Radar Principles - extended עקרונות מכ"ם - מורחב

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October 2021 - January 2022

Radar Books Introductory and General

N. Levanon: "*Radar Principles*", New York, Wiley, 1988.

N. Levanon and **E. Mozeson** "*Radar Signals*", Wiley, 2004.

M.A. Richards "*Fund. Radar Signal Processing*", 2nd ed., McGraw-Hill, 2014.

P.E. Pace "*Detecting and Classifying LPI Radar*"
2nd ed. Artech, 2009.

M. Skolnik: "*Introduction to Radar Systems*",
3rd ed. New York, McGraw-Hill, 2002.

M.A. Richards: "*Principles of Modern Radar - Basic Principles* ",
SciTech, 2010.

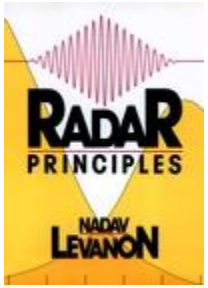
W.L. Melvin: "*Principles of Modern Radar - Advanced Techniques*",
SciTech, 2012.

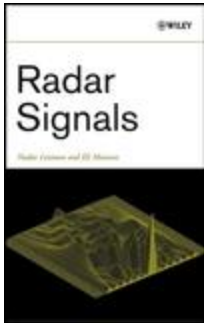
G.W. Stimson: "Stimson's *Introduction to Airborne Radar*",
Hughes Aircraft Co., 1983(2nd edition, SciTech, 1990).
Updated 3rd ed., SciTech 2014, H.D. Griffiths, C. J. Baker, D. Adamy

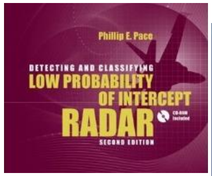
A.W. Rihaczek: "*Principles of High Resolution Radar*",
New York, McGraw-Hill, 1969 (Artech, 1996)

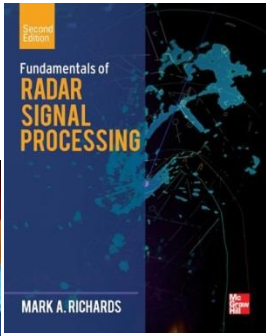
B. R. Mahafza: "*Radar Systems Analysis and Design Using MATLAB* ", 3rd edition, CRC Press, 2013.

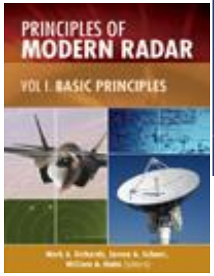
M.C. Budge Jr. and **S.R. German**: "Basic Radar Analysis",
Norwood, MA, Artech House, 2015.

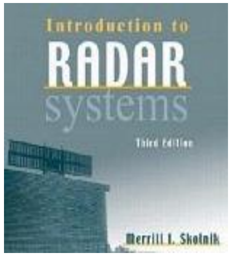


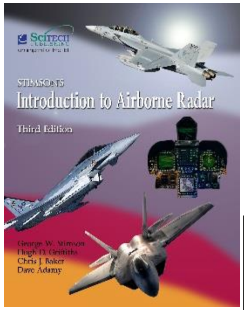


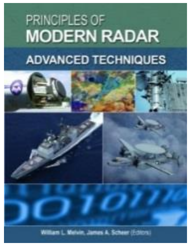


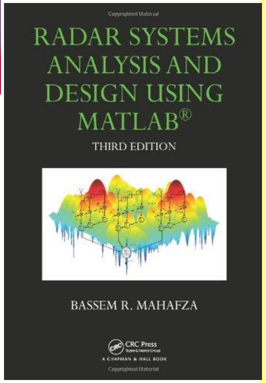


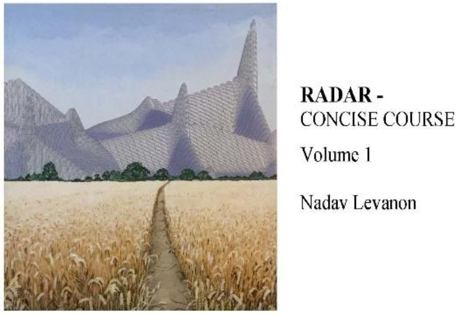


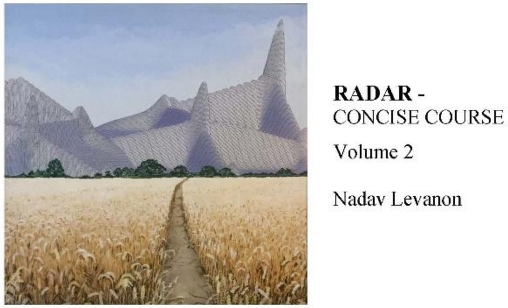












RAdio Detection And Ranging

or

(RAdio Direction And Range)

מגלה כיוון ומרחק

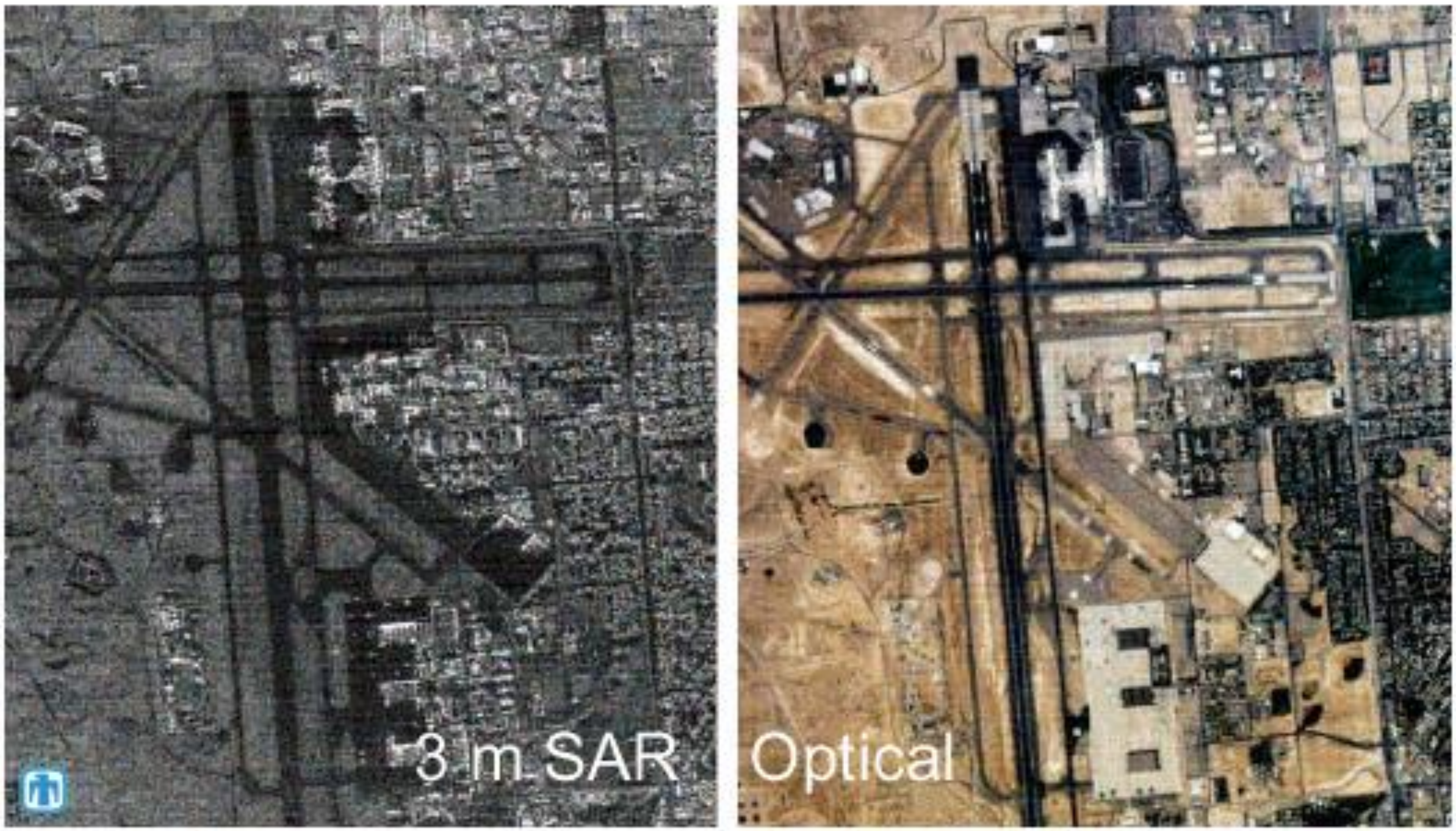
Basic Radar Functions

- Detection
  - determine if observed signal represents noise only, or noise plus the echo of a transmitted signal
    - implies reflection off an object
- Tracking
  - measure and track the location and velocity of a detected object
- Imaging
  - form a 2- D or 3- D image of an area or a volume

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

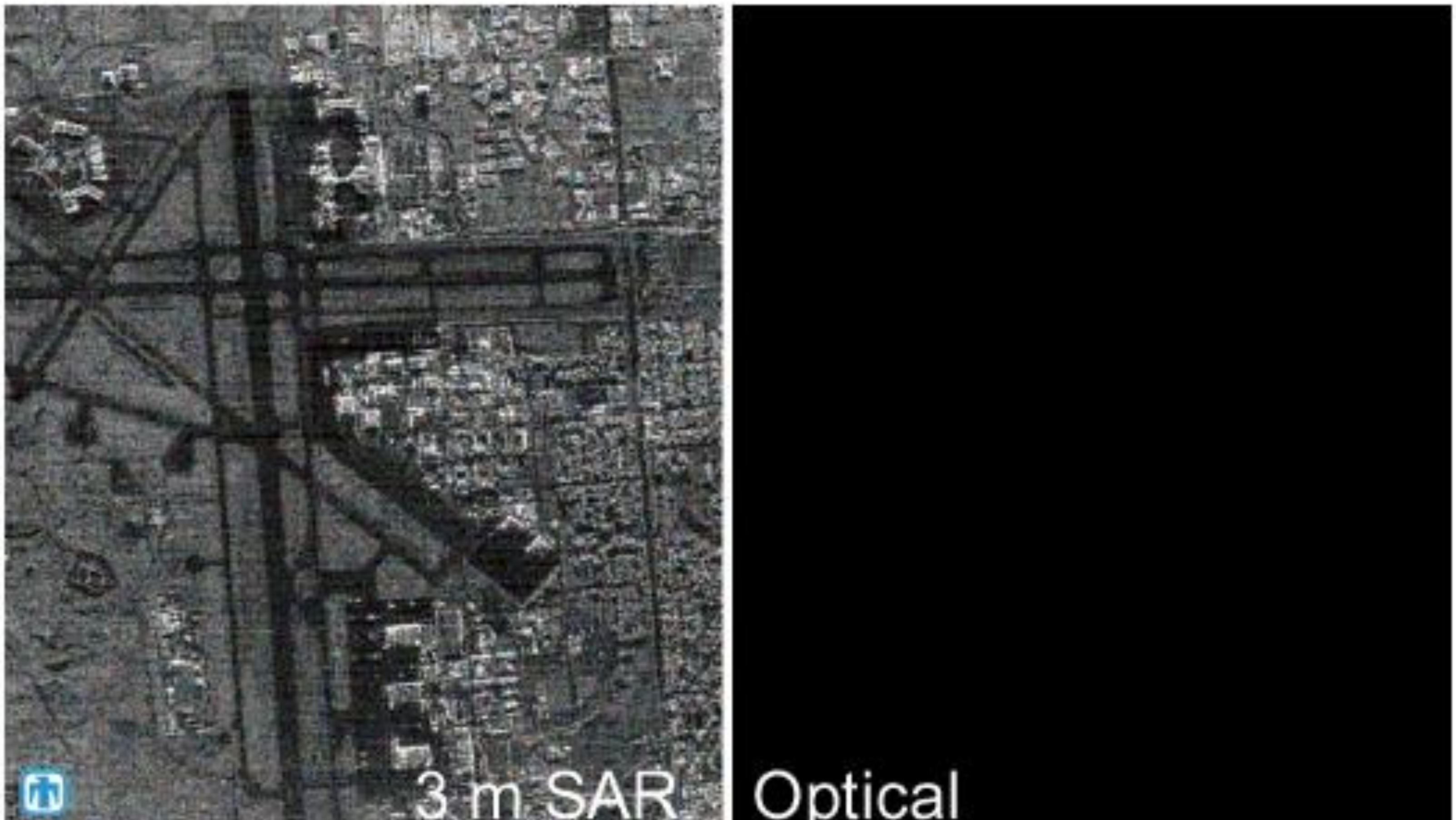
Albuquerque Airport

- 3 meter resolution, K_u band (15 GHz)
- **http://www.sandia.gov/RADAR/sar_sub/images/**



Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

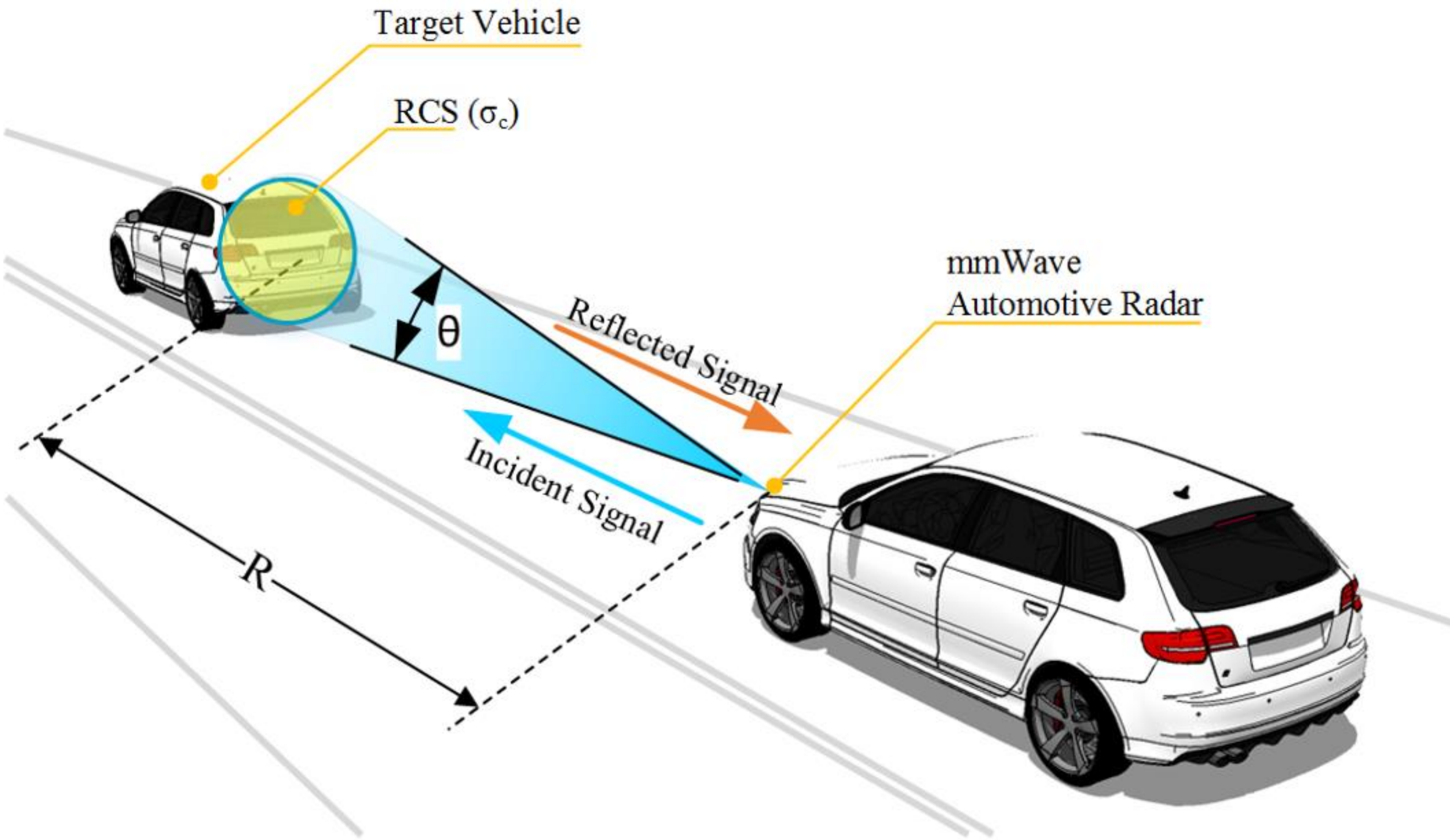
What the Albuquerque Airport Might Look Like on a Cloudy Night

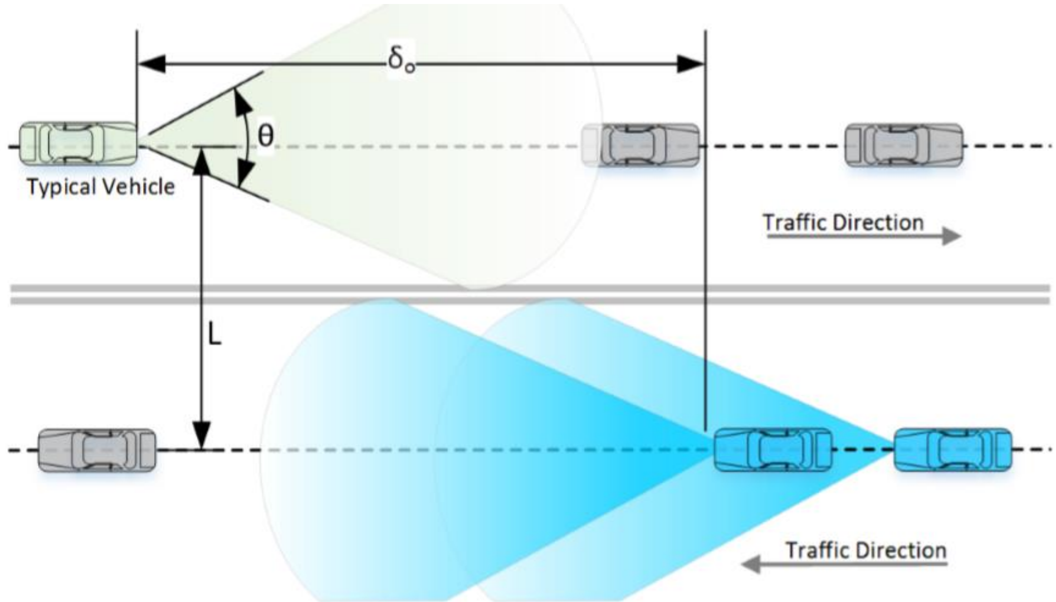


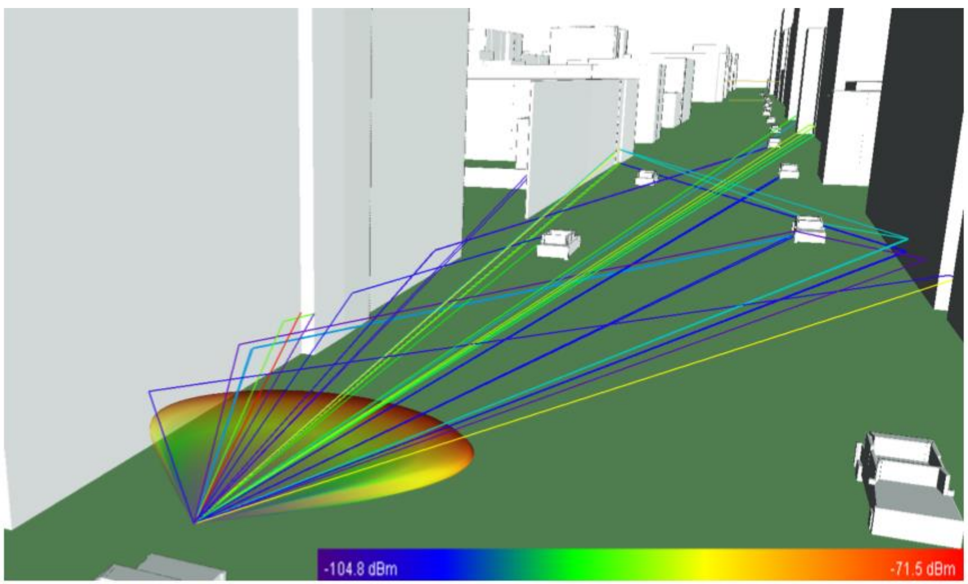
From M.A. Richards, Georgia Tech

IFSAR of University of Michigan Stadium

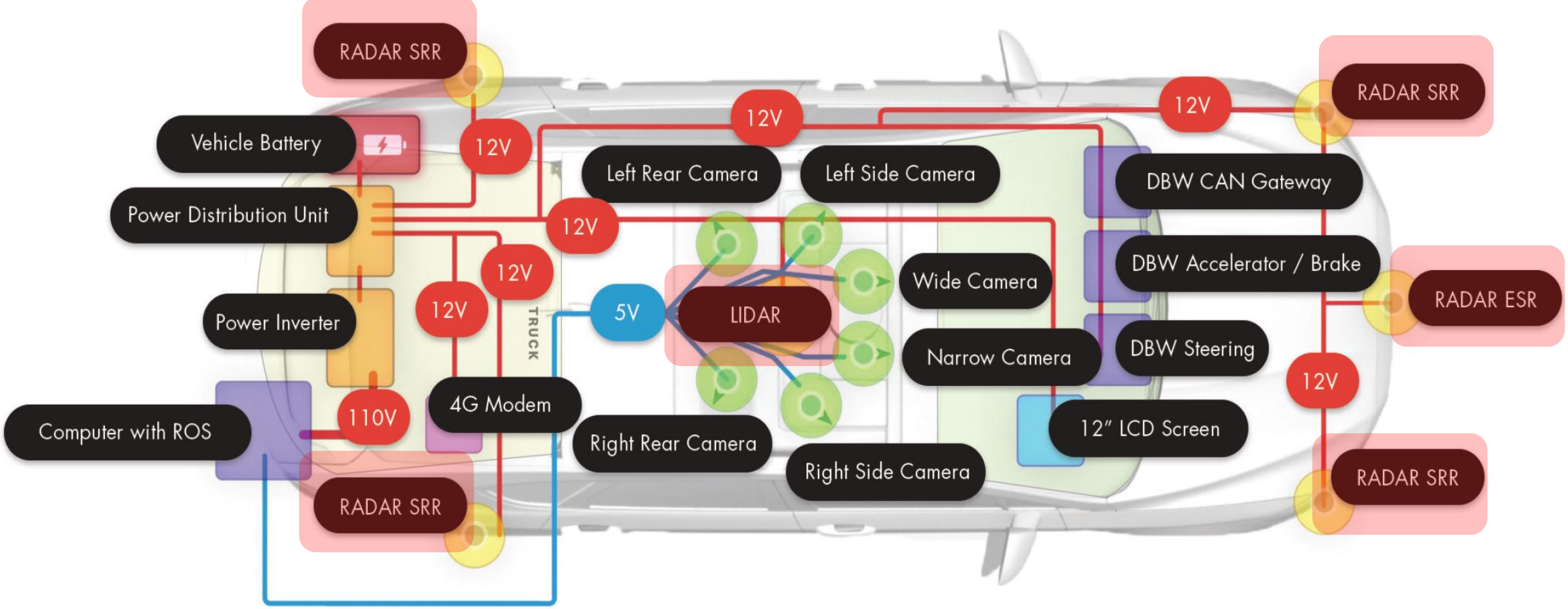
Collected with ERIM/DARPA/TEC IFSARE system


Target Vehicle
RCS (σ_c)
θ
Reflected Signal
Incident Signal
mmWave
Automotive Radar
R





One temporal snapshot of radar ray-tracing, showing the reflected-back rays



A Voyage self-driving taxi on the road at The Villages community in Florida.

IEEE SIGNAL PROCESSING MAGAZINE

September 2019

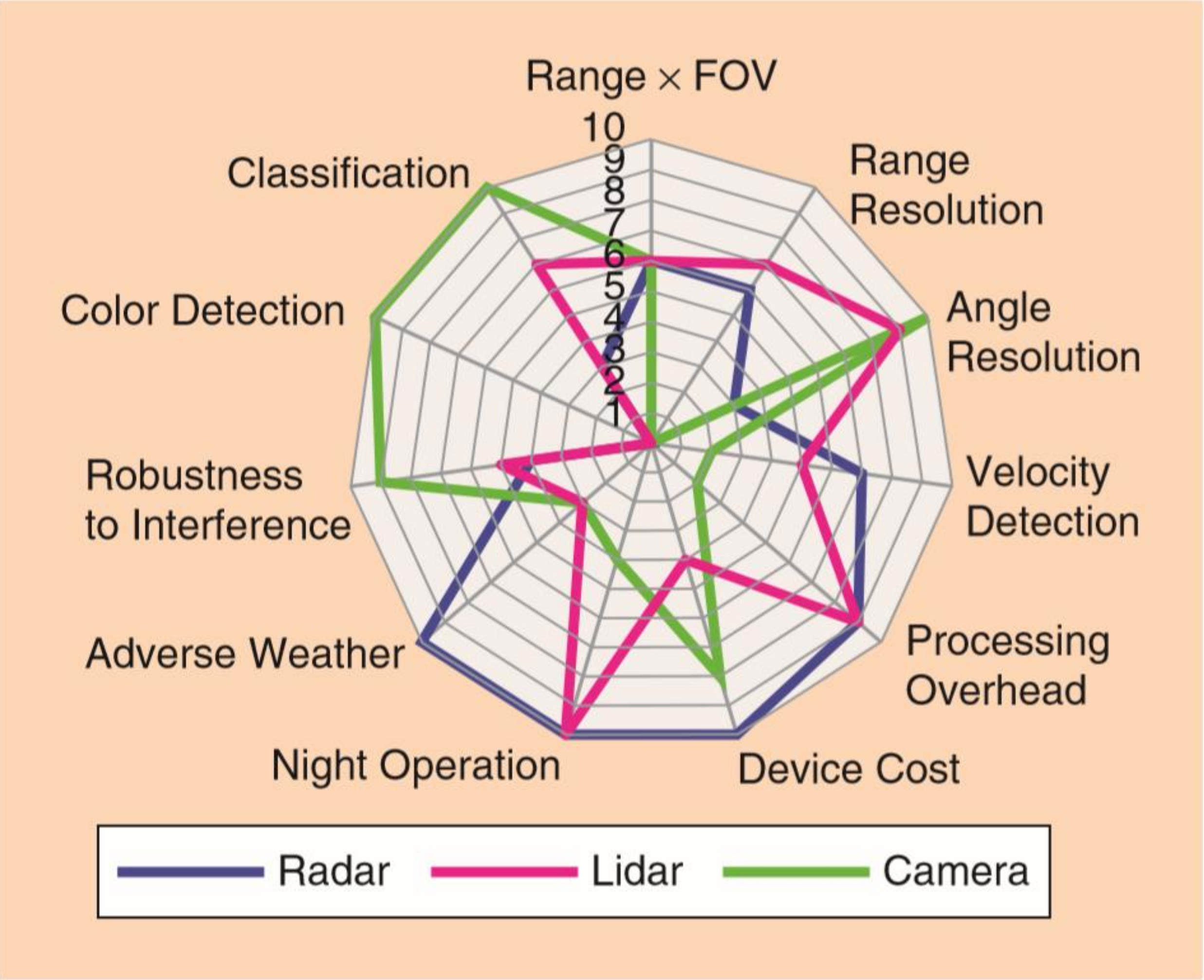


FIGURE 1. A comparison of automotive sensors.

Stephen Alland, Wayne Stark, Murtaza Ali, and Manju Hegde

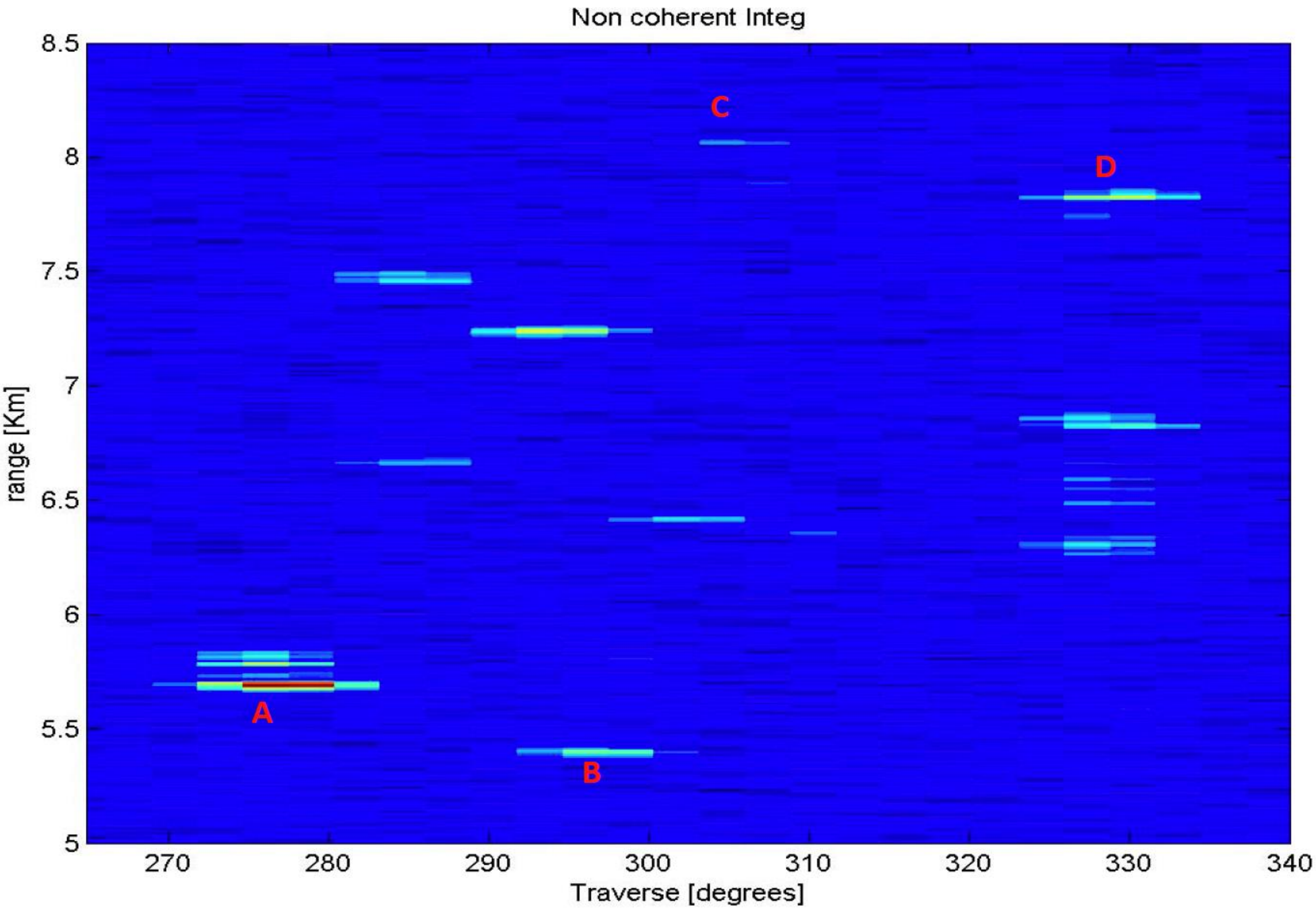
Interference in Automotive Radar Systems

Characteristics, mitigation techniques, and current and future research

Low-cost civil marine radar (~1500 US$)

Intensity on Range/Azimuth Map

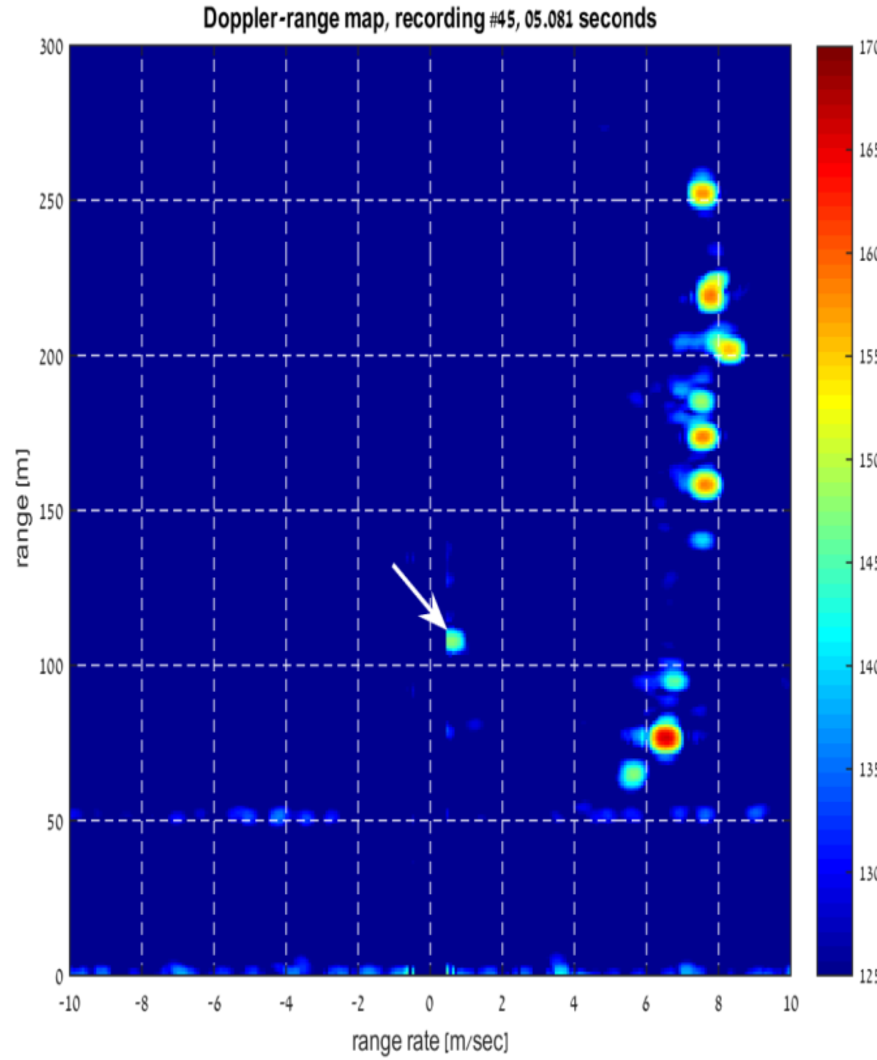
Ships facing the port of Ashdod



Nadav Levanon, Tel-Aviv University

Cars receding in an alley in Bnei-Brak

Intensity on Range/Range-Rate Map



RADAR HISTORY

Germany

Christian Hulsmeyer – 1904 (Radar)
Hans E. Hollmann – 1925 (Magnetron)

