DETERMINISTIC TARGETS

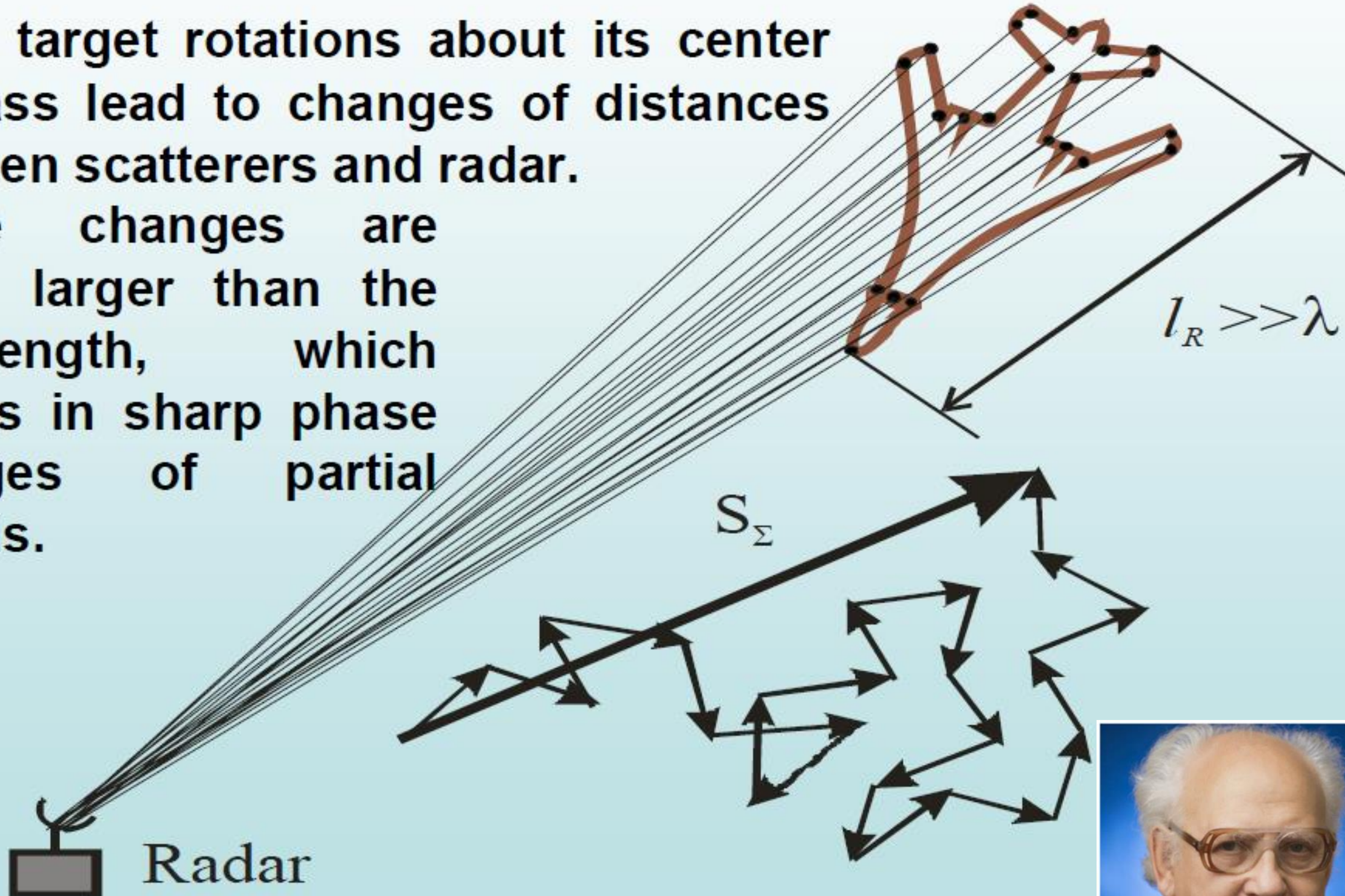
Sphere, plane, corner reflector, antenna

STATISTICAL TARGETS

Multiple scatterers, fluctuating targets, distributions, Swerling models

Small target rotations about its center of mass lead to changes of distances between scatterers and radar.

These changes are much larger than the wavelength, which results in sharp phase changes of partial signals.




Professor Viktor Chernyak

1929 - 2020

Victor Chernyak: ***Tutorial at the RADAR 2010, 10 May, 2010, Arlington VA, USA***

Signal from a Multi-Scatterer Target

- Assume N scatterers
  - individual RCS σ_i
  - fixed individual ranges R_i
- To within a constant, we have by superposition, in field units

$$y(t) = \sum_{i=1}^{N} \sqrt{\sigma_i} \exp\left[j2\pi f \left(t - 2R_i / C\right)\right] = \exp\left(j2\pi ft\right) \sum_{i=1}^{N} \sqrt{\sigma_i} \exp\left(-j4\pi R_i / \lambda\right)$$

The voltage of the envelope detected field is $\varsigma = |y|$

The power, which reflects the RCS is given by

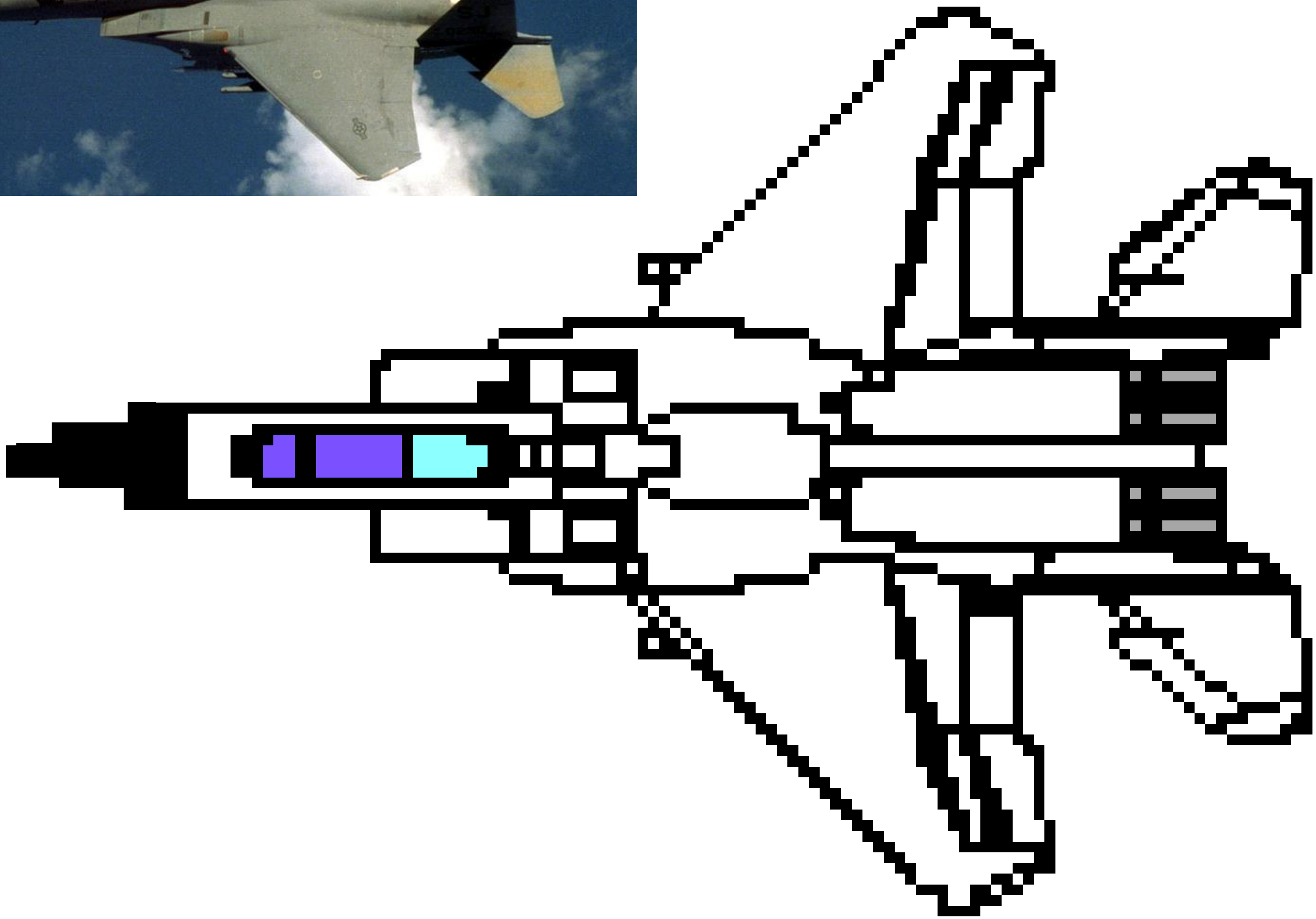
$$\sigma = \varsigma^2 = \left| \sum_{i=1}^{N} \sqrt{\sigma_i} \exp\left(-j4\pi R_i / \lambda\right) \right|^2$$

RCS of Multi-Scatterer Target

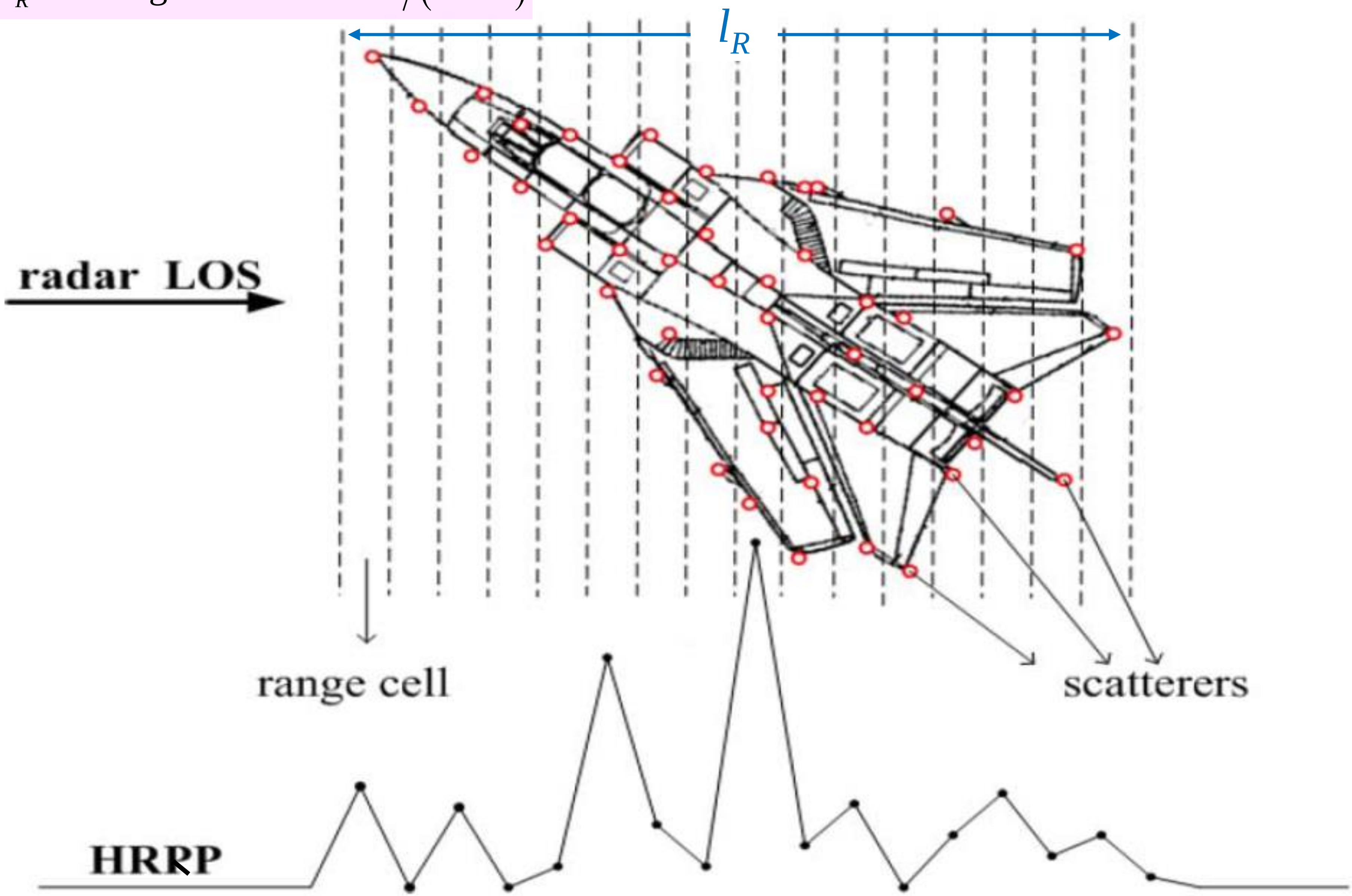
- RCS is proportional to $\left|\bar{y}(t)\right|^2$
- Define the "voltage" $\varsigma \equiv \left|\bar{y}\right| = \left|\sum_{i=1}^{N} \sqrt{\sigma_i}\, e^{-j4\pi R_i/\lambda}\right|$

and "power" (RCS) $\boxed{\sigma = \varsigma^2 = \left|\sum_{i=1}^{N} \sqrt{\sigma_i}\, e^{-j4\pi R_i/\lambda}\right|^2}$

