / t_b


Piecewise NLFM, TBW=40, P=0.75, Q=0.50
X: 17
Y: -26.72
Autocorrelation [dB]
0
-10
-20
-30
-40
-50
-60
-70
0
20
40
60
80
100
120
140
160
180
200
τ/t_b


LFM , TBW=40
1
0.8
0.6
0.4
0.2
0
|χ(τ,ν)|
5
4
3
2
1
0
ν Mt_b
-150
-100
-50
0
50
100
150
τ / t_b


Piecewise NLFM, TBW=40, P=0.75, Q=0.5
1
0.8
0.6
0.4
0.2
0
|χ(τ,ν)|
5
4
3
2
1
0
ν Mt_b
-150
-100
-50
0
50
100
150
τ / t_b