England

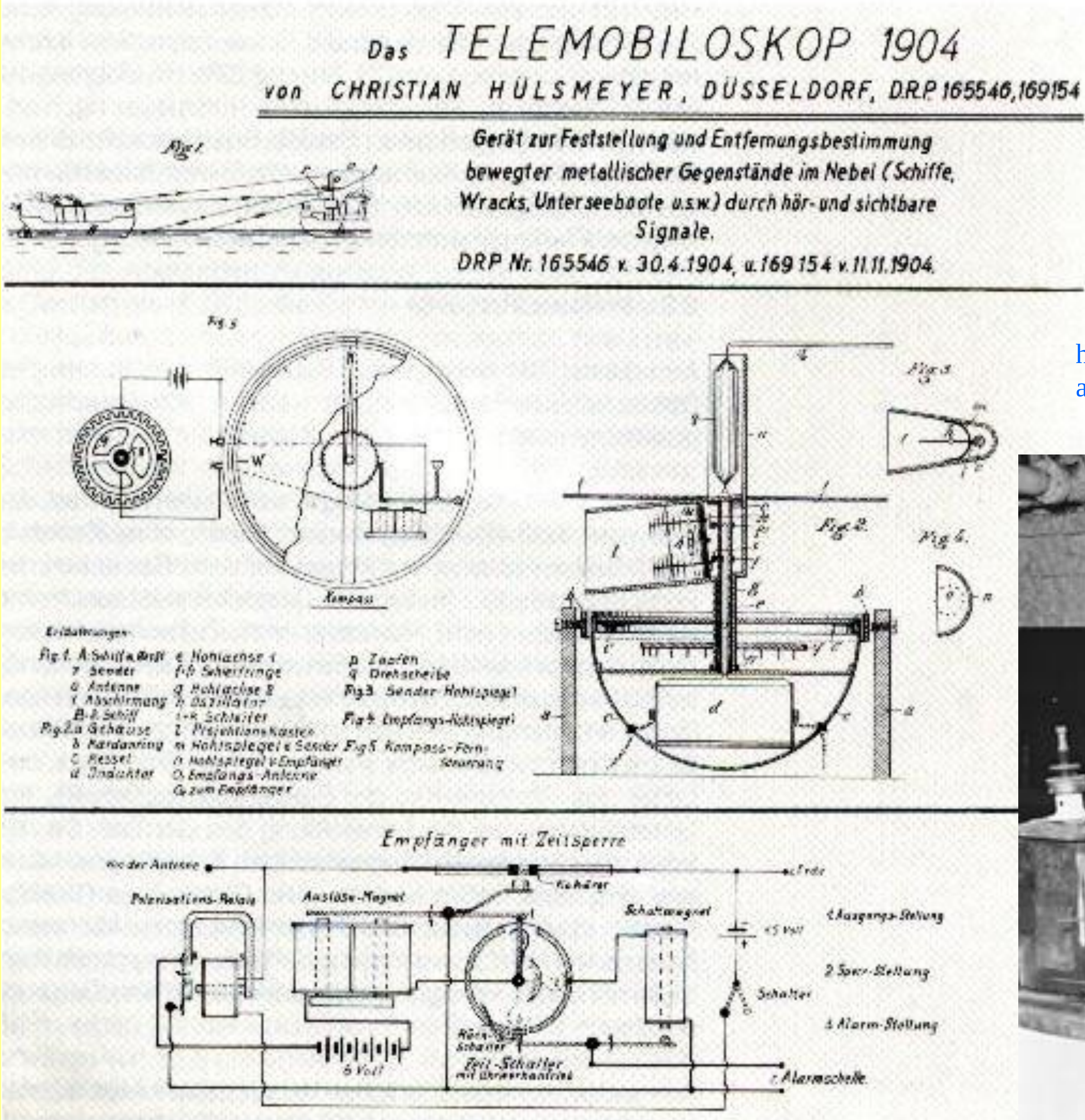
Robert Watson-Watt - 1937 (Chain Home radar)
John Randall & Harry Boot – 1940 (High power magnetron)

USA

Albert Hoyt Taylor - 1922 (NRL, Bi-static radar)
Robert W. Page - 1937 (NRL, Naval radar, CXAM)

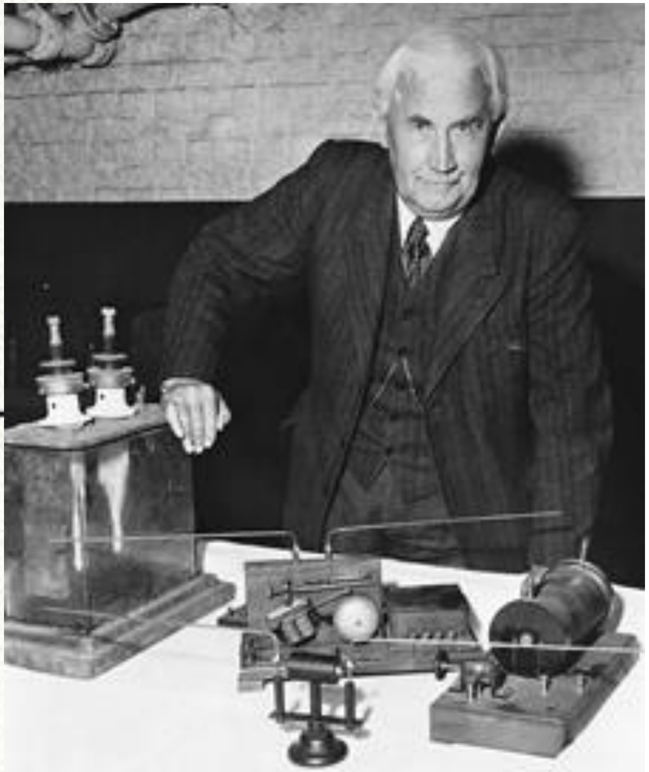
Italy

Guglielmo Marconi - 1934

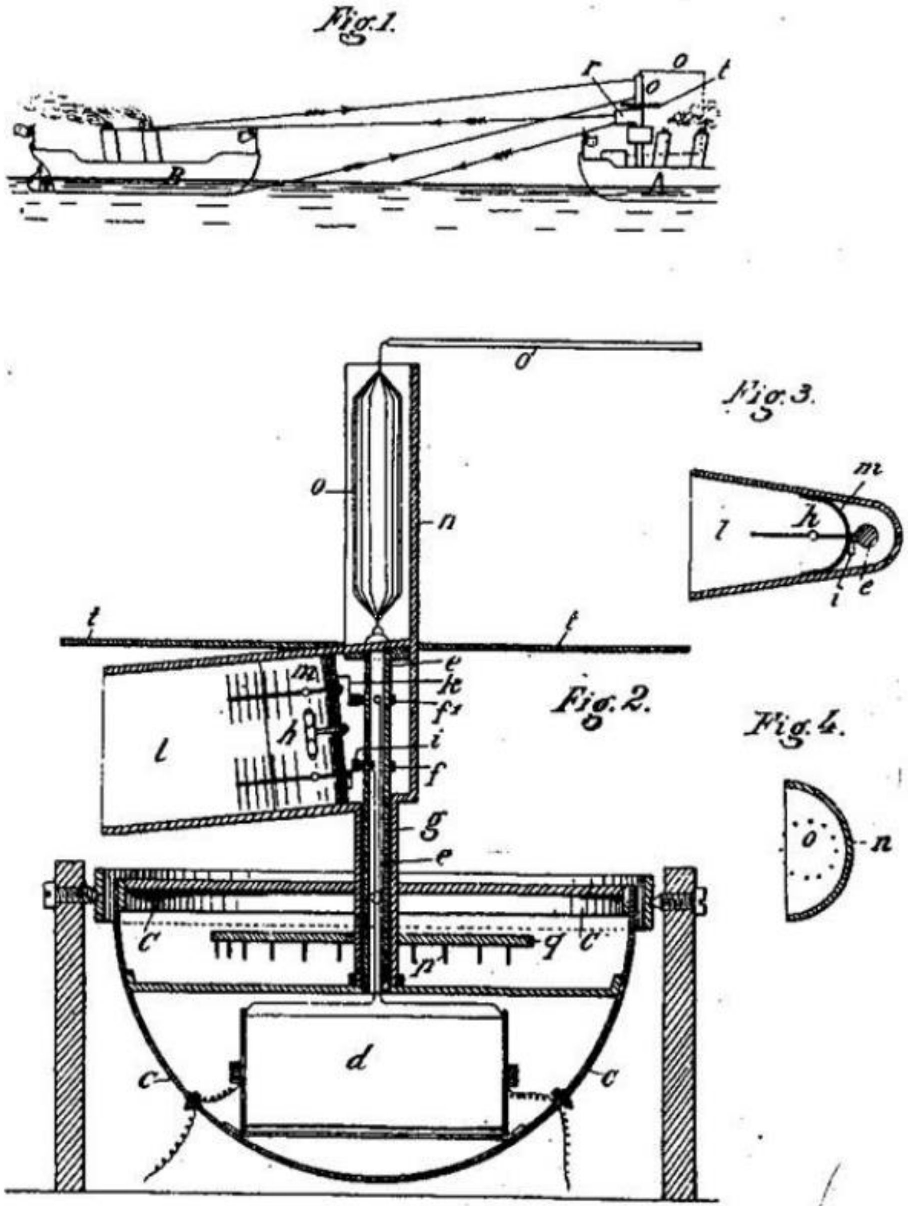


The German radar patent

http://ieee-aess.org/sites/ieee-aess.org/files/documents/paper_v4.pdf



The British patent



N° 13,170 A.D. 1904

Date of Application, 10th June, 1904—Accepted, 22nd Sept., 1904

COMPLETE SPECIFICATION.

"Hertzian-wave Projecting and Receiving Apparatus Adapted to Indicate or Give Warning of the Presence of a Metallic Body, such as a Ship or a Train, in the Line of Projection of such Waves".—

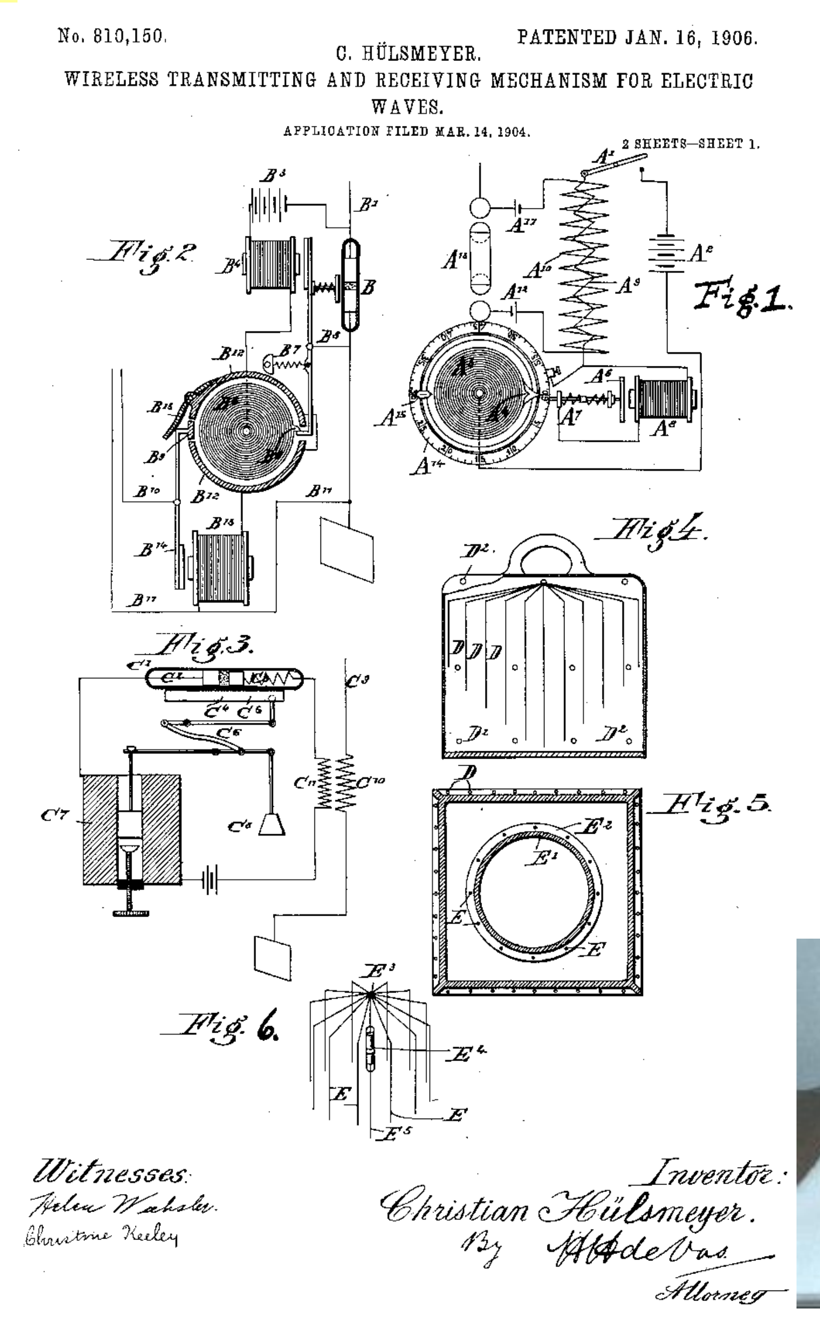
I, CHRISTIAN HÜLSMEYER, of 3 Grabenstrasse, Düsseldorf, Germany, Engineer do hereby declare the nature of this invention and in what manner the same is to be performed to be particularly described and ascertained in and by the following statement:—

This invention consists, broadly, of improved apparatus for projecting electric waves in any desired direction combined with improved apparatus for receiving said waves when reflected back from any metallic body, such as a ship or a train, said receiving apparatus being adapted to put into action an audible or a visible signal and thus give warning of the presence of such metallic body in the line of projection of the waves.

Hülsmeyer and Watson-Watt actually met, at a radar conference in Frankfurt in 1953 (Figure 6). This must have been a remarkable meeting. Watson-Watt is recorded as saying: 'I am the father of radar. You may be its grandfather' – which in hindsight seems a shade arrogant. The expression on Hülsmeyer's face shows a certain confidence in his own claims.

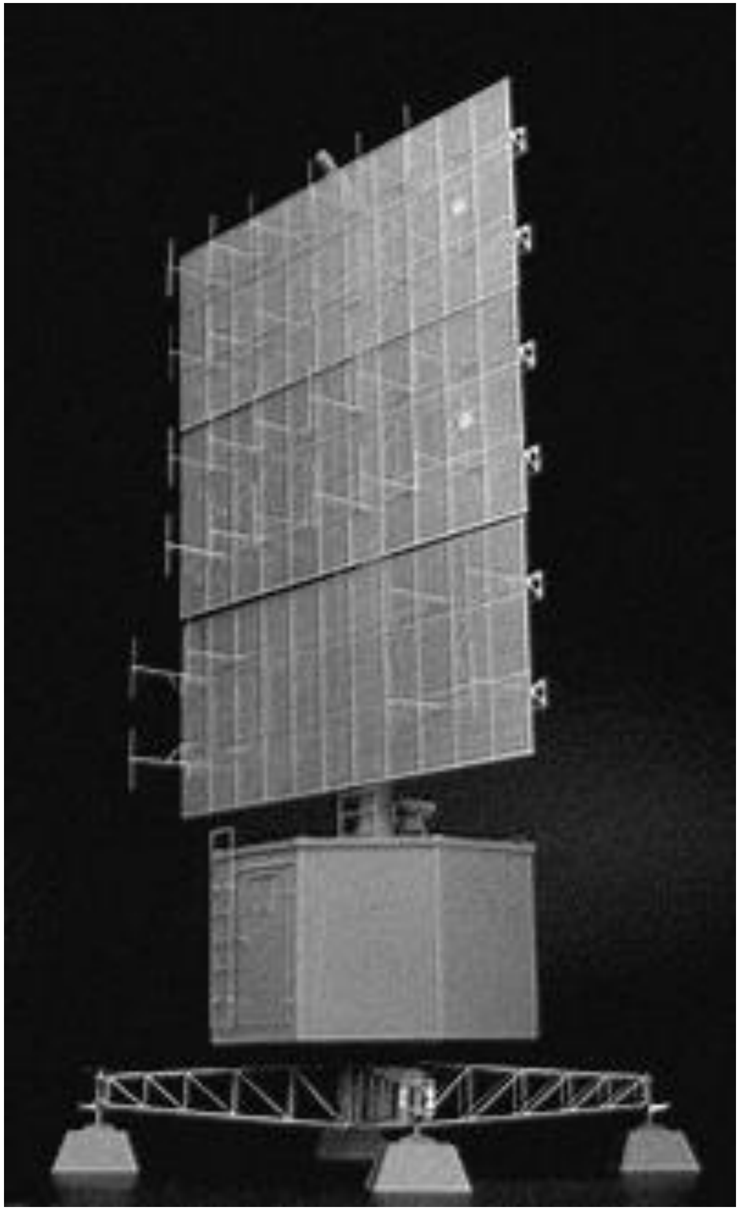
Hülsmeyer died on 31 January 1957, at the age of 75, and was buried in the North Cemetery at Düsseldorf.

Figure 6: Hülsmeyer (centre) and Watson-Watt (left) meeting in 1953.

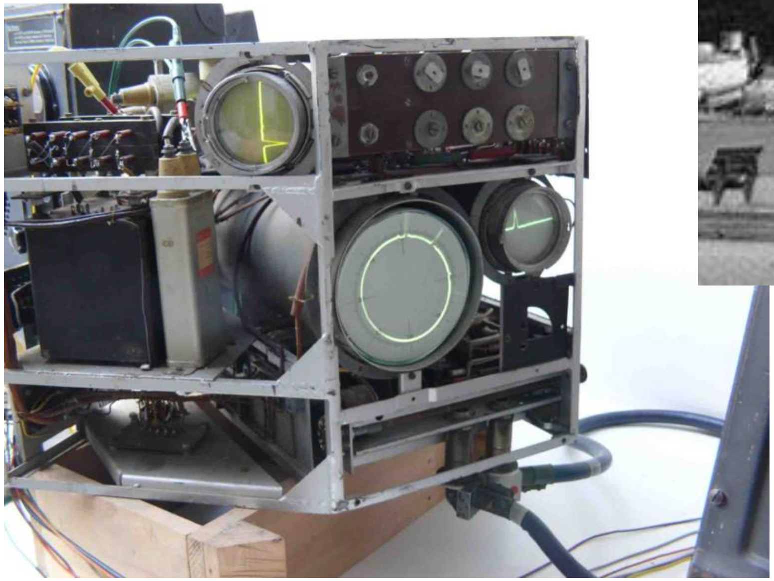

No. 810,150.
PATENTED JAN. 16, 1906.
C. HÜLSMEYER.
WIRELESS TRANSMITTING AND RECEIVING MECHANISM FOR ELECTRIC WAVES.
APPLICATION FILED MAR. 14, 1904.
2 SHEETS—SHEET 1.
Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Witnesses:
Helen Wahsler.
Christine Keeley
Inventor:
Christian Hülsmeyer.
By
Attorney


IN·DIESEM·HAUSE·LEBTE·VON
1930·BIS·ZU·SEINEM·TODE·1957
DER·UR-RADAR·ERFINDER
ING. CHR. HÜLSMEYER
GEB. 25.12.1881·IN·EYDELSTEDT
DAS·HÜLSMEYER-PATENT·WAR
DAS·ALLERERSTE
RADAR-PATENT·DER·WELT
DRP·165546 v. 30.4.1904
DIESE·TAFEL·ENTSTAND·ANLÄSSLICH·DER·VOR
UND·AUF·DEM·INTERN. SCHIFFAHRTS-
KONGRESS·IN·ROTTERDAM·ERFOLGREICH
VORGEFÜHRTEN·PRAKTISCHEN·VERSUCHE

Hans E. Hollmann - In 1928 Hollmann started a company called GEMA. GEMA built the first radar in the autumn of 1934 for naval use. It used a 50cm wave-length and could find ships up to 10 km away. By 1935, they had developed the technology into two applications. For naval use, the "Seetakt" system used a wavelength of 80 cm. A land based version at 120 cm wavelength was also developed as "Freya".



Telefunken set up a radar business in 1933 based on Hollmann's work and developed a much shorter-range gun-laying system called "Würzburg." During World War II, Freya and Würzburg worked in pairs. Freya would spot the incoming aircraft while the Würzburg calculated the distance and height.



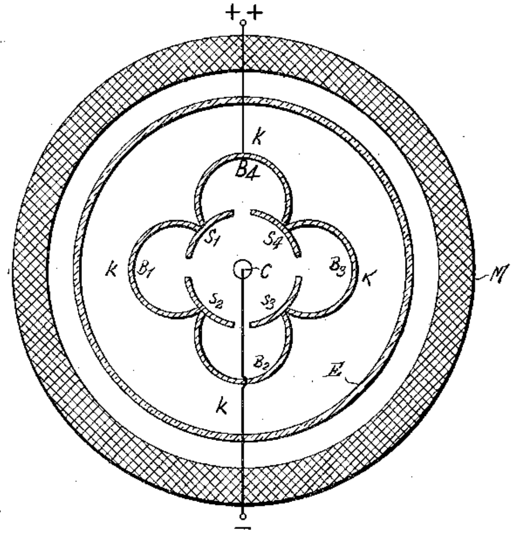
The German military considered the Magnetron's frequency drift to be undesirable and based their radar systems on the klystron instead. It was primarily for this reason that German night fighter radars were not a match for their British counterparts.

July 12, 1938. H. E. HOLLMANN 2,123,728

MAGNETRON

Filed Nov. 27, 1936

Fig. 1



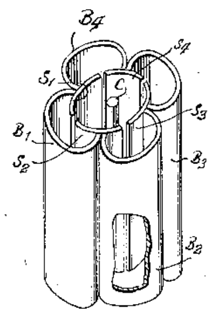
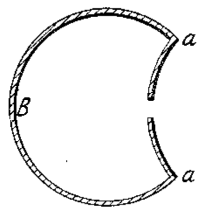


Fig. 3

Fig. 2



INVENTOR
HANS ERICH HOLLMANN
BY Charles McClair
ATTORNEY

March 28, 1939. H. E. HOLLMANN 2,151,766

MAGNETRON

Filed Sept. 8, 1938

Fig. 1

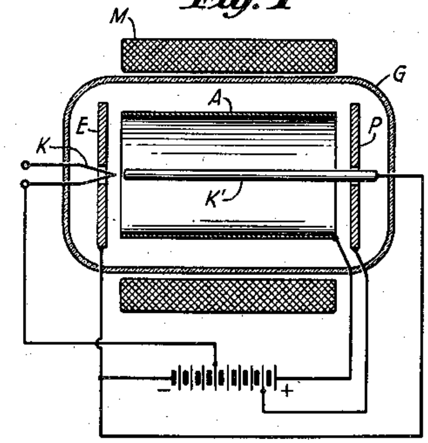
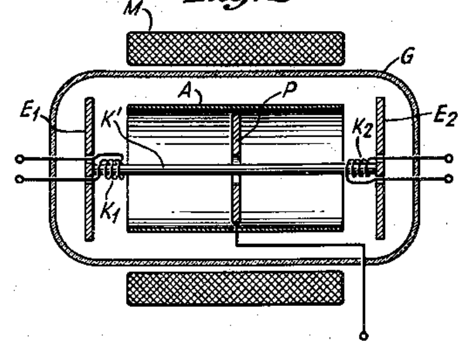
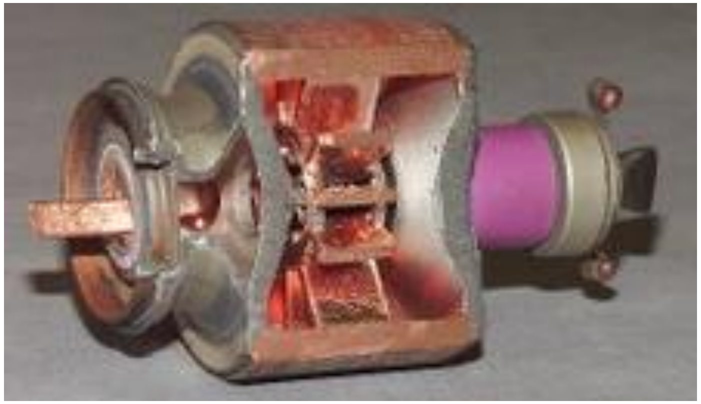


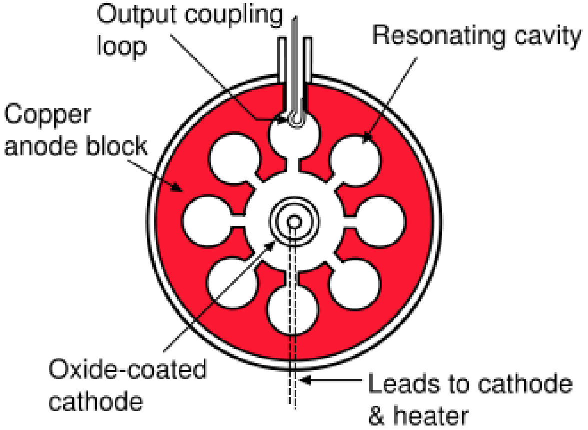
Fig. 2



INVENTOR.
HANS ERICH HOLLMANN
BY Charles McClair
ATTORNEY.

In 1940, at the University of Birmingham in the UK, John Randall and Dr. Harry Boot produced a working prototype similar to Hollman's cavity magnetron, but added liquid cooling and a stronger cavity. Randall and Boot soon managed to increase its power output 100 fold. Instead of giving up on the magnetron due to its frequency inaccuracy, they sampled the output signal and synced their receiver to whatever frequency was actually being generated.





Resonant cavity magnetron high-power high-frequency oscillator

Sir Harry Boot
1917- 1983

Sir John Randall
1905-1984

OBITUARY

DR HARRY BOOT

Invention of 10 centimetre radar

Dr Harry Boot, who died on February 8 was a physicist who made an outstanding contribution to the successful application of British science during the Second World War. With J. T. (later Sir John) Randall he produced the first 10 centimetre radar through the cavity magnetron, a discovery which had a profound impact on the waging of the war in several important spheres, the most vital of which was perhaps, the Battle of the Atlantic.

Here, centimetric radar was able to provide the Allies with a means of locating with accuracy, surfaced U-Boats in any weather, night or day, an advance which led to the defeat of Admiral Doenitz's wolf pack system of deploying his U-boats. This was to turn the tide in the Atlantic with surprising speed. Other major areas in which centimetric radar was decisive was the defeat of German night bombers in the "Little Blitz" of 1943-44, and in the improvement in the accuracy of the Allies' own night bombing.

The discovery that the magnetron could be a source of short wave radiation of great power came after no lengthy research. Boot and Randall were among others in Professor M. L. E. Oliphant's laboratory at Birmingham University where the Problems of producing shortwave radar were being researched. Here the klystron, a recent American invention, was under consideration as a producer of centrimetric radar pulses and Randall and Boot were given the unglamorous task of making Barkhausen-Kurz tubes as possible receivers.

They were not successful in this task and, in the corner of an elementary teaching laboratory they were left much to their own devices. They therefore turned their own attention to the problem of generating centimetric radar waves. They eventually dismissed the klystron as a viable source of such radiation and in their consideration of other sources of power, thought of the more favourable geometry of the magnetron.

In February 1940 after three months work a "sealing wax and string" model of their own cavity magnetron was switched on and after several false starts they were able to measure the wavelength of the pulses they were transmitting. These were found to be 9.8 centimetres, a figure of an order which was to usher in a wholly new dimension in the usefulness of radar.

The Battle of Britain had to be fought with large, crude, and moreover vulnerable radar stations in chains, which through their numbers and position could enable approaching enemy air forces to be detected and roughly located. But to do this the sky had, in crude terms, to be virtually 'flooded' with radar transmissions. Centimetric radar brought in the era of the precise radar beam, and the small, lightweight radar transmitter.

First used in the aircraft of Bomber Command on their night raids it soon became the property of anti-aircraft units, convoy escorts and night fighters, making a decisive difference in the quality of radar detection available to the Allies, from the middle of the war on.

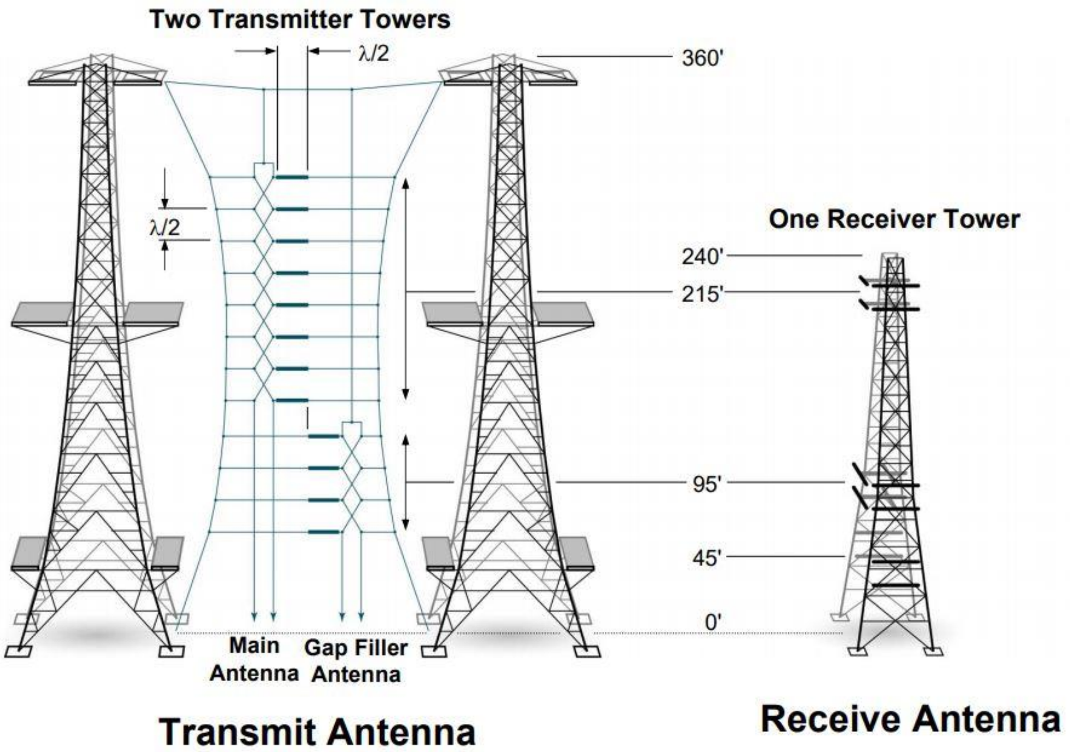
Boot and Randall were intially rewarded with a prize of £50 from the Royal Society of Arts for "improving the safety of life at sea". Coming on the same day as the dropping of the atomic bomb on Hiroshima this modest announcement made little impression. It was only after the war that they applied to the Royal Commission for Awards for Inventions, and, with James Sayers, who did much of the work on improving their original design, received a reward more commensurate with their decisive effort – £36,000.

Typical Chain Home Radar Site

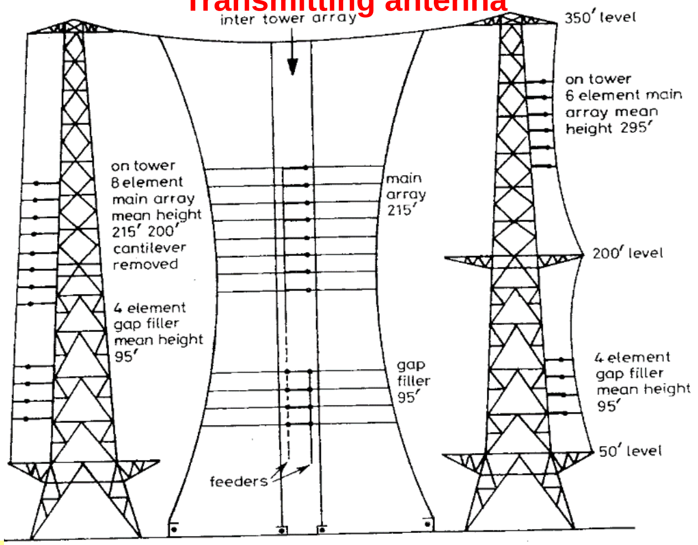
Radar Parameters

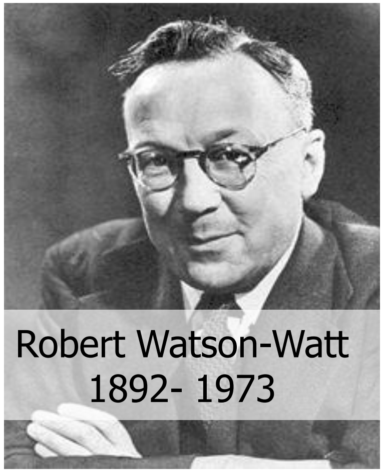
- **Frequency**
  - **20-30 MHz**
- **Wavelength**
  - **10-15 m**
- **Antenna**
  - **Dipole Array on Transmit**
  - **Crossed Dipoles on Receive**
- **Azimuth Beamwidth**
  - **About 100°**
- **Peak Power**
  - **350 kW**
- **Detection Range**
  - **~160 nmi on German Bomber**

The British Chain Home radar. The transmit antennas are suspended between the towers on the right; the receive antennas are on the four wooden towers on the left, with each tower initially operating on a separate frequency (MIT radar course).