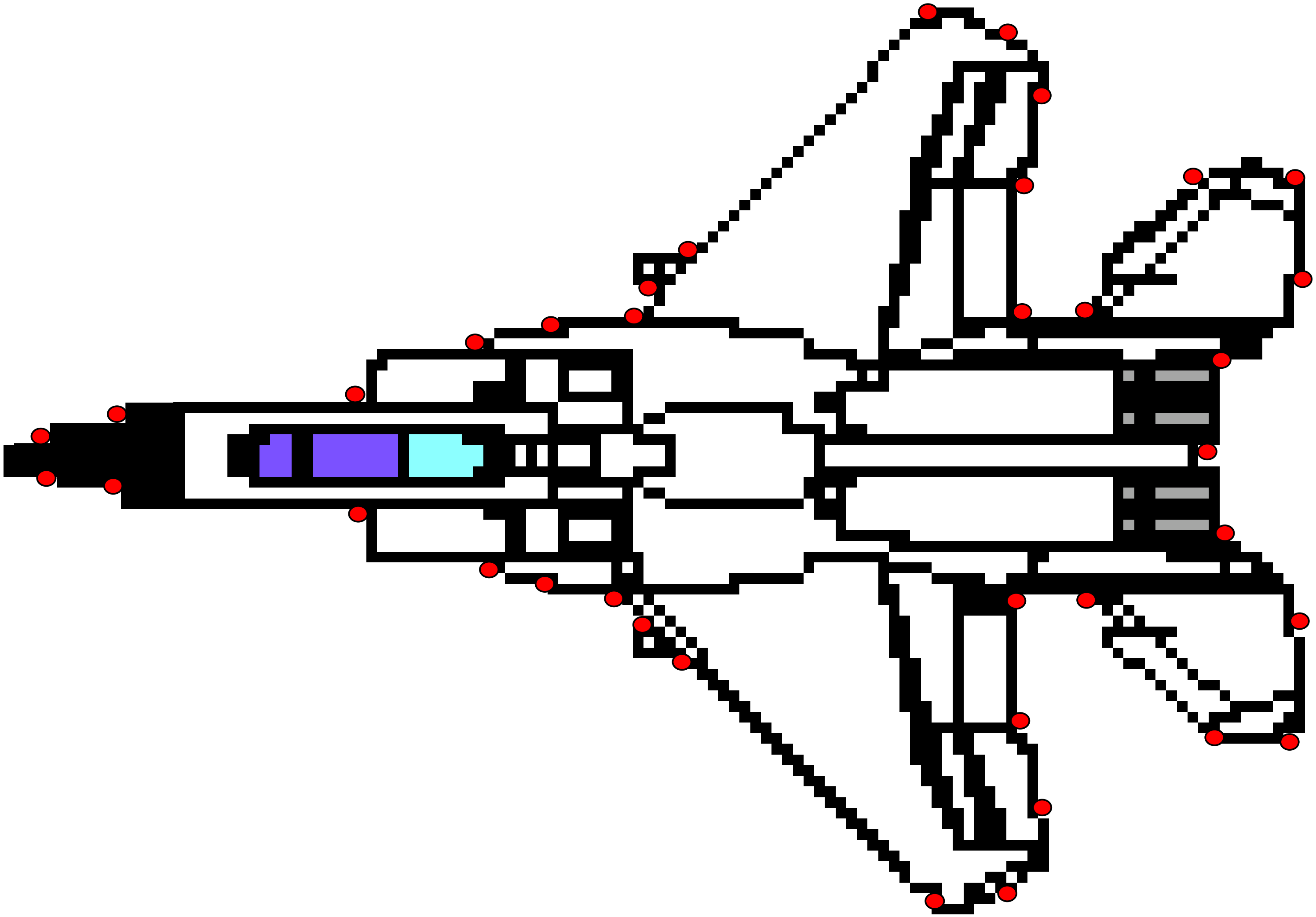

F-15

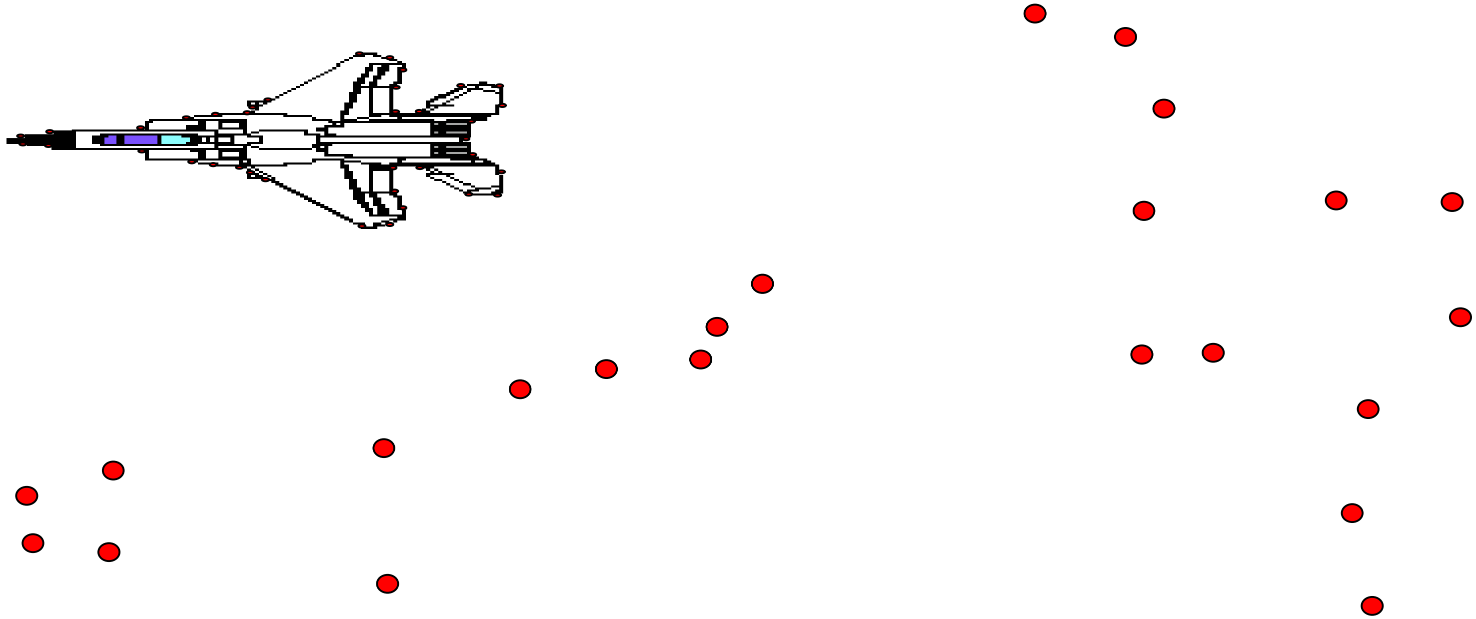


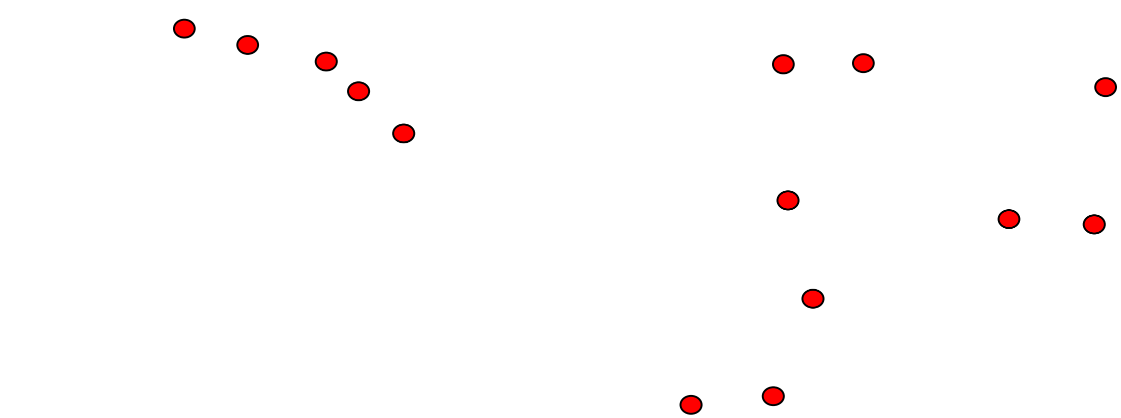
$l_R >> \text{Range resolution} \approx C/(2BW)$



HRRP – High Resolution Range Profile







Assumptions (not realistic but simplify the model):

Isotropical reflectors of identical σ, no shadowing, no multiple reflections

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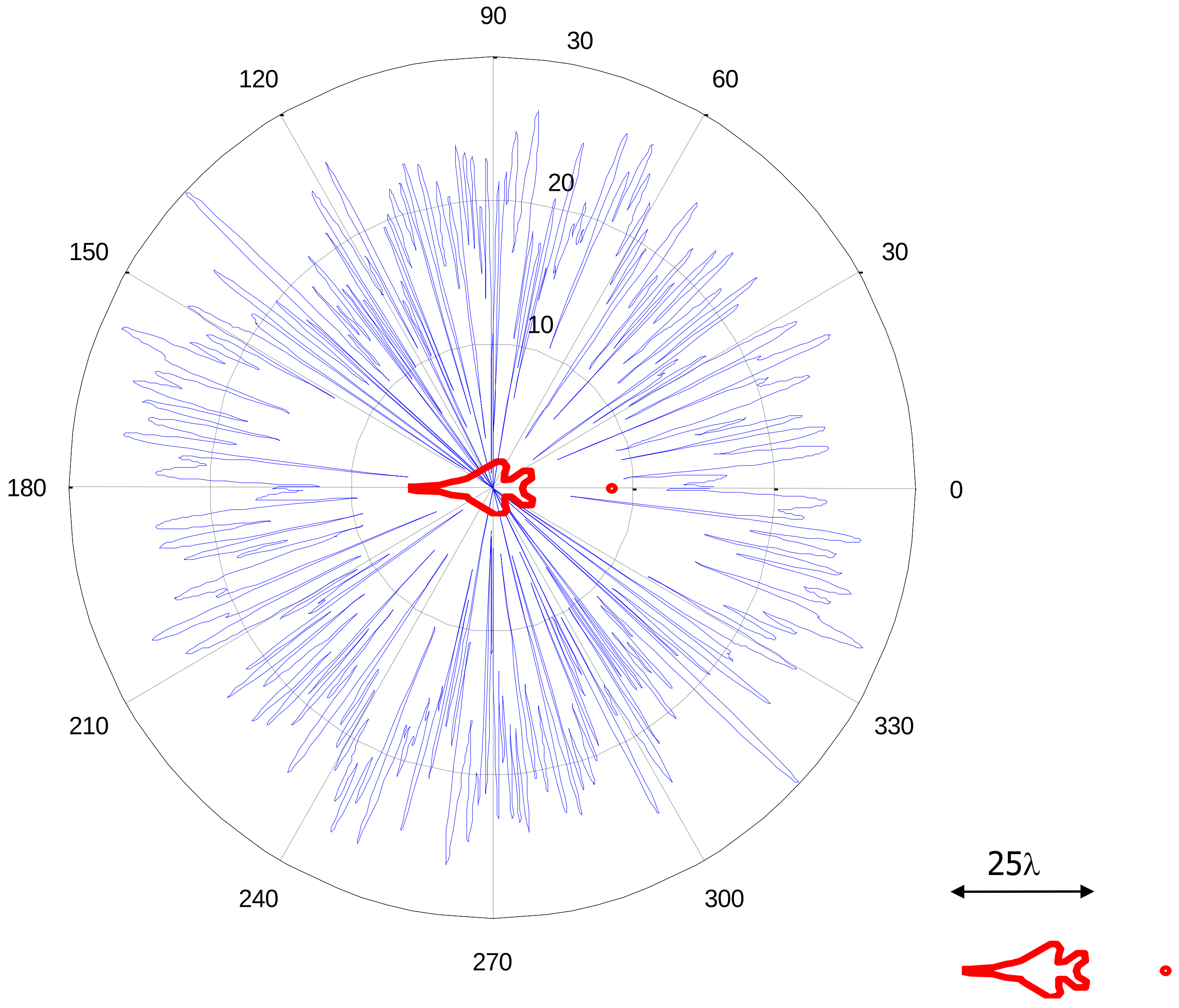
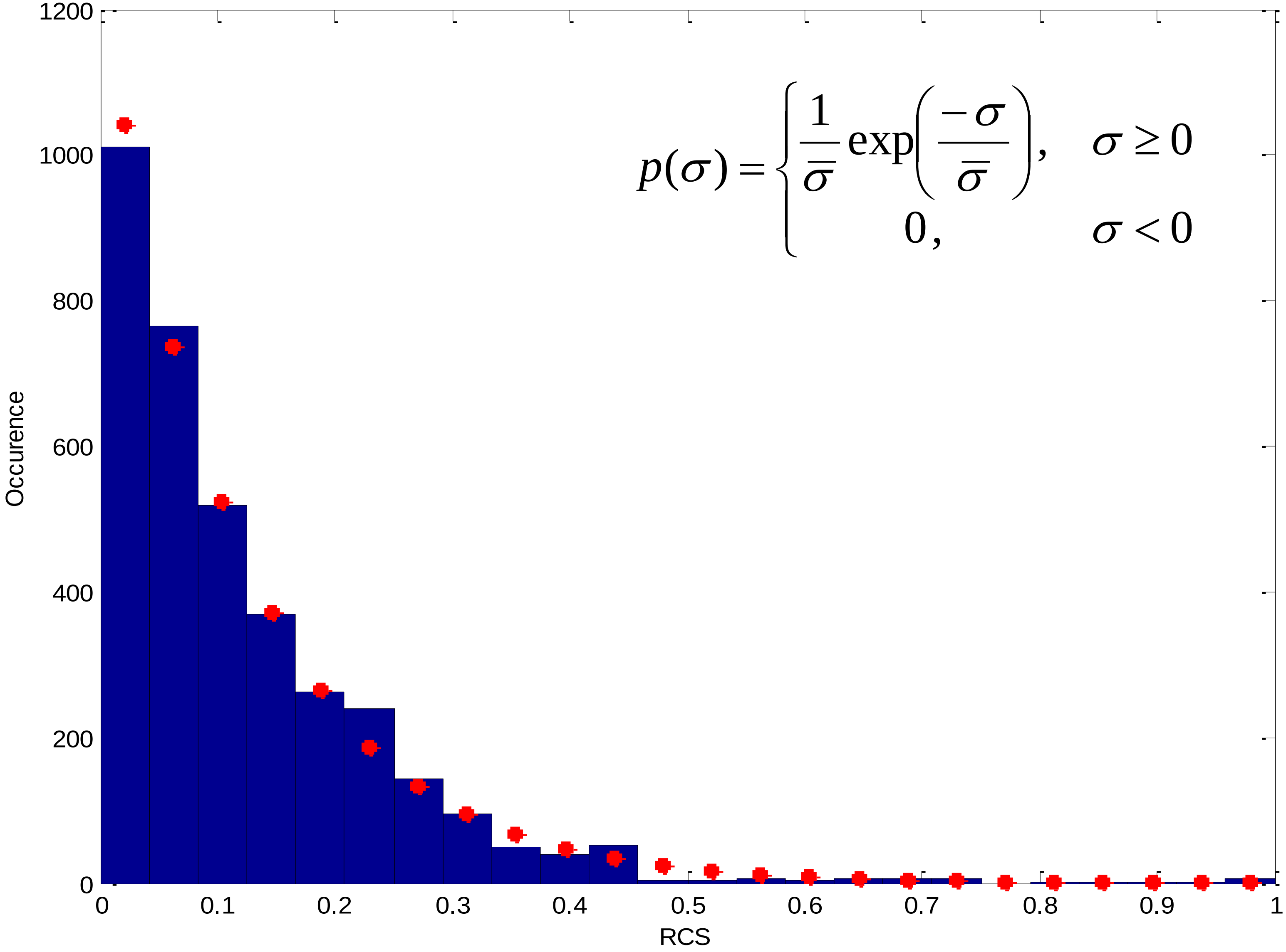




$$p(\sigma) = \begin{cases} \frac{1}{\overline{\sigma}} \exp\left(\frac{-\sigma}{\overline{\sigma}}\right), & \sigma \geq 0 \\ 0, & \sigma < 0 \end{cases}$$

Exponential PDF (power) or **Rayleigh PDF** (voltage)

$$p(\sigma) = \begin{cases} \dfrac{1}{\bar{\sigma}} \exp\left(\dfrac{-\sigma}{\bar{\sigma}} \right), & \sigma \ge 0 \\ 0, & \sigma < 0 \end{cases}$$

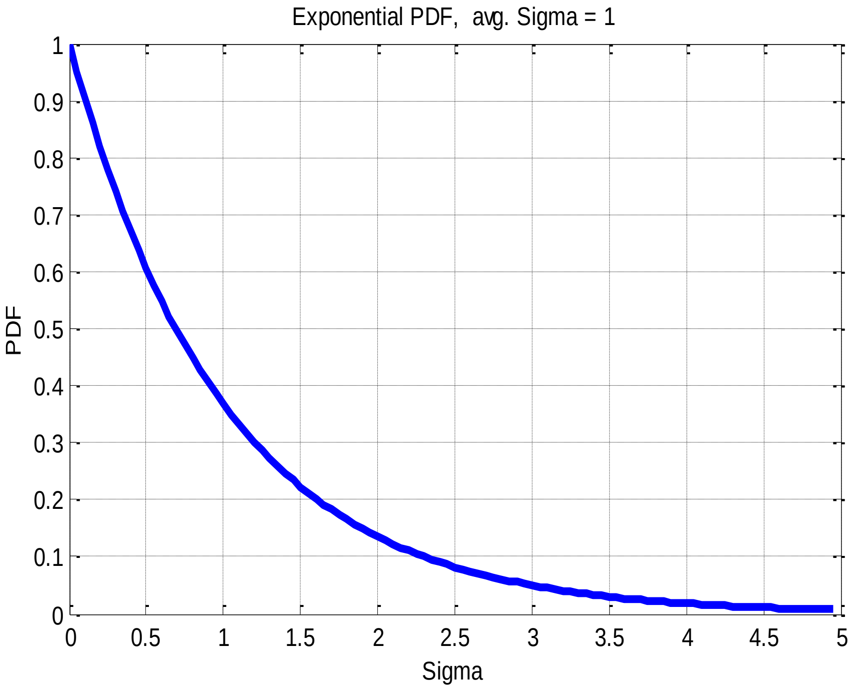
$\boldsymbol{\sigma} \propto \text{RCS} \propto \text{power}$

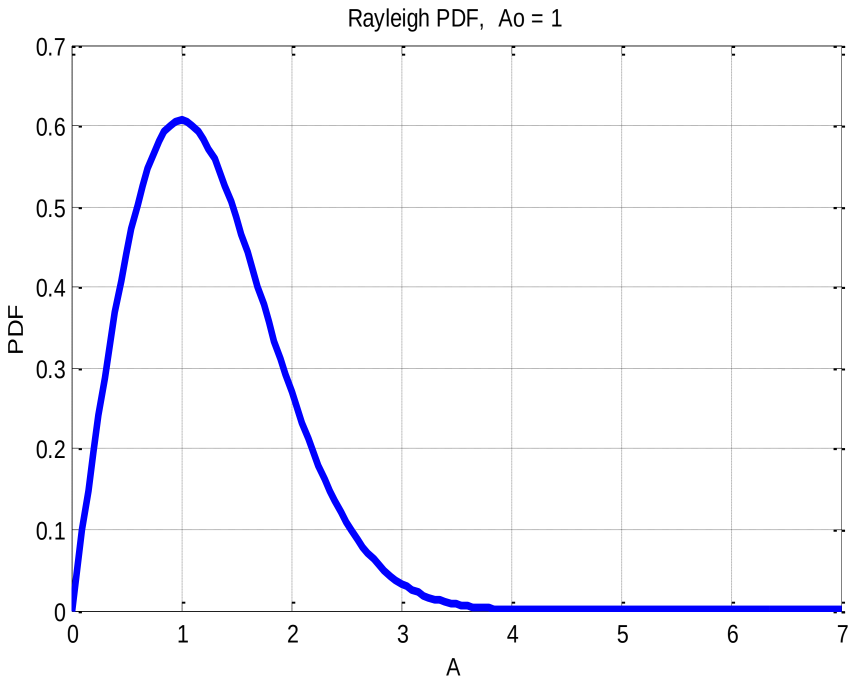
$\boldsymbol{A} \propto \text{amplitude} \propto \text{field}$

$$p(A) = \begin{cases} \dfrac{A}{A_0^2} \exp\left(\dfrac{-A^2}{2A_0^2} \right), & A \ge 0 \\ 0, & A < 0 \end{cases}$$

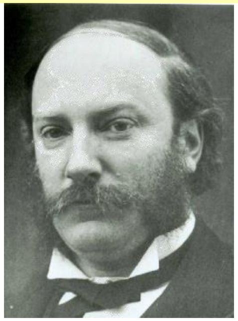
$$\sigma = \frac{A^2}{2} \quad , \quad A_0^2 = \bar{\sigma}$$

$$\bar{A} = A_0 \sqrt{\frac{\pi}{2}} \, , \quad \overline{A^2} = 2A_0^2 \, , \quad A_M = A_0 \sqrt{\ln 4} \, , \quad \text{Var}\{A\} = A_0^2 \left(2 - \pi/2\right)$$





Why Rayleigh ?



Signal reflected from M scatterers of <u>similar</u> size

The range differences between the scatterers >> $\lambda \Rightarrow \phi_k >> 2\pi$

The modulo 2π nature of phase

$\therefore \ \phi_k$ uniformly distributed between 0 and 2π

The Rayleigh PDF was not developed by John William Strutt (Lord Rayleigh III) but was named in his honor.

$$y(t) = \mathrm{Re}\left\{ \left[\exp\left(j\omega_c t + j\phi_0 \right) \right] \sum_{k=1}^{M} a_k \exp\left(j\phi_k \right) \right\}$$

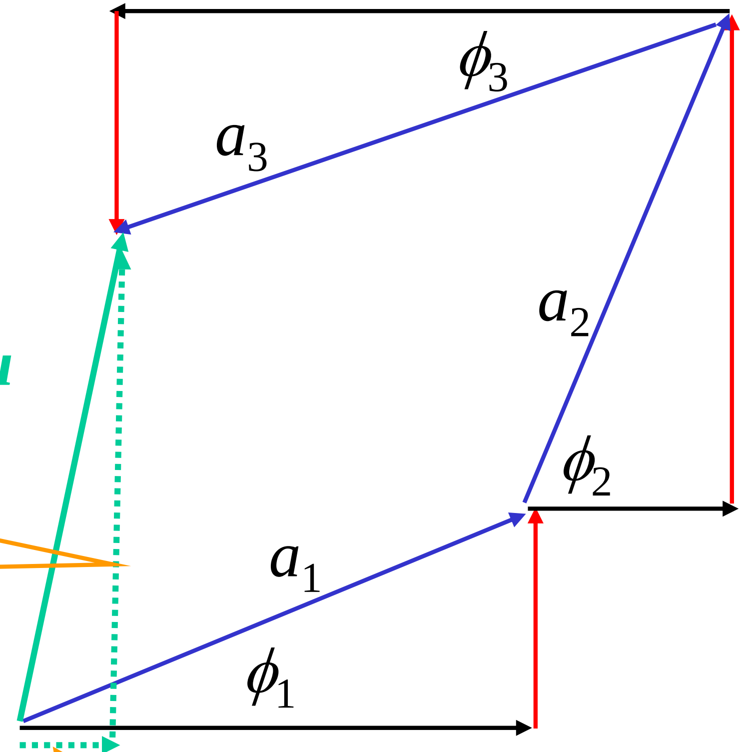
The received signal

$$\sum_{k=1}^{M} a_k \exp\left(j\phi_k \right) = u = r \exp\left(j\theta \right)$$

The complex envelope of the signal

$$u = \sum_{k=1}^{M} a_k \cos\left(\phi_k \right) + j \sum_{k=1}^{M} a_k \sin\left(\phi_k \right)$$

$$u = \sum_{k=1}^{M} a_k \cos(\phi_k) + j \sum_{k=1}^{M} a_k \sin(\phi_k)$$



$$\sum_{k=1}^{M} a_k \sin(\phi_k)$$

$$\sum_{k=1}^{M} a_k \cos(\phi_k)$$

$$\frac{u}{a} = \sum_{k=1}^{M} \cos(\phi_k) + j\sum_{k=1}^{M} \sin(\phi_k) = X + jY$$

To simplify the analysis assume

$$a_k = a$$

$$p(\phi_k) = \begin{cases} \frac{1}{2\pi} & 0 \le \phi_k \le 2\pi \\ 0 & \text{elsewhere} \end{cases} \Rightarrow \overline{\sin(\phi_k)} = \overline{\cos(\phi_k)} = 0$$

E(Y) E(X)

For M>>1, the central limit theory implies that X and Y are Gaussian distributed with zero mean and variance equal to M times the variance of $\cos(\phi_k)$

$$\mathrm{Var}\,Y = \mathrm{Var}\,X = M \int_0^{2\pi} \frac{1}{2\pi} \cos^2\phi \, d\phi = \frac{M}{2}$$

Next slide

$$\mathrm{E}\{XY\} = 0 = \mathrm{E}\{X\}\mathrm{E}\{Y\} \Rightarrow$$ X and Y uncorrelated

X and Y uncorrelated and Gaussian $\Rightarrow$ independent

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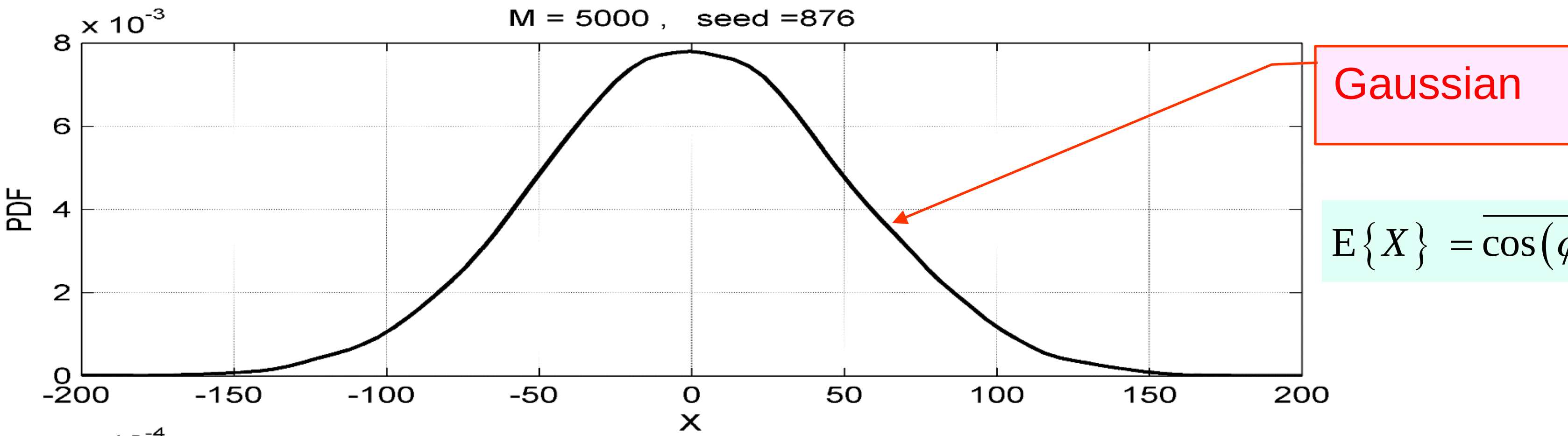
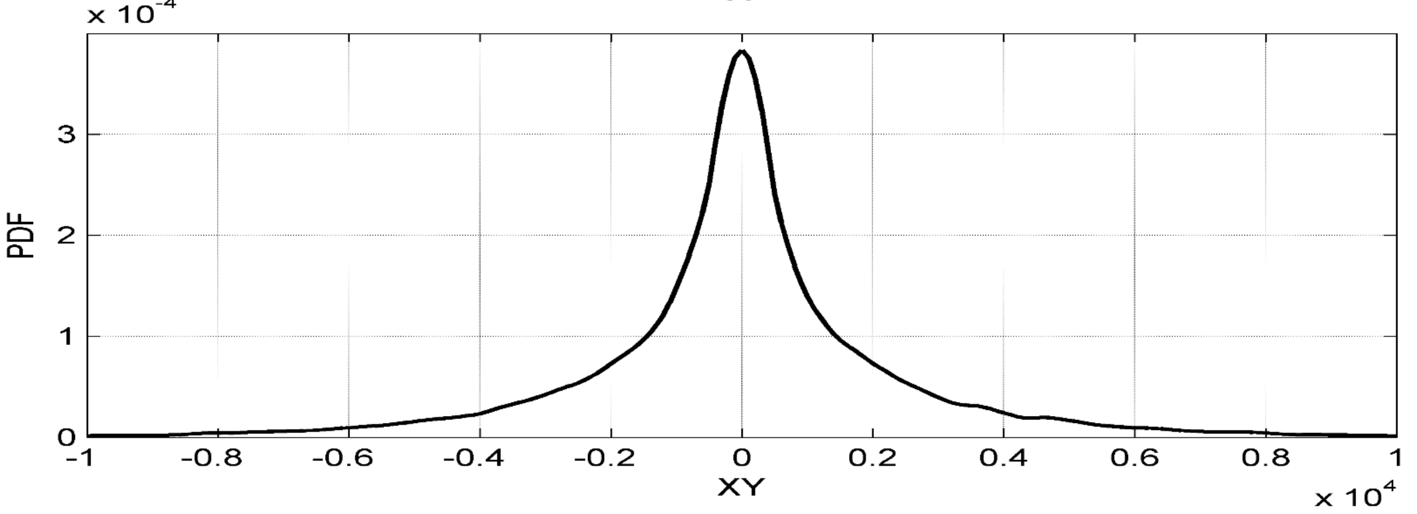


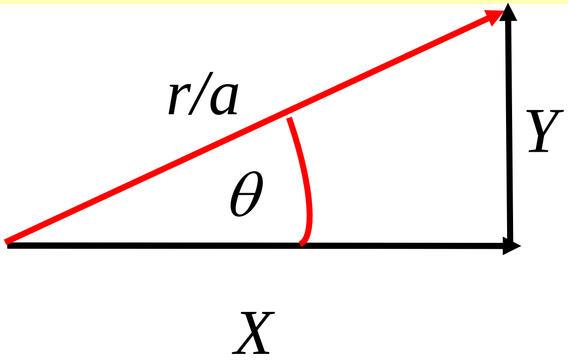
$$\mathrm{E}\{X\} = \overline{\cos(\varphi_k)} = 0$$



$$\mathrm{E}\{XY\} = 0$$

$\mathrm{E}\{XY\} = 0 = \mathrm{E}\{X\}\mathrm{E}\{Y\} \Rightarrow X$ and Y uncorrelated

X and Y uncorrelated and Gaussian $\Rightarrow$ independent



$$\left(\frac{r}{a}\right)^2 = X^2 + Y^2 \quad , \quad \theta = \tan^{-1}\left(\frac{Y}{X}\right)$$

$$\mathrm{Var}\,Y = \mathrm{Var}\,X = \frac{M}{2}$$

$$p(X,Y) = p(X)p(Y) = \frac{1}{M\pi}\exp\left[\frac{-(X^2+Y^2)}{M}\right]$$

$$\frac{M}{2}a^2 = \mathrm{Var}(Xa)$$

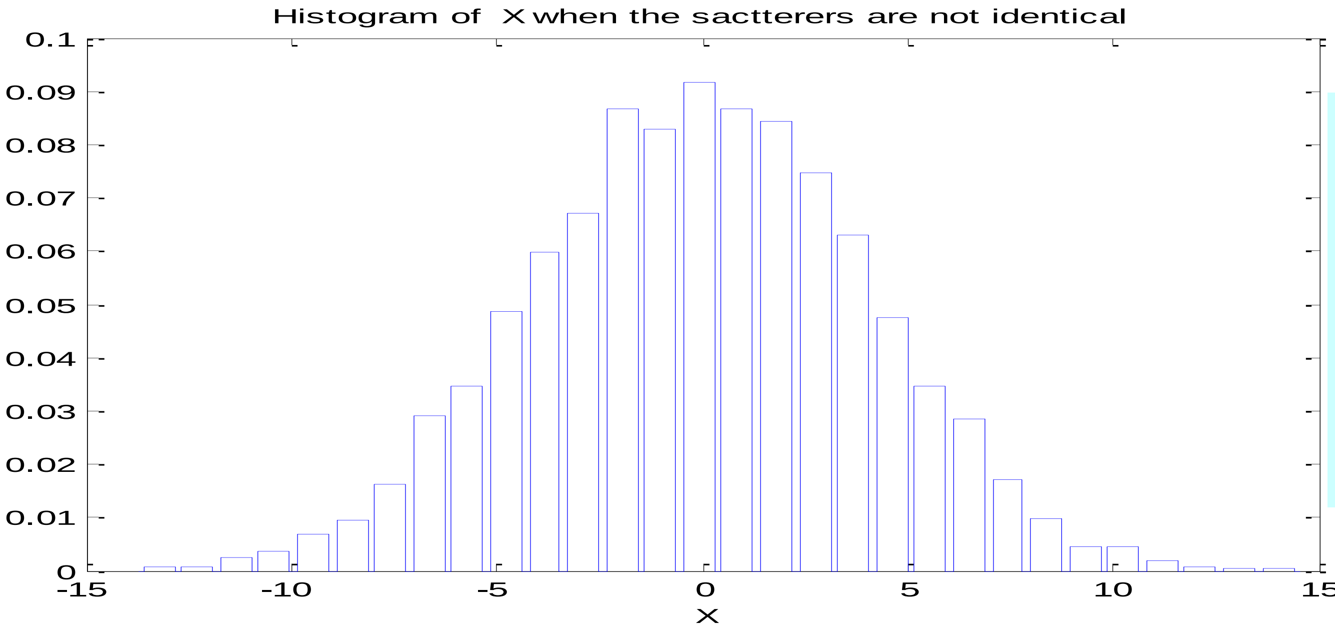
$$p(r) = \frac{2r}{Ma^2}\exp\left(\frac{-r^2}{Ma^2}\right)$$

$$p(\theta) = \begin{cases} \dfrac{1}{2\pi} & 0 \le \theta \le 2\pi \\ 0 & \text{elswhere} \end{cases}$$

$$A = r\sqrt{2} \quad , \quad A_0^2 = Ma^2$$

$$p(A) = \frac{A}{A_0^2}\exp\left(\frac{-A^2}{2A_0^2}\right) \quad , \quad 0 \le A$$

With the assumption that a_k=a, we got Rayleigh PDF of A. What happens when this assumption does not hold?