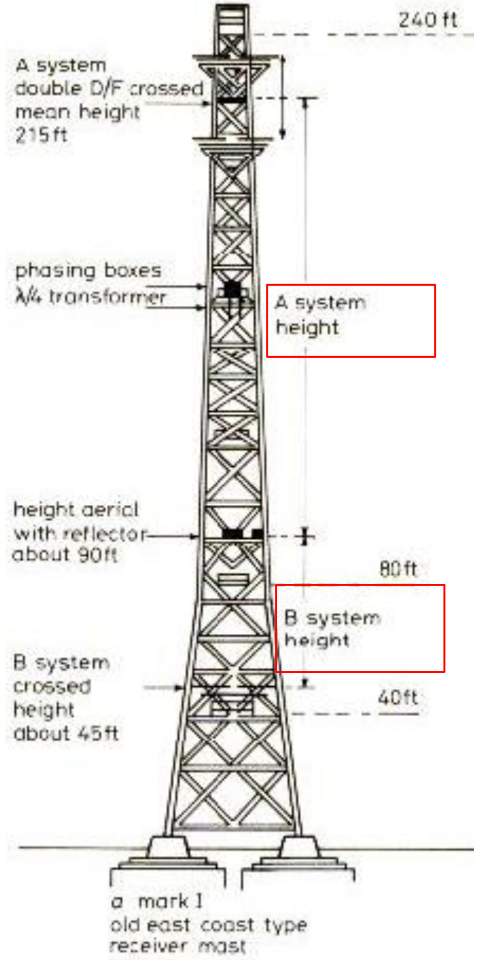


Transmitting antenna



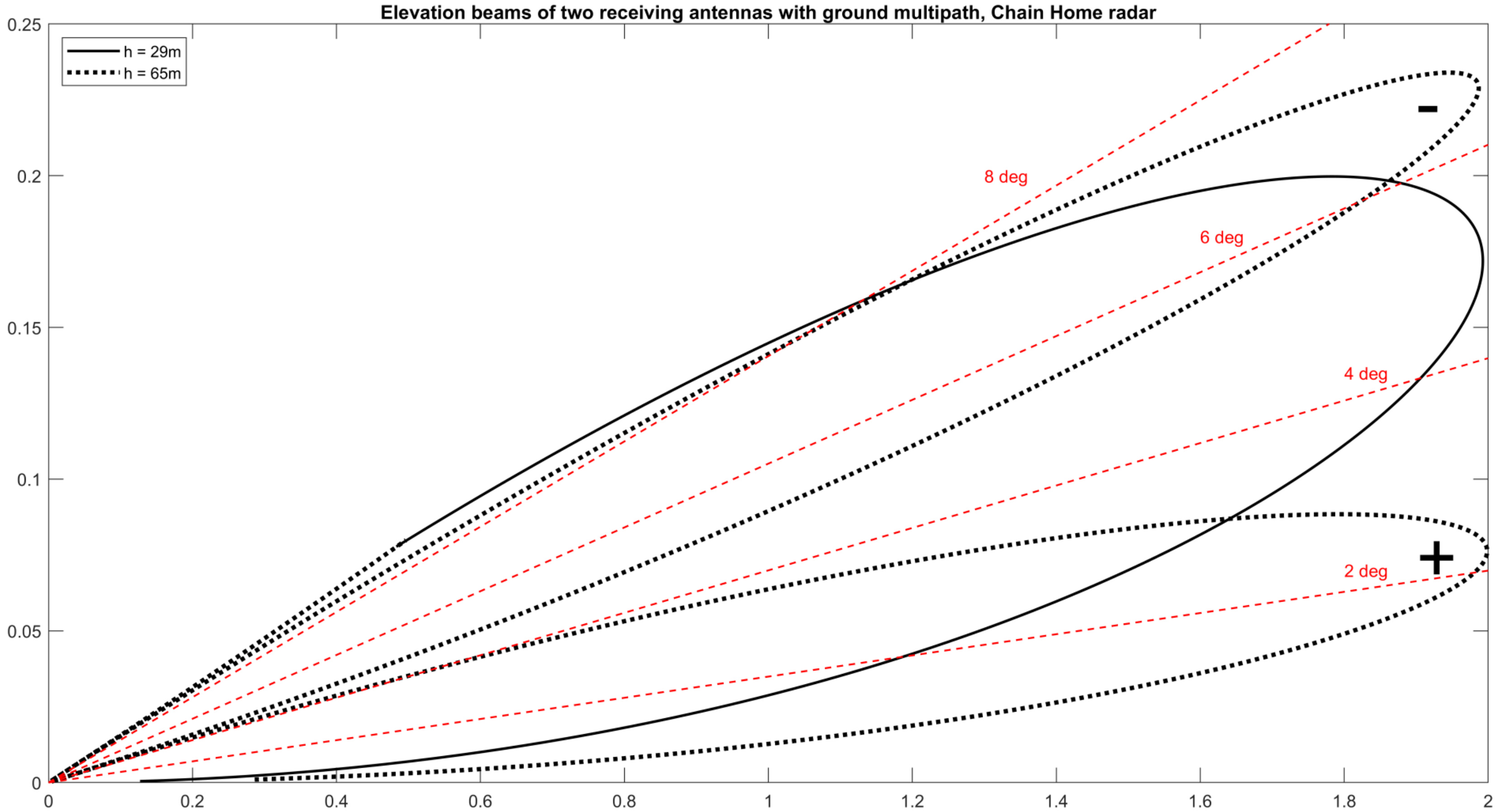

Robert Watson-Watt
1892- 1973

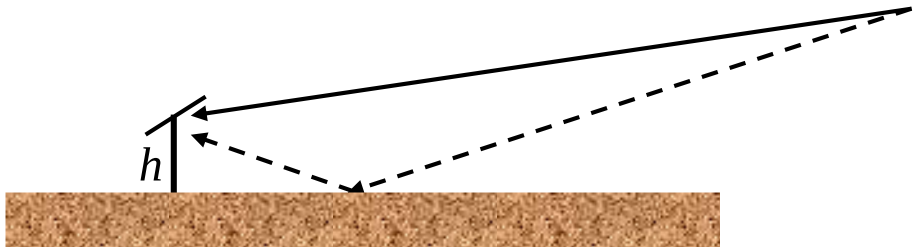
Receiving antenna



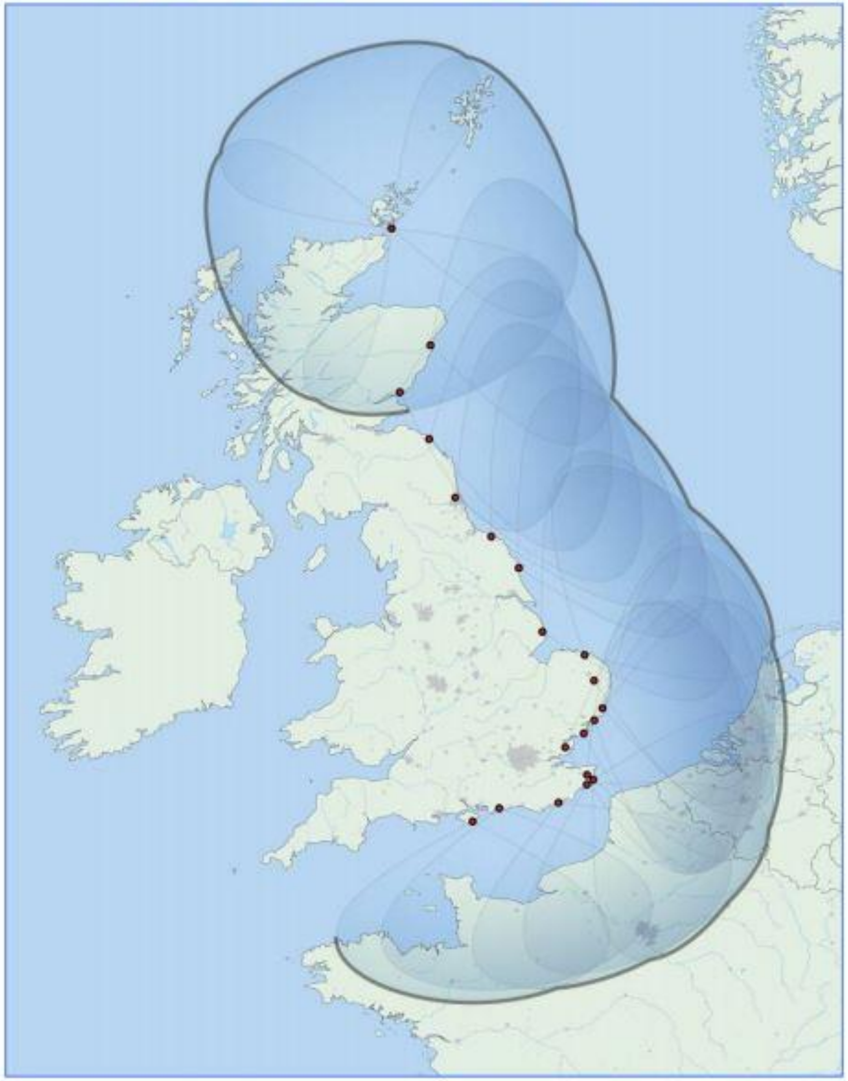
Nadav Levanon, Tel-Aviv University

Obtaining good elevation resolution on-receive using multipath from two different antenna heights

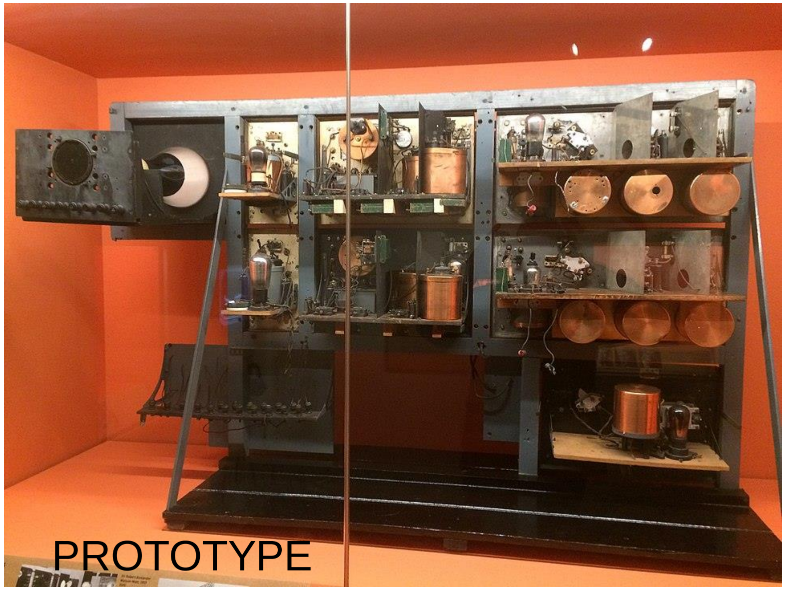




Chain Home Radar Coverage circa 1940 (21 Early Warning Radar Sites)



CHAIN HOME RADAR



Klein Heidelberg—The First Modern Bistatic Radar System

HUGH GRIFFITHS, Fellow, IEEE
University College London

NICHOLAS WILLIS
Consultant

We present a description and analysis of the German WW2 bistatic radar system Klein Heidelberg (KH). A brief account is given of the nature of the electronic war between the Allied bombers and the German air defense system, to show the context in which the KH system evolved. This is followed by a description of the development of KH, a technical description, and an assessment of its performance. Next, a discussion of its operational significance, of what happened after WW2, and finally some conclusions and some lessons learned that may be relevant to the development of present-day bistatic radar systems. In particular, we show that its performance was impressive, yielding detection ranges of Allied bombers in excess of 300 km, but that it became operational too late in WW2 to significantly improve German air defense operations.

Manuscript received March 28, 2010; revised July 22, 2010; released for publication August 16, 2010.

IEEE Log No. T-AES/46/4/938744.

Refereeing of this contribution was handled by P. Willett.

Authors' addresses: H. Griffiths, Dept. of Electronic and Electrical Engineering, University College London, Gower Street, London, WC1E 6BT, UK, E-mail: (h.griffiths@ee.ucl.ac.uk); N. Willis, 7009 Valley Greens Circle, Carmel, CA 93923.

0018-9251/10/$26.00 © 2010 IEEE

I. INTRODUCTION

The term bistatic refers to a radar in which the transmitter and receiver are in separate locations (Fig. 1). In practice this means that they are separated by a considerable distance, usually understood to be of the order of the target range, so as to distinguish it from smaller separations designed only for receiver isolation from the transmit signal, and this gives bistatic radars some different and distinct properties compared with conventional monostatic radars [1, 2]. Bistatic radar is presently a subject of significant interest and research in many countries worldwide, which is reflected in the large volume of publications in journals and at conferences.

The purpose of this paper is to present and analyse information on a German WW2 bistatic radar system called Klein Heidelberg[1] (hereafter denoted KH) which was used to enhance German air defenses by exploiting transmissions from the British Chain Home (CH) radar transmitter, as shown in Fig. 1. Whilst some scattered information about this system has been known for many years, recently-discovered material has greatly increased our knowledge about KH and its performance, and the relevance to present-day systems means that this discussion may be timely.

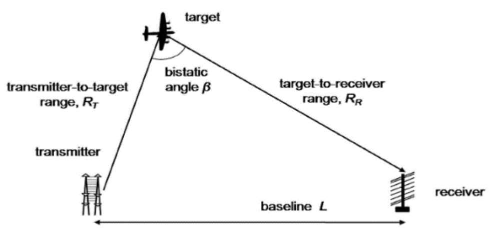
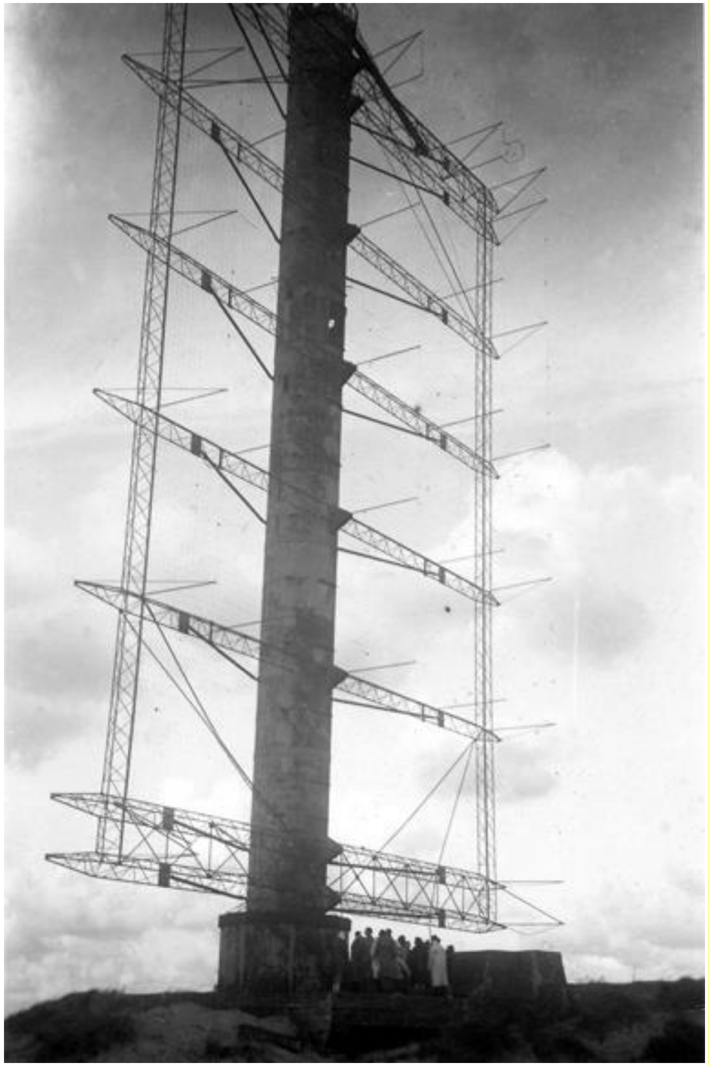
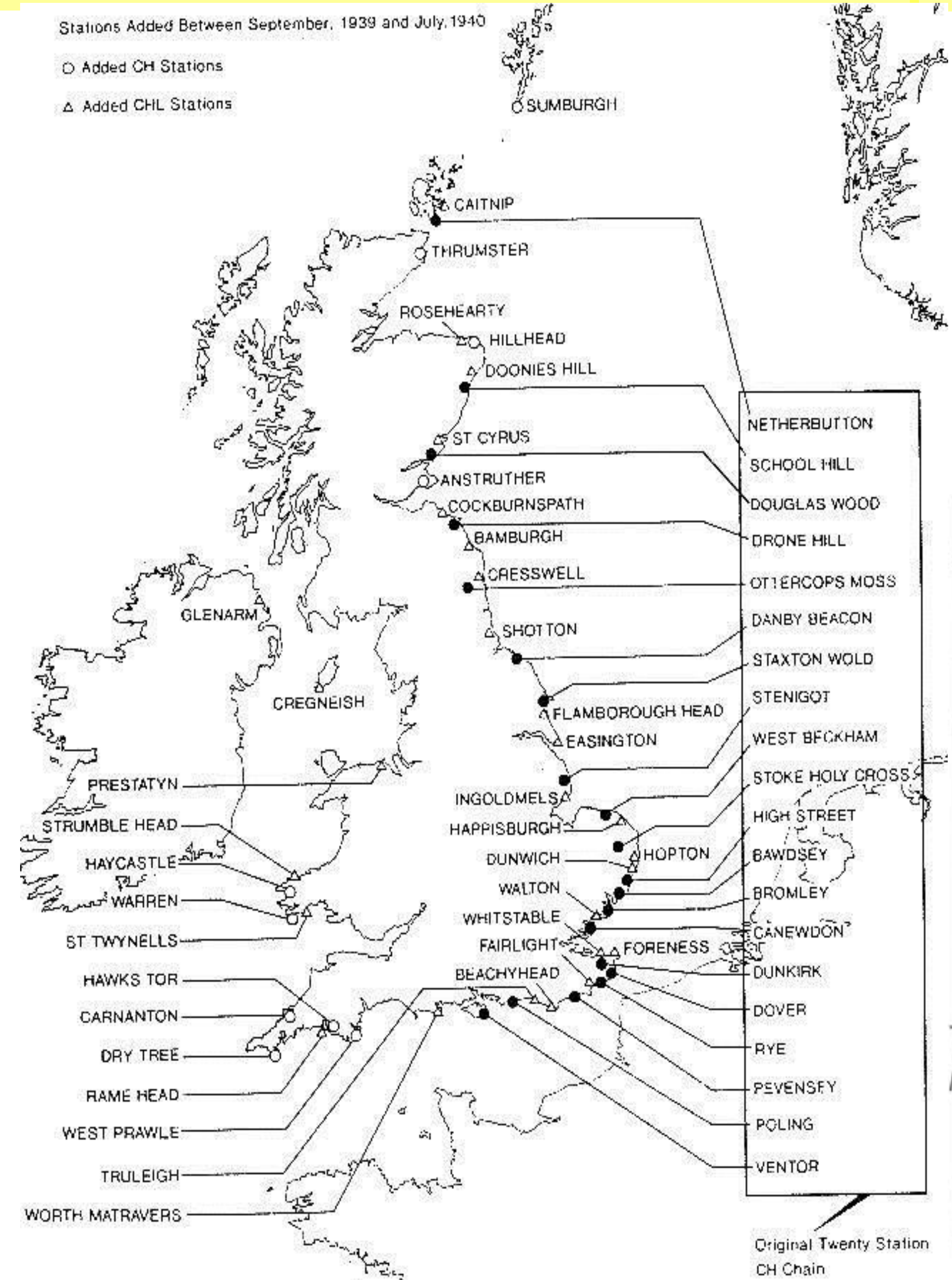


Fig. 1. Bistatic radar. Many of the properties are a function of the bistatic triangle formed by the transmitter, target and receiver.

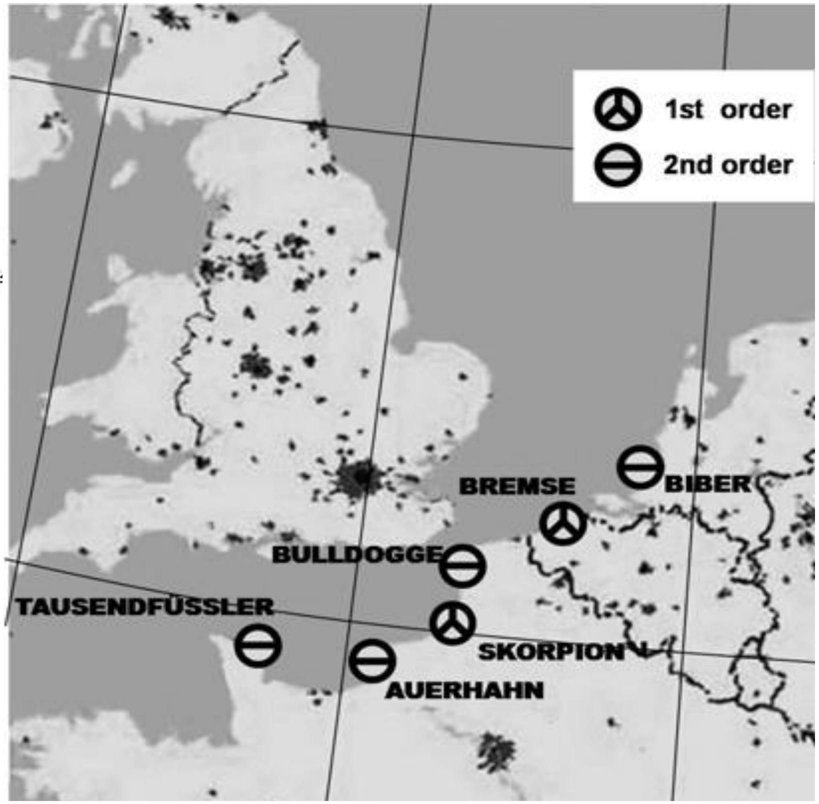
The paper begins with a review of the properties of bistatic radar, to explain the present interest and to set the context for what follows. Next we give a short summary of German WW2 air defense radar and electronic warfare. This is followed by an updated account of the development of KH, a technical description, and an assessment of its performance. We then discuss its operational significance during WW2, what happened after WW2, and finally we present some conclusions and lessons learned, particularly with respect to present-day bistatic radar systems.

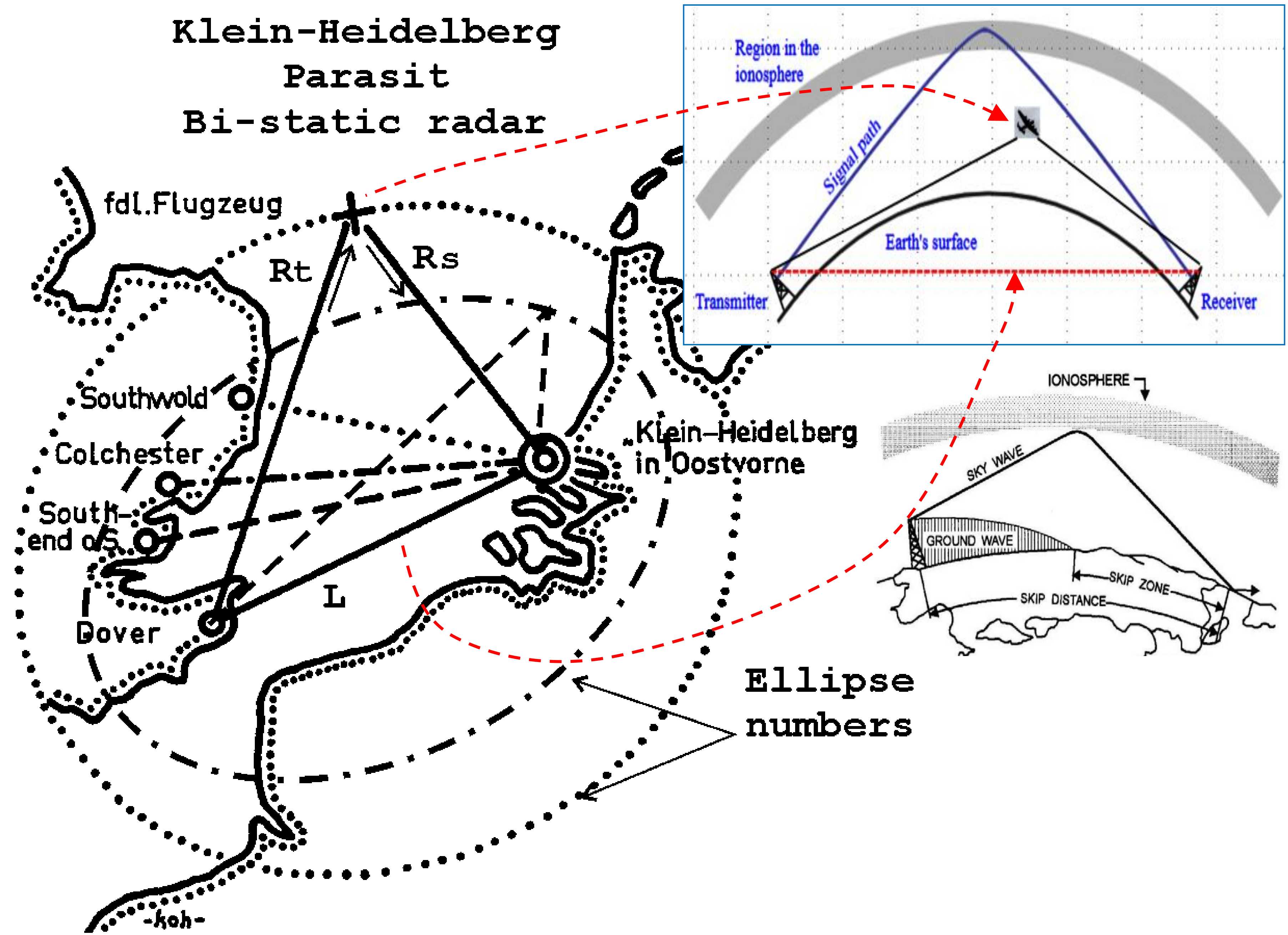
[1]The name comes from Heidelberg Versuche, which was the codename for the German research program in HF radar.

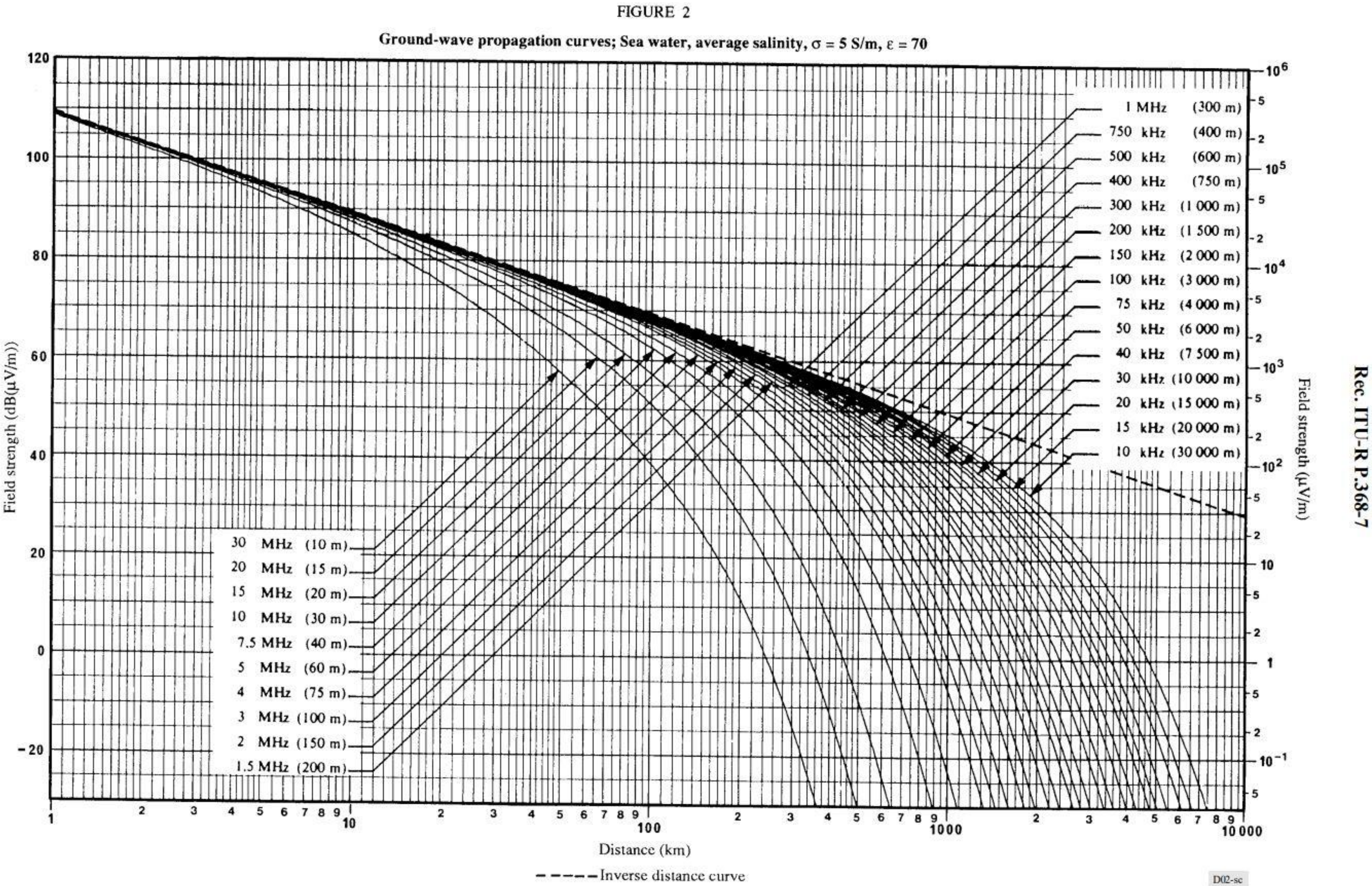




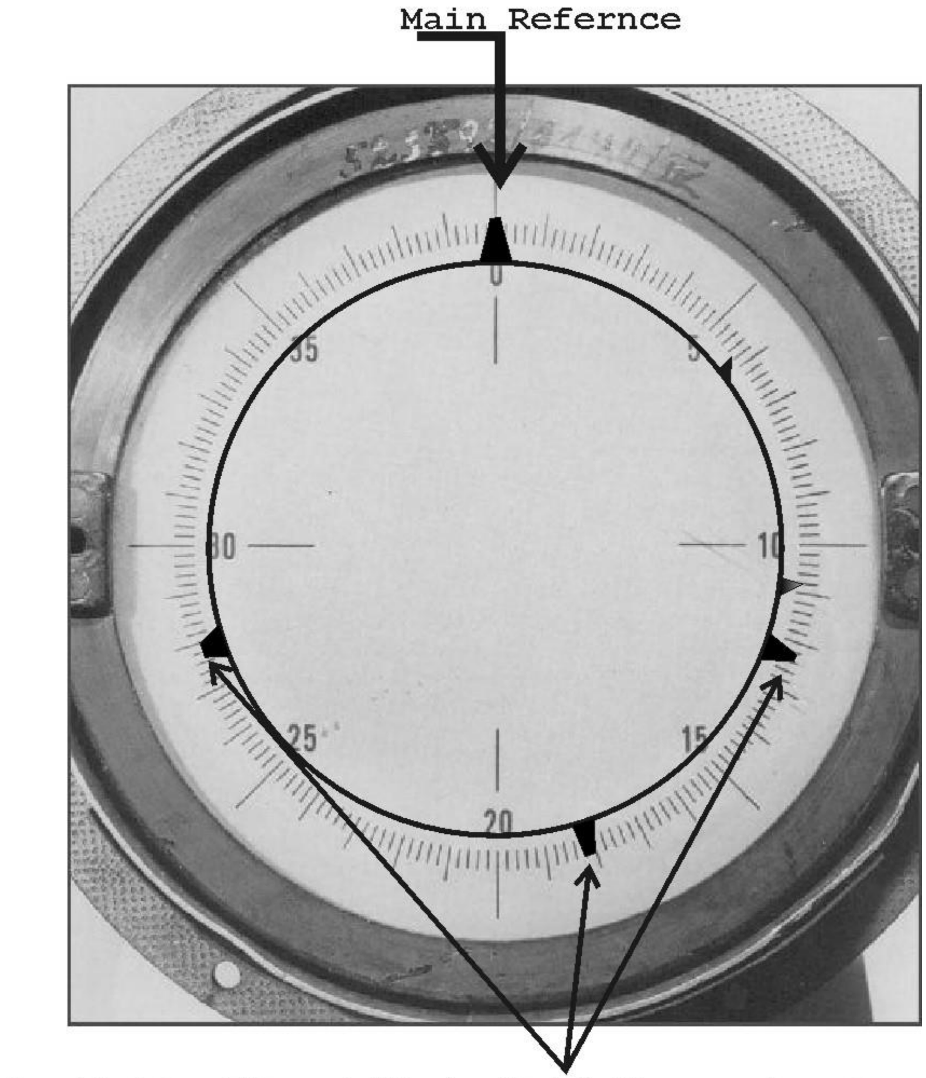
Klein Heidelberg (passive bi-static German radar)

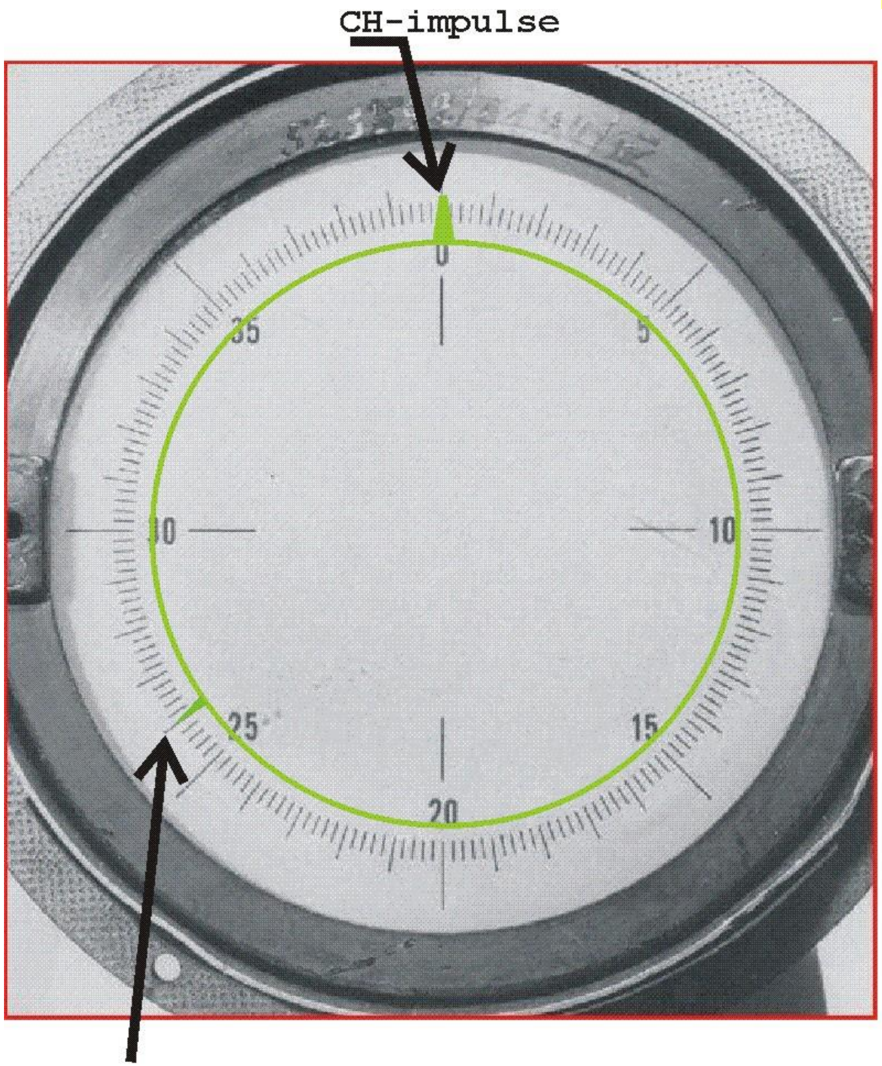



Klein-Heidelberg
Parasit
Bi-static radar
fdl.Flugzeug
Rt
Rs
Southwold
Colchester
South-end o/S.
Dover
L
Klein-Heidelberg in Oostvorne
Ellipse numbers
-koh-
Region in the ionosphere
Signal path
Earth's surface
Transmitter
Receiver
IONOSPHERE
SKY WAVE
GROUND WAVE
SKIP ZONE
SKIP DISTANCE



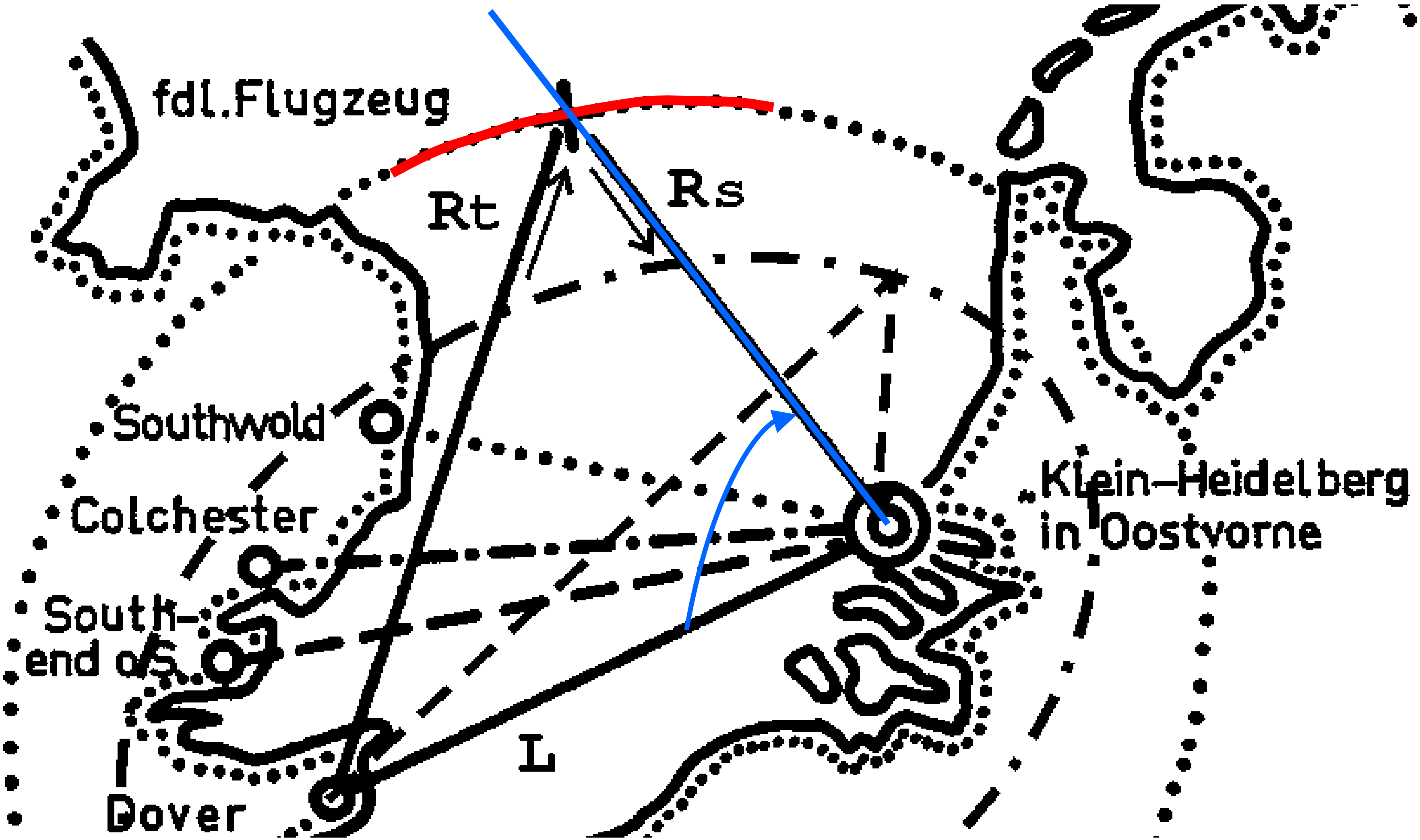
Ground wave propagation over sea water (ITU report)

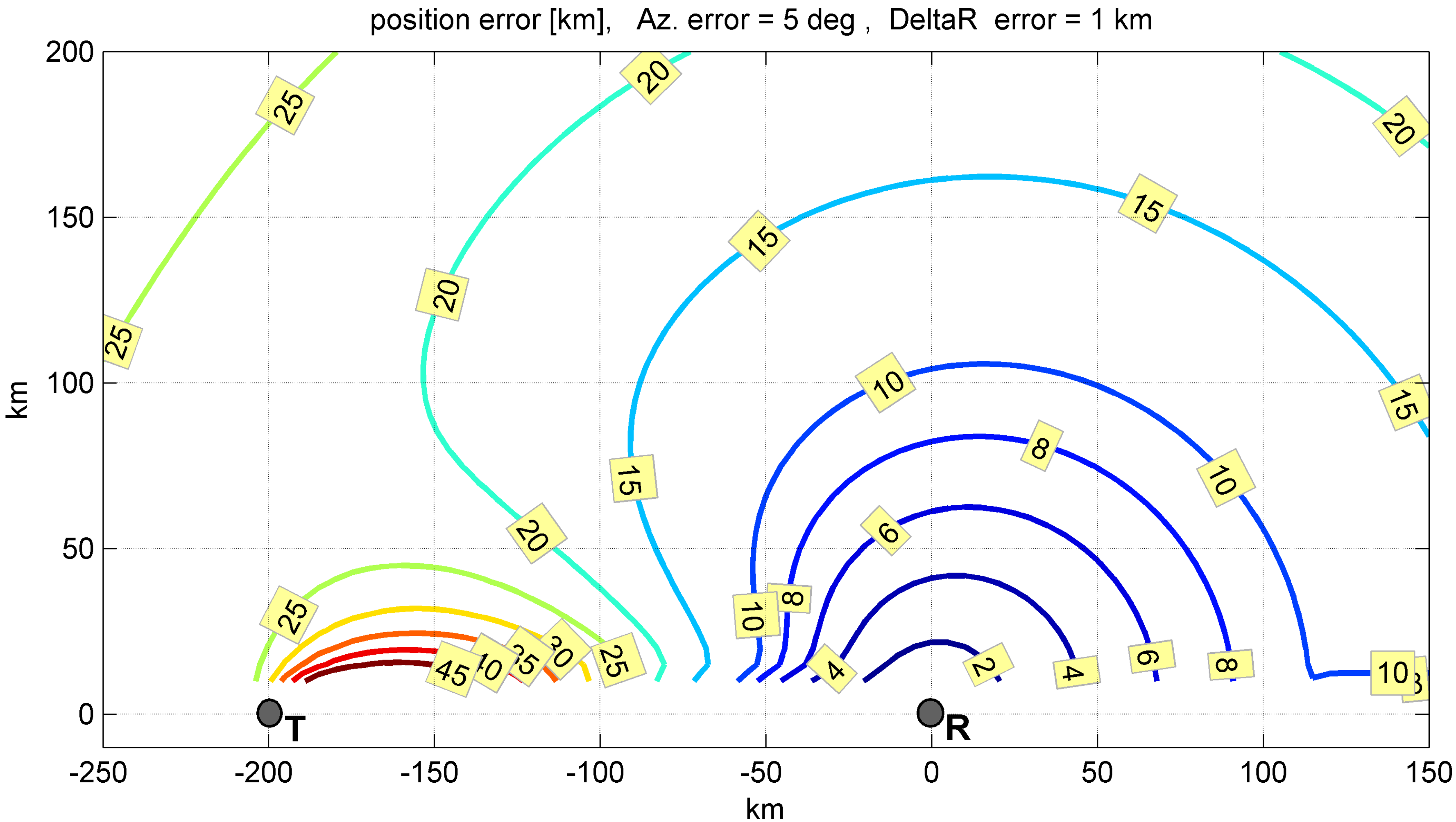


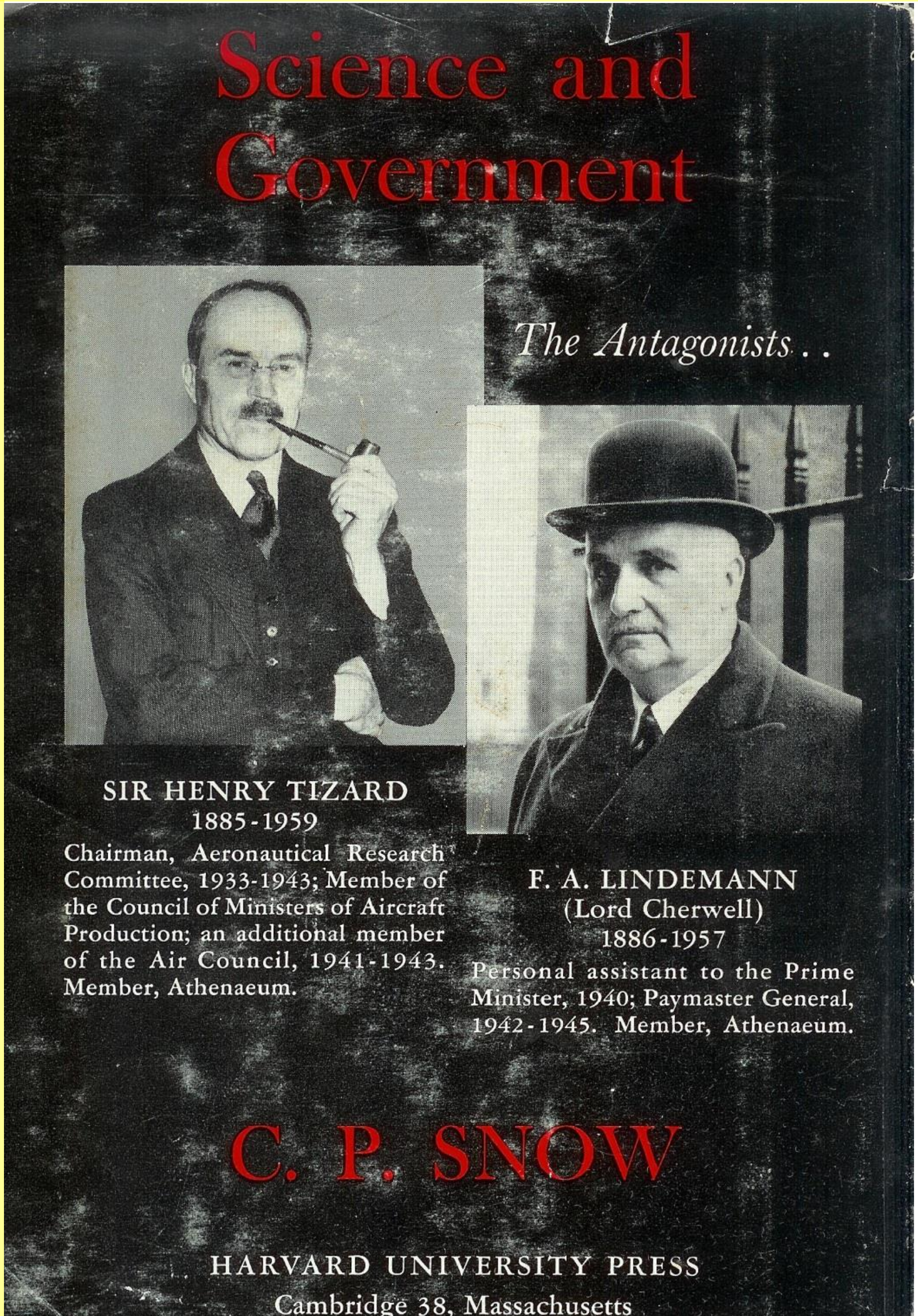


Available CH and Klein-Heidelberg signals at a certain frequency. Some pulses might show fluctuations due to ionospheric propagation and/or shifting signal phase

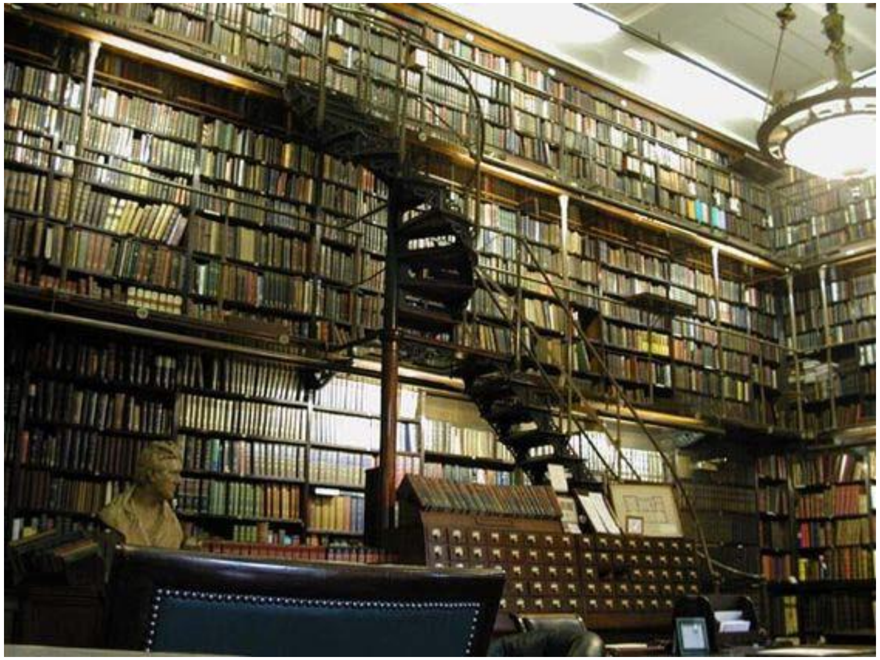
Target ellipse number 26, delay-time Rt + Rs - L. With the bearing vector the target is confined. Its trace was painted only for 2 ms out of 40 ms (25 Hz prf of CH). Trace blanking 38 ms


fdl.Flugzeug
Rt
Rs
Southwold
Colchester
South-
end of S.
Dover
L
Klein-Heidelberg
in Oostvorne

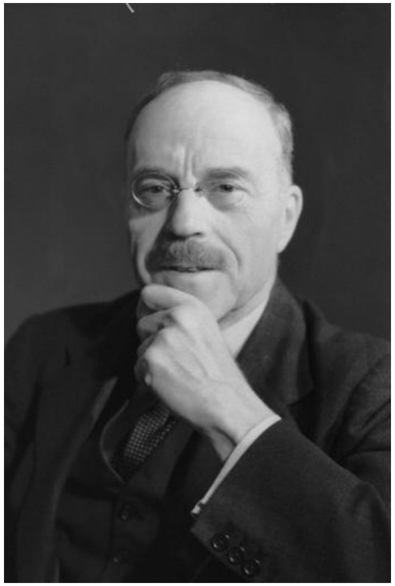

position error [km], Az. error = 5 deg , DeltaR error = 1 km
200
150
100
50
0
km
-250
-200
-150
-100
-50
0
50
100
150
km
T
R
25
20
15
10
8
6
4
2
45



The Athenaeum



Henry Tizard headed the Committee for the Scientific Study of Air Defence, 1935-1940



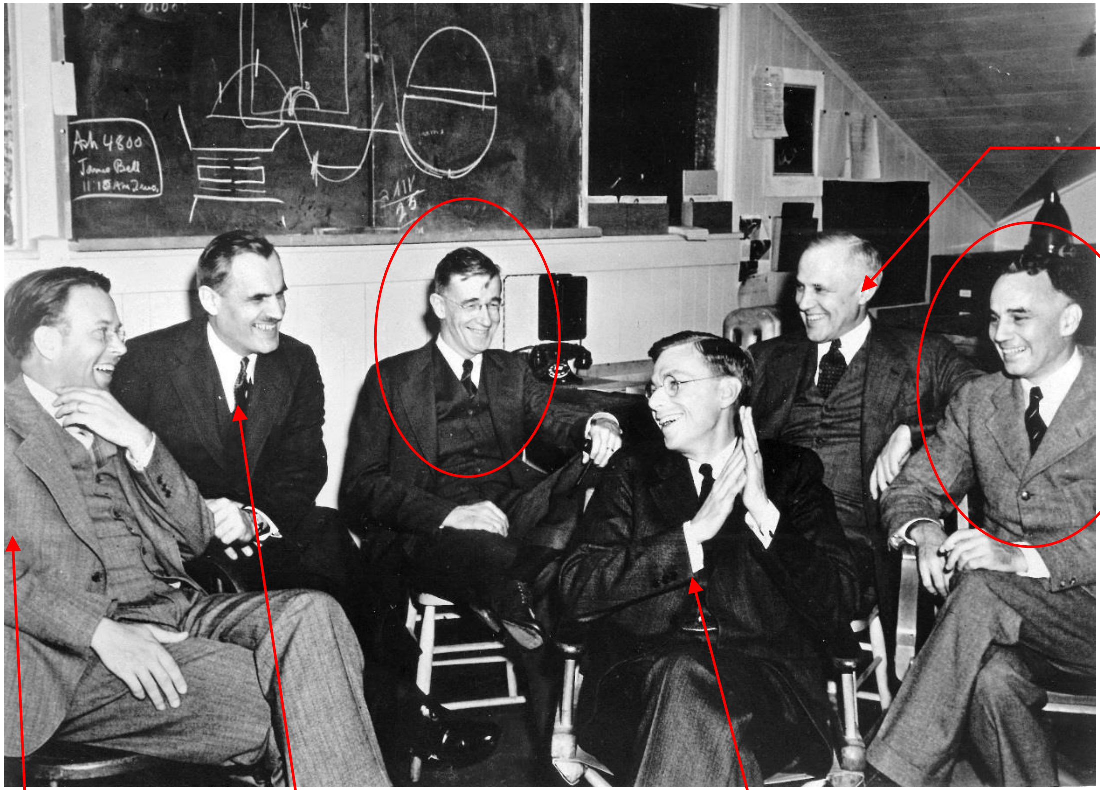
Sir Henry Tizard

The Tizard mission to the US and Canada
(September 1940)
Sir Henry Tizard (Mission Leader)
Brigadier F.C. Wallace (Army)
Captain H.W. Faulkner (Navy)
Group Captain F.L. Pearce (RAF)
Professor John Cockcroft (Army Research)
Dr. E.G. Bowen (Radar)
A.E. Woodward Nutt (Secretary)

"Tizard carried a black leather suitcase which contained nearly all the important new English war devices – and, of a different order of importance from the rest, the new cavity magnetron - 'The most valuable cargo ever brought to our shores' ".

"Tizard mission, on which John Cockcroft was his second in command, was one of the successes of both their lives. American scientists, both at the time and since, have spoken with extreme generosity, of the effect that visit made. It is true that, mainly because the English had been forced to think in order to survive at all, in most military scientific fields they were ahead. This was preeminently true of radar. Although English, American, and German scientists had all begun developing radar at about the same time, by 1940 the English had carried it further"

C.P. Snow "Science and Government", Harvard University Press, 1961



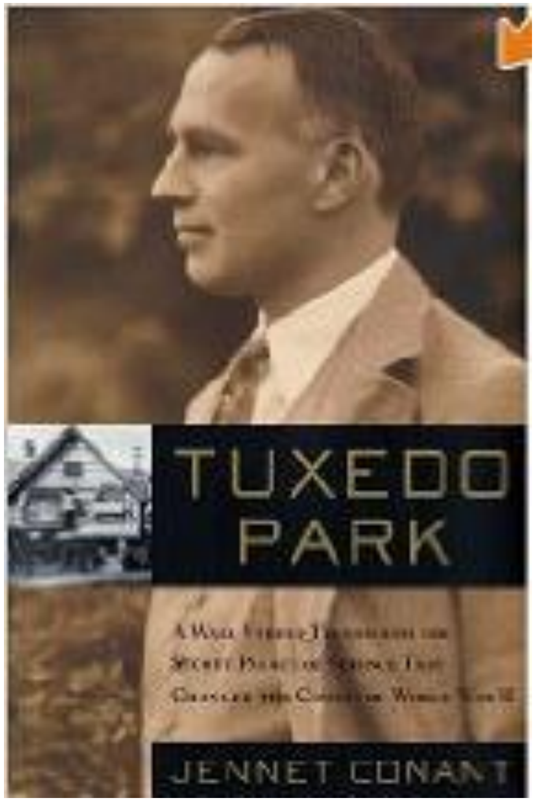
MIT president

L-R: Ernest O. Lawrence, Arthur H. Compton, **Vannevar Bush**, James B. Conant, Karl T. Compton, **Alfred L. Loomis**.

1927 Nobel, Physics

Harvard president

1936 Nobel, Physics



Alfred Lee Loomis
1887-1975

An American attorney, investment banker and patron of scientific research.
Played a key roll in establishing the Radiation Lab at MIT.

Vannevar Bush, MIT EE Prof., Head - Nat'l Res. Defense Committee. His students: Claude Shannon, Fredric Terman

In October 18, 1940 the Radiation Laboratory at the Massachusetts Institute of Technology with Lee A. DuBridge as its technical director was set up with the prime purpose of developing radar for the war effort. It had three primary goals:

1. Develop a 10 cm Aircraft Interception (AI) radar.
2. Develop a gun laying radar.
3. Aircraft navigation. In March 1941, a 10 cm radar was tested on a B-18 bomber.

In its 6 years of existence, over 2.1 billion dollars were spent on the development of radar. This was about as much as was spent on the development of the atom bomb

Lee DuBridge, director of MIT Radiation Lab (left) and **John Trump**, in Paris during WW II.

In 1943 the Rad Lab established a British Branch (BBRL), which created a direct channel between the lab and the European Theater. Trump became the head of BBRL in February 1944. By November he had begun shuttling back and forth between Great Britain and forward military positions on the continent, serving a dual role with BBRL and with Bowles's contingent working for the US Army Air Forces in Europe.

In the autumn of 1922, Albert H. Taylor and Leo C. Young of the U.S. Naval Research Laboratory (NRL) were conducting communication experiments when they noticed that a wooden ship in the Potomac River was interfering with their signals; in effect, they had demonstrated the first continuous wave (CW) interference radar with separated transmitting and receiving antennas. In June, 1930, Lawrence A. Hyland of the NRL in the U.S. detected an airplane with this type of radar operating on 33 MHz.

Taylor instructed an assistant, Robert M. Page to construct a working prototype - a problem solved by 1934. By 1937 his team had developed a practical shipboard radar that became known as CXAM radar - a technology very similar to that of Britain's Chain Home radar system.

Albert Hoyt Taylor
1879-1961

Robert M. Page 1903-1992
Was central to the development of US Naval Radar systems which were completed prior to World War II. His successful demonstration of pulse radar was critical to the funding which made rapid development of the naval radar systems possible.

Patented Nov. 27, 1934 **1,981,884**

UNITED STATES PATENT OFFICE

1,981,884

SYSTEM FOR DETECTING OBJECTS BY RADIO

Albert H. Taylor, Leo C. Young, and Lawrence A. Hyland, Washington, D. C.

Application June 13, 1933, Serial No. 675,624

12 Claims. (Cl. 250—2)

(Granted under the act of March 3, 1883, as amended April 30, 1928; 370 O. G. 757)

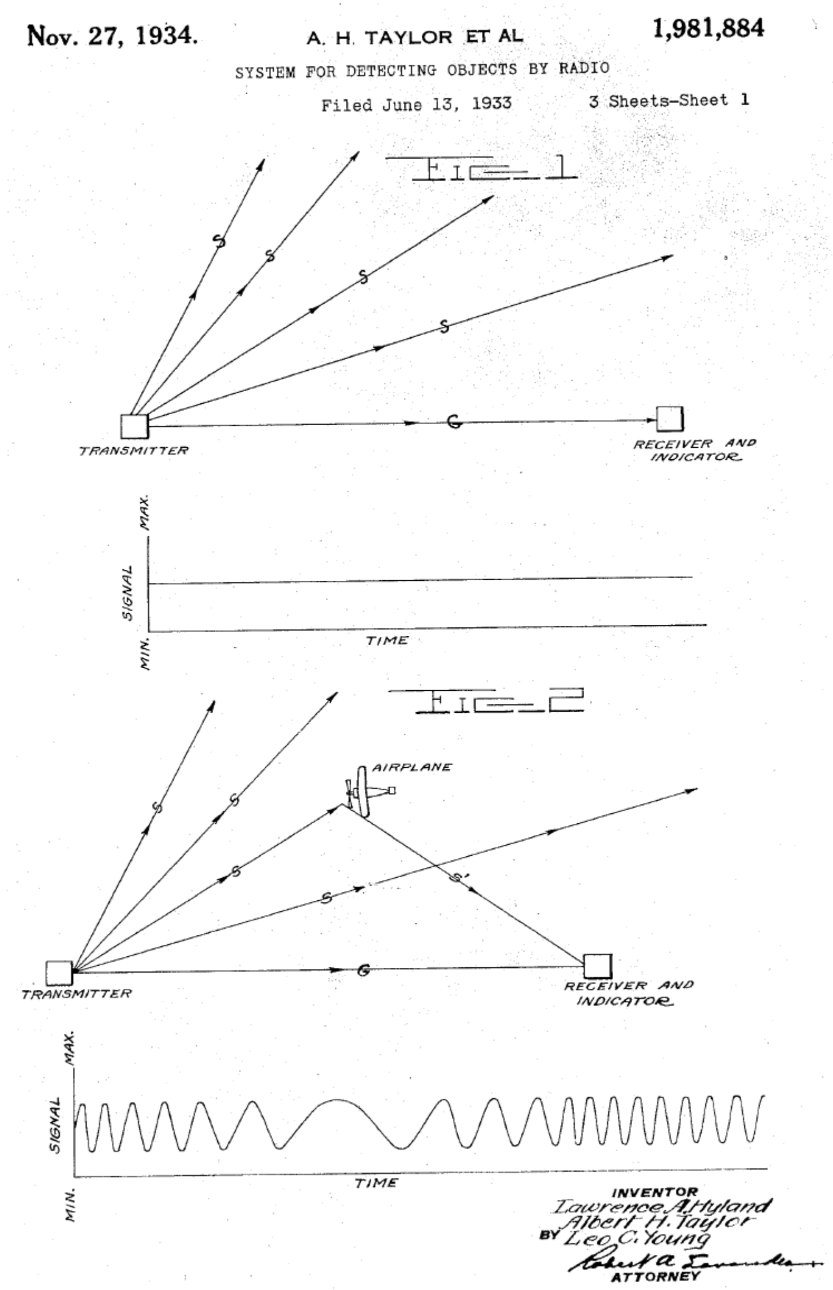
This invention relates to a method of and means by which moving objects in the air or on the surface of the earth may be detected by the employment of radio receiving and transmitting equipment.

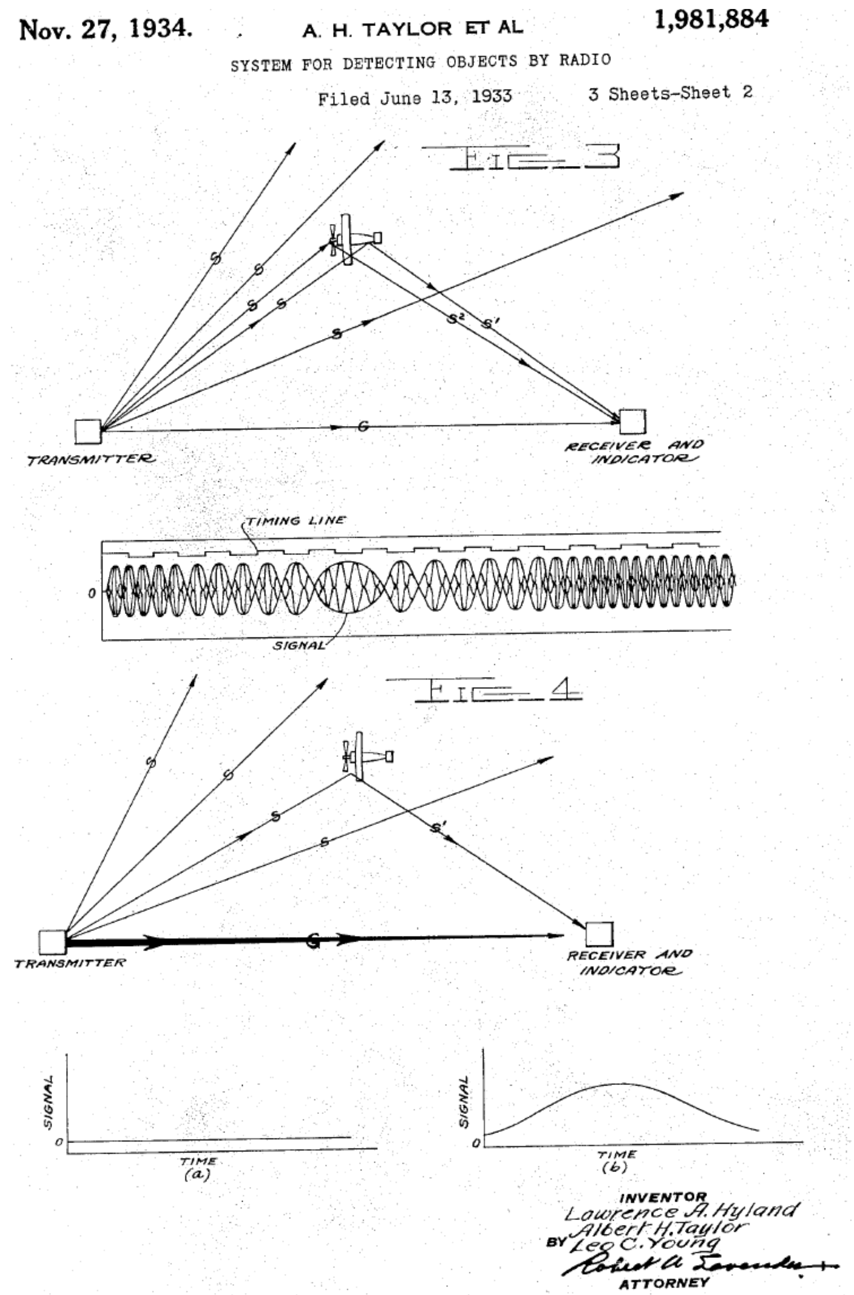
An object of this invention is to utilize the ground and other components of radiation commonly known as sky waves emitted by a radio transmitter and received in an indicating radio receiver so as to indicate the presence of an airplane or other motive vehicle when within the vicinity of the transmitter-receiver or when within the electromagnetic field created by said transmitter.

Another object of this invention is to so induce currents in intervening objects of a size comparable to the half wave length of the wave emitted by a radio transmitter that the radiated energy commonly called reradiation resulting from said induced currents and the energy received directly from the transmitter create a wave interference pattern which will be indicated by a characteristic signal in the receiver.

The phenomena upon which this invention operates is based upon the transmission of radio waves which may or may not be directional, the reradiation of those waves by an intervening object and the reception of the primary, as well as the reradiated waves by a receiver remotely situated with respect to the transmitter. As will be shown, the detection of intervening objects is accomplished either by properly receiving and interpreting the interference pattern created by the interaction of the ground waves as sent out from the transmitter and the reradiated waves from the intervening objects, such as an airplane, motive vehicle, or vessel, or by eliminating the said ground waves and adjusting the receiver to actuation only by the reradiated sky waves.

In connection with the foregoing, it is desired to point out the distinction between reradiation and reflection. In the previously issued patents directed along similar lines, reflection has been the basic phenomenon. These patents, in every case, have to do with the location of objects or the determination of altitude by means of re-

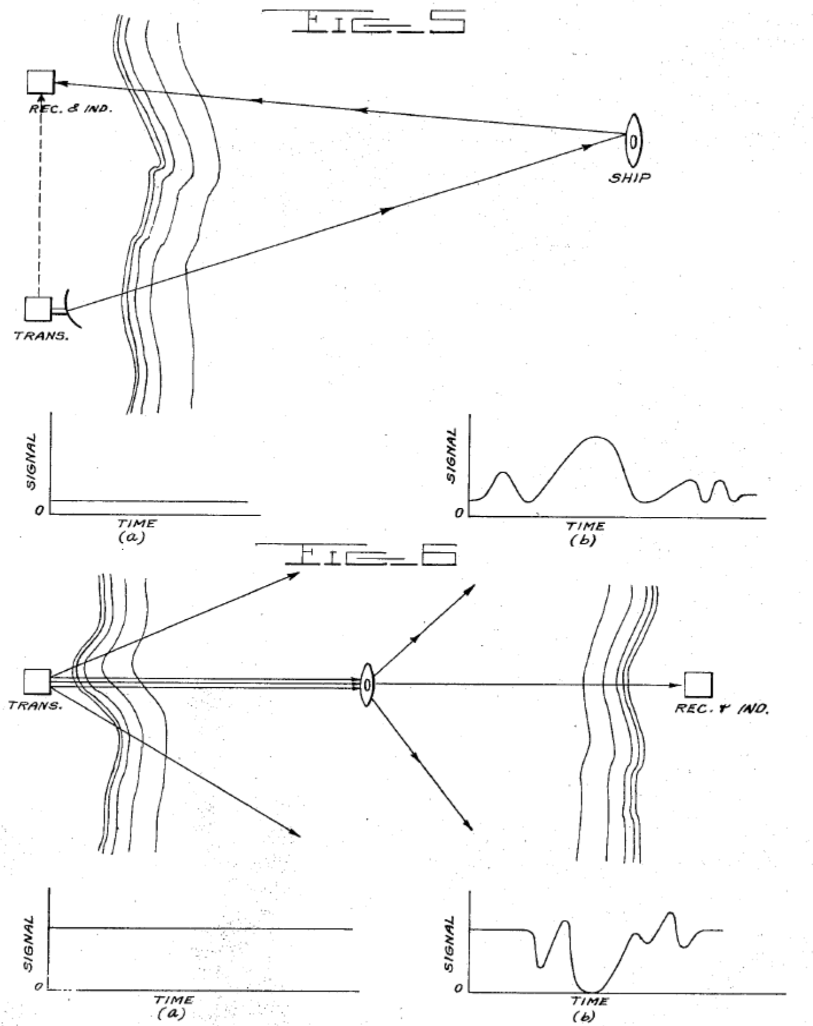

Nov. 27, 1934.
A. H. TAYLOR ET AL
1,981,884
SYSTEM FOR DETECTING OBJECTS BY RADIO
Filed June 13, 1933
3 Sheets-Sheet 1
FIG. 1
TRANSMITTER
RECEIVER AND INDICATOR
MAX.
SIGNAL
MIN.
TIME
FIG. 2
AIRPLANE
TRANSMITTER
RECEIVER AND INDICATOR
MAX.
SIGNAL
MIN.
TIME
INVENTOR
BY
ATTORNEY


Nov. 27, 1934.
A. H. TAYLOR ET AL
1,981,884
SYSTEM FOR DETECTING OBJECTS BY RADIO
Filed June 13, 1933
3 Sheets-Sheet 2
FIG. 3
TRANSMITTER
RECEIVER AND INDICATOR
TIMING LINE
SIGNAL
FIG. 4
TRANSMITTER
RECEIVER AND INDICATOR
SIGNAL
TIME (a)
TIME (b)
INVENTOR
BY
ATTORNEY

Nov. 27, 1934. A. H. TAYLOR ET AL 1,981,884

SYSTEM FOR DETECTING OBJECTS BY RADIO

Filed June 13, 1933 3 Sheets-Sheet 3



INVENTOR
Lawrence A. Hyland
Albert H Taylor
BY Leo C. Young

ATTORNEY

The United States Army Signal Corps started developing radar as early as 1930. In 1935, tests on microwave propagation using Hollmann built valves, RCA magnetron operating at 9 cm, RCA acorn valves were performed. In 1937 the test radar unit was demonstrated. Based on this test unit, in 1940, the SCR-270 became available for coastal defense and it was first deployed in Panama in the Fall of 1940 as an early warning for the Air Corps, Pursuit Squadron. This unit operated on a frequency of 205 MHz (l=1.5 m) and had a range of 23 miles, had an angular accuracy of 1 deg. 18 units were built by the Army Signal Corps Laboratory for training purposes. By June of 1941 a total of 85 sets had been delivered by Western Electric. A total of 794 were produced between 1939 and 1944.

SCR-271 3 m air warning radar at New Caledonia in May 1943. The SCR-271 was an early version of the SRC-270.

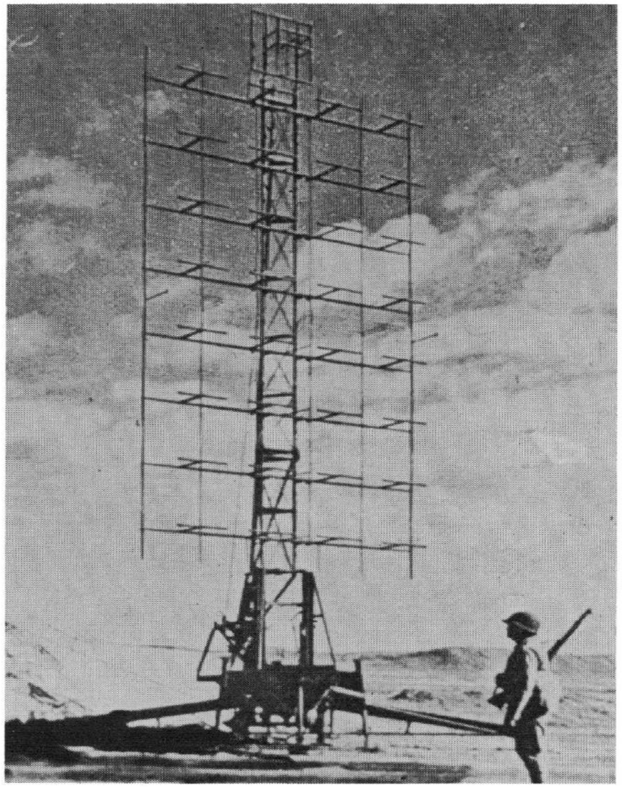
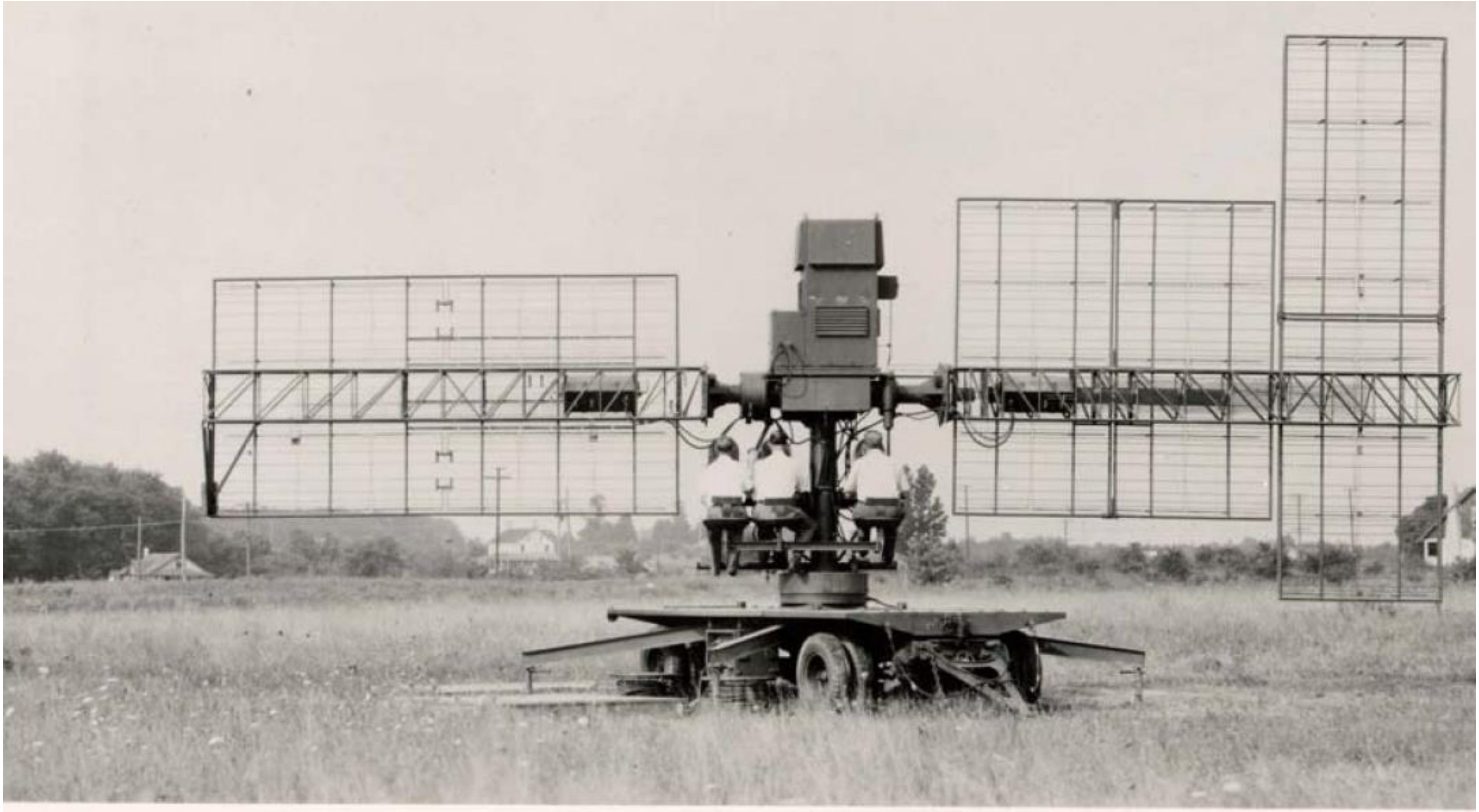
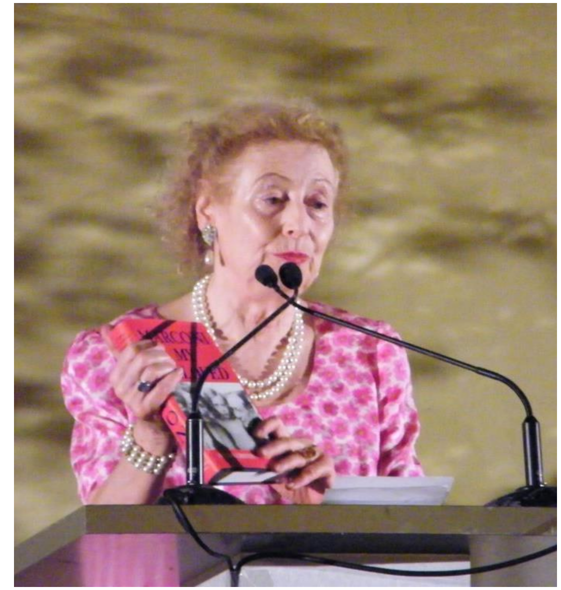


Fig. 1—AN/SCR-270.