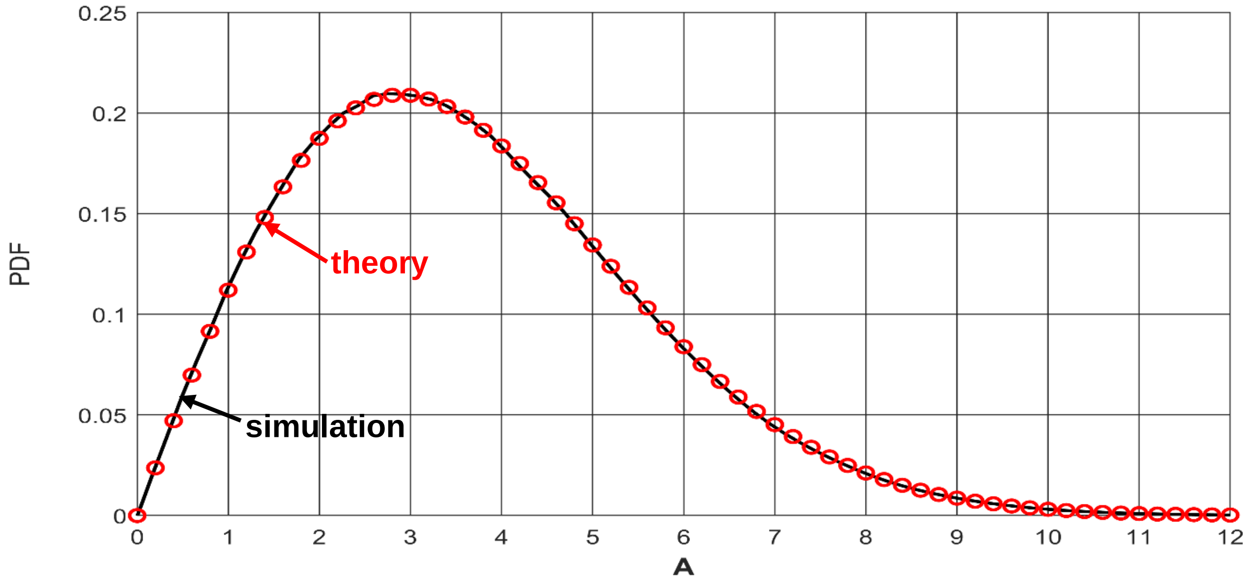
$$p(a_k)=1,\ \ 0\le a_k \le 1$$

$$X=\sum_{k=1}^{M} a_k \cos(\varphi_k)$$

$$Y=\sum_{k=1}^{M} a_k \cos(\varphi_k)$$

$$r=\sqrt{X^2+Y^2}$$

M = 100
mean x = -0.0643
mean xy = 0.1860
STD x = 4.0739
Var x = 16.5966



With the assumption above, we also got Rayleigh PDF of ***A*** (or ***r***).

To plot the PDF of the simulation result, we used MALAB's `ksdensity` function

COHERENT and NON-COHERENT RADAR

For a radar system that transmits a pulse train, to be called coherent, requires:

- The transmitted pulses must be coherent.
- The receiver needs to perform synchronous detection.
- The target must maintain coherence for the duration of the pulse train.

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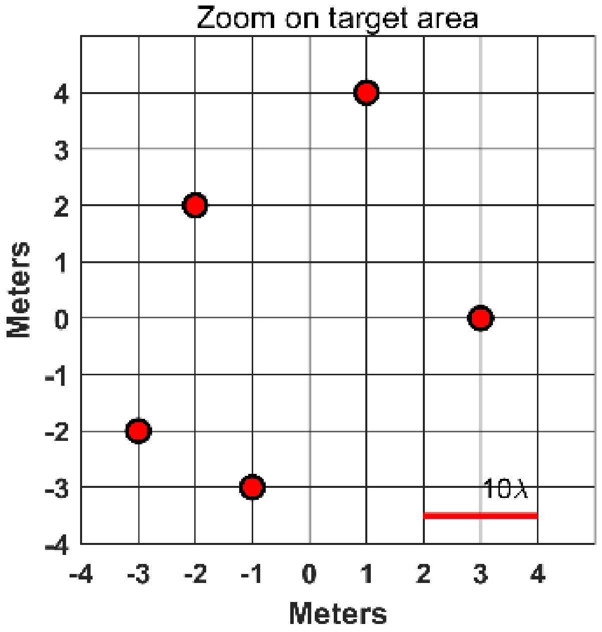
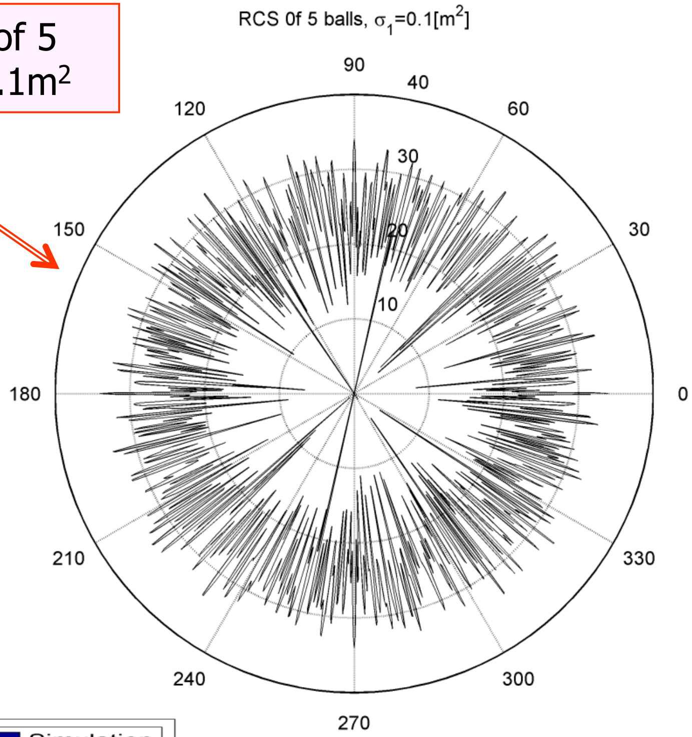
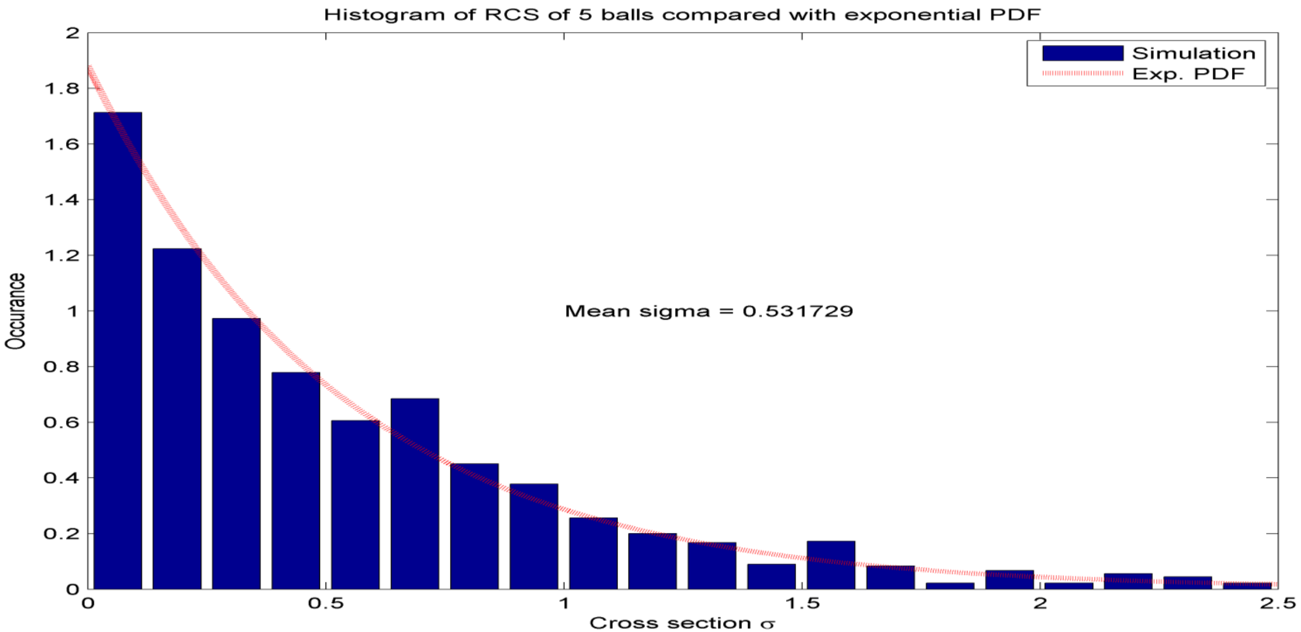


RCS angular dependence of 5 identical balls, each σ is 0.1m²

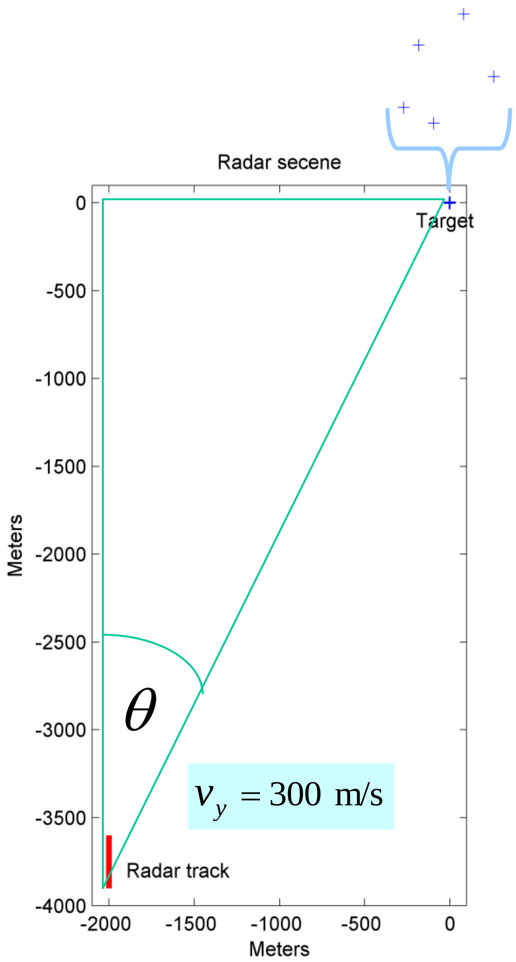
If such a target is moving, hence changing aspect angle toward the radar, can it be a coherent target ?

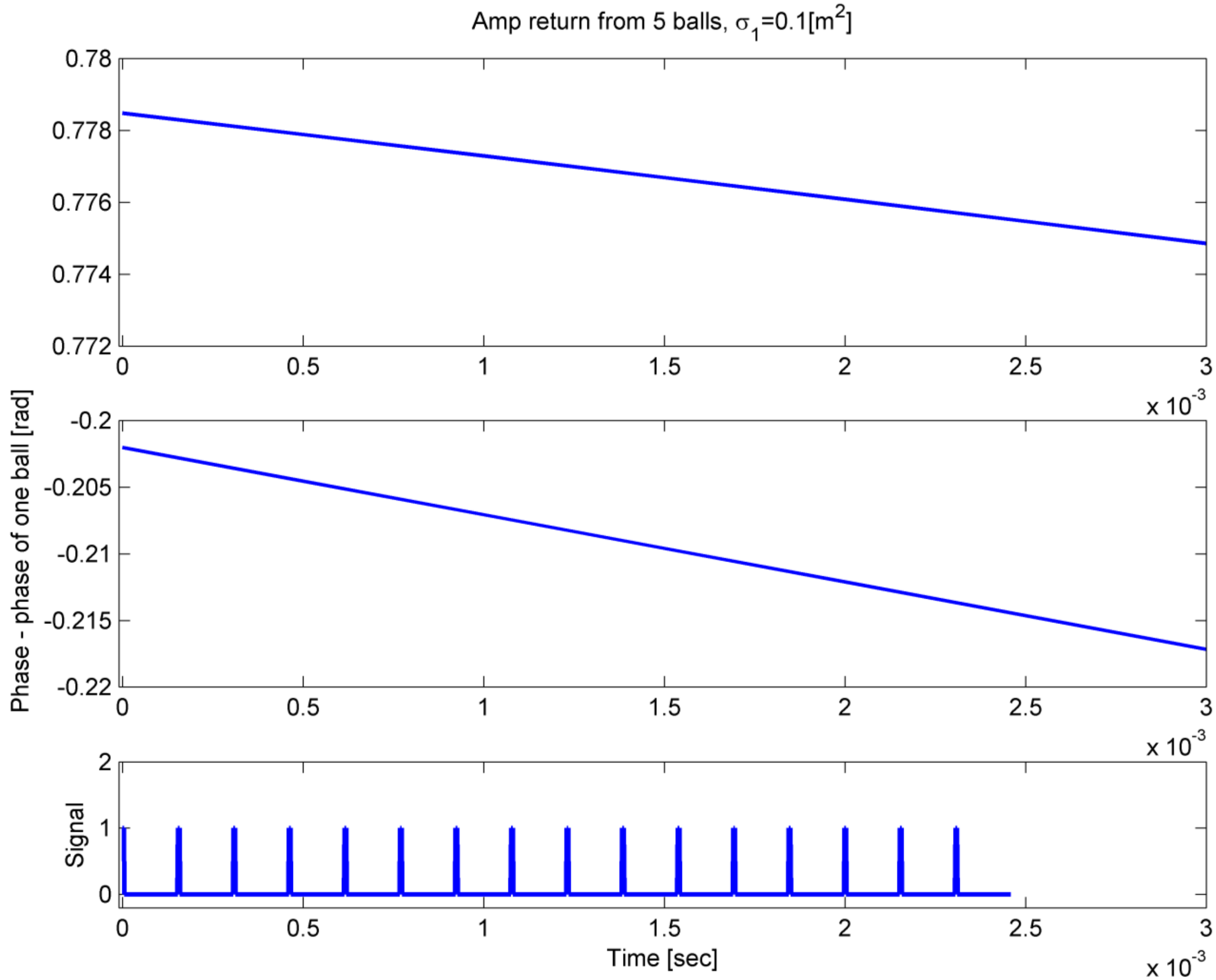




$$p(\sigma)=\begin{cases}\dfrac{1}{\bar{\sigma}}\exp\left(\dfrac{-\sigma}{\bar{\sigma}}\right), & \sigma\ge 0\\ 0, & \sigma<0\end{cases}$$

The target must maintain coherence for the duration of the pulse train.

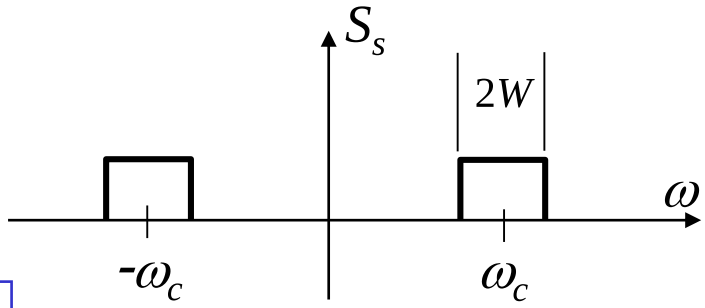




Narrow bandpass signal

Natural envelope

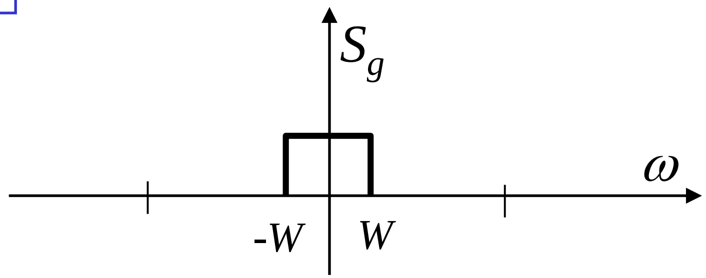
$$s(t) = g(t)\cos[\omega_c t + \phi(t)]$$