FIRST AIRCRAFT DETECTION RADAR - The first aircraft detection radar in the United States was this SCR-268, developed and built at Fort Monmouth in 1938. Aircraft detection radars were vital to Allied victory in World War II in both the Pacific and European threaters.

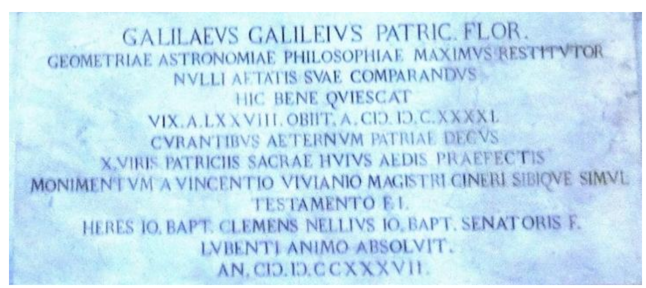
Guglielmo Marconi
Nobel prize, 1909

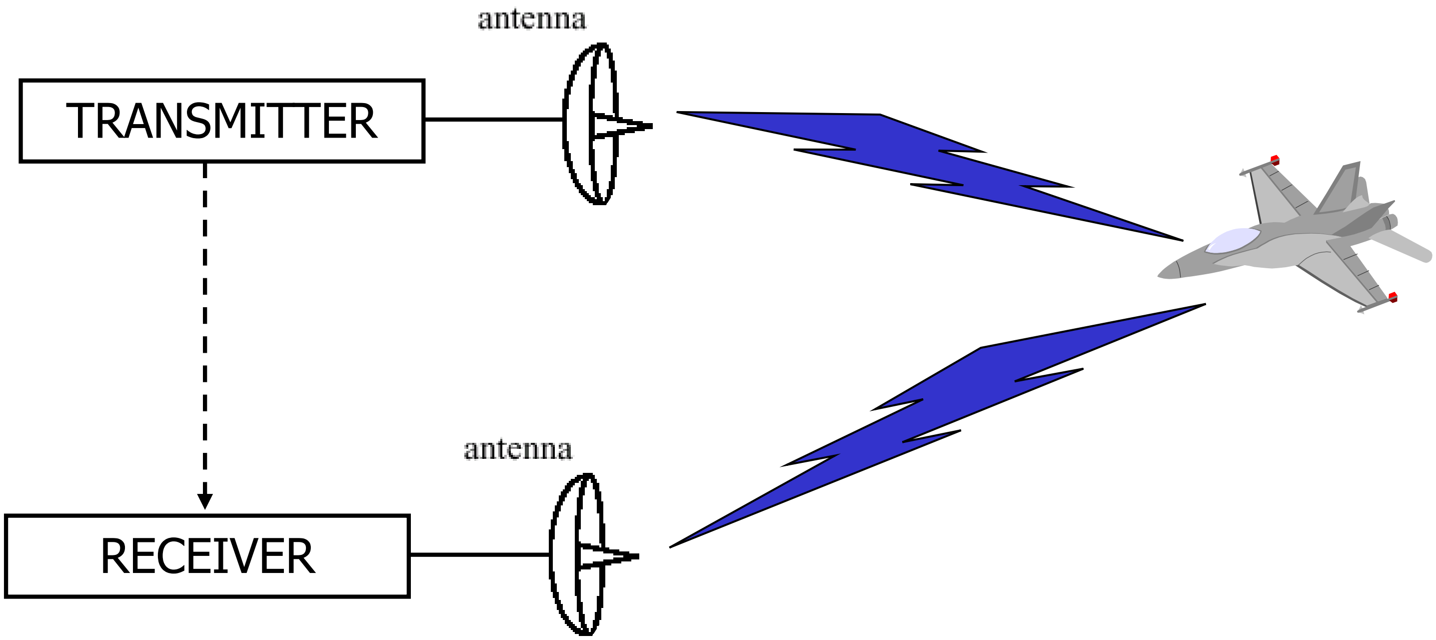


Princess Elletra Marconi
(Guglielmo's daughter) speaking at
RADAR 2008 Conference in Rome

In 1922 Marconi foretold the discovery of radar in a lecture at the American Institute of Radio Engineers in New York.

In Italy in 1935 he gave a practical demonstration of radar.




antenna
TRANSMITTER
antenna
RECEIVER

Range ⇔ Delay

$$R = \tfrac{1}{2} C_p \tau$$

Range

2-way

Velocity of propagation

Delay

Velocity ⇔ Doppler shift

$$f_D \approx -2\frac{\dot{R}}{\lambda}$$

Range rate

Wavelength

2-way

Doppler shift

$$\lambda = \frac{C_p}{f_0}$$

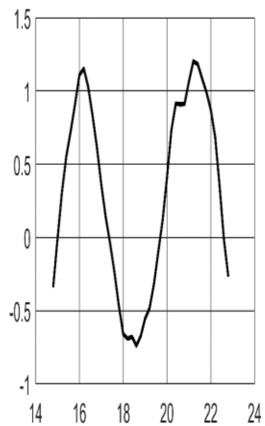
Velocity of propagation

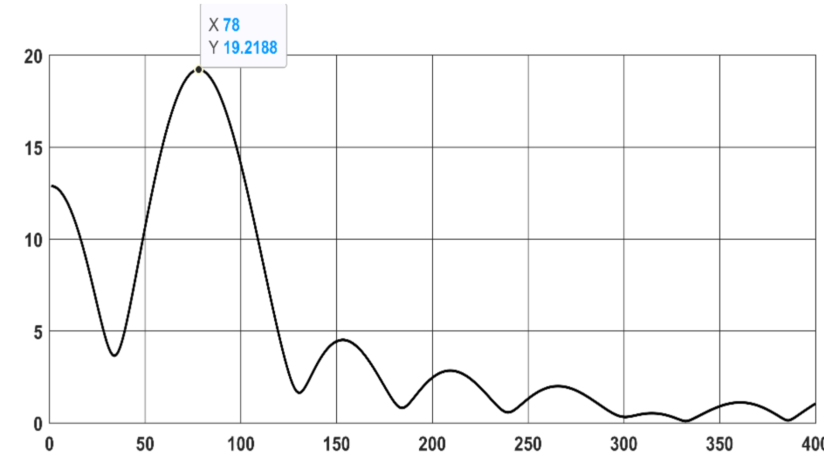
Carrier frequency

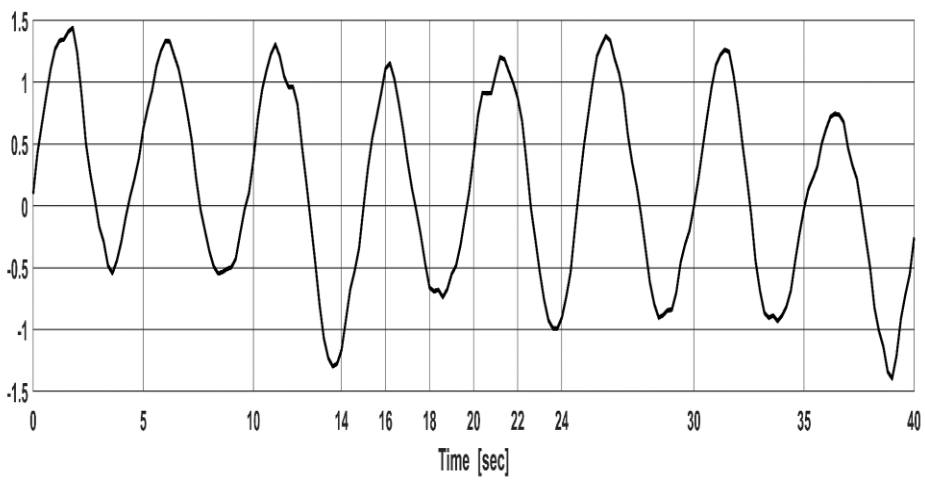
Comments on implementation of delay and frequency measurement

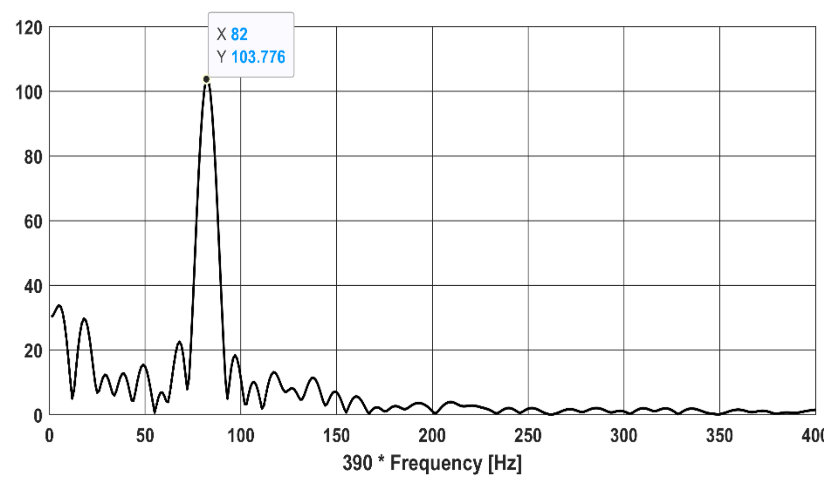
(Covered in details in the lecture on “Radar Measurements” and in Ch. 13 of “Radar – Concise Course”)

- **Delay measurement** – Involves detecting the instant when the returning pulse arrives. (A relatively simple and instant measurement).
- **Frequency measurement** – Not instant. Requires time duration to perform. Frequency estimation accuracy improves with measurement duration and signal-to-noise ratio.



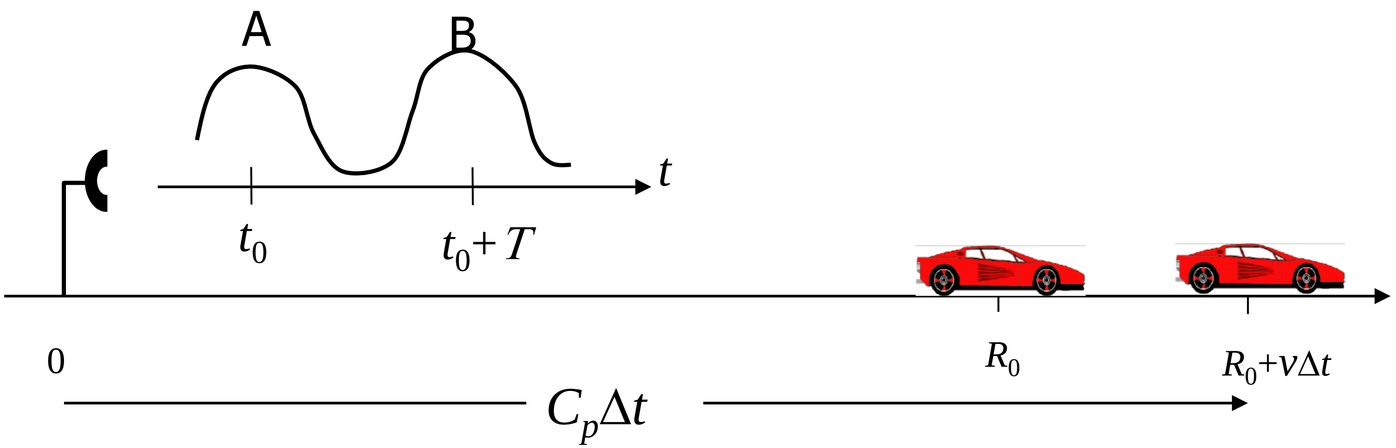






Christian Doppler's birthplace

Salzburg, Austria



R_o = Target location when peak **A** leaves the radar (t=t_o)

Δt = Travel time of peak **A** to reach the target

$v\Delta t$ = Distance traveled by target during Δt

t_1 = Time of return of peak **A** to radar

R_1 = Target location when peak **B** leaves the radar (at $t = t_0 + T$)

t_2 = Time of return of peak **B** to radar

T = Period of transmitted waveform

T_R = Period of received waveform

$$C_p \Delta t = R_0 + v\Delta t$$

$$\Delta t = \frac{R_0}{C_p - v}$$

$$t_1 = t_0 + 2\Delta t = t_0 + \frac{2R_0}{C_p - v}$$

$$t_2 = t_0 + T + \frac{2R_1}{C_p - v}$$

$$R_1 = R_0 + vT$$

$$t_2 = t_0 + T + \frac{2(R_0 + vT)}{C_p - v}$$

$$T_R = t_2 - t_1 = t_0 + T + \frac{2(R_0 + vT)}{C_p - v} - \left(t_0 + \frac{2R_0}{C_p - v}\right) = T\,\frac{C_p + v}{C_p - v}$$

$$T_R = T\,\frac{C_p + v}{C_p - v}$$

$$\frac{T_R}{T} = \frac{C_p + v}{C_p - v}$$

$$\frac{f_R}{f_0} = \frac{C_p - v}{C_p + v} = \frac{1 - \frac{v}{C_p}}{1 + \frac{v}{C_p}}$$

$$f_R = f_0\,\frac{1 - \frac{v}{C_p}}{1 + \frac{v}{C_p}}$$

$$v \ll C_p \Rightarrow \frac{1}{1 + \frac{v}{C_p}} = 1 - \frac{v}{C_p} + \frac{v^2}{C_p^2} - \ldots$$

$$f_R = f_0\left(1 - \frac{v}{C_p}\right)\left(1 - \frac{v}{C_p} + \frac{v^2}{C_p^2} - \ldots\right) = f_0\left(1 - \frac{2v}{C_p} + \ldots\right) \approx f_0\left(1 - \frac{2v}{C_p}\right)$$

$$f_R \approx f_0 - \frac{2v}{C_p/f_0} = f_0 - \frac{2v}{\lambda}$$

$$f_D = f_R - f_0 \approx -\frac{2v}{\lambda}$$

$$v = \dot{R}$$

$$f_D \approx -\frac{2\dot{R}}{\lambda}$$

Two assumptions were made:

$$f_R \approx f_0 - \frac{2v}{C_p/f_0} = f_0 - f_D$$

$v << C_p$ (assumption valid in EM propagation, not necessarily in acoustics or sonar)

Narrow band signal - All the frequencies of the signal are almost equal to f_0,
Hence, all the frequencies are shifted by the same f_D .

$$s(t) = g(t)\cos\left[2\pi f_0 t + \phi(t)\right]$$
$$s_R(t) = g(t-\tau_0)\cos\left[2\pi\left(f_0 + f_D\right)\left(t-\tau_0\right) + \phi(t-\tau_0)\right]$$

When the signal is wideband (namely $g(t)$ and $\phi(t)$ are wideband) they are changed and not only delayed.

$$T = T_R \frac{C_p - v}{C_p + v} \underset{v<<C_p}{\approx} \left(1 - \frac{2v}{C_p}\right) T_R$$

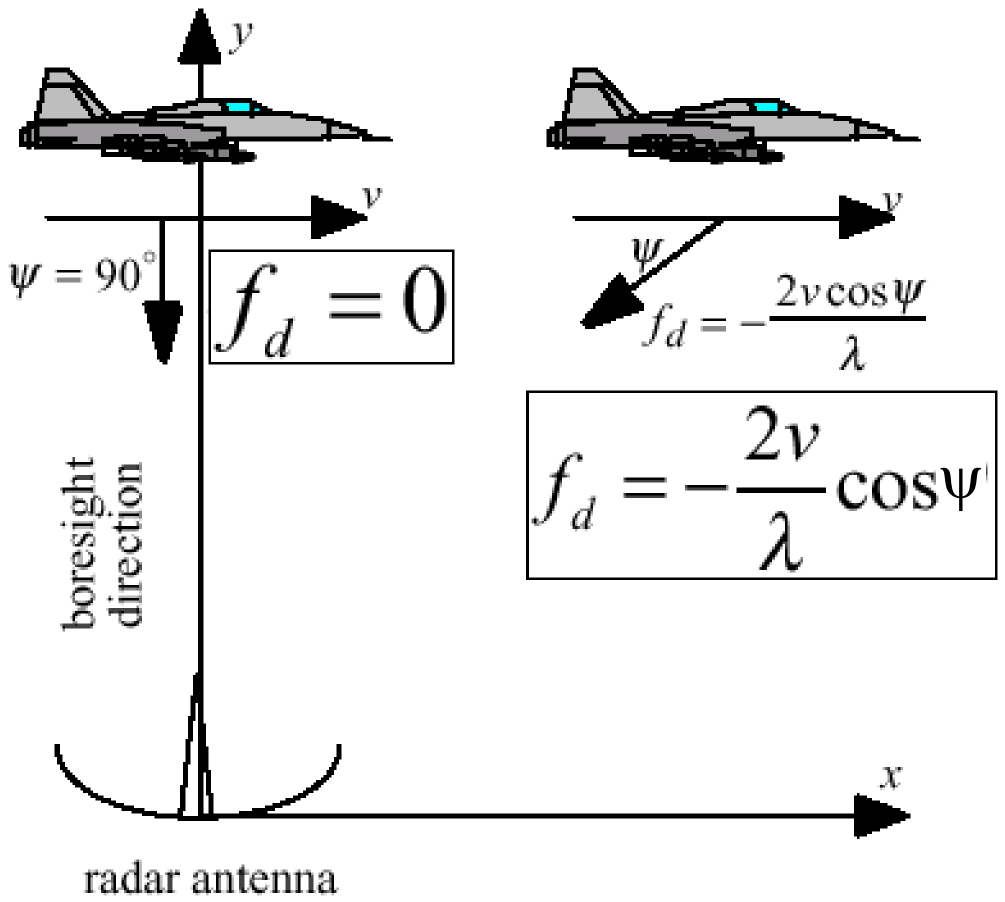
$$s_R(t) = s\left[\left(\frac{C_p - v}{C_p + v}\right)t - \tau_0\right]$$
$$s_R(t) \approx s\left[\left(1 - \frac{2v}{C_p}\right)t - \tau_0\right]$$

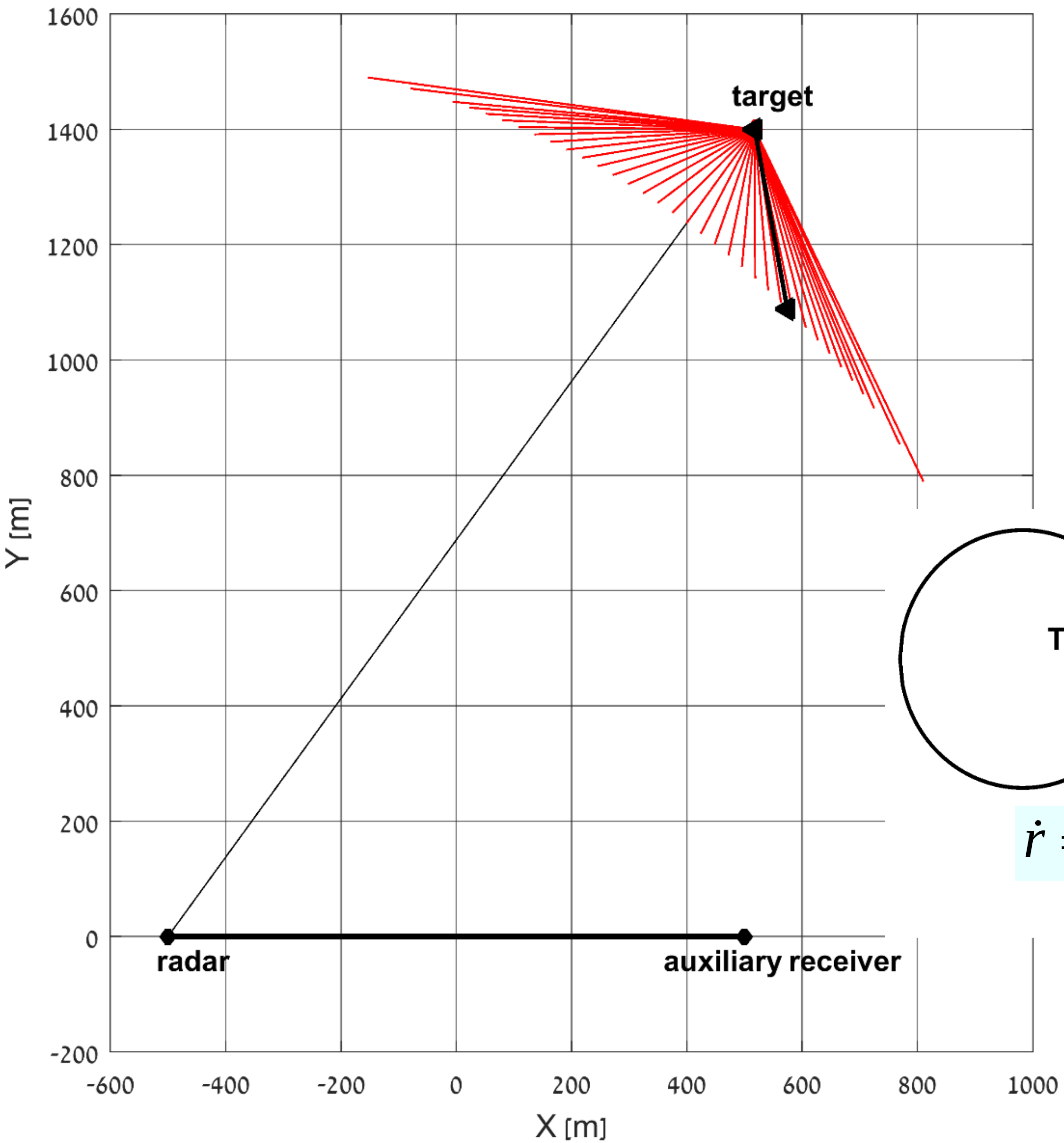
The received signal is both delayed and time-scaled.

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

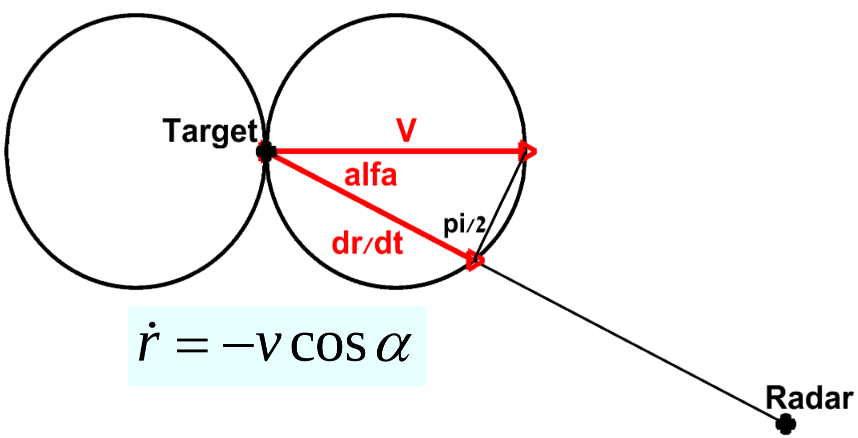
Radial Velocity

For a monostatic radar the Doppler shift is determined by the radial component of the velocity





All the target's velocity vectors (red lines) have the same range-rate toward the radar.



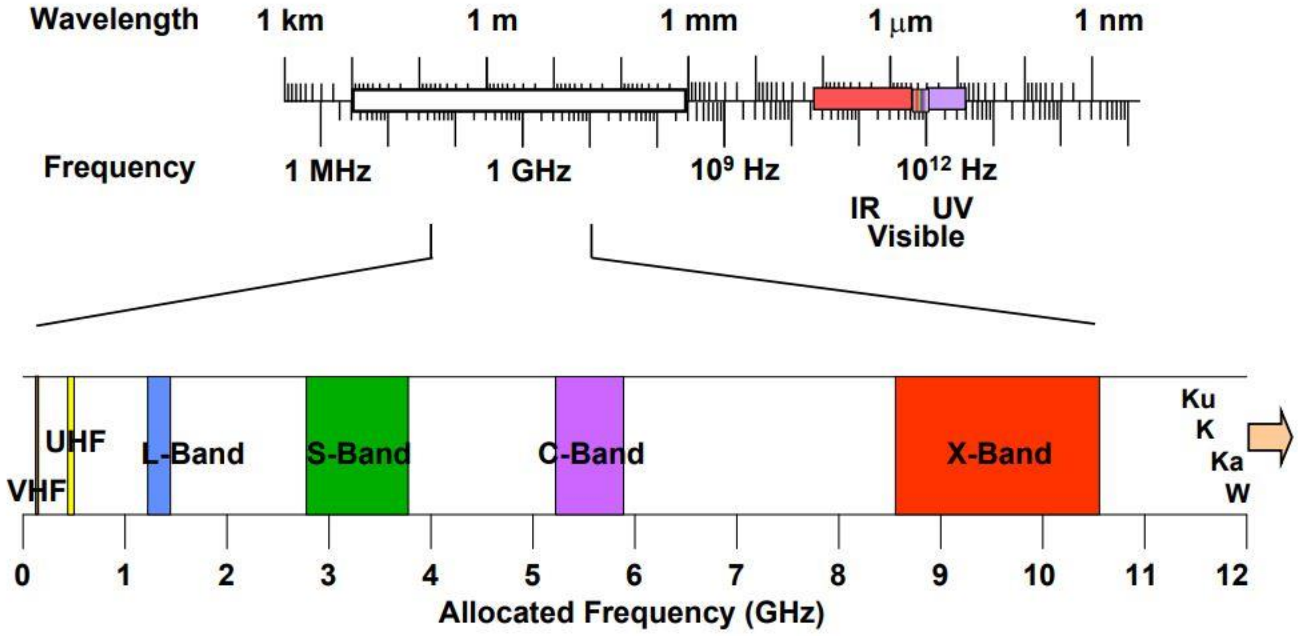
$$\dot{r} = -v\cos\alpha$$

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Radar Frequency Bands

Band	*Frequencies*	*Wavelengths*
HF	3 - 30 MHz	100 m - 10 m
VHF	30 - 300 MHz	10 m - 1 m
UHF	300 MHz - 1 GHz	1 m - 30 cm
L	1 - 2 GHz	30 cm - 15 cm
S	2 - 4 GHz	15 cm - 7.5 cm
C	4 - 8 GHz	7.5 cm - 3.75 cm
X	8 - 12 GHz	3.75 cm - 2.5 cm
Ku	12 - 18 GHz	2.5 cm - 1.67 cm
K	18 - 27 GHz	1.67 cm - 1.11 cm
Ka	27 - 40 GHz	1.11 cm - 7.5 mm
mm	40 - 300 GHz	7.5 mm - 1 mm

$$c = \lambda f$$



Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Values of Doppler Shift

- Mach 2 aircraft causes 4.4 kHz shift at L band (1 GHz)

Band	*Frequency (GHz)*	*Doppler shift (Hz) for v = 1 m/s*
L	1	6.67
C	6	40.0
X	10	66.7
K_a	35	233
W	95	633

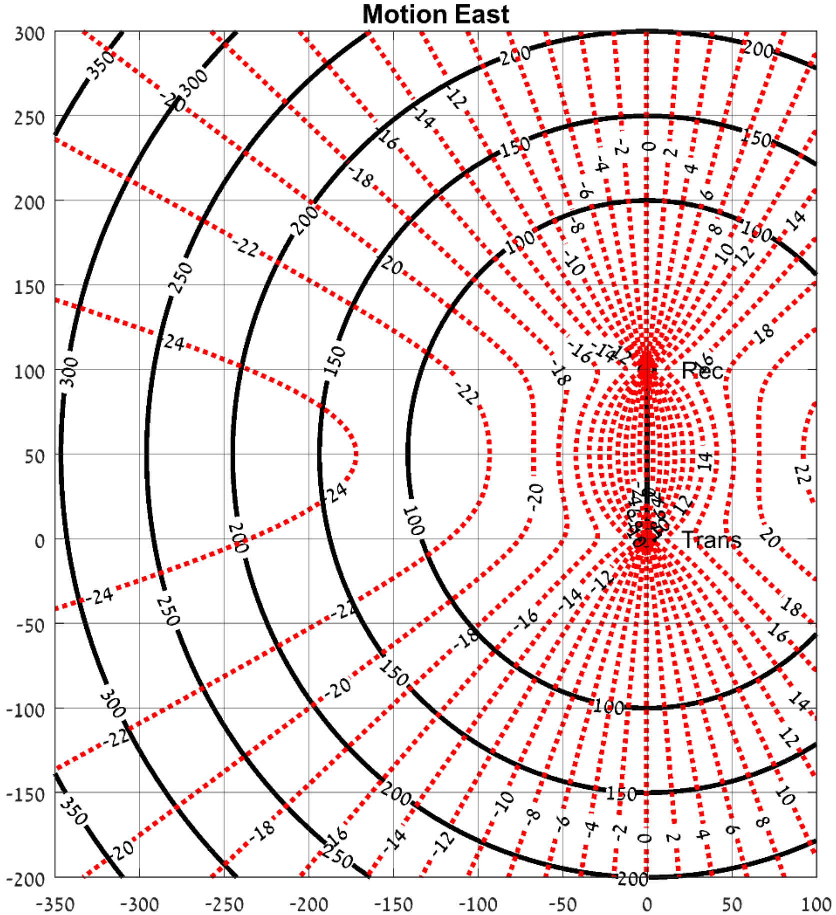
Bi-static scene (2-D)

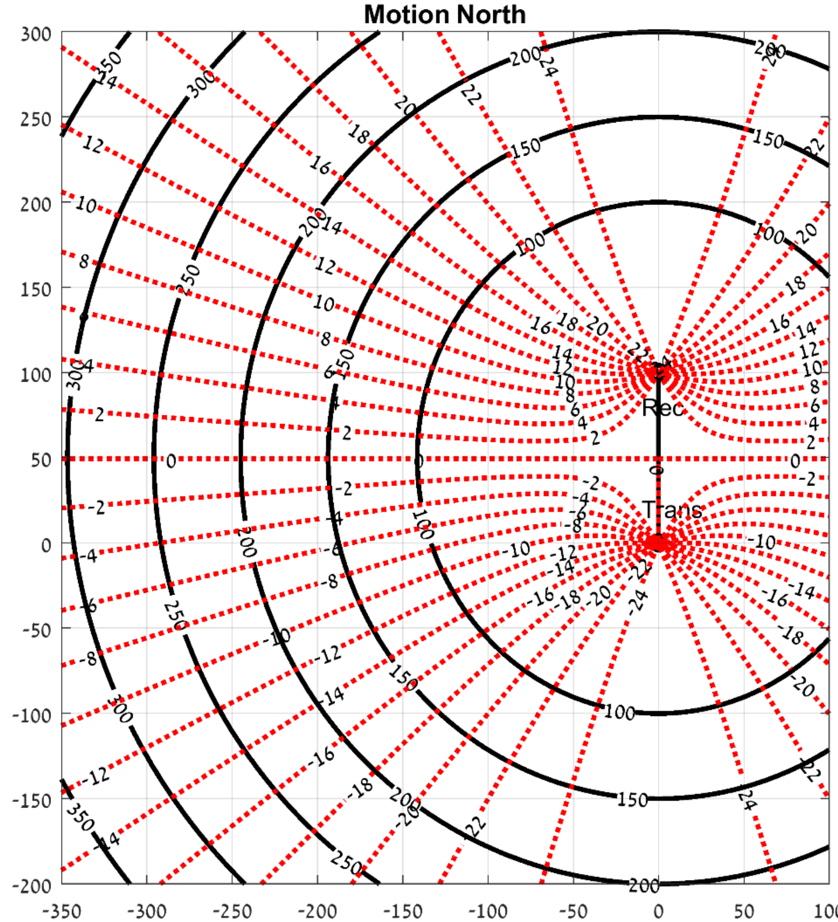
Iso-range and iso-Doppler contours in bi-static scene (2-D)

(Passive radar is a special case of Bi-static radar)

$$\text{Iso-range contour value} = \frac{1}{2}\left(R_{\text{target}-\text{trans}} + R_{\text{target}-\text{rec}} - b\right)$$

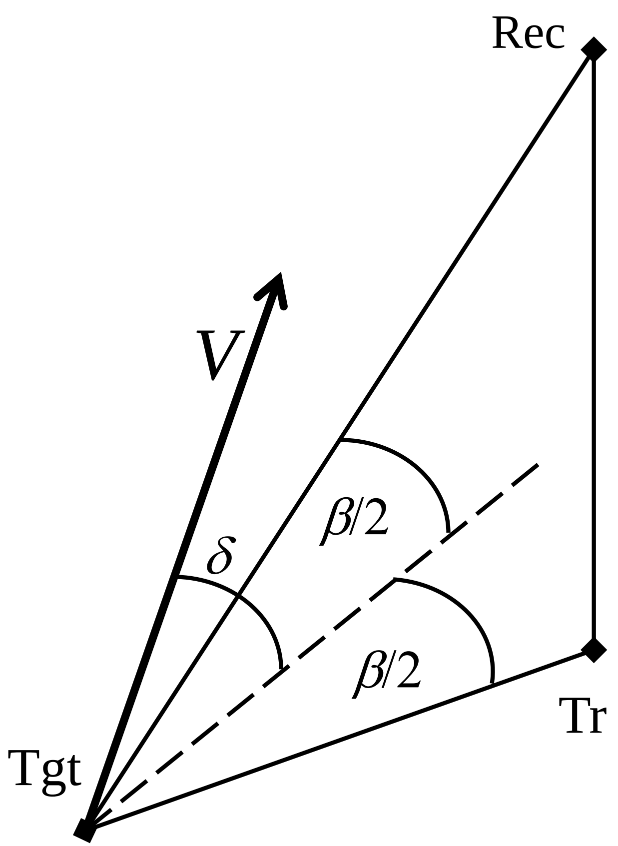
$$\text{baseline}: b = R_{\text{trans}-\text{rec}}$$





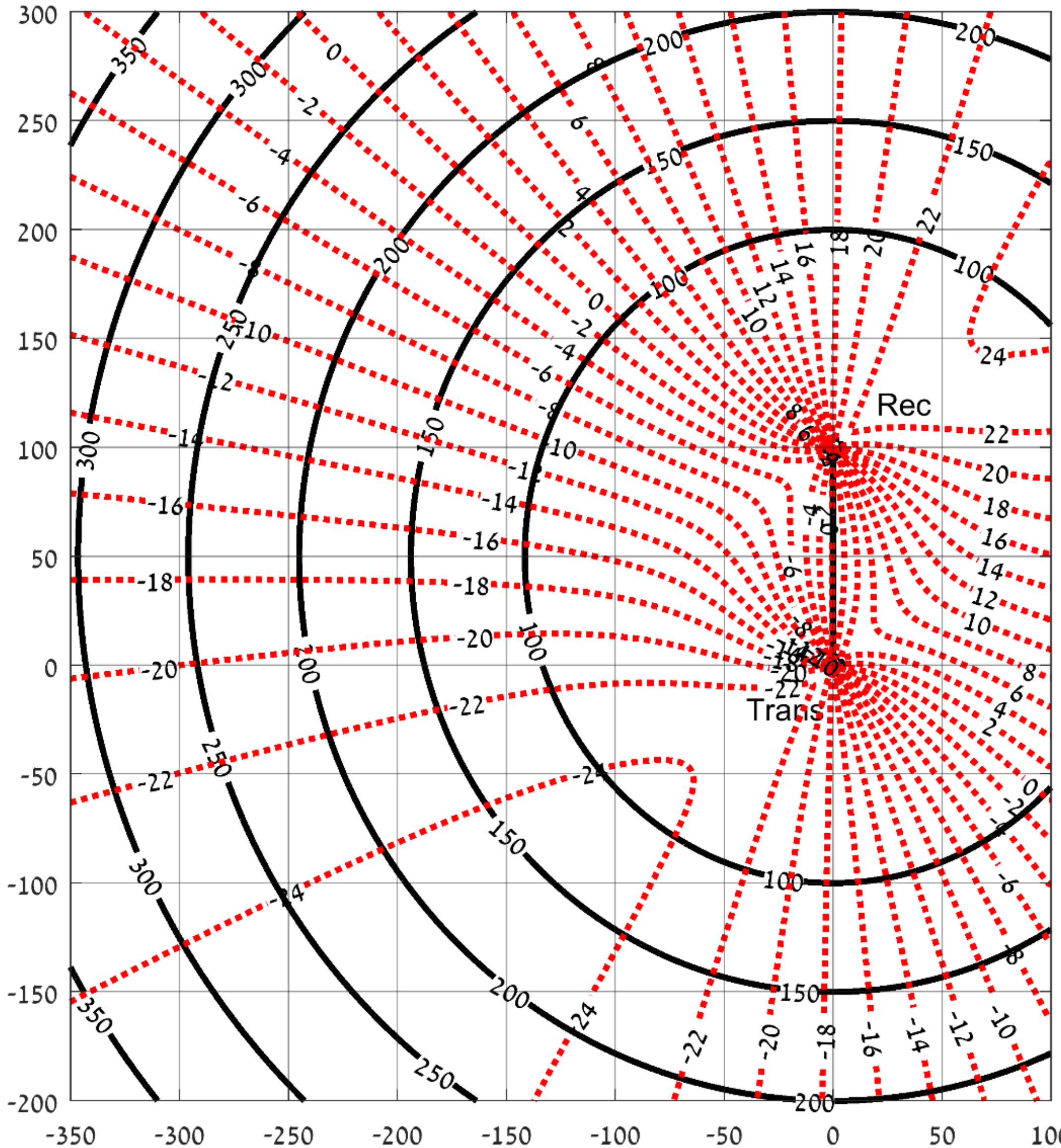
Bi-static scene (2-D)

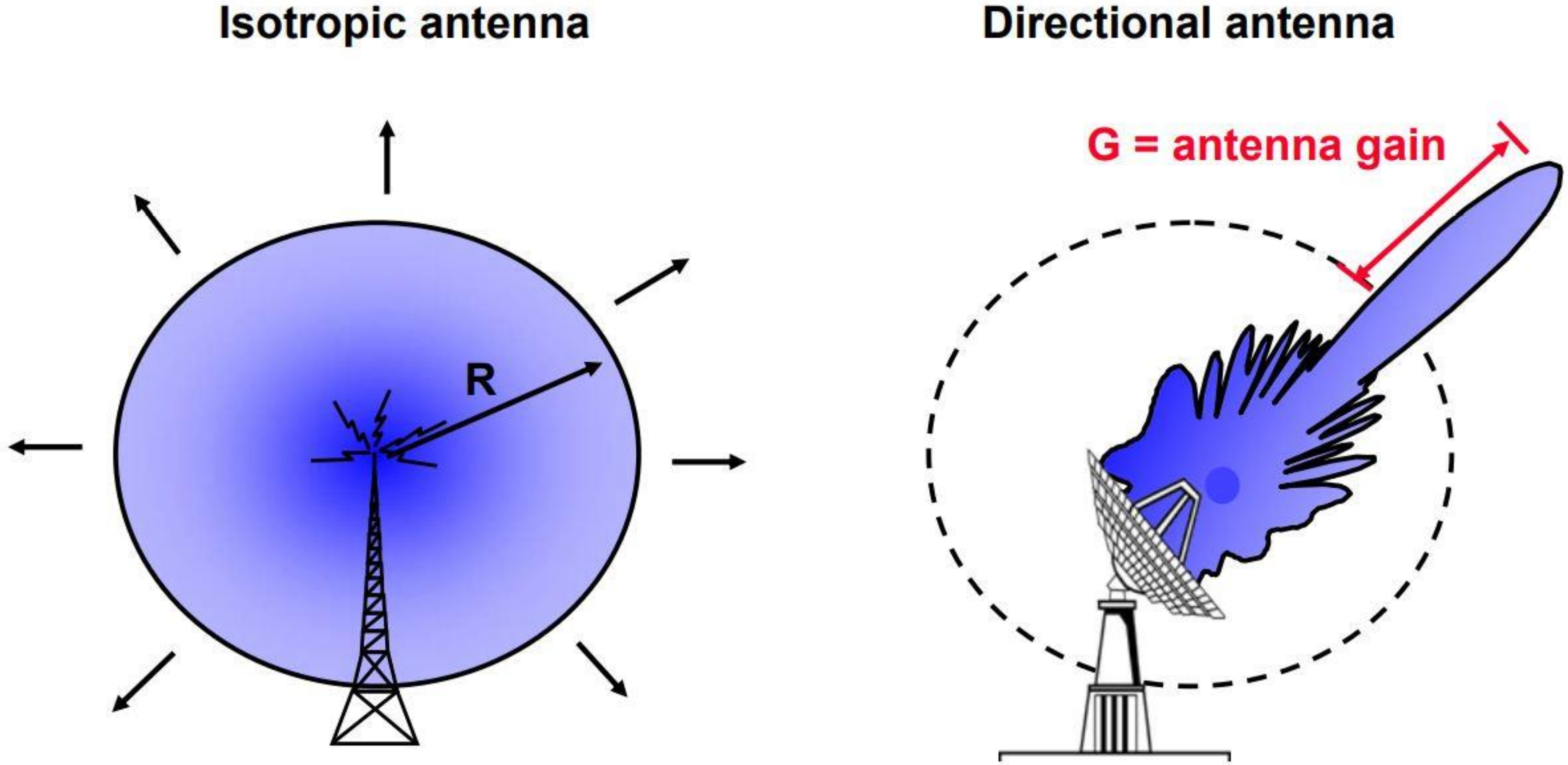
Iso-range and iso-Doppler contours in a bi-static scene (2-D)



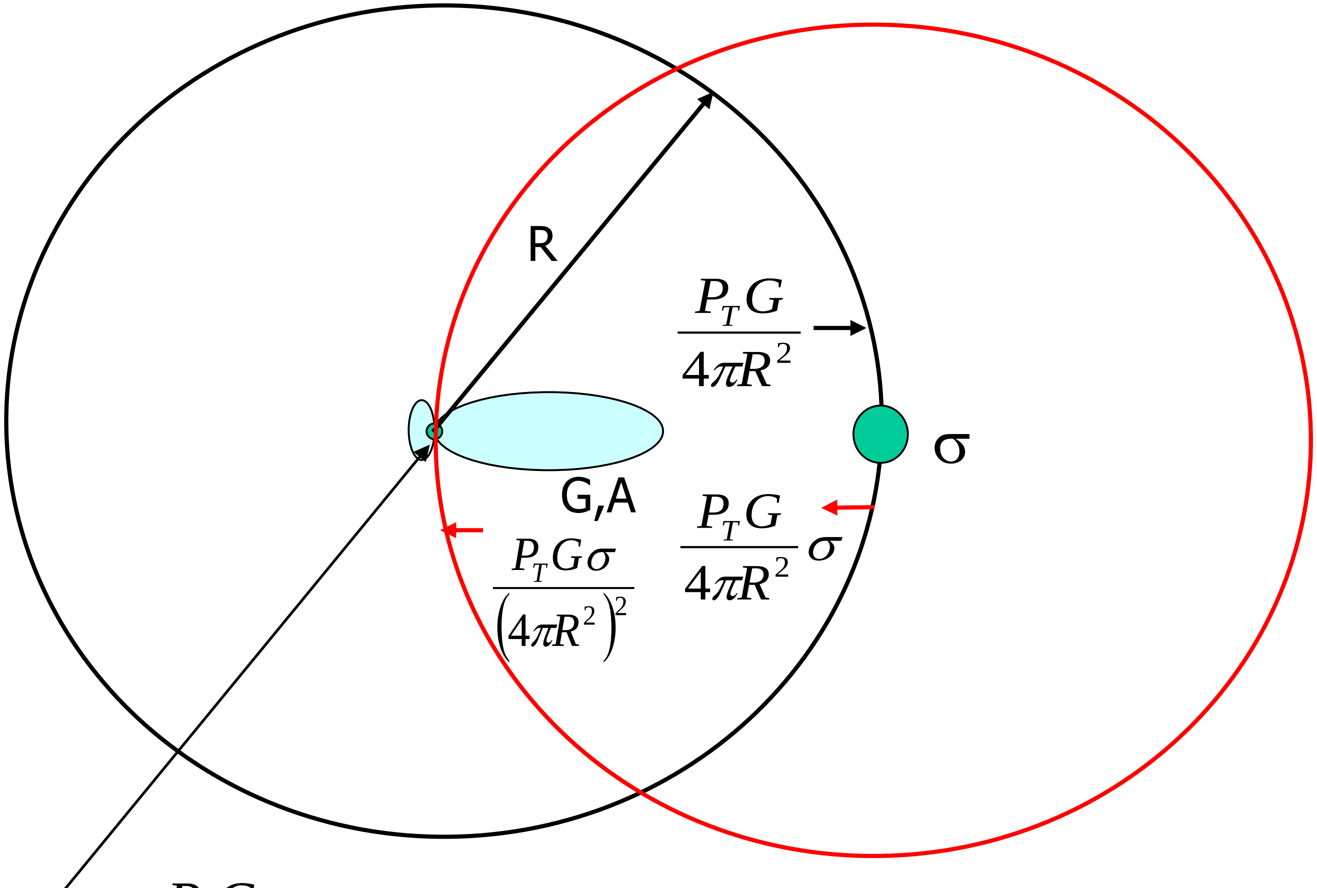
$$f_D = \frac{-2V}{\lambda}\cos(\delta)\cos\left(\frac{\beta}{2}\right)$$

Motion North-East, Iso-range (solid) , Iso-Doppler (dotted)




Isotropic antenna
R
Directional antenna
G = antenna gain

RADAR EQUATION



$$P_R = \frac{P_T G \sigma}{\left(4\pi R^2\right)^2} A$$

$$A = \frac{G\lambda^2}{4\pi}$$

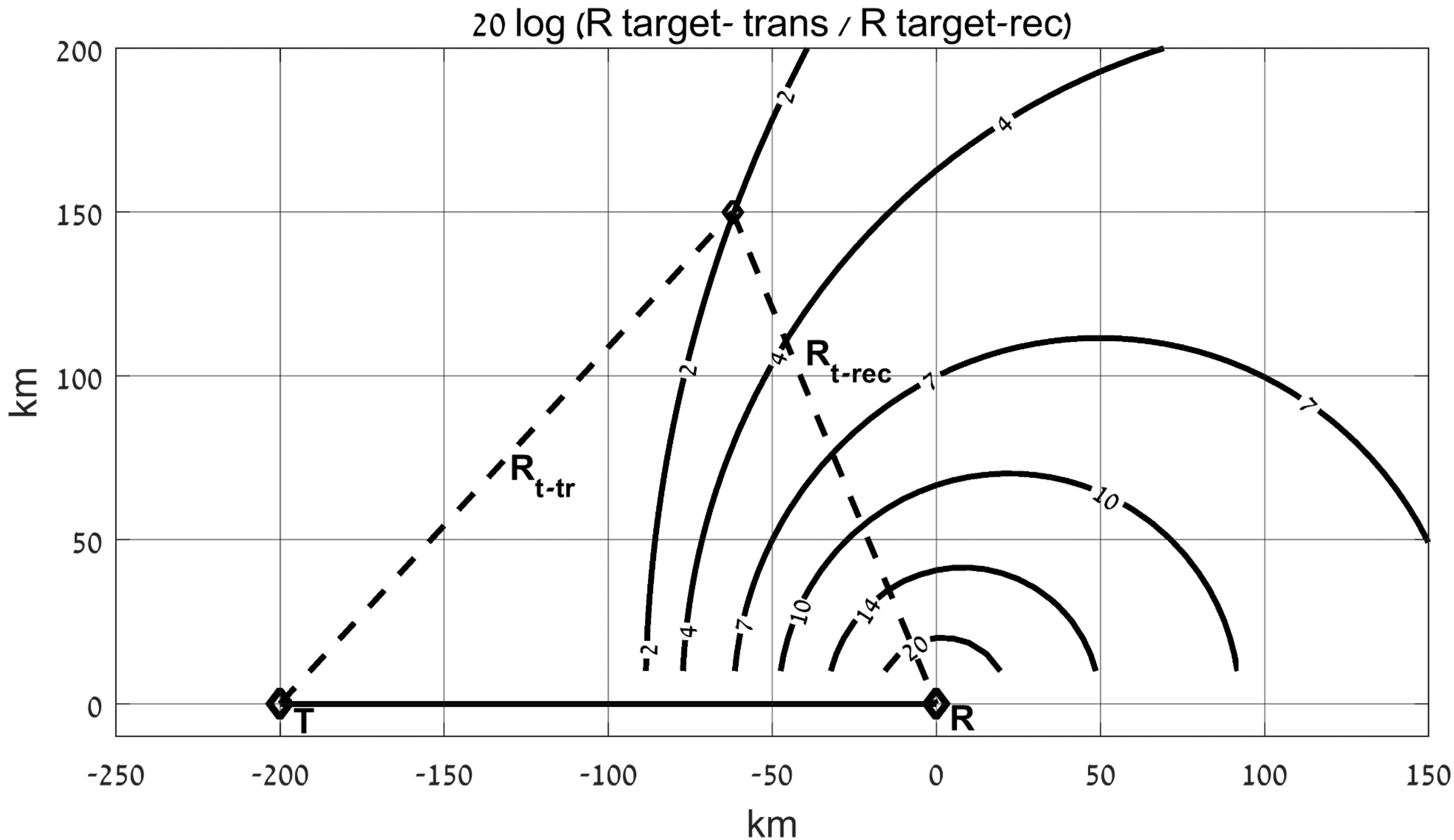
$$P_R = \frac{P_T G^2 \lambda^2}{\left(4\pi\right)^3 R^4}\sigma$$

Radar cross section of a target - σ

σ = The area of a virtual target which reflects back isotropically, that would have caused the same return as the actual target.

Dimension of $\boldsymbol{\sigma}$ is m^2 (i.e., area)

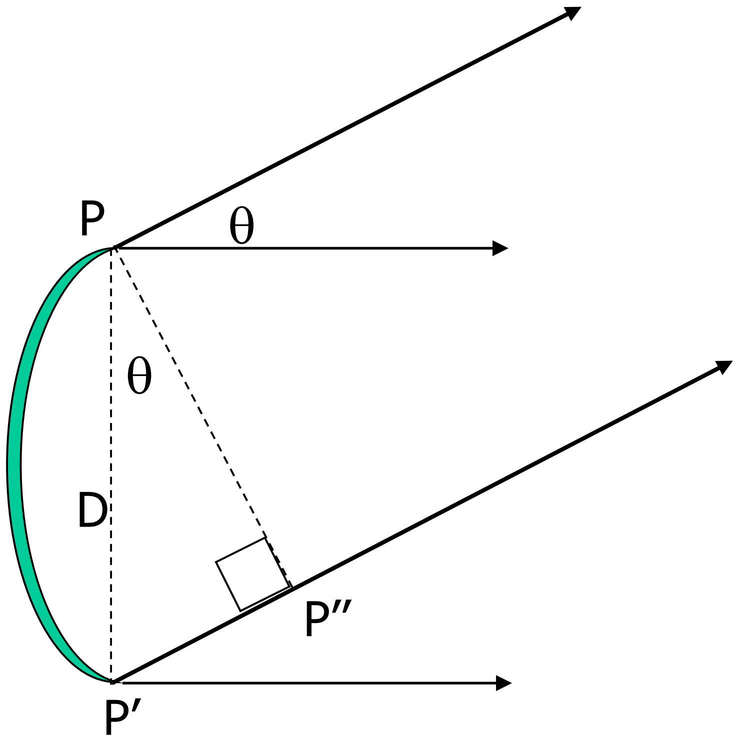
Bistatic radar equation



$$\text{Power ratio[dB]} = 10\log \frac{R_{t-tr}^4}{R_{t-tr}^2 \, R_{t-rec}^2} = 20\log \frac{R_{t-tr}}{R_{t-rec}}$$

Antenna aperture ⇔ antenna gain

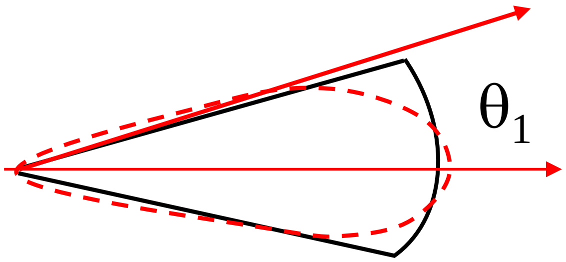
$$A = \frac{G\lambda^2}{4\pi}$$



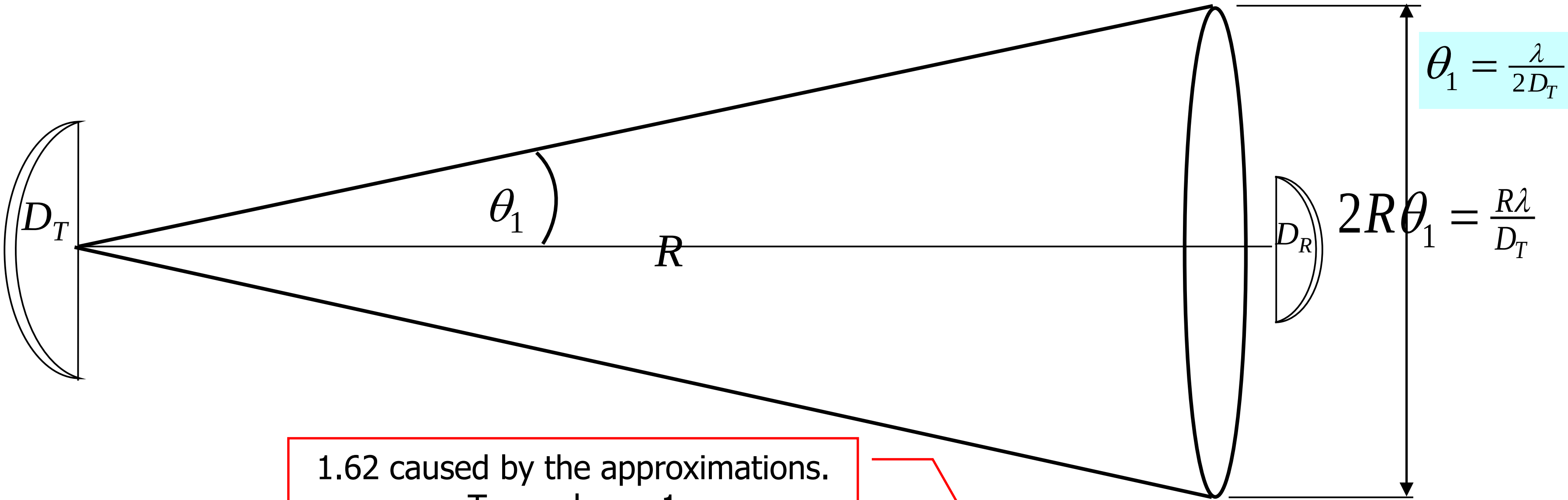
When $P'P'' = \frac{\lambda}{2}$ the radiation from P and P′ are out of phase and cancel. Approximate:

$$\theta_1 = \frac{\lambda}{2D}$$

$$G(\theta) = \begin{cases} G_0 & |\theta| < \theta_1 \\ 0 & \text{elsewhere} \end{cases}$$



$$P'P'' = D\sin\theta \approx D\theta \quad , \quad \theta << 1\text{rad}$$



1.62 caused by the approximations.
True value = 1

$$\frac{P_R}{P_T} \approx \frac{D_R^2}{\left(R\lambda/D_T\right)^2} = \frac{D_R^2 D_T^2}{\left(R\lambda\right)^2}$$

$$D^2 = \frac{4A}{\pi}$$

$$\frac{P_R}{P_T} \approx \frac{16 A_R A_T}{\pi^2 \left(R\lambda\right)^2} = 1.62 \frac{A_R A_T}{\left(R\lambda\right)^2}$$

$$\frac{P_R}{P_T} = \frac{A_R A_T}{\left(R\lambda\right)^2}$$

From the radar equation development, the power received (and reflected) by a target of area $A_R = \sigma$ is:

$$\frac{P_R}{P_T} = \frac{A_R G_T}{4\pi R^2}$$

$$\therefore A_T = \frac{G_T \lambda^2}{4\pi}$$

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From M.A. Richards, Georgia Tech

Typical RCS Values - 1

Target	*RCS, m^2*	*RCS, dBsm*
Conventional unmanned winged missile	0.5	-3
Small single-engine aircraft	1	0
Small fighter aircraft or 4-passenger jet	2	3
Large fighter aircraft	6	8
Medium bomber or jet airliner	20	13
Large bomber or jet airliner	40	16
Jumbo jet	100	20
Small open boat	0.02	-17
Small pleasure boat	2	3

Typical RCS Values - 2

Target	*RCS, m^2*	*RCS, dBsm*
Cabin cruiser	10	10
Large ship at zero grazing angle	10,000+	40+
Pickup truck	200	23
Automobile	100	20
Bicycle	2	3
Human	1	0
Bird	0.01	-20
Insect	0.00001	-50

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RCS of a Conducting Sphere

- Three regions, depending on relative size of sphere and wavelength

- For radius $a >> \lambda$

$$\sigma = \pi a^2$$

- Aspect and frequency independence makes it a good calibration target

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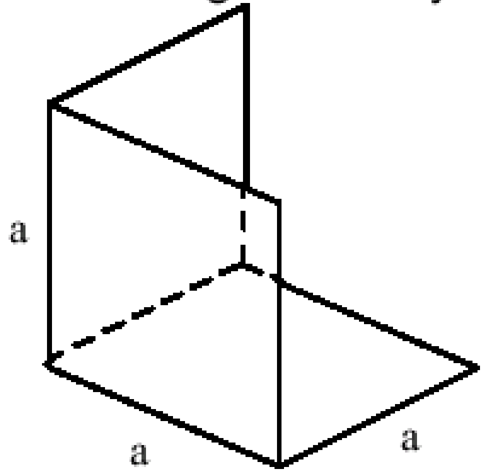
RCS of Conducting Trihedral

- RCS is not a simple scalar in general
  - depends on aspect angle, frequency, polarization
- Example 1: trihedral ("corner reflector")
  - RCS can be determined theoretically when looking along axis of symmetry ("into the corner")



$$\sigma = \frac{12\pi a^4}{\lambda^2}$$

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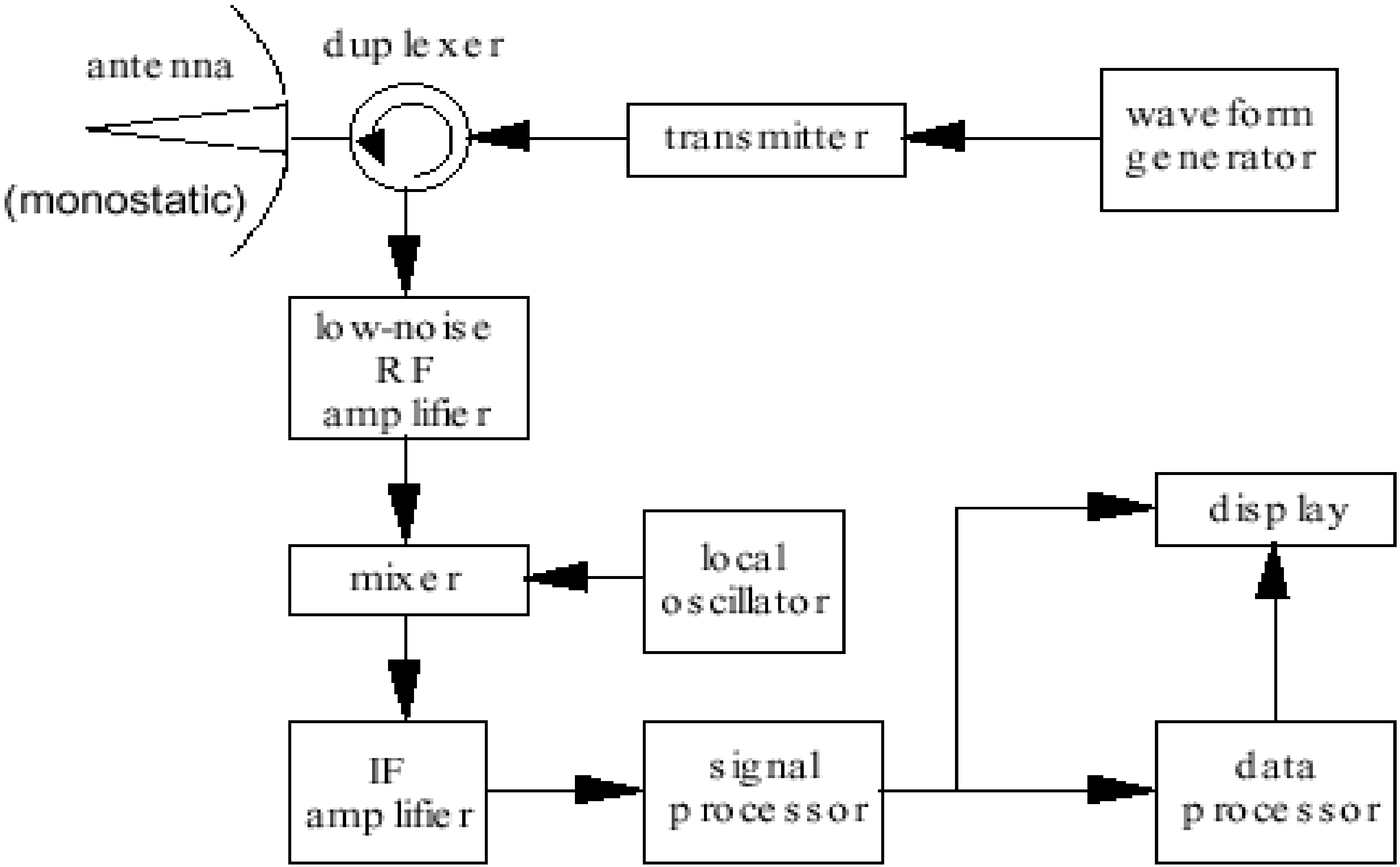
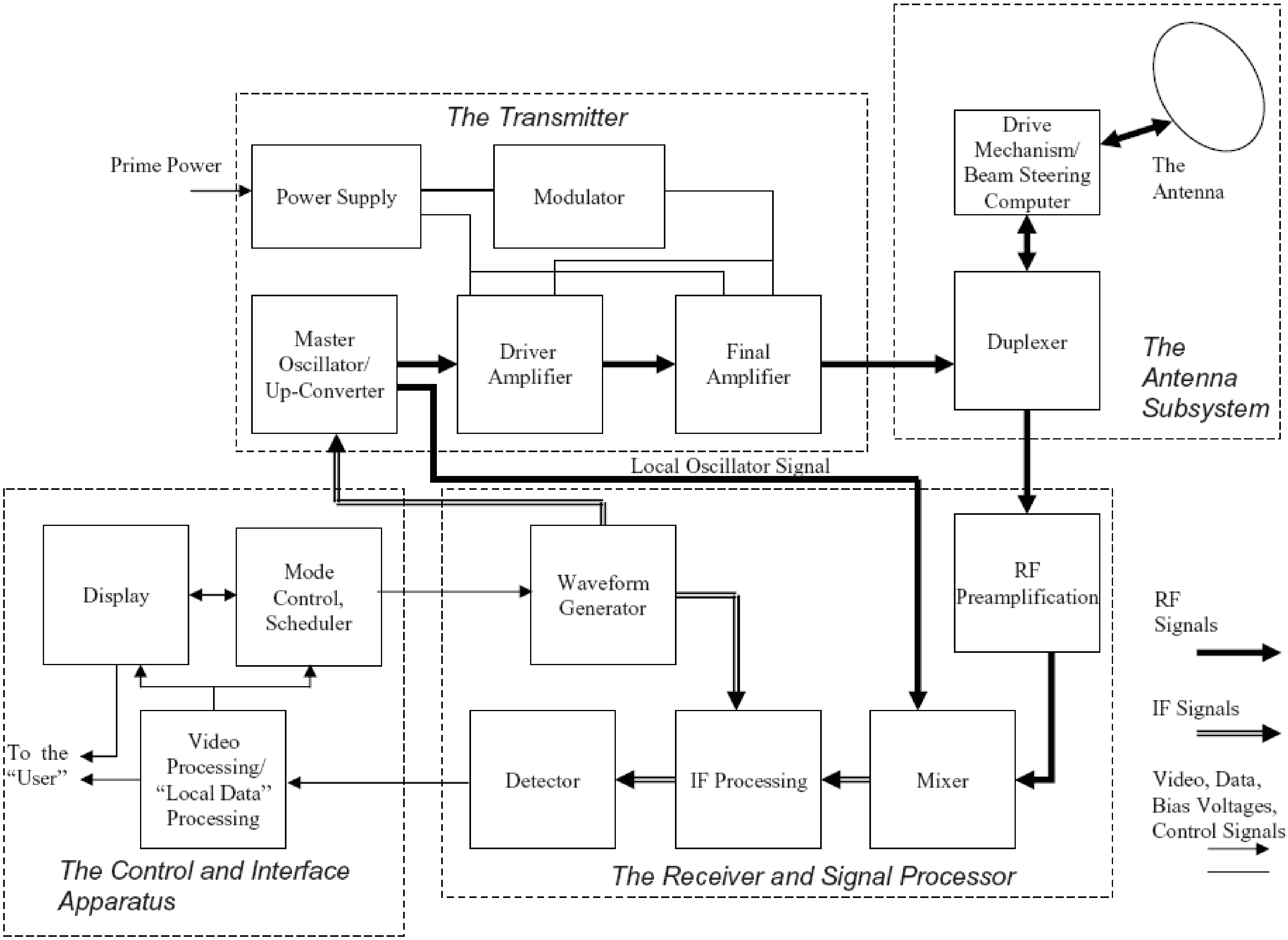
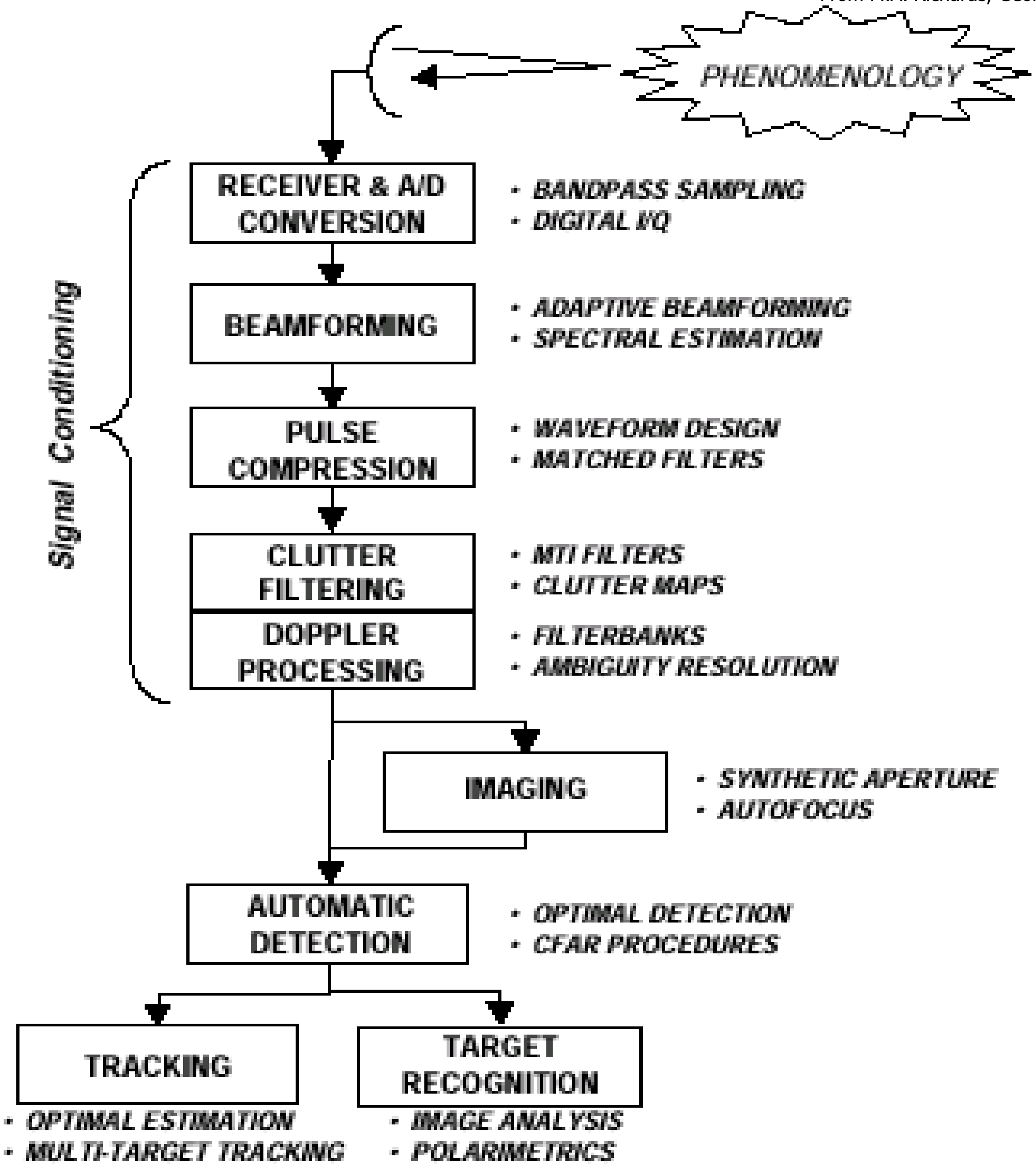
Elements of a Pulsed Radar





An elementary radar block diagram showing composition of the four principal subsystems