In-phase component

Quadrature component

$$s(t) = g_c(t)\cos\omega_c t - g_s(t)\sin\omega_c t$$



$$g_c(t) = g(t)\cos\phi(t)$$

$$g_s(t) = g(t)\sin\phi(t)$$

$$u(t) = g_c(t) + jg_s(t) = \boldsymbol{I}(t) + j\boldsymbol{Q}(t)$$

Complex envelope of $s(t)$

$$s(t) = \mathrm{Re}\left\{u(t)e^{j\omega_c t}\right\}$$

$$g(t) = |u(t)|$$

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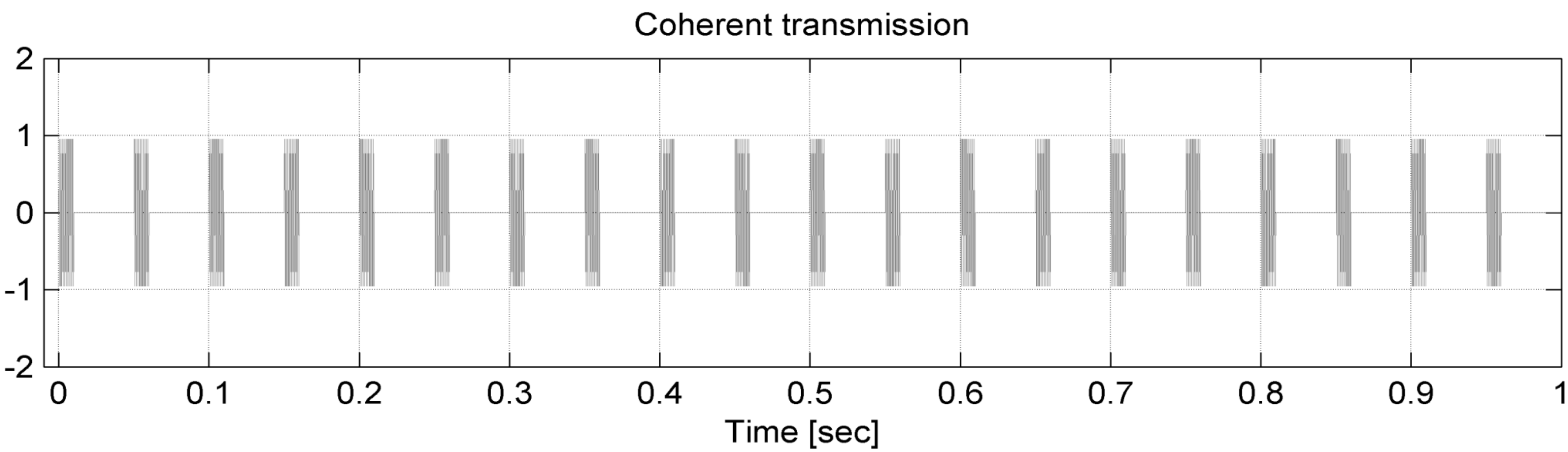
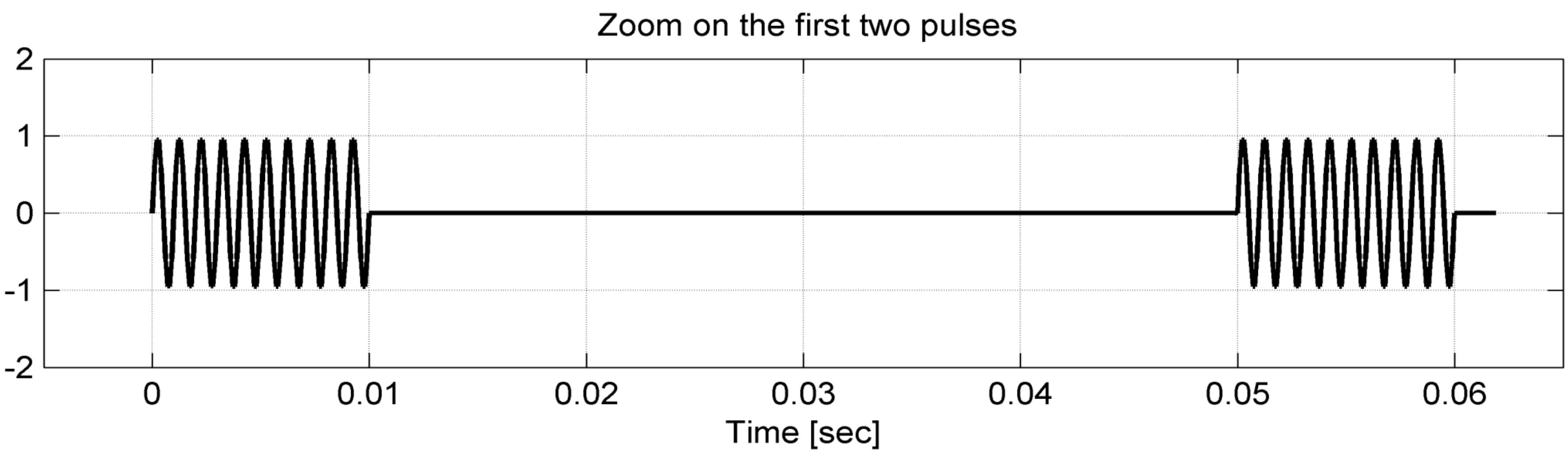
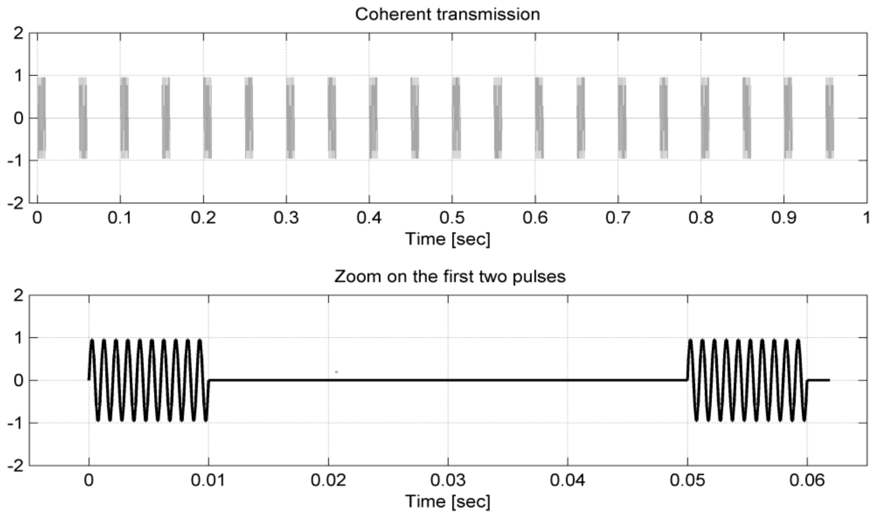
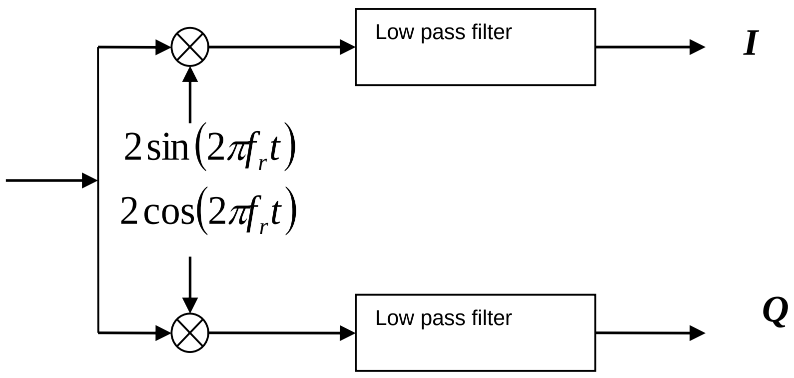
The transmitted pulses must be coherent.

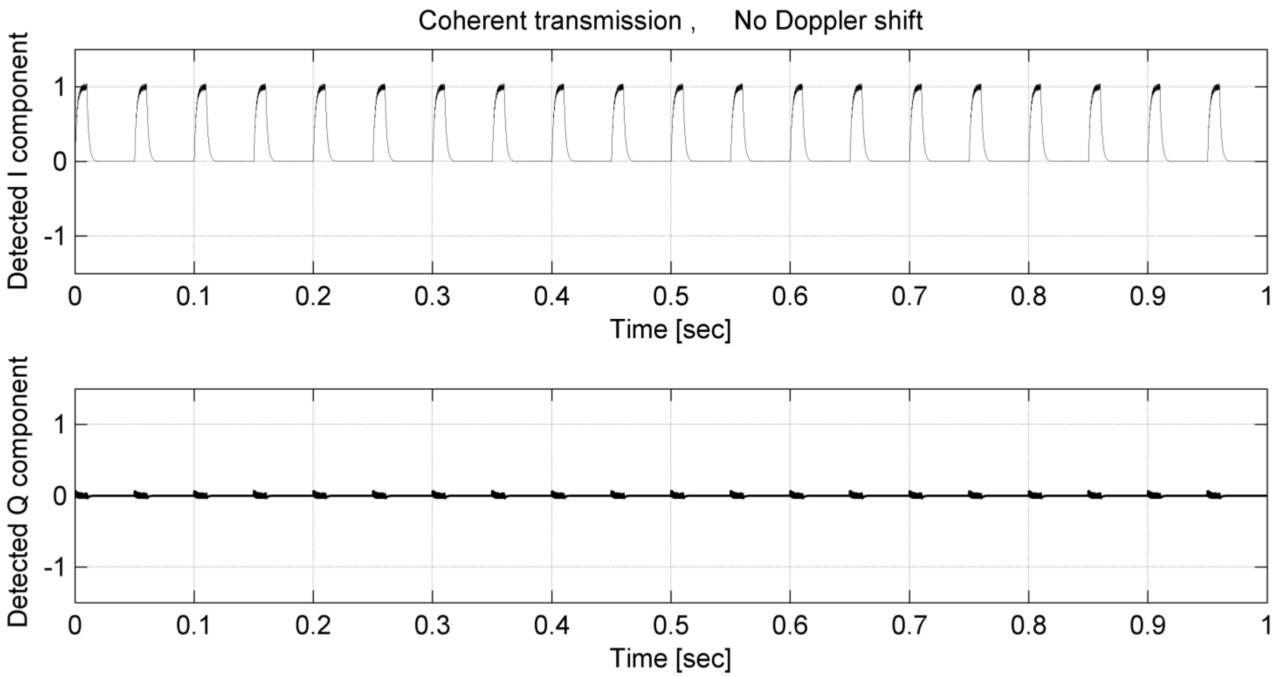


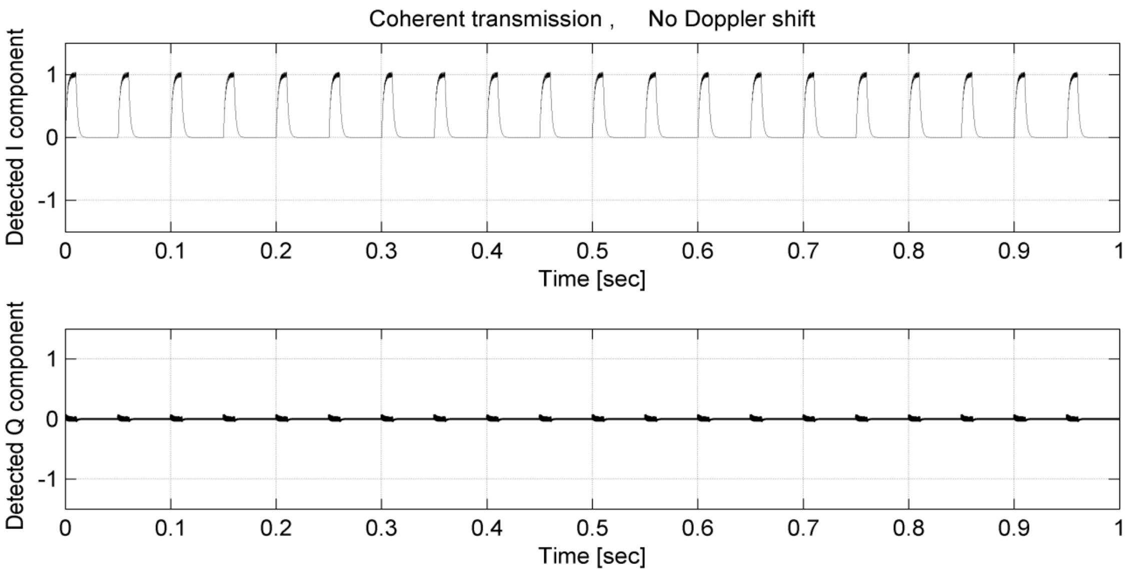


The receiver needs to perform synchronous detection.

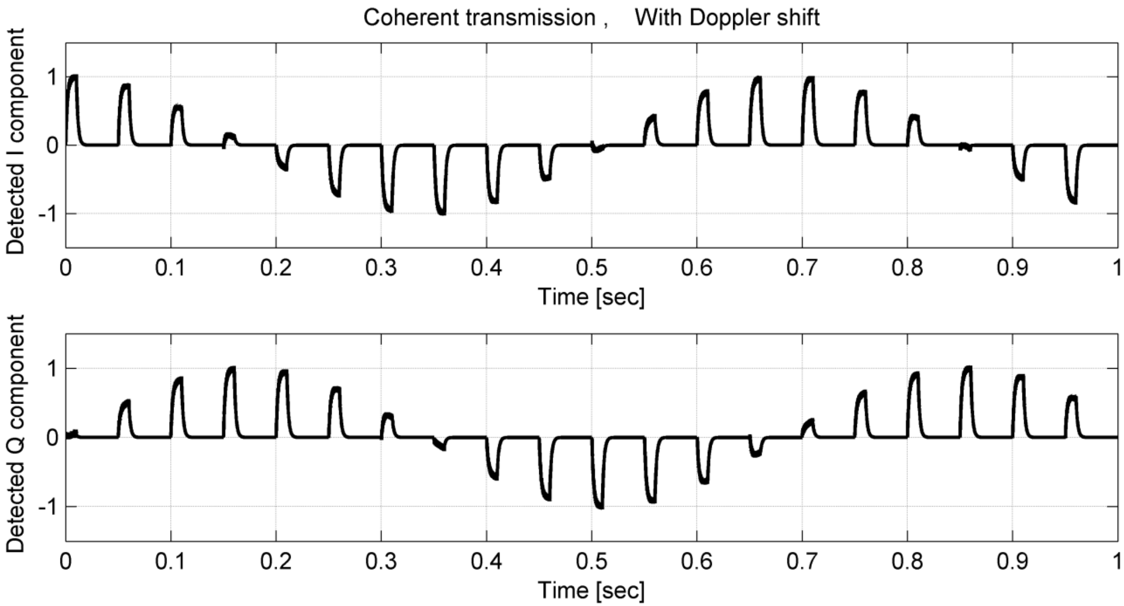




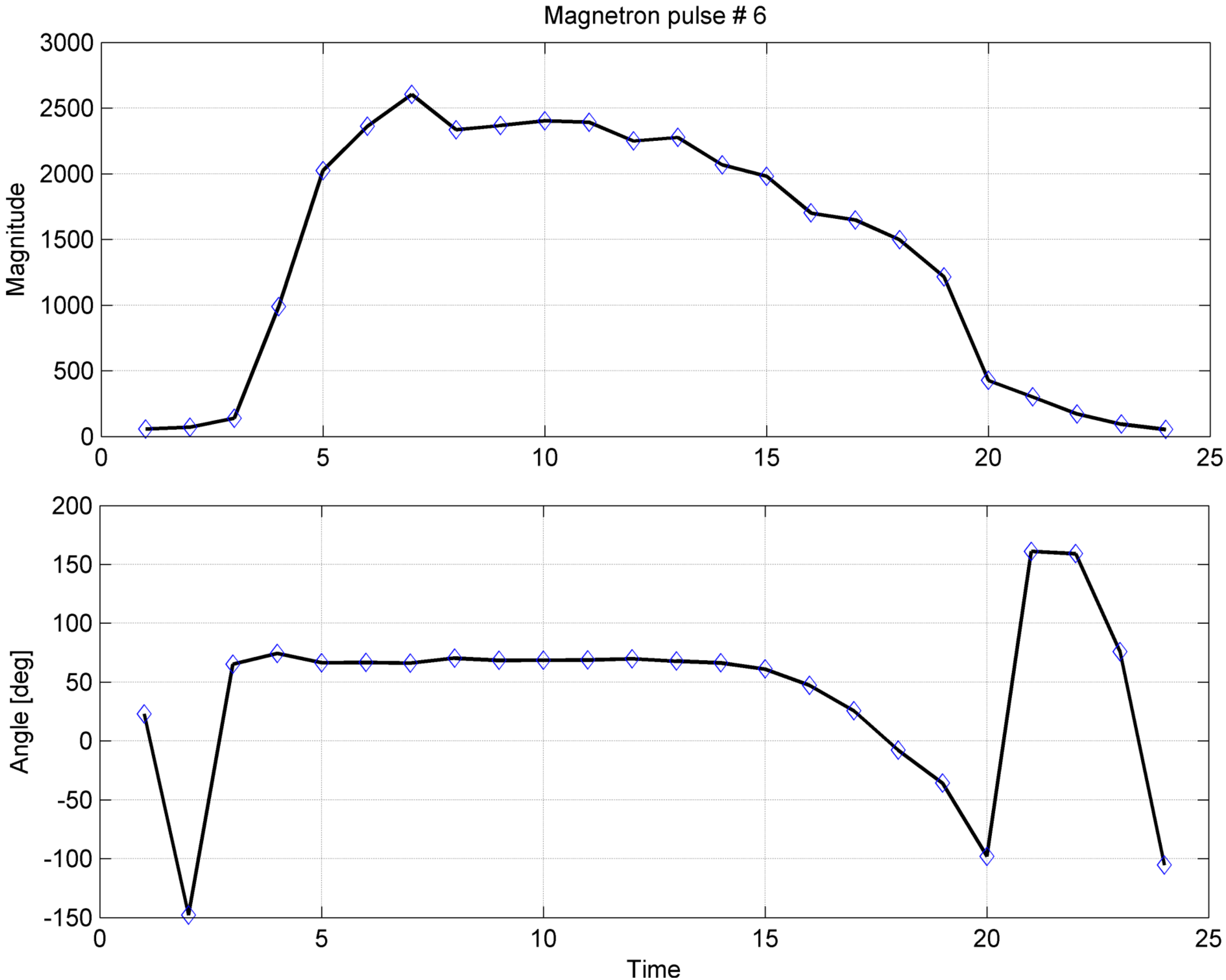


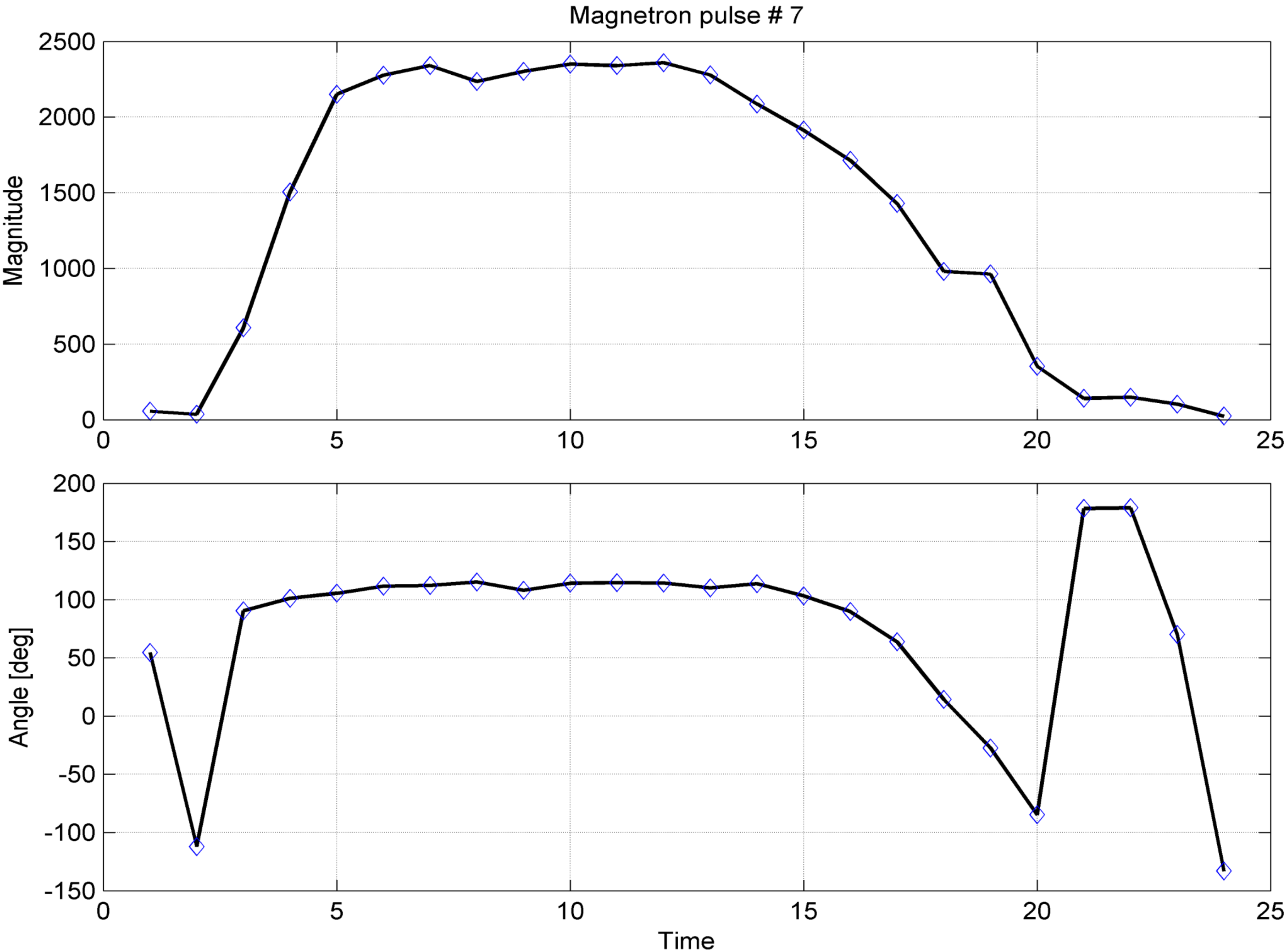


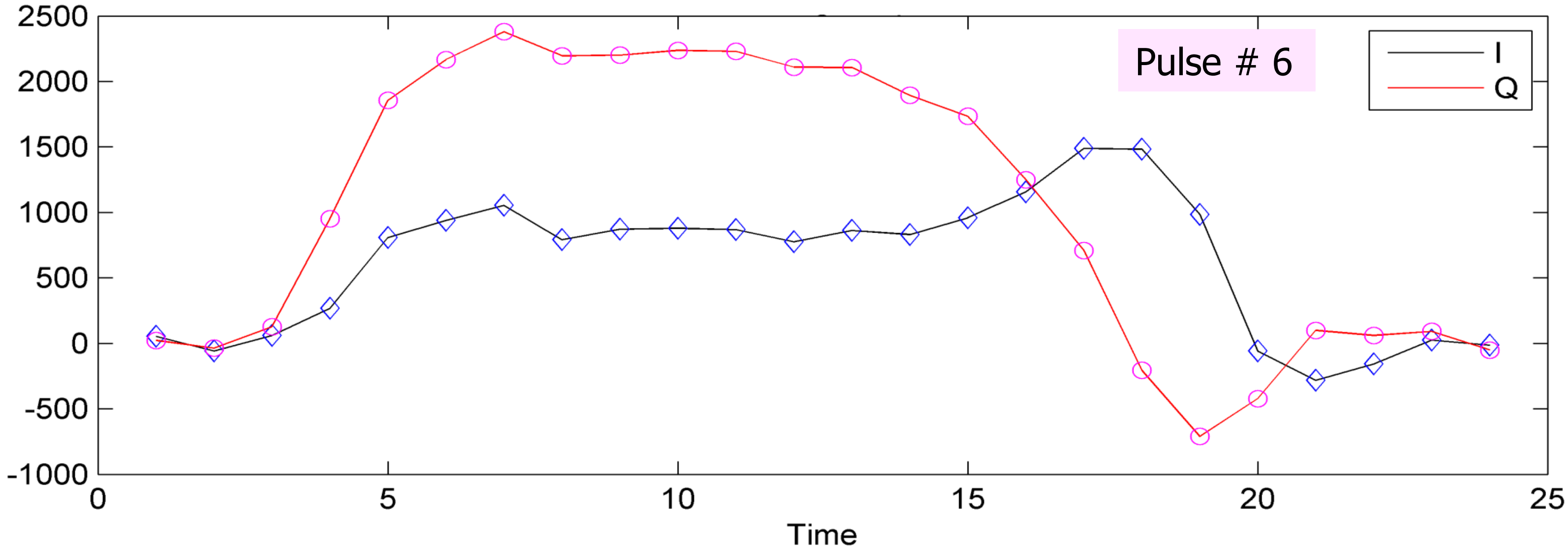
Coherent radar allows Doppler processing.

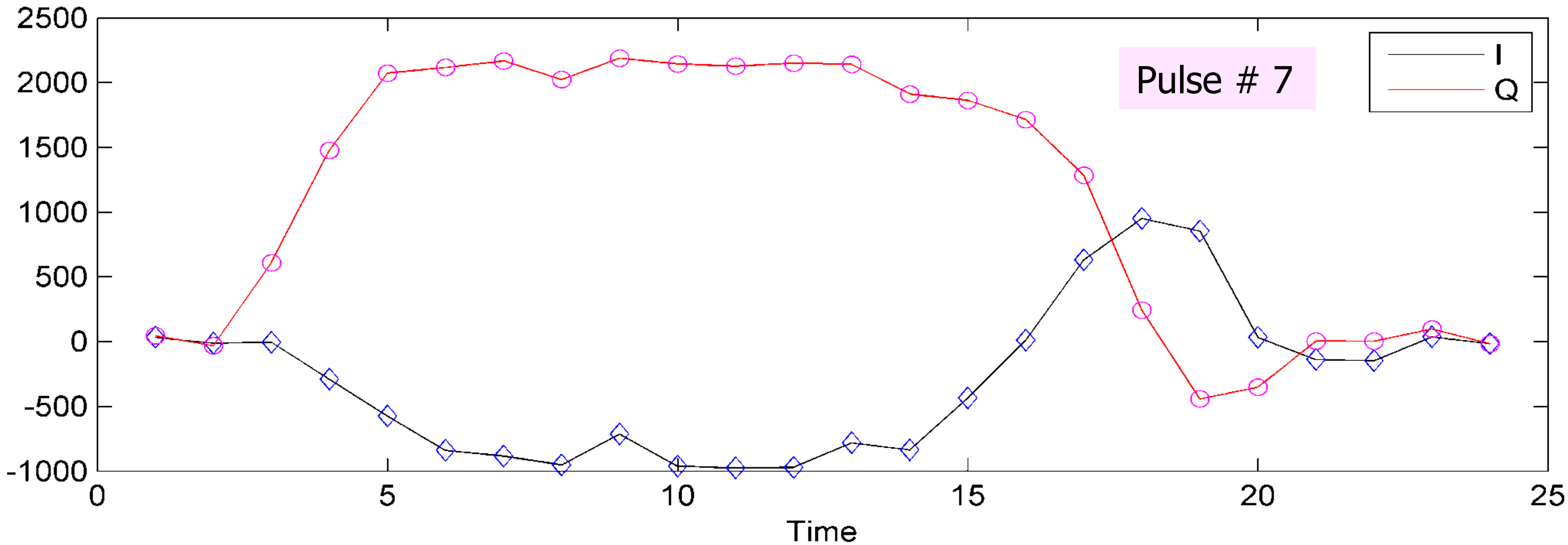


AMPLITUDE AND PHASE OF **MAGNETRON** PULSES


Magnetron pulse # 6
Magnitude
3000
2500
2000
1500
1000
500
0
0
5
10
15
20
25
Angle [deg]
200
150
100
50
0
-50
-100
-150
0
5
10
15
20
25
Time


Magnetron pulse # 7
Magnitude
2500
2000
1500
1000
500
0
0
5
10
15
20
25
Angle [deg]
200
150
100
50
0
-50
-100
-150
Time


Pulse # 6
I
Q
2500
2000
1500
1000
500
0
-500
-1000
0
5
10
15
20
25
Time


Pulse # 7
I
Q
2500
2000
1500
1000
500
0
-500
-1000
0
5
10
15
20
25
Time

- Magnetron is a low-cost source of high-power microwave pulses
- Magnetron pulses are not coherent
- Magnetron-based radar cannot produce Doppler information and cannot separate moving targets from stationary clutter

(unless the phase of each individual transmitted pulse is measured and stored)

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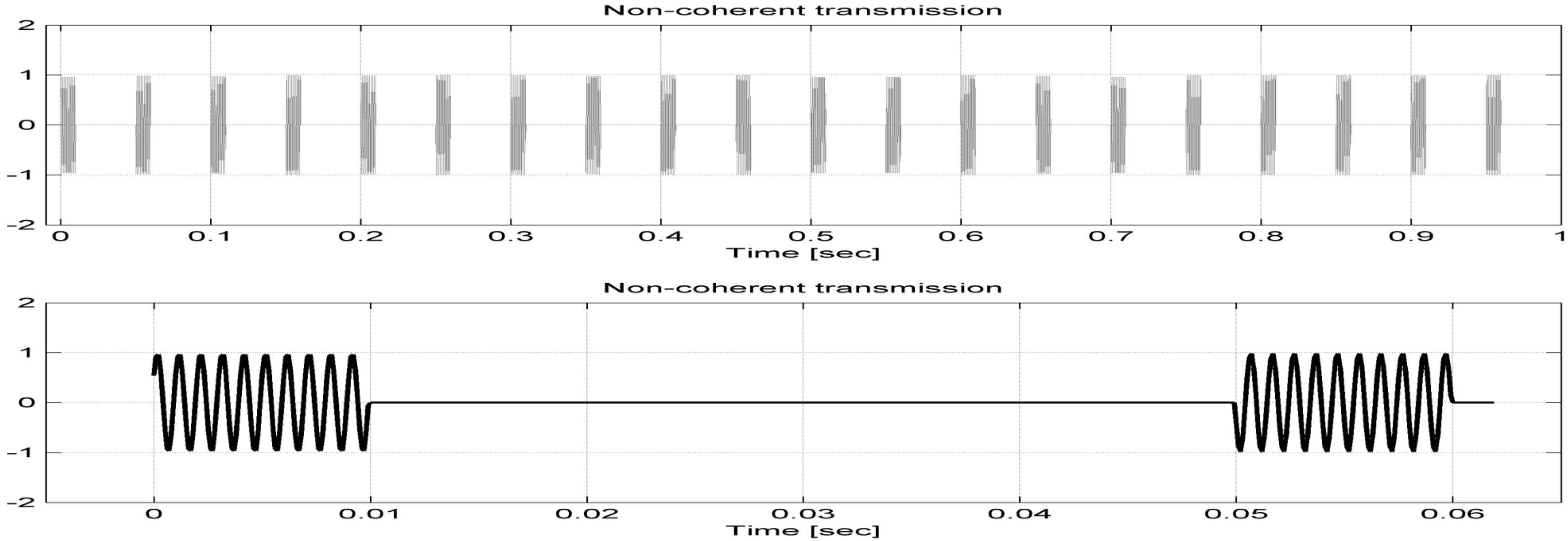
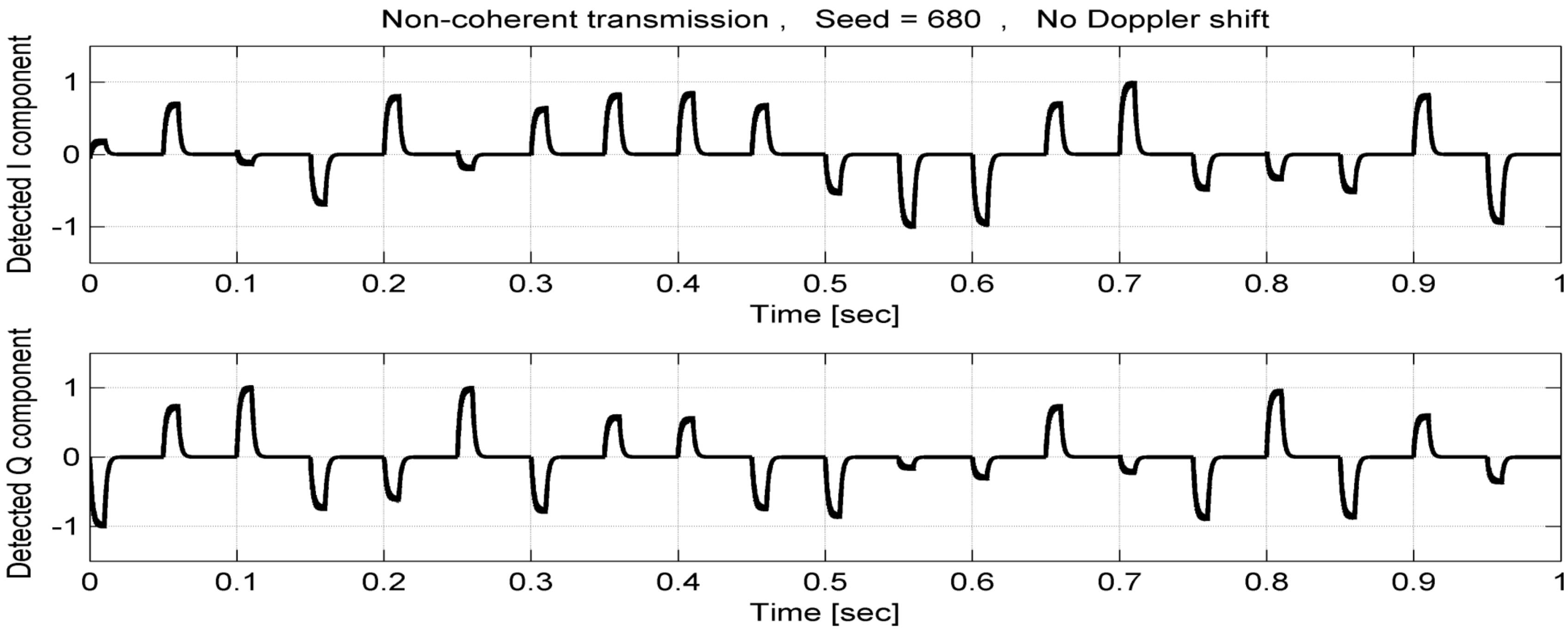
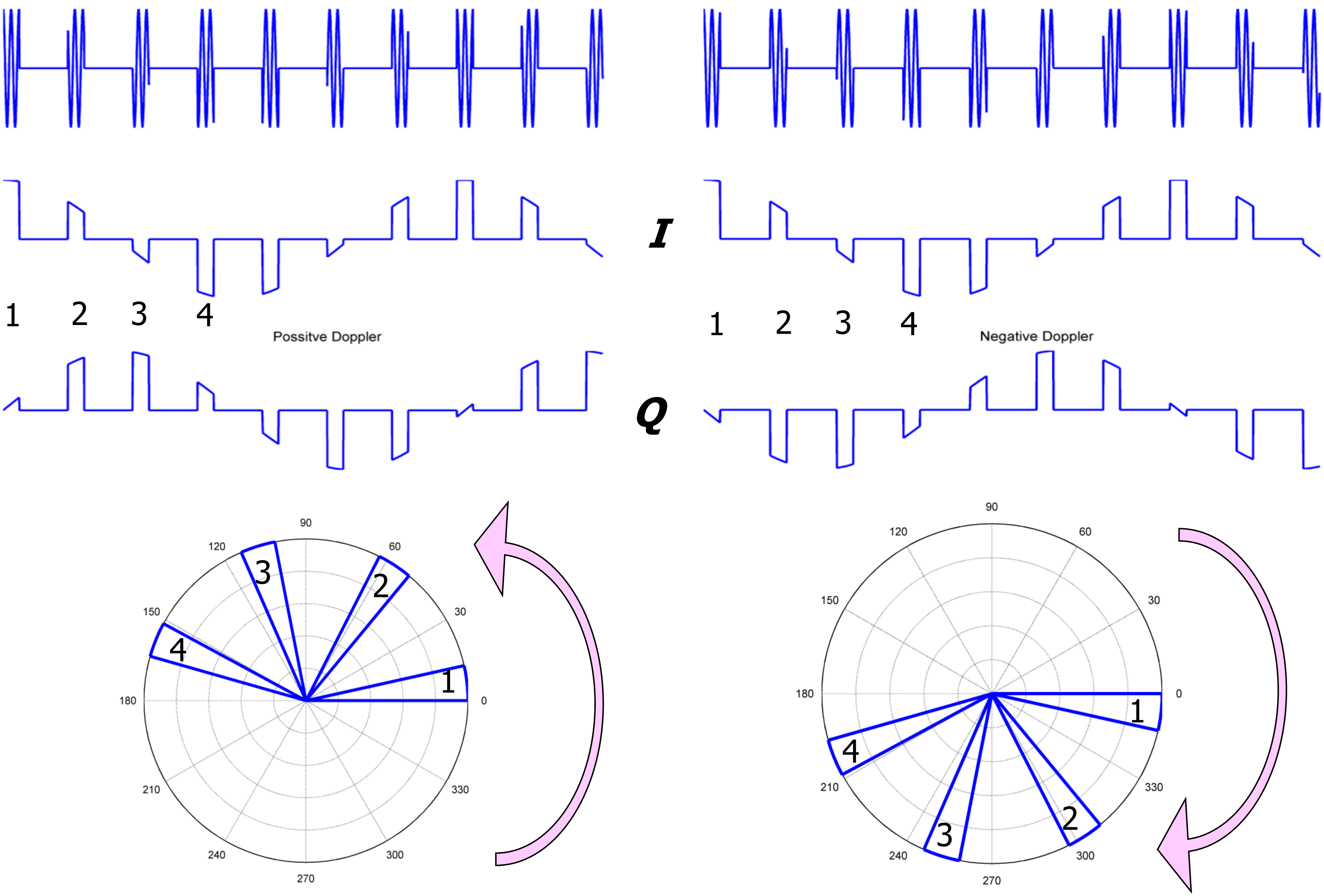