A Tour of Basic Radar Signal Processing Operations



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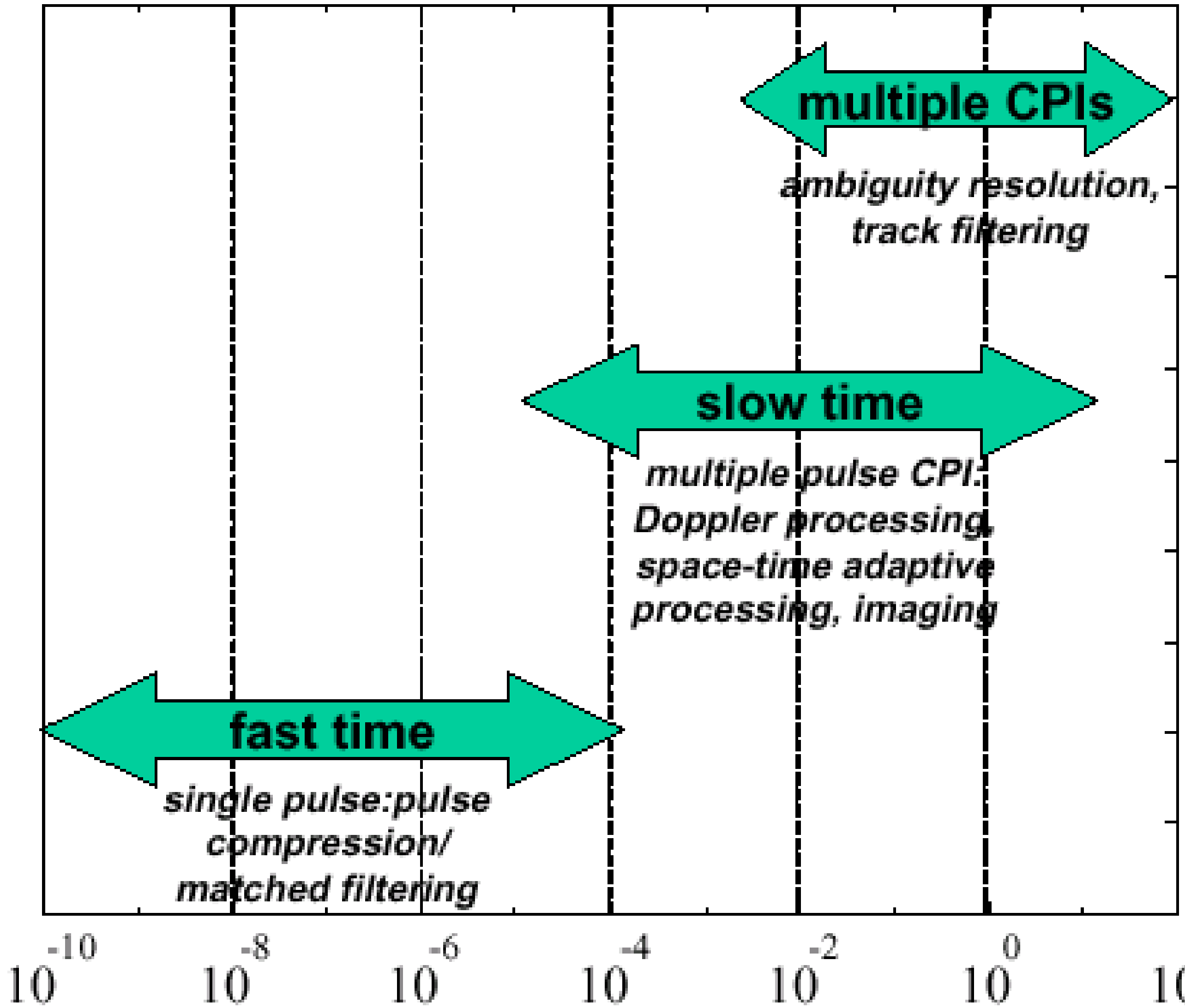
Radar Time Scales

Processing occurs on time scales varying by 10-12 orders of magnitude



Two Loss Sources

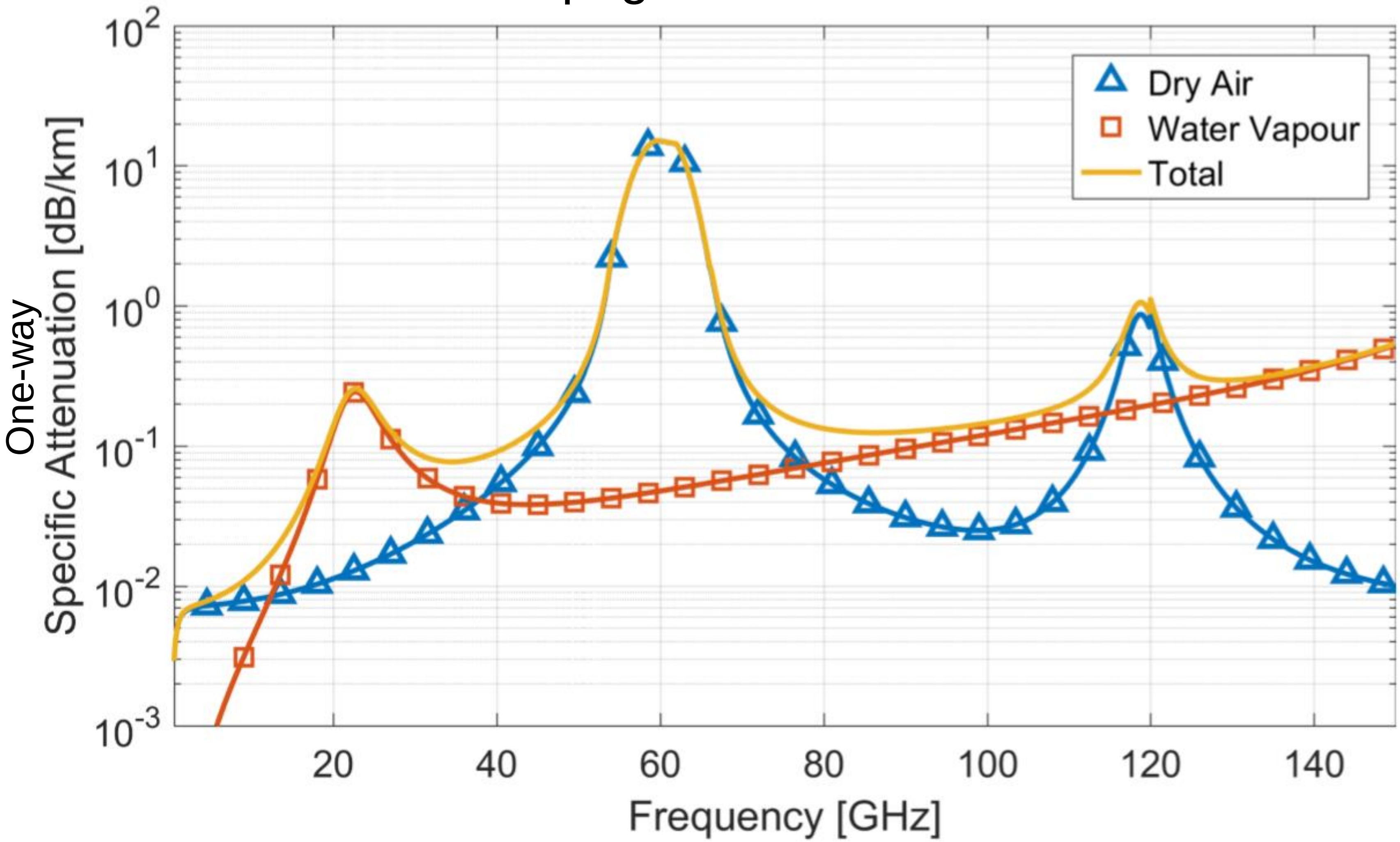
- Received power calculation so far is for an ideal radar in free space with no processing to increase sensitivity
- Real systems suffer losses in duplexers, waveguide, power dividers, radome, *etc.* represented by a *system loss factor* L_s
- Also suffer atmospheric propagation losses
  - function of range
  - with R in meters, loss factor α in dB/km, we have

$$L_a(R) = 10^{\alpha R/5000}$$

$$L_a\left(R_{[\mathrm{m}]}\right)_{[\mathrm{dB}]} = 10\log_{10}\left[L_a\left(R_{[\mathrm{m}]}\right)\right] = 10\,\alpha_{[\mathrm{dB/km}]}\,\frac{R_{[\mathrm{m}]}}{5000} = \alpha_{[\mathrm{dB/km}]}\,2\,\frac{R_{[\mathrm{m}]}}{1000} = \alpha_{[\mathrm{dB/km}]}\,2\,R_{[\mathrm{km}]}$$

$$L_a\left(R_{[\mathrm{km}]}\right)_{[\mathrm{dB}]} = \alpha_{[\mathrm{dB/km}]}\,2\,R_{[\mathrm{km}]}$$

Propagation Losses



ITU-R, Rec. ITU-R P.676-9 Attenuation by Atmospheric Gases, P Series, Radiowave propagation.

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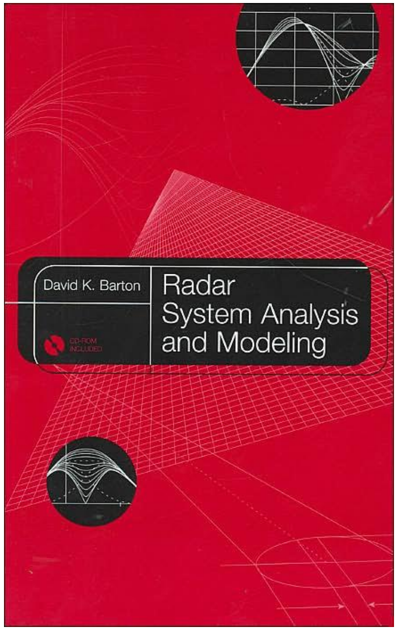
Radar Range Equation for a Point Target

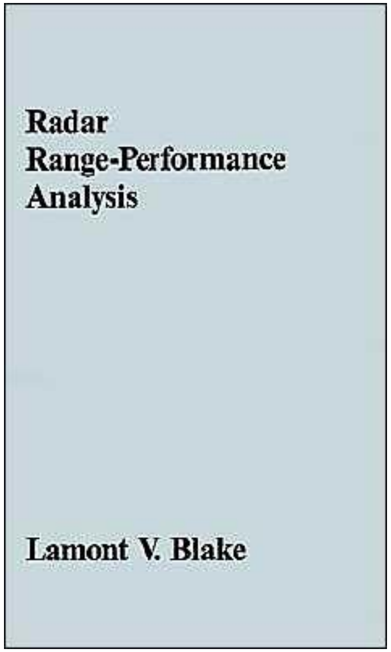
- Adding in loss factors gives:

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_s L_a(R)}$$

- Note for a point target, received power decreases as R^4:
  - Doubling range requires
    - 16x (12 dB) transmitted power increase, *or*
    - 4x (6 dB) antenna gain increase ⇒ 4x antenna area increase
  - Stealth: Halving detection range requires 16x decrease in RCS σ

David K. Barton: "*Radar Systems Analysis and Modeling* "
Artech House, 2004





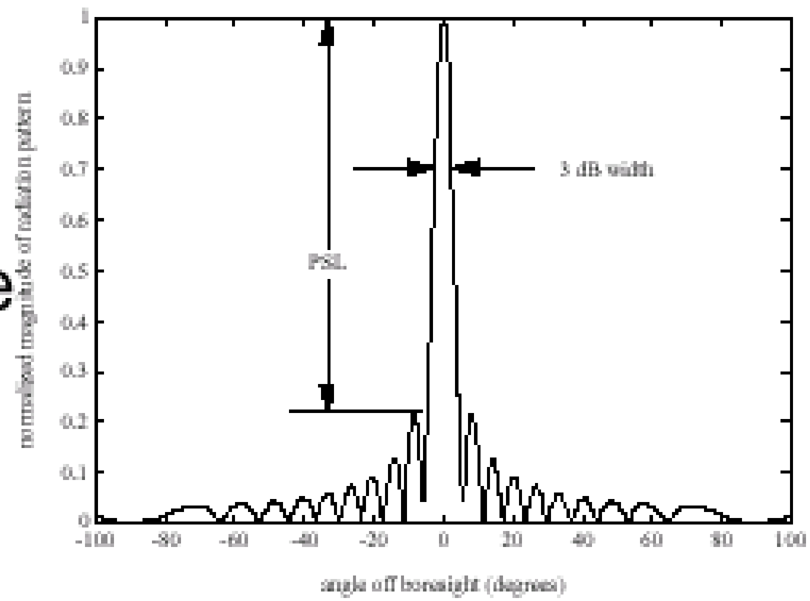
Lamont V. Blake: "*Radar Range Performance Analysis* "
D. C. Heath 1980 (Artech House, 1986)

Aperture Antennas

- Parabolic dishes, flat plates, *etc.*
- Far-field antenna radiation pattern is the Fourier transform of the aperture current distribution:

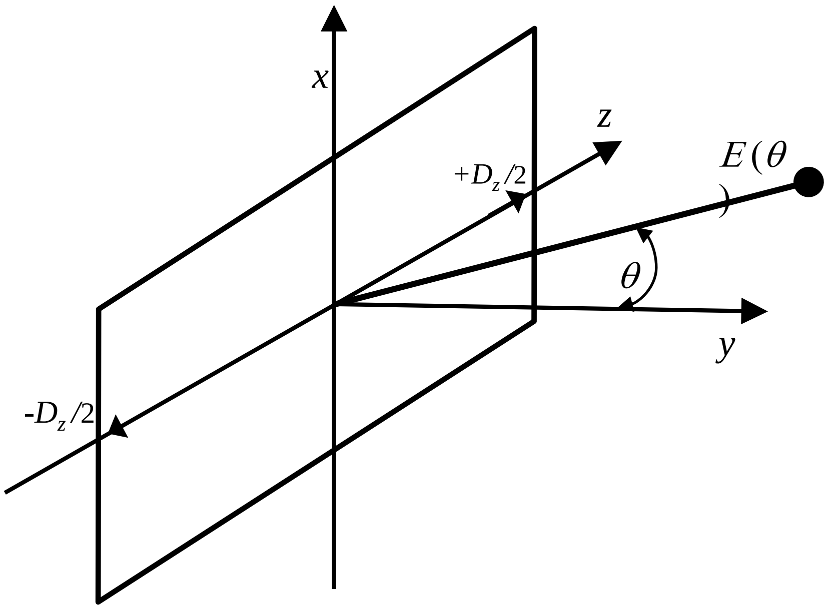
$$E(\theta)=\int_{-D_z/2}^{D_z/2} A(z)e^{\left[j\frac{2\pi z}{\lambda}\sin\theta\right]}dz$$

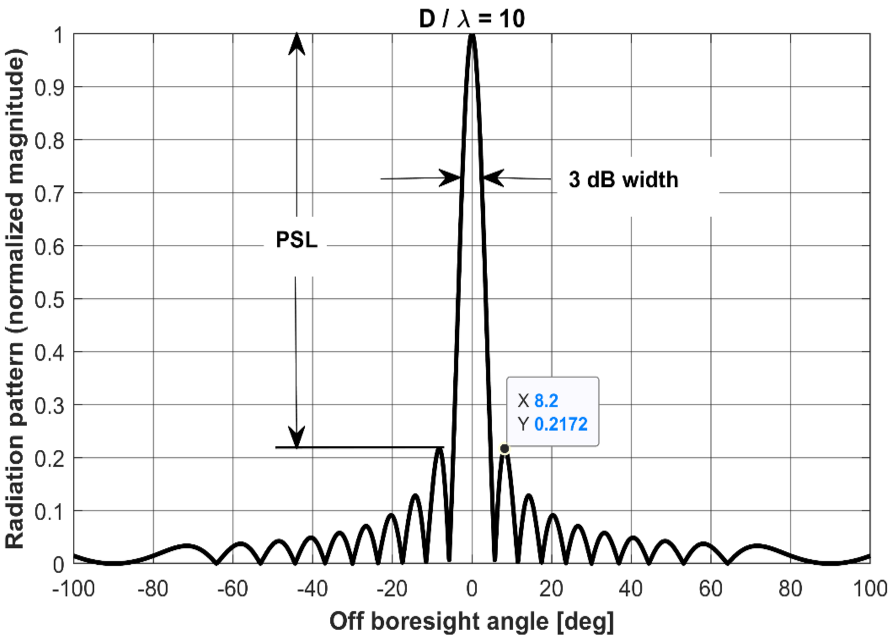
- Important parameters are
  - peak gain
  - peak sidelobe
  - mainbeam (3 dB) width



Uniform Current Distribution Antenna

Simplest case





$$E(\theta)=\frac{\sin\left[\pi\left(D_z/\lambda\right)\sin\theta\right]}{\pi\left(D_z/\lambda\right)\sin\theta}$$

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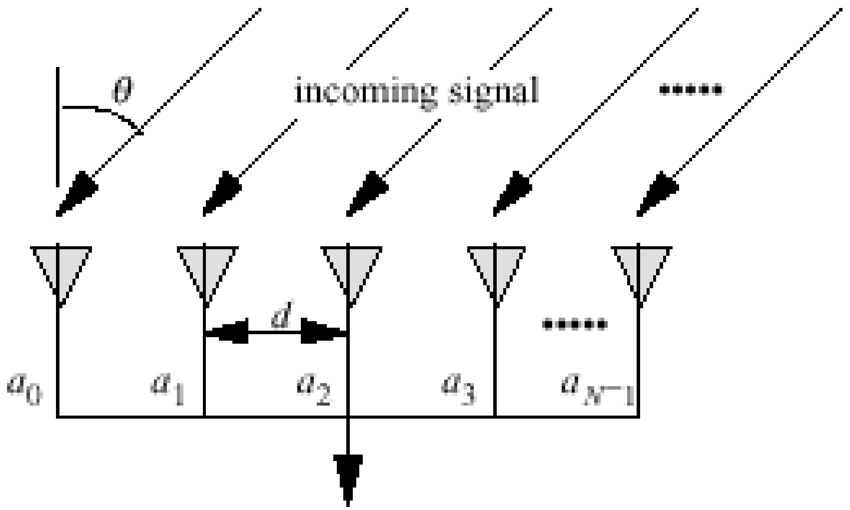
Array Antennas

- Uniform array of individual elements
- Conducive to digital beamforming



- Antenna pattern is product of *array factor* and *element pattern:*

$$E(\theta) = AF(\theta)E_{el}(\theta)$$

$$AF(\theta) = E_0 \sum_{n=0}^{N-1} a_n e^{j(2\pi/\lambda)nd\sin\theta}$$

$$E_{el}(\theta) \cong \cos\theta$$

- Linear array case: $\left|AF(\theta)\right| = E_0 \left| \dfrac{\sin\left[N(\pi d/\lambda)\sin\theta\right]}{\sin\left[(\pi d/\lambda)\sin\theta\right]} \right|$

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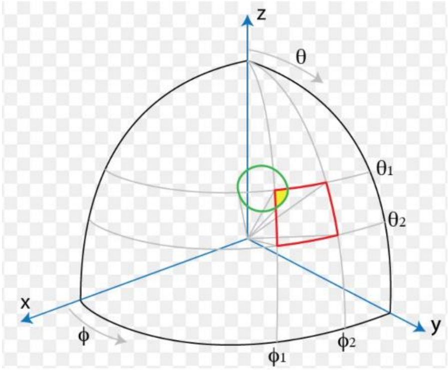
Approximations to Antenna Parameters



$$G \approx \frac{20,000}{\theta_3 \phi_3} \quad (\theta_3, \phi_3 \text{ in degrees})$$

$$= \frac{6.1}{\theta_3 \phi_3} \quad (\theta_3, \phi_3 \text{ in radians})$$

Number of square degrees in a sphere: $4\pi(180/\pi)^2 = 41253$

Typical ant. Efficiency= 0.5

$41253*0.5 \cong 20000$

- Beamwidth: $\theta_3 = 2\sin^{-1}\left(\frac{1.4\lambda}{\pi D_z}\right) \approx 0.89\frac{\lambda}{D_z}$ radians

$$l_{circle} = 2\pi r, \quad l^0 = 360 \Rightarrow r = \frac{l^0}{2\pi} = \frac{360}{2\pi} = \frac{180}{\pi}$$

$$s_{sphere} = 4\pi r^2, \; s^0_{sphere} = 4\pi\left(\frac{180}{\pi}\right)^2 = 41253$$

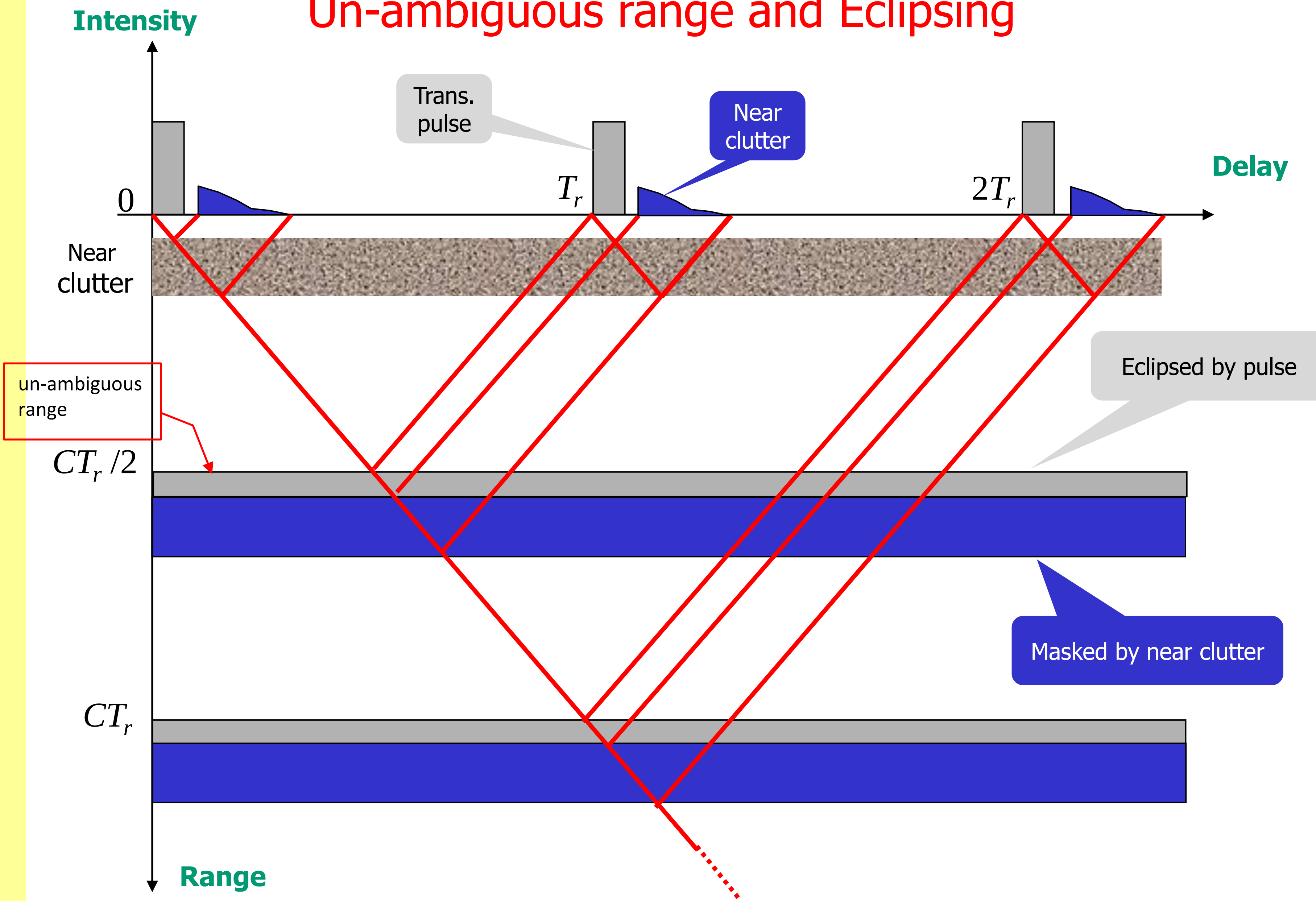
Range Equation Example

- X band (10 GHz)
  - $\lambda = 3\text{x}10^8/10\text{x}10^9 = 3$ cm
- Transmitted power $P_t = 1$ kW
- Beamwidth (azimuth and elevation) = 1°
  - $G = 20{,}000/(1)(1) = 20{,}000 = 43$ dB
- Jumbo jet aircraft: RCS = 100 m²
- Range $R = 10$ km
- What is received power P_r?

$$P_r = \frac{(1{,}000)(20{,}000)^2(0.03)^2(100)}{(4\pi)^3(10{,}000)^4} = 1.814\text{x}10^{-9}$$

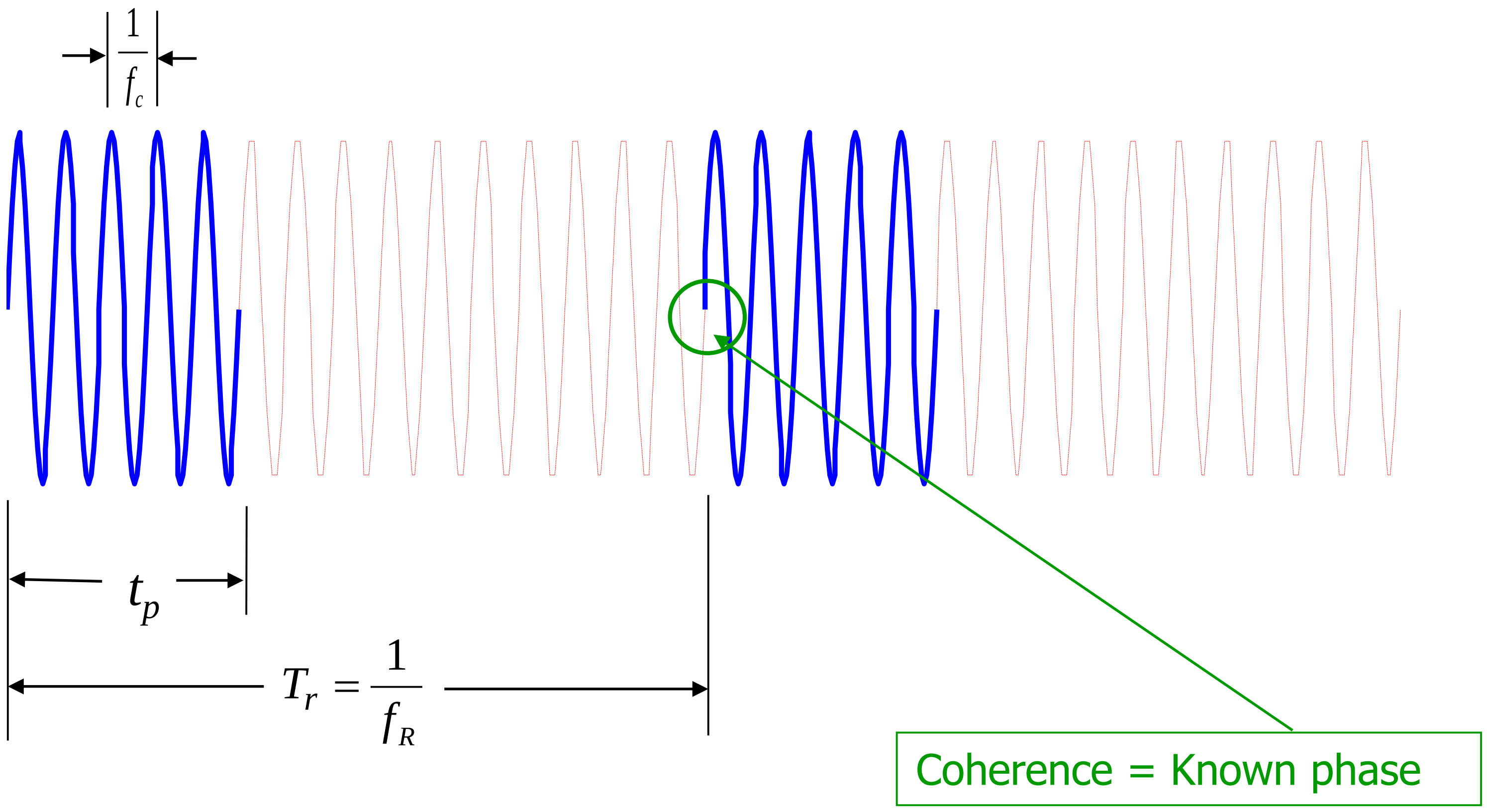
12 orders of magnitude!

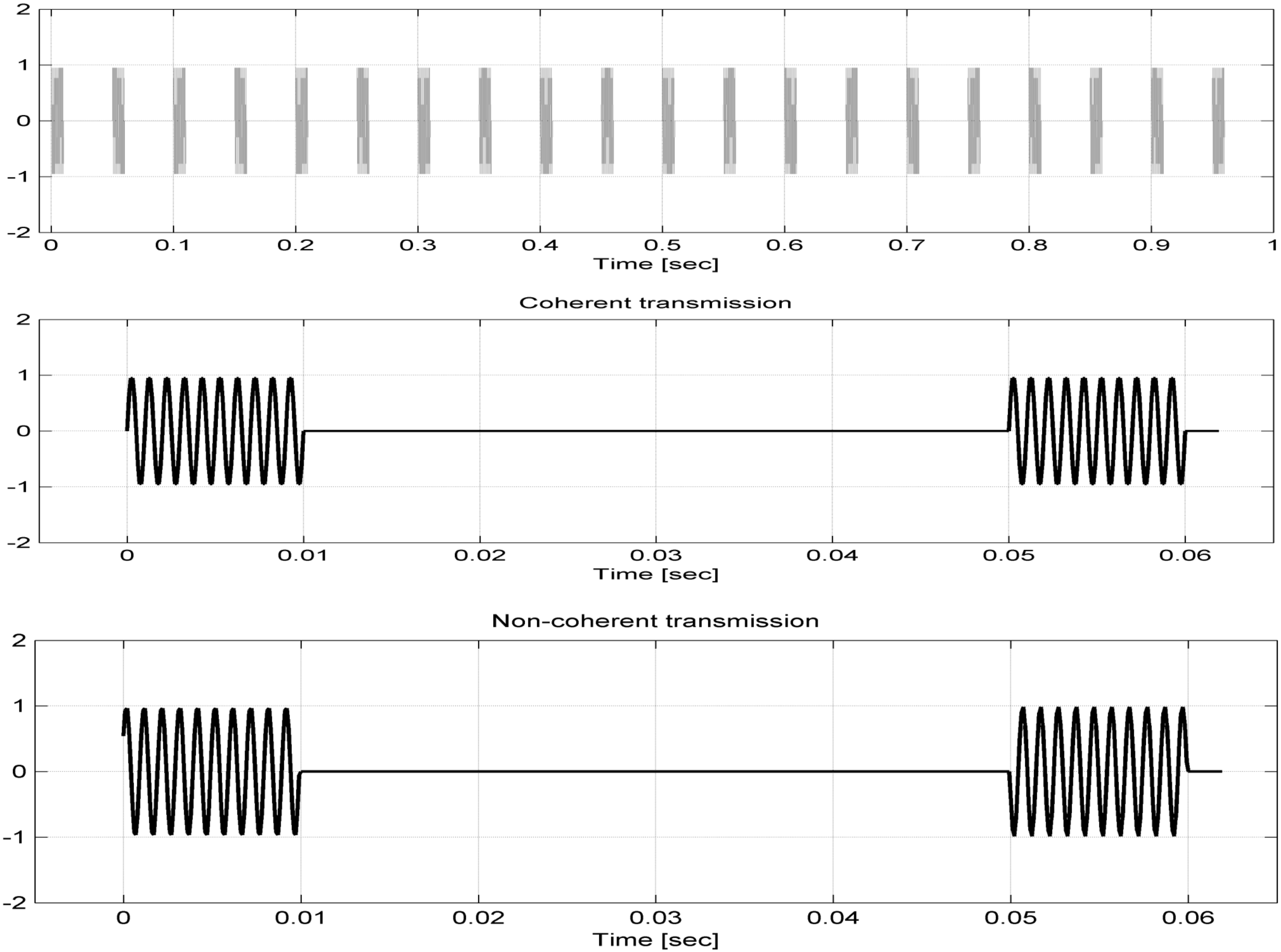
Un-ambiguous range and Eclipsing



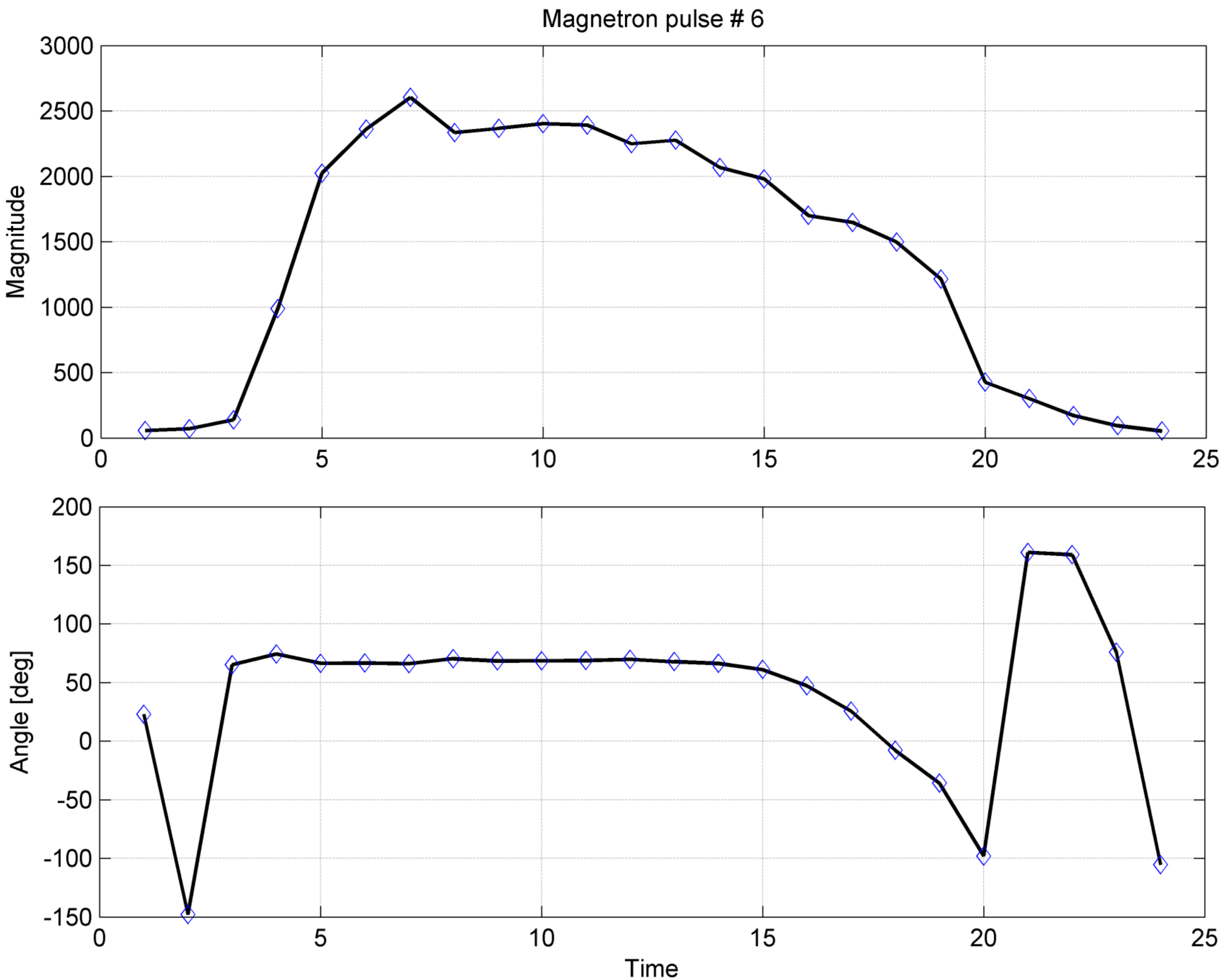
SNR version of the radar equation

(Will be demonstrated on a coherent pulse train, but true for all coherent signals)

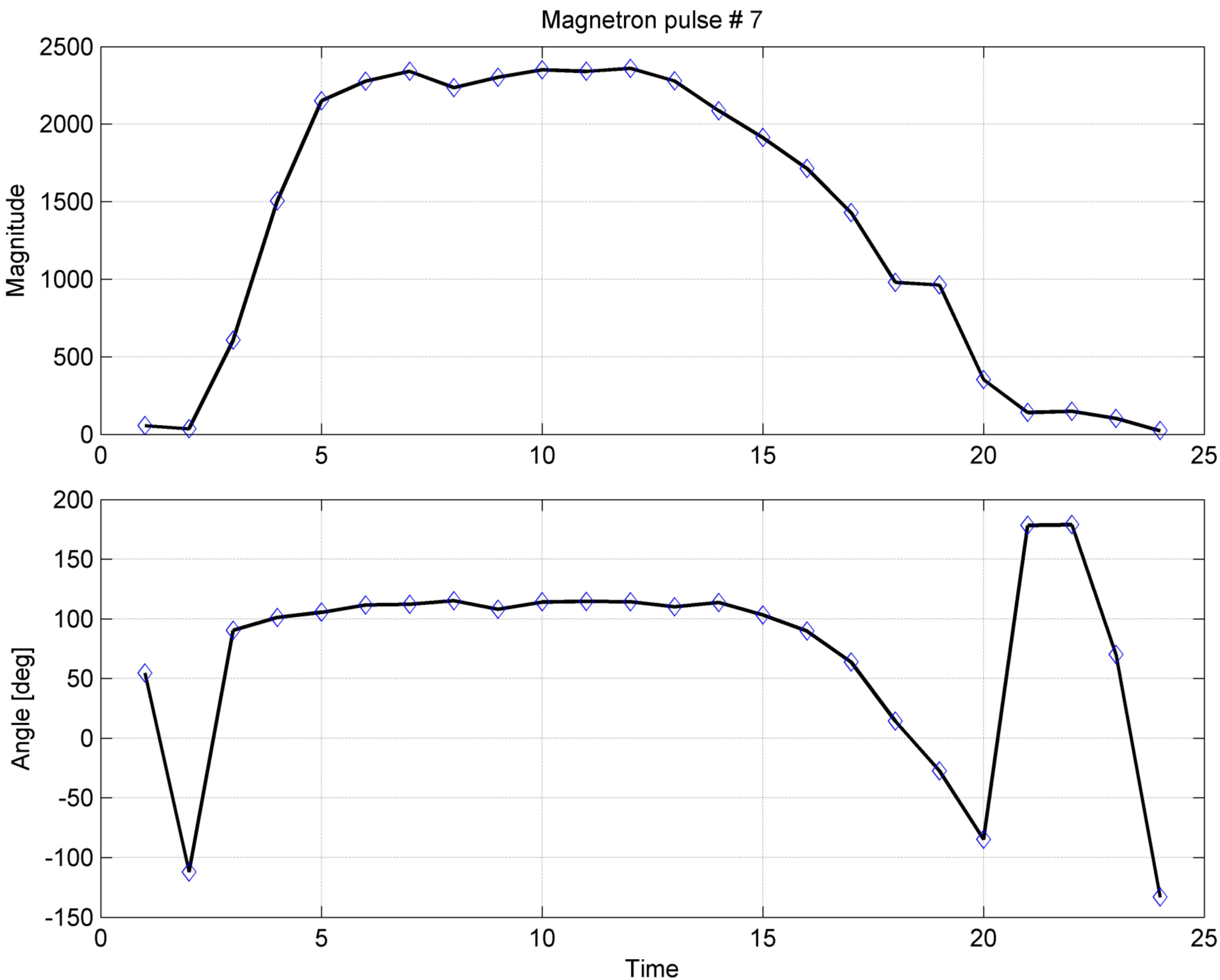



2
1
0
-1
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time [sec]
Coherent transmission
0
0.01
0.02
0.03
0.04
0.05
0.06
Time [sec]
Non-coherent transmission
Time [sec]