Why do we need both *I* and *Q* ?



Checking the coherency of the reflection from the same 5 identical balls, each with a σ of $0.1m^2$,
when observed from an approaching aircraft

The change in phase and amplitude, of the reflection,
after removing the common Doppler effect

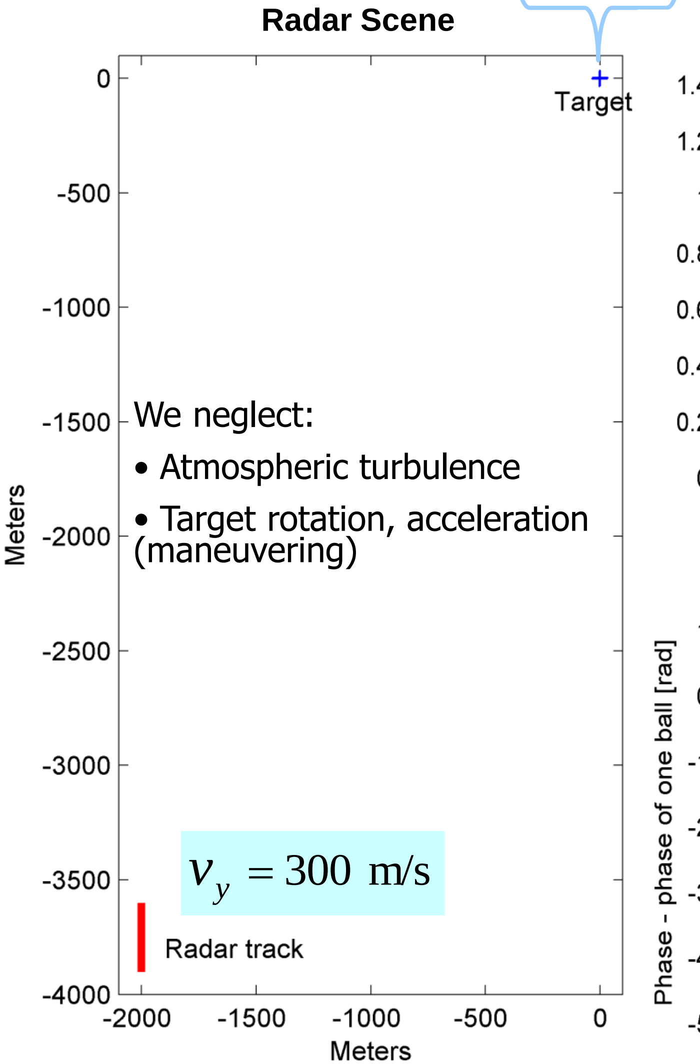
Nadav Levanon, Tel-Aviv University

$$\sigma_k = \sigma_1 = 0.1 \text{ m}^2$$

$$\lambda = 0.2 \text{ m}$$

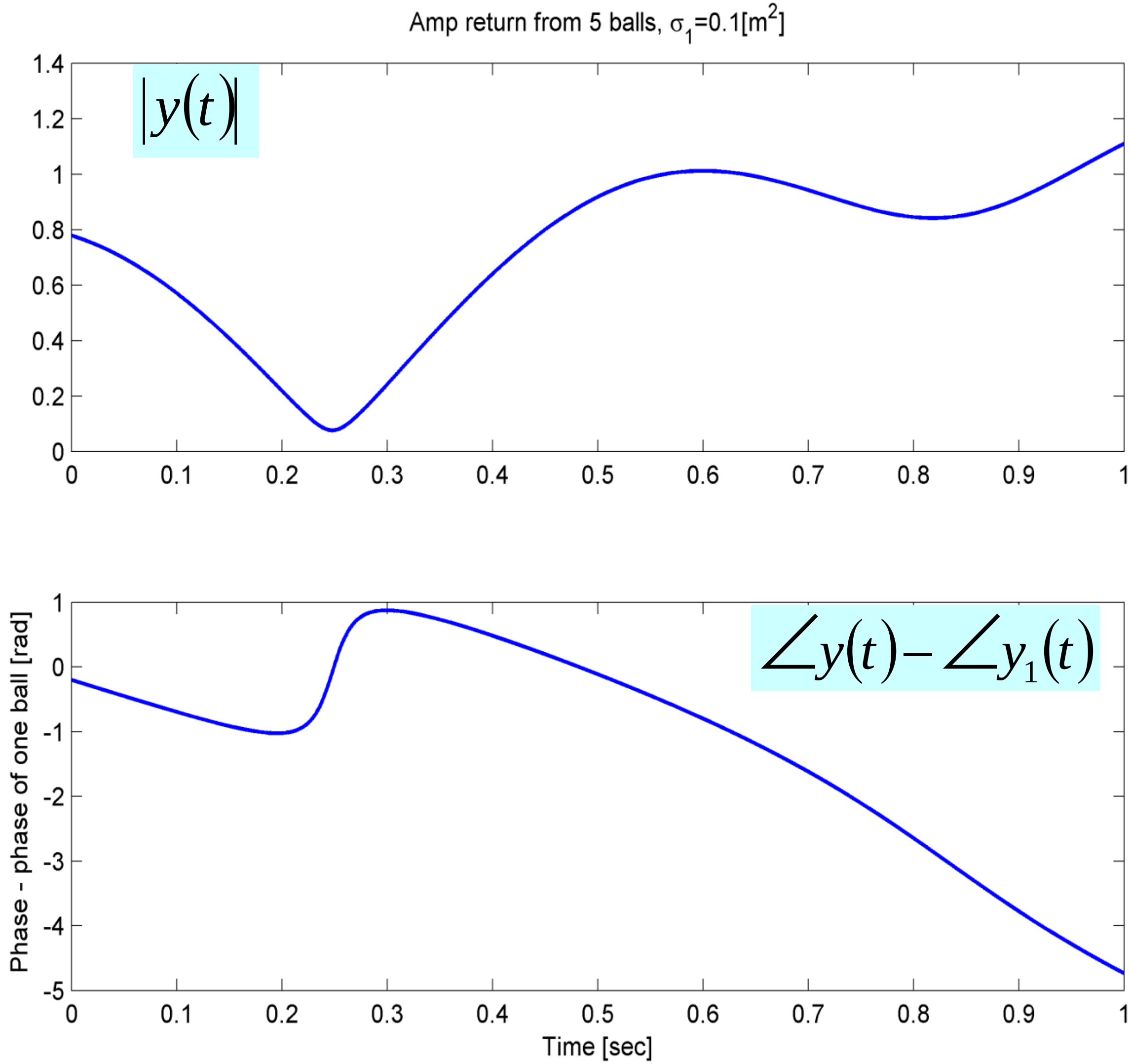
$$y(t) = \sum_{k=1}^{5} \sqrt{\sigma_k} \exp[-j4\pi R_k(t)/\lambda]$$

$$y_1(t) = \sqrt{\sigma_1} \exp[-j4\pi R_1(t)/\lambda]$$

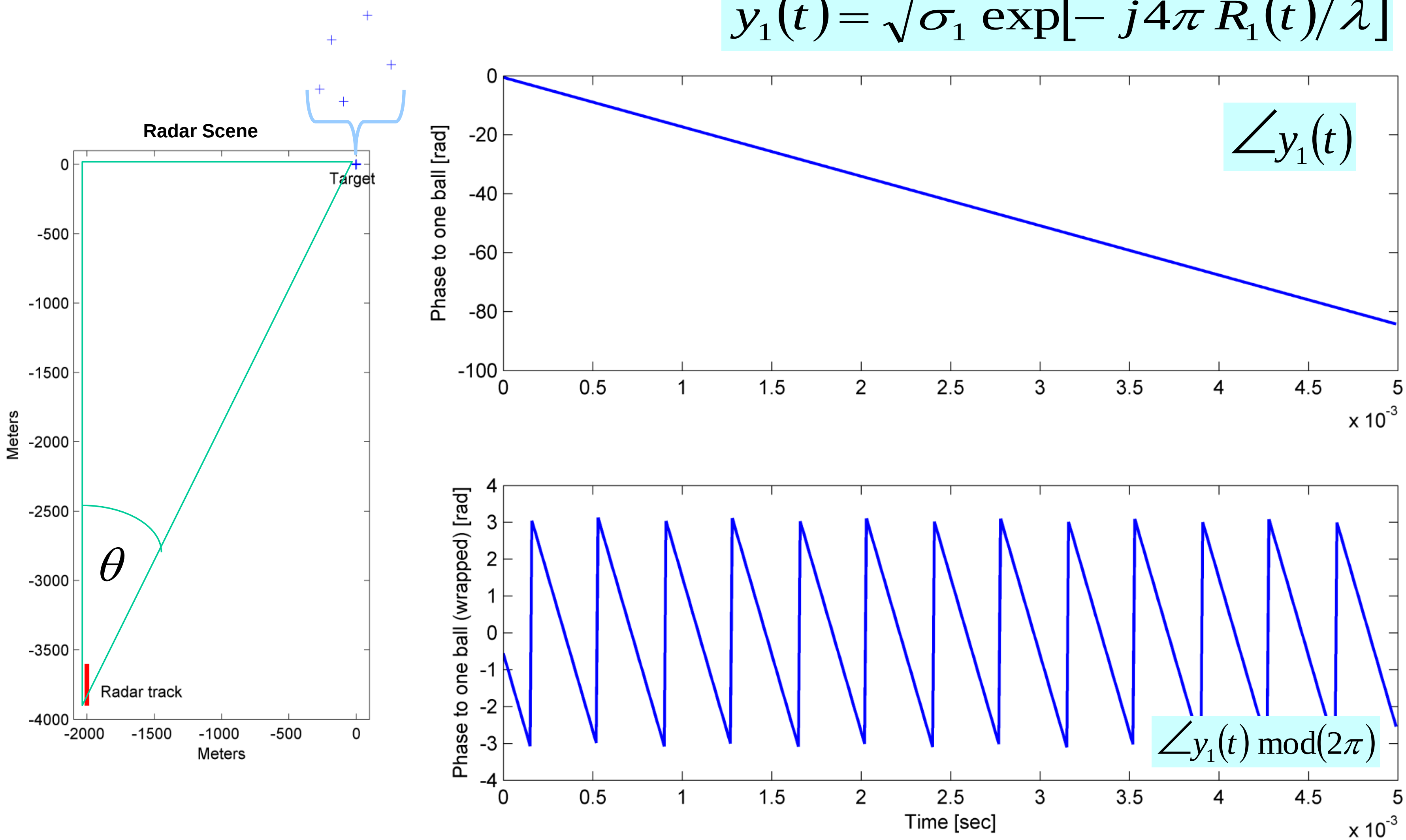


We neglect:

- Atmospheric turbulence
- Target rotation, acceleration (maneuvering)



$$y_1(t) = \sqrt{\sigma_1}\exp[-j4\pi R_1(t)/\lambda]$$



$$f_D = \frac{2v}{\lambda}\cos\theta = \frac{2\cdot 300}{0.2}\,\frac{3900}{\sqrt{2000^2+3900^2}} = 2670\,\text{Hz}$$

$$f_{D\ \text{measured}} = \frac{13.3\,\text{cycles}}{0.005\,\text{sec}} = 2660\,\text{Hz}$$