Example of non-coherent magnetron pulses



ITT-Gilfillan
GCA – Mark 5


Magnetron pulse # 7
Magnitude
2500
2000
1500
1000
500
0
0
5
10
15
20
25
Angle [deg]
200
150
100
50
0
-50
-100
-150
0
5
10
15
20
25
Time

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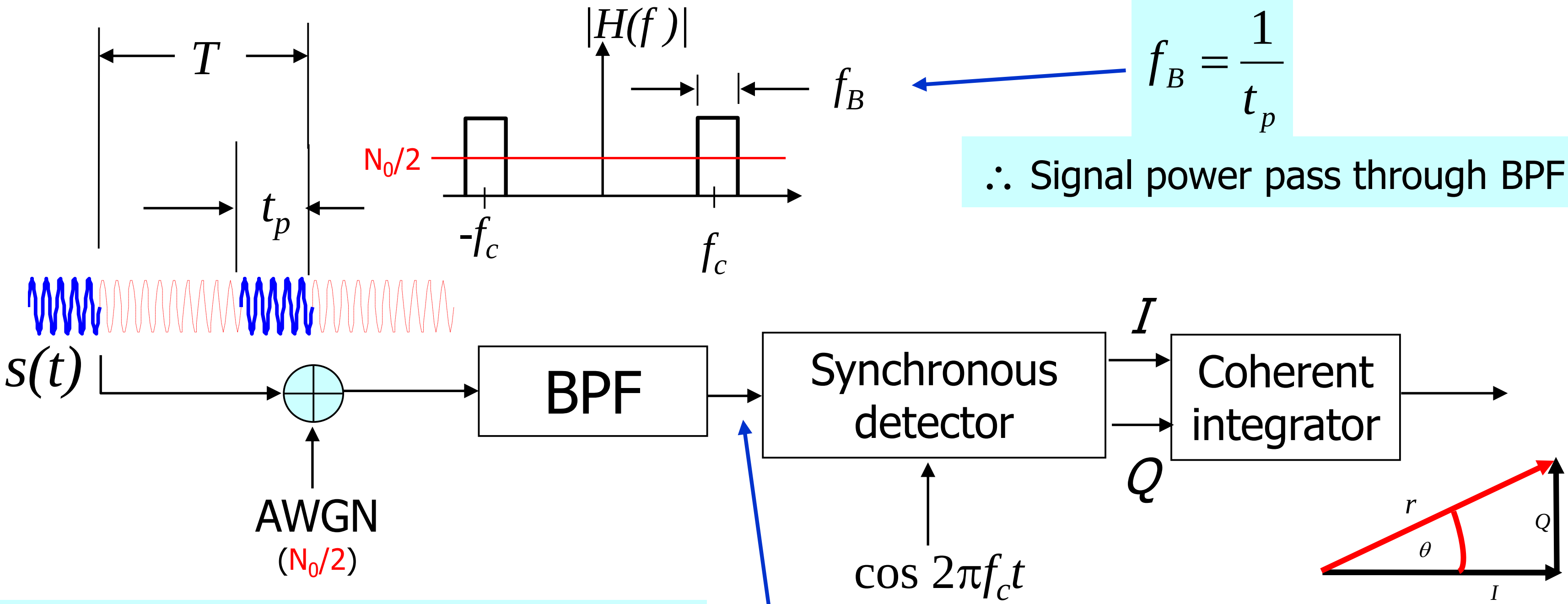


$$f_B = \frac{1}{t_p}$$

∴ Signal power pass through BPF

SNR_p = for a single pulse

SNR = for integration time

T_I = Coherent integration interval
= **C**oherent **P**rocessing **I**nterval (**CPI**)

M = number of pulses in T_I

$$SNR_p = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 N_0 f_B}$$

$$M = \frac{T_I}{T} = T_I f_R$$

$$SNR = M\, SNR_p$$

$$P_{ave} = \frac{P_T t_p}{T} = \frac{P_T M}{f_B T_I}$$

$$SNR = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \frac{P_{ave} T_I}{N_0}$$

F - Receiver's noise figure

K - Boltzman coefficient = 1.38E-23 $\left[\text{Watt}\cdot\text{sec}/{}^{0}\text{K}\right]$

T_e - Effective temperature $\left[{}^{0}\text{K}\right]$

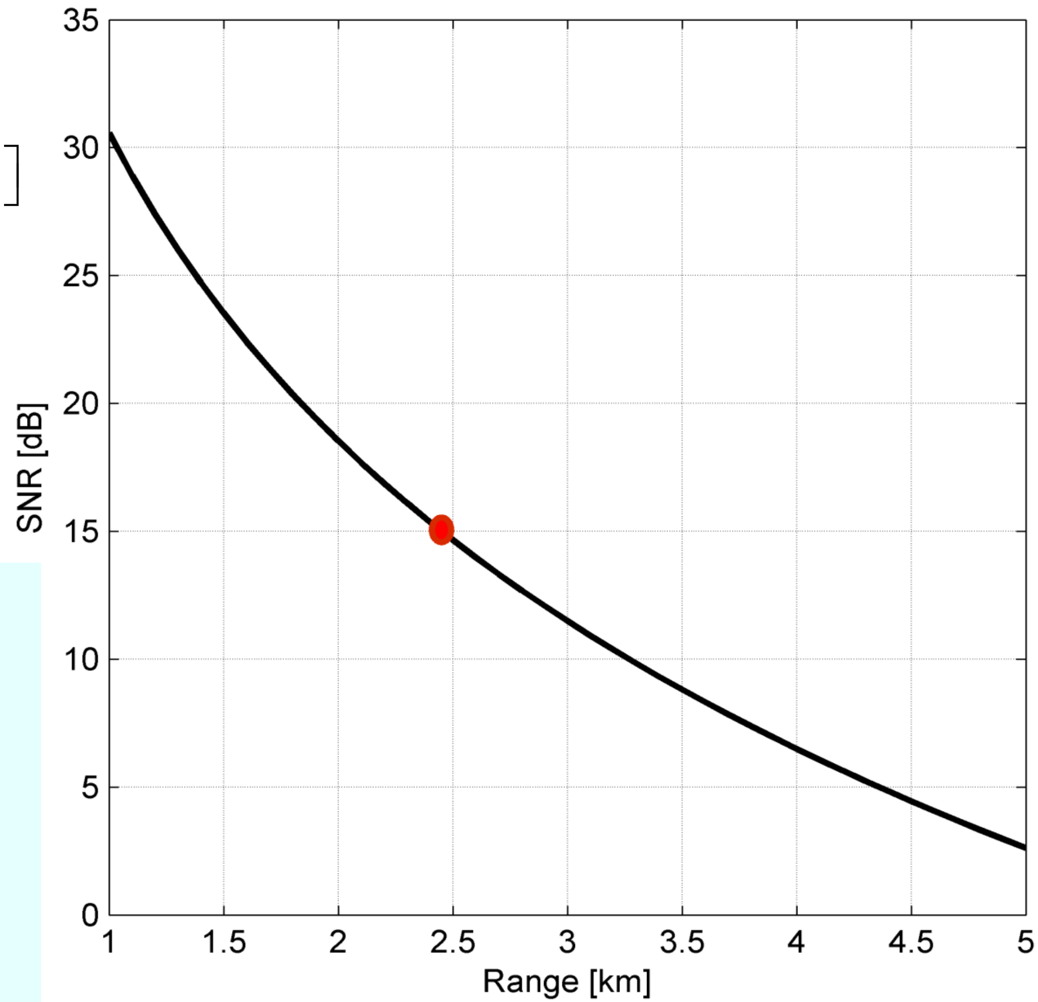
$$N_0 = F\,K\,T_e$$

```
pt=3;% transmitted power in W
g=50; % antenna gain
rcs=100; % target RCS in square meters
fc=3e9; % carrier frequency in Hz
c= 3e8; % velocity of propagation in m/s
k= 1.38e-23; % Boltzman coeff in Watt*sec/degK
te=300; % effective temperature in Kelvin =273+27
f=1.6; % noise figure (good LNA)
tp=2e-6; % pulse width in sec

fb=1/tp; % receiver bandwidth (one sided)
lambda=c/fc;
n0=f*k*te;

r=1000:100:5000;
snr=pt * g^2 * lambda^2 * rcs / ((4*pi)^3 * n0 * fb)
*r.^-4;
snrdb=10*log10(snr);

figure(1)
plot(r/1000,snrdb,'k', 'linewidth',2)
xlabel(' Range [km] ')
ylabel(' SNR [dB] ')
grid on
```



$$SNR_p = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 N_0 f_B}$$

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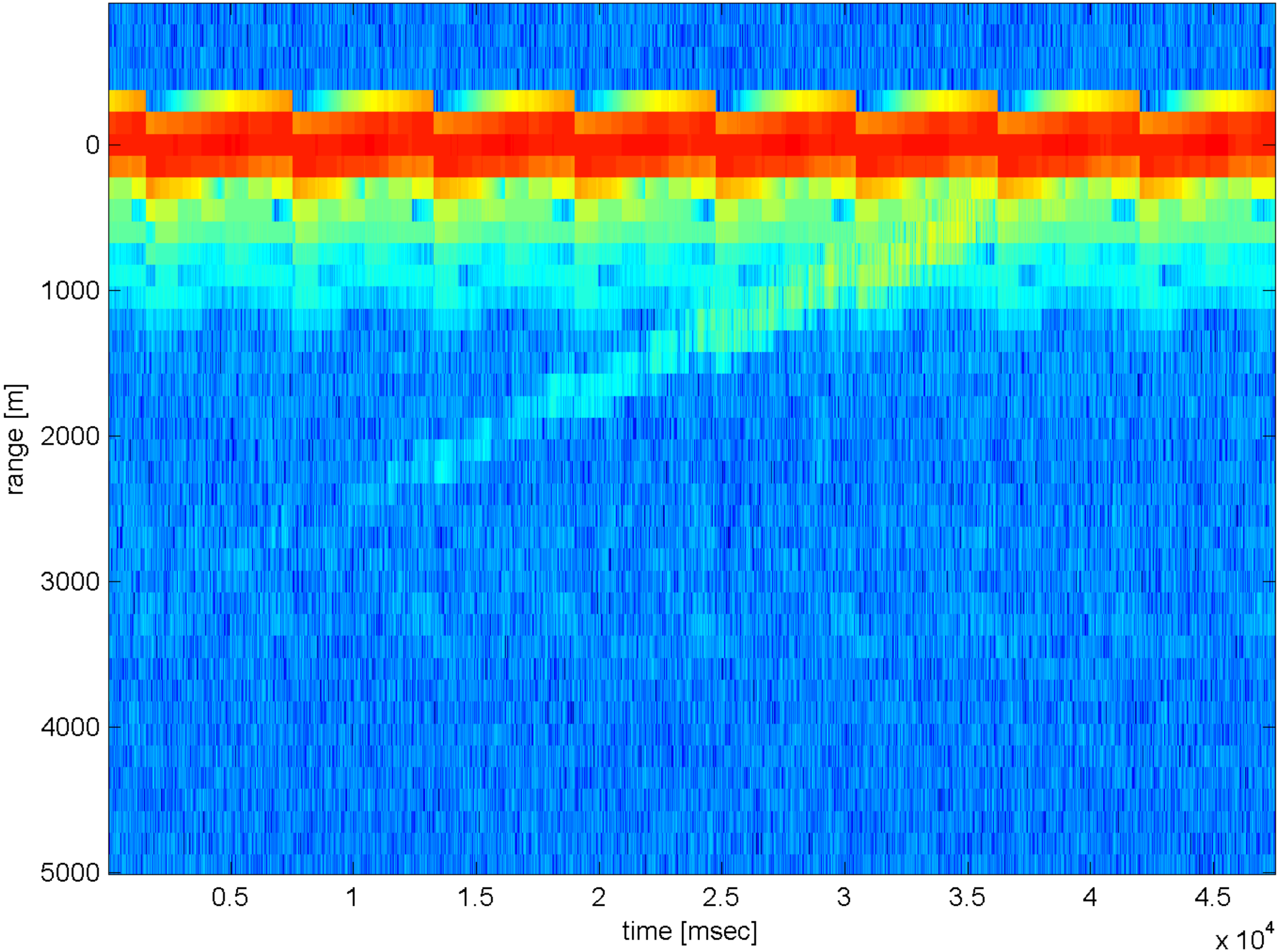
Intensity on Range/Time Map

```
pt=3;% transmitted power in W
g=50; % antenna gain
fc=3e9; % carrier frequency in Hz
f=1.6; % noise figure (good LNA)
tp=2e-6; % pulse width in sec
```

Recorded reflection from a jetliner approaching Ben-Gurion airport



$$SNR = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \frac{P_{ave} T_I}{N_0}$$

SNR determines the detection probability

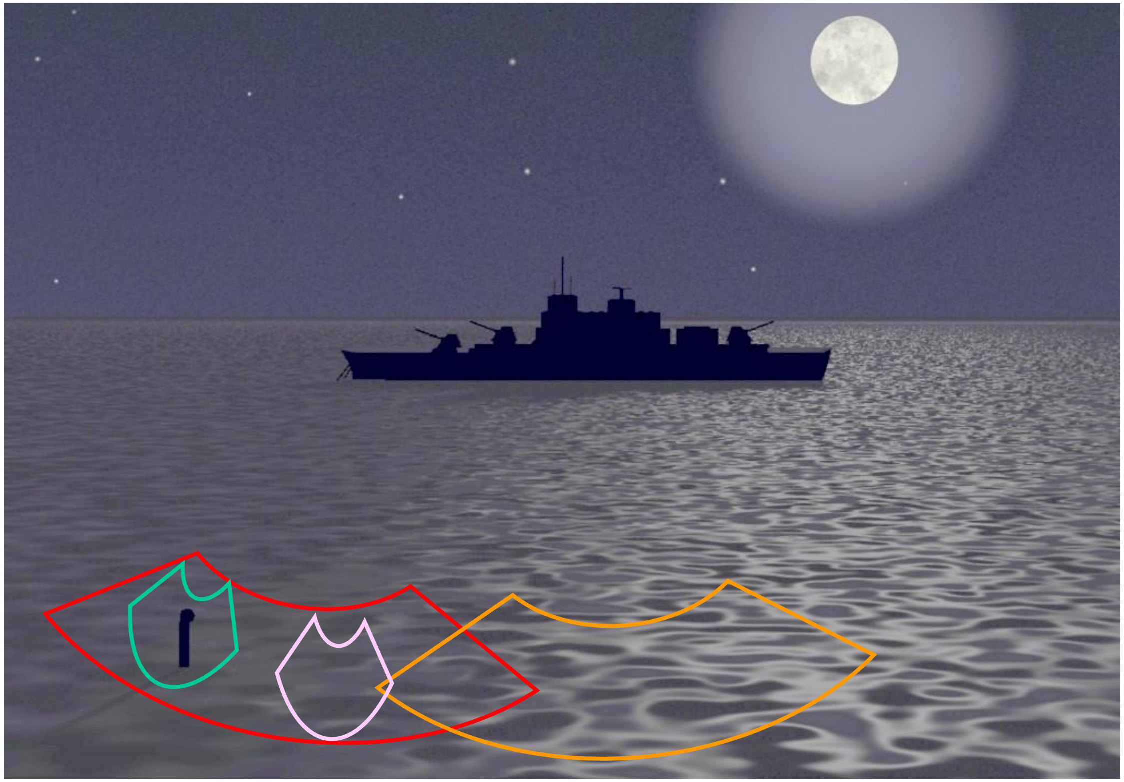
SNR determines the accuracy of measurement (range, velocity, angle)

The noise in the expression of SNR was thermal noise

$SN_{thermal}R$ can be improved by increasing the transmitted power

Returns from ground or sea are also “noise” and can undermine detection or measurements

$SC_{lutter}R$ can be improved by decreasing range, Doppler and/or azimuth resolution



Basic Topics

The nature of the target: RCS, RCS statistics, location, motion.

The competition (background): Multipath, ground reflection (clutter), noise.

Clutter: Normalized RCS, angular dependence, statistics.

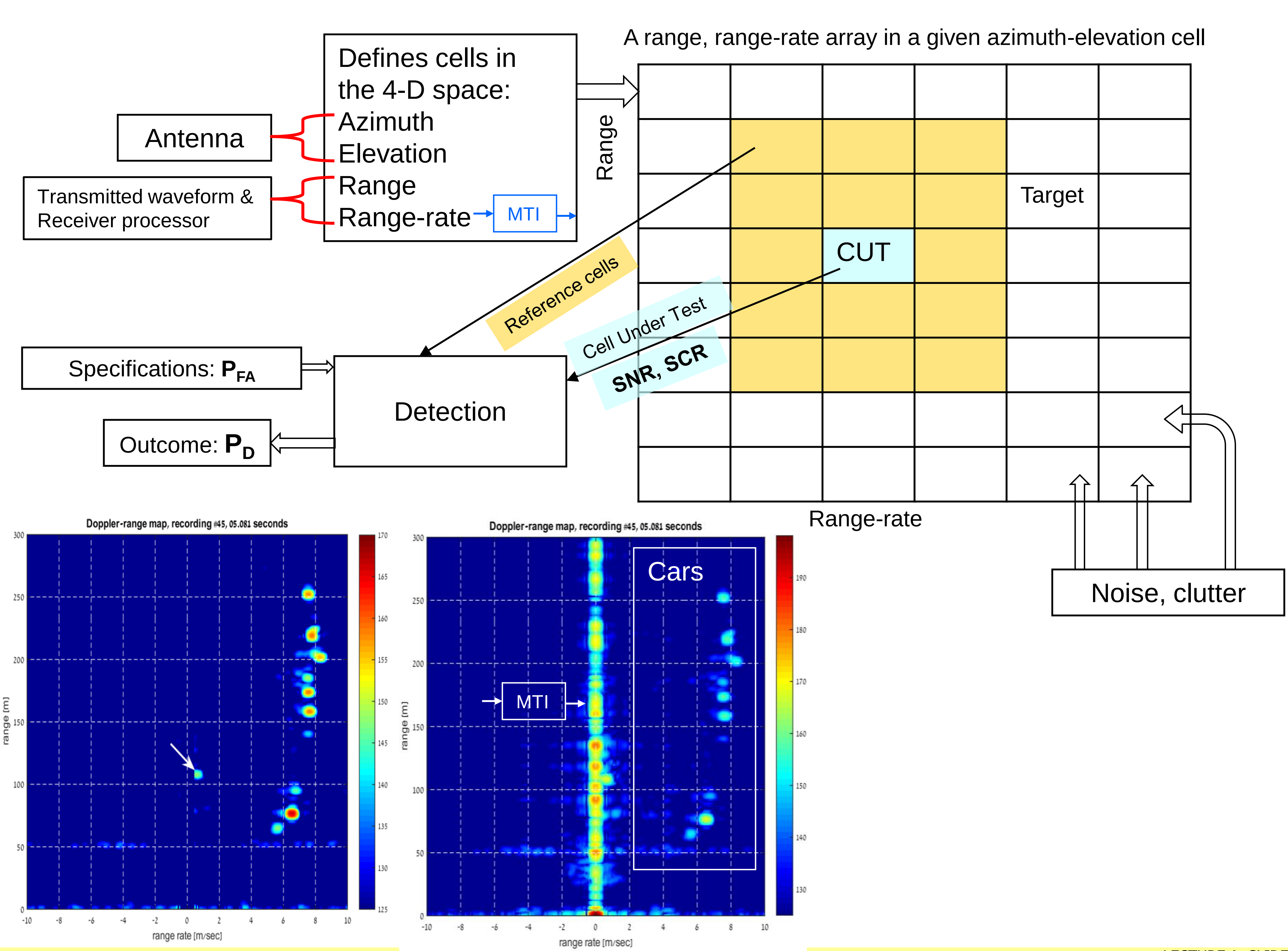
Selecting (defining) **a cell location in the 4-D space**: Azimuth, elevation, range, range-rate , or in the 6-D space: $\{x, y, z, \dot{x}, \dot{y}, \dot{z}\}$

Minimizing the cell size: In order to minimize the competition (From: clutter, other targets, noise).

Minimizing ambiguities: Preventing returns from other cells to add erroneously to the return in the desired cell.

Detection: Declare if there is a target in the defined cell, with high probability of detection P_D and low probability of false alarm P_{FA} .

Improving the detection performances using `integration` (coherent or non-coherent) of returns from many pulses, using possible `motion` of the target or the radar, using reference `neighbor cells` to define an adaptive detection threshold.


A range, range-rate array in a given azimuth-elevation cell
Defines cells in the 4-D space:
Azimuth
Elevation
Range
Range-rate
MTI
Antenna
Transmitted waveform & Receiver processor
Range
Target
CUT
Reference cells
Cell Under Test
SNR, SCR
Specifications: P_{FA}
Detection
Outcome: P_D
Range-rate
Noise, clutter
Doppler-range map, recording #45, 05.081 seconds
range [m]
range rate [m/sec]
Doppler-range map, recording #45, 05.081 seconds
Cars
MTI
range [m]
range rate [m/sec]

Electromagnetic propagation in the Earth atmosphere

$$C_p = \frac{C}{n}$$

C = Speed of light in vacuum = 2.997925x10^8 m/s ≈3x10^8 m/s

n = refractive index

$$N = (n-1)10^6$$

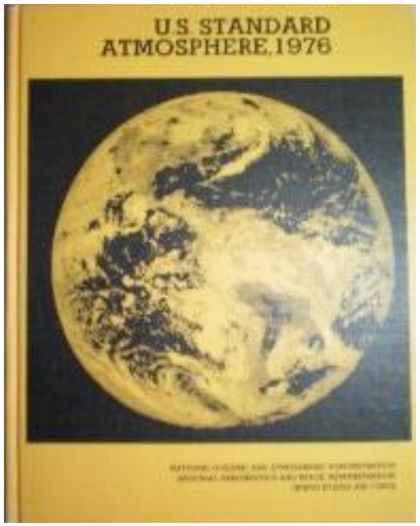
$$N = \frac{77.6}{T}\left(P + 4810\frac{e}{T}\right)$$

T - air temperature in ^{o}K

P - air pressure in milibar

e - water vapor presure in milibar

Refraction index as function of height for standard atmosphere



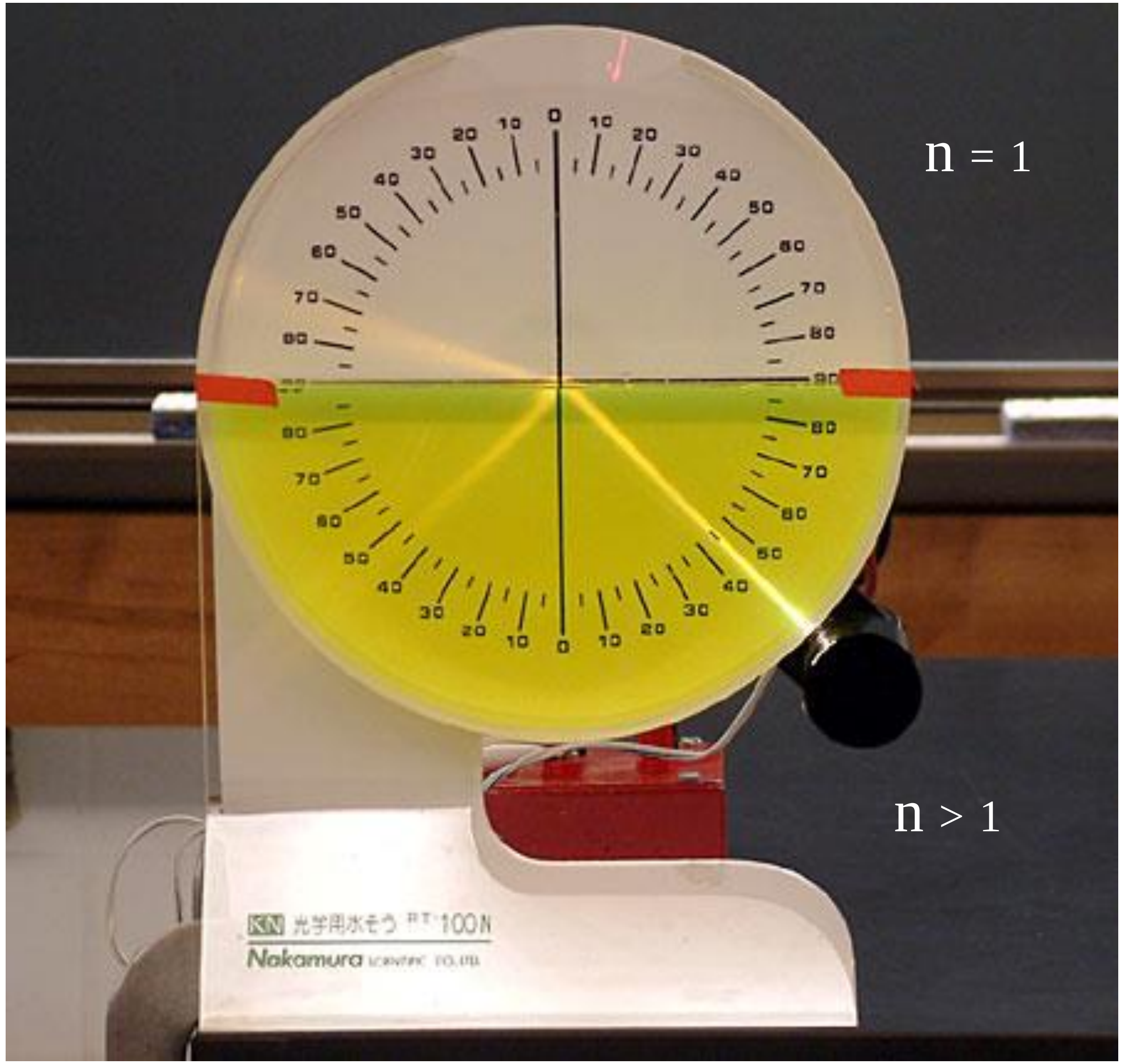
H [km]	N		N_s-[h/(4a)]10^6
0	319 = N_s		319
1	277	≈	279
3	216		201
10	92		-
20	20		-
50	0.2		-

$$N = (n-1)10^6$$

$$n = 1 + \frac{N}{10^6}$$

$$n(h) \approx n_s - \frac{h}{4a} \quad , \quad h < 5\,\text{km}$$

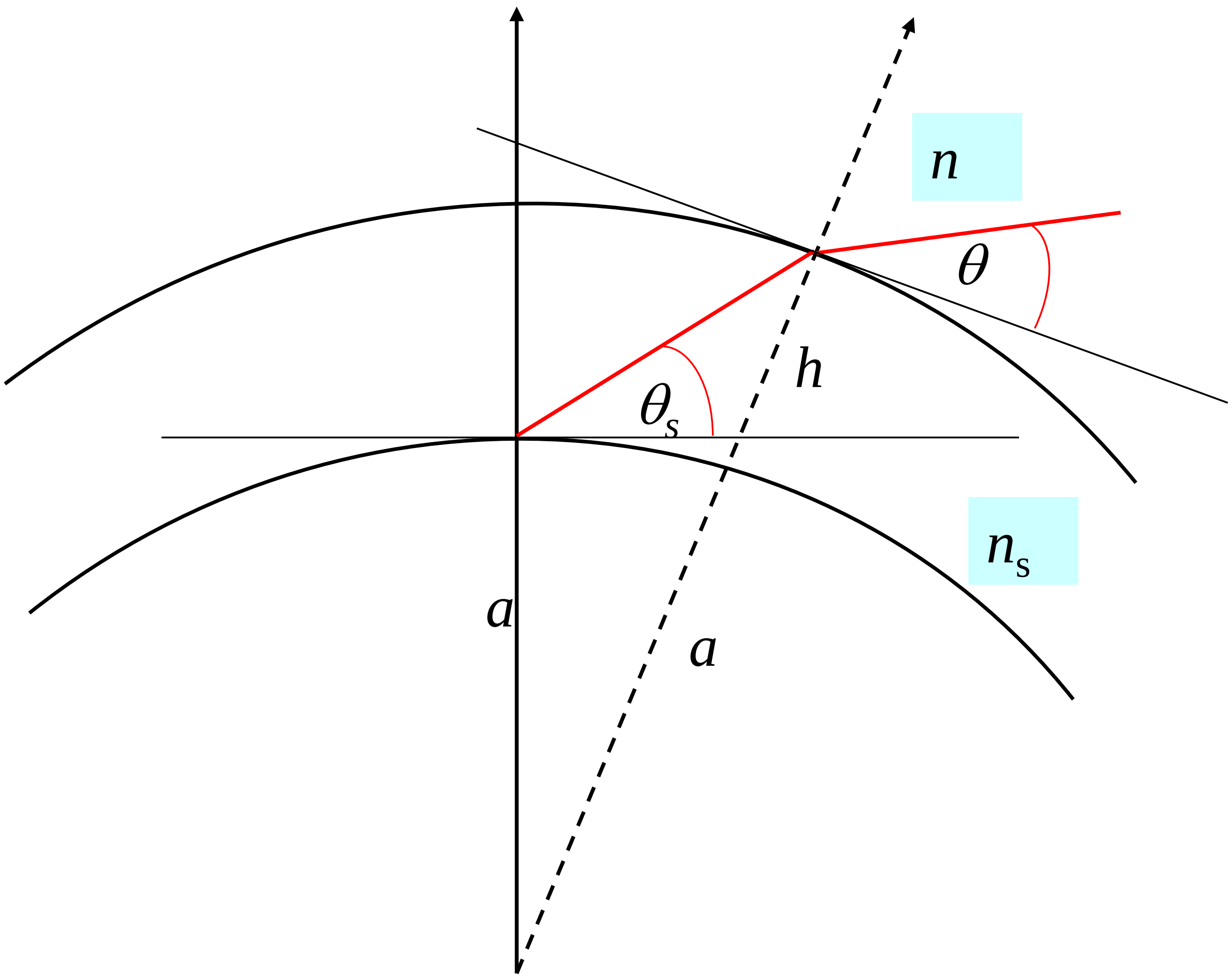
a = Earth radius = 6370 km


n = 1
n > 1
KN 光学用水そう RT-100N
Nakamura

Nadav Levanon, Tel-Aviv University

Snell's law in polar coordinates:

$$n_s a \cos\theta_s = n(a+h)\cos\theta$$



$$n_s a \cos\theta_s = n(a+h)\cos\theta$$

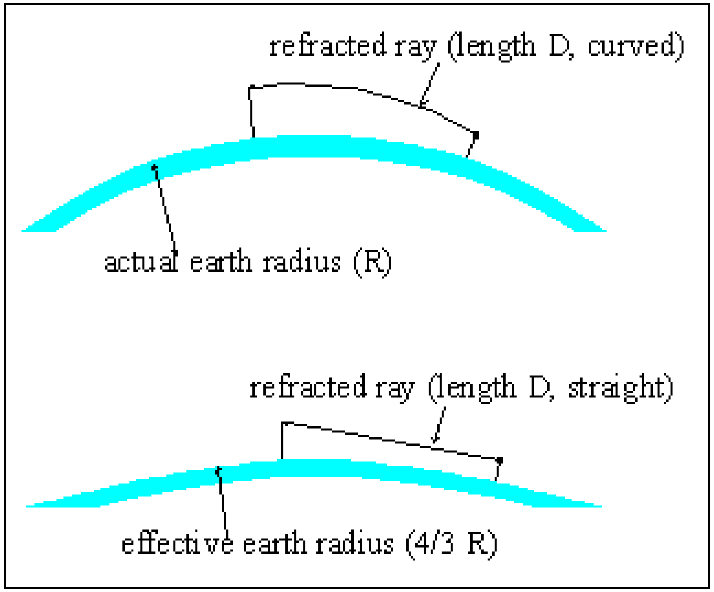
$$\cos\theta = \frac{a}{a+h}\frac{n_s}{n}\cos\theta_s$$

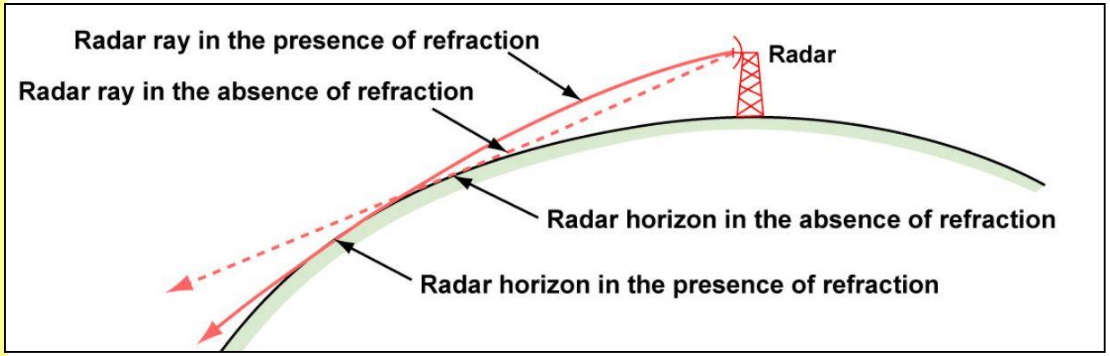
$$n_s \approx 1$$
$$h << a$$

$$n = n_s - \frac{h}{4a}$$

$$\cos\theta \approx \frac{\frac{4}{3}a}{\frac{4}{3}a+h}\cos\theta_s$$

$\therefore$ If we assume an Earth radius of $\frac{4}{3}a$ we can ignore the change in n with height, i.e., ignore refractivity.





$$\frac{\cos\theta_s}{\cos\theta} = \frac{(a+h)\left(n_s - \frac{h}{4a}\right)}{a\,n_s} = \frac{a+h}{a}\left(1 - \frac{h}{4an_s}\right) = \frac{a+h}{a} - \frac{a+h}{a}\frac{h}{4an_s}$$

$$\underset{n_s \approx 1}{\approx} \frac{a+h}{a} - \frac{a+h}{a}\frac{h}{4a} = \frac{a+h}{a}\left(1 - \frac{h}{4a}\right) = \frac{4a^2 + 4ah - ah - h^2}{4a^2}$$

$$= \frac{4a^2 + 3ah - h^2}{4a^2} = 1 + \frac{3}{4}\frac{h}{a} - \frac{h^2}{4a^2} \underset{h<<a}{\approx} 1 + \frac{3}{4}\frac{h}{a} = \frac{4a+3h}{4a} = \frac{\frac{4}{3}a + h}{\frac{4}{3}a}$$

$$\frac{\cos\theta}{\cos\theta_s} = \frac{\frac{4}{3}a}{\frac{4}{3}a + h}$$