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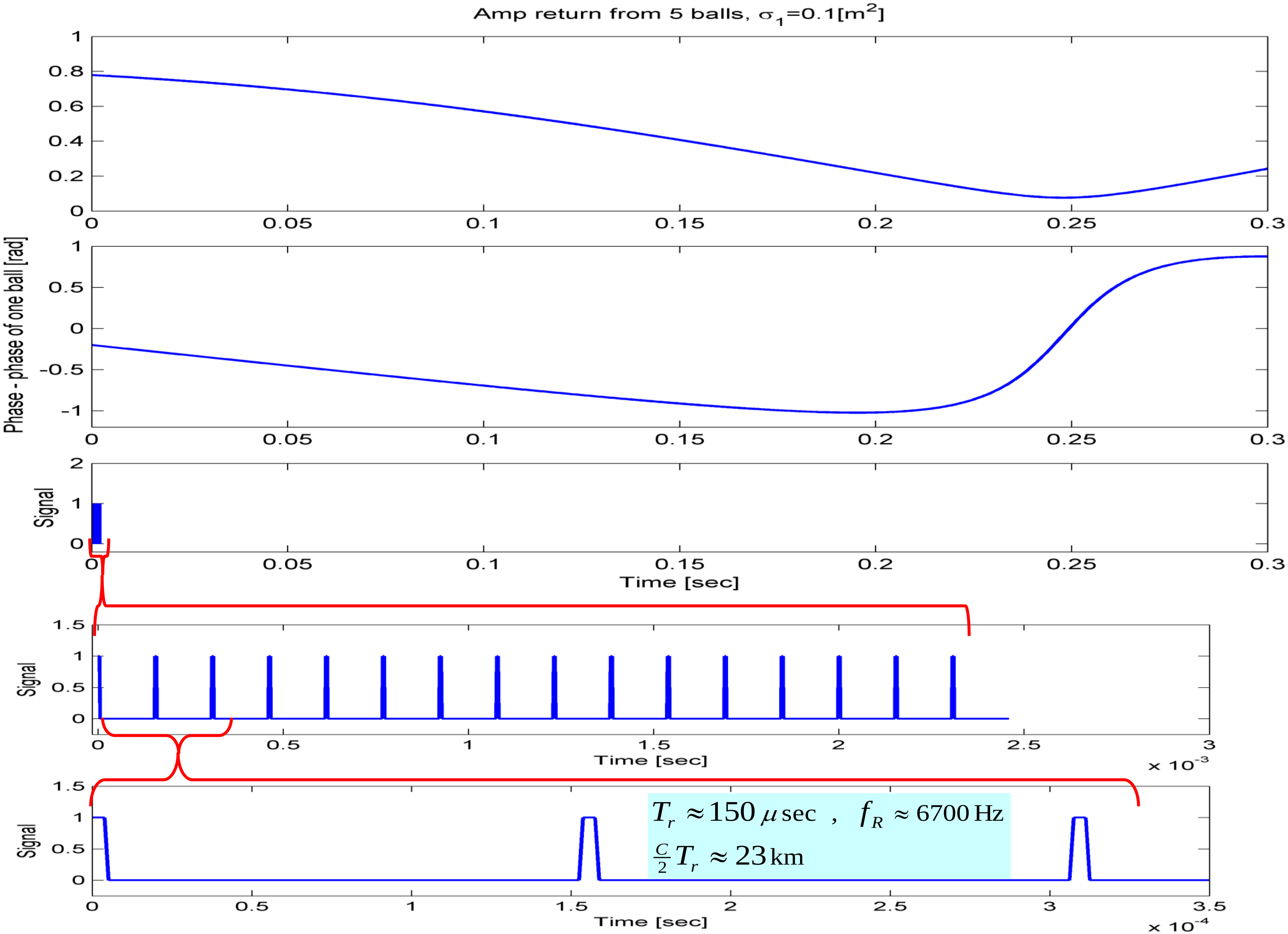
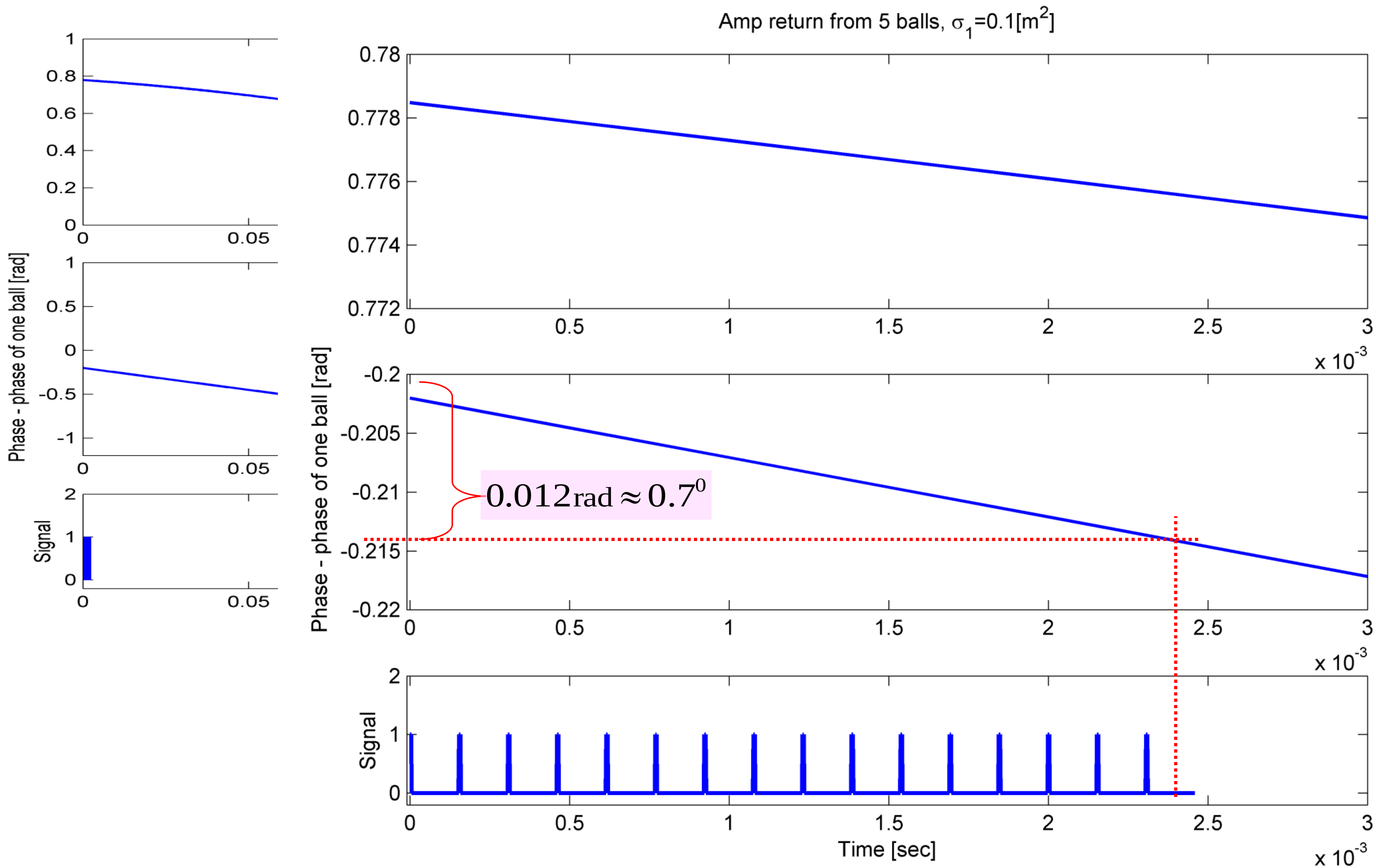
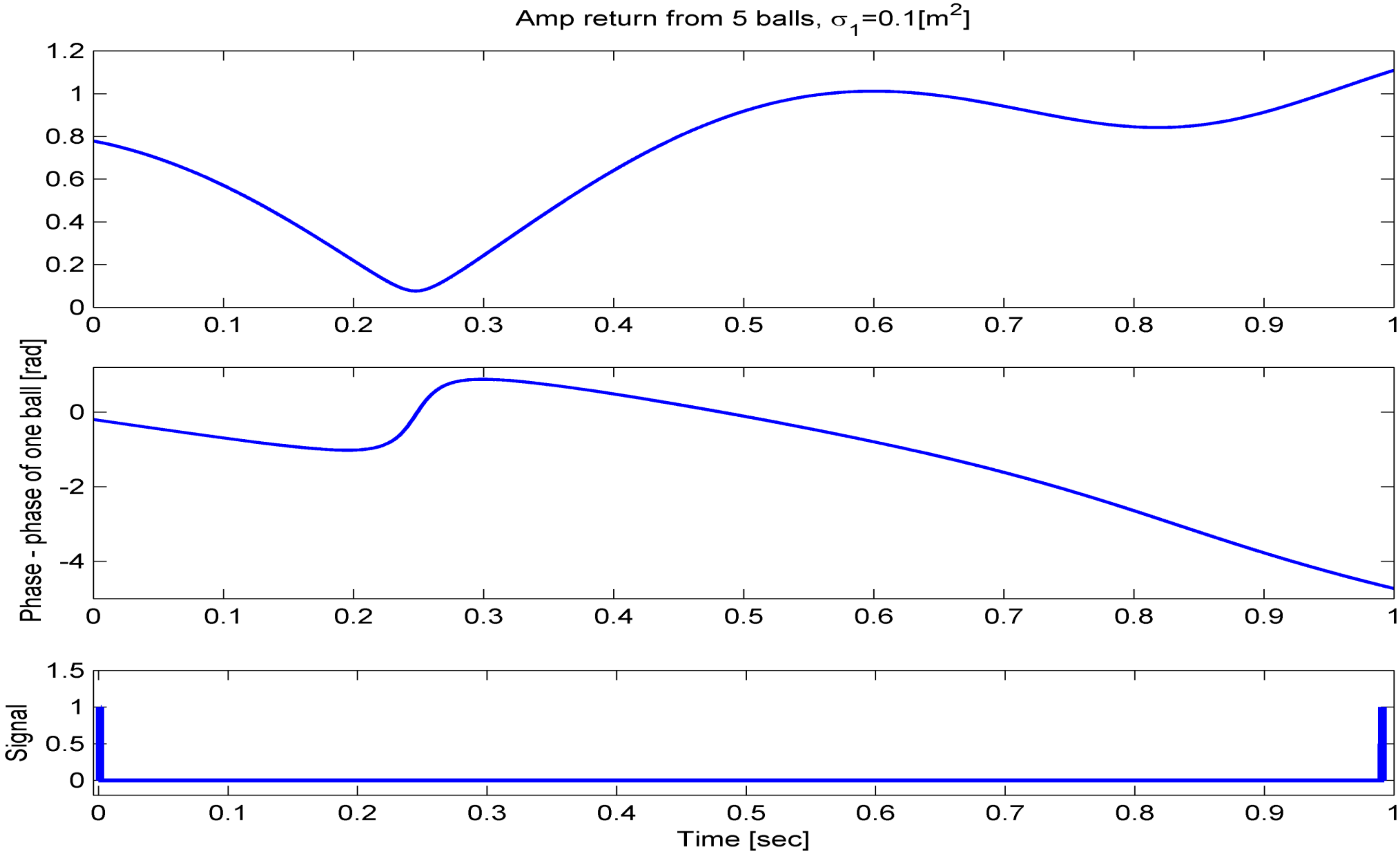


$T_r \approx 150\,\mu\text{sec}$, $f_R \approx 6700\,\text{Hz}$

$\frac{C}{2} T_r \approx 23\,\text{km}$



Within the pulse-train the amplitude and phase do not change from pulse-to-pulse (except for a ramp phase change due to Doppler) implying coherence.
The common amplitude is drawn from a Rayleigh PDF. The common phase is drawn from a uniform PDF.



From pulse-train to pulse-train (scan-to-scan) the amplitude and phase change and are practically independent.

The amplitudes are drawn from a Rayleigh PDF. The phases are drawn from a uniform PDF.

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COHERENCE ESTIMATION FOR A RADAR SIGNAL REFLECTED FROM AN AIR TARGET

M. M. Chernykh and O. V. Vasil'ev

Coherence estimation is considered for a centimeter-range radar signal as reflected from actual air targets; the coherence interval is determined, and numerical values are obtained for various types of air target.

Experimental results. The general method was to record signals from actual air targets at the intermediate frequency for subsequent spectral processing by a ground-level recording system based on a highly coherent Doppler-pulse radar working in the centimeter range. The single aircraft flew in straight stationary lines over the radar at distances of 50-100 km at heights of 5000-6000 m with speeds of 800-1000 km/h under simple weather conditions.

CONCLUSIONS

An experimental analysis has been performed for centimeter-range signals reflected from actual aircraft, which indicates the following regularities. When one observes a target of fighter class from the forward hemisphere, the reflected signal has a strong dominant component. The coherence in the reflected signal is largely determined by the stability of the high-frequency structure in that component. The length of the coherence interval governs the maximum coherent processing time and is 300-400 msec, while the correlation interval for the amplitude of the dominant component lies in the range 450-600 msec.

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Other RCS PDFs

- Many radar targets are *not* well-modeled as “many equal-strength scatterers”
- Many distributions have been proposed for modeling radar targets based on empirical measurements

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Model Name	pdf for RCS σ	Comment
Nonfluctuating, Marcum, Swerling 0, or Swerling 5	$p(\sigma) = \delta_D(\sigma - \bar{\sigma})$ $\mathrm{var}(\sigma) = 0$	Constant echo power, *e.g.* calibration sphere or perfectly stationary reflector with no radar or target motion.
Rayleigh / Exponential, Chi-square of degree 2	$p(\sigma) = \frac{1}{\bar{\sigma}} \exp\left[\frac{-\sigma}{\bar{\sigma}}\right]$ $\mathrm{var}(\sigma) = \bar{\sigma}^2$	Many scatterers, randomly distributed, none dominant. Used in Swerling case 1 and 2 models.
Chi-square of degree 4	$p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp\left[\frac{-2\sigma}{\bar{\sigma}}\right]$ $\mathrm{var}(\sigma) = \bar{\sigma}^2 / 2$	Approximation to case of many small scatterers + one dominant, with RCS of dominant equal ≈3x sum of RCS of others. Used in Swerling case 3 and 4 models.

$$\sigma = \frac{A^2}{2} \; , \; \bar{\sigma} = A_0^2$$

$$p(A) = \frac{A}{A_0^2} \exp\left(\frac{-A^2}{2A_0^2}\right) \; , \quad 0 \le A$$

$$p(A) = \frac{2A^3}{A_0^4} \exp\left(\frac{-A^2}{A_0^2}\right) \; , \quad 0 \le A$$

RCS σ is related to the received power. The amplitude A is related to $\sqrt{\sigma}$. But if we use a square-law detector that yields A^2 we are back to the PDF of σ .

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Model Name	***pdf for RCS σ***	***Comment***
Chi-square of degree 2*m*, Weinstock	$p(\sigma) = \frac{m}{\Gamma(m)\bar{\sigma}}\left[\frac{m\sigma}{\bar{\sigma}}\right]^{m-1} \exp\left[-\frac{m\sigma}{\bar{\sigma}}\right]$ $\operatorname{var}(\sigma) = \bar{\sigma}^2 / m$	Generalization of the two preceding cases. Weinstock cases correspond to $0.6 \le 2m \le 4$. Higher degrees correspond to presence of a more dominant single scatterer.
Rice or Rician, non-central chi-square of degree 2	$p(\sigma) = \frac{1}{\bar{\sigma}}\left(1+a^2\right)\exp\left[-a^2 - \frac{\sigma}{\bar{\sigma}}\left(1+a^2\right)\right] \bullet I_0\left[2a\sqrt{\left(1+a^2\right)\left(\sigma/\bar{\sigma}\right)}\right]$ $\operatorname{var}(\sigma) = \frac{\left(1+2a^2\right)}{\left(1+a^2\right)^2}\bar{\sigma}^2$	Exact solution for one dominant scatterer plus many small ones. Ratio of dominant RCS to sum of small RCS is a^2.

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Model Name	pdf for RCS σ	Comment
Weibull	$p(\sigma) = CB\sigma^{C-1}\exp\left[-B\sigma^{C}\right]$ $\bar{\sigma} = \Gamma(1+1/C)B^{-1/C}$ $\mathrm{var}(\sigma) = B^{-2/C}\left[\Gamma(1+2/C) - \Gamma^2(1+1/C)\right]$	Empirical fit to many measured target and clutter distributions. Can have longer "tail" than above cases.
Log-normal	$p(\sigma) = \frac{1}{\sqrt{2\pi}\, s\sigma}\exp\left[-\ln^2(\sigma/\sigma_m)/2s^2\right]$ $\bar{\sigma} = \sigma_m \exp\left(s^2/2\right)$ $\mathrm{var}(\sigma) = \sigma_m^2 \exp\left(s^2\right)\left[\exp\left(s^2\right) - 1\right]$	Empirical fit to many measured target and clutter distributions. "Tail" is longest of above cases. σ_m is the median value of σ.

$$p(A) = \frac{2}{\Gamma(C)}\left(\frac{C}{B}\right)^{C} A^{2C-1}\exp\left(\frac{-C}{B}A^2\right),\quad 0 < A$$

Generalized Chi

$$a = A^2$$

$$p(a) = \frac{a^{C-1}\exp\left(\frac{-C}{B}a\right)}{\left(\frac{B}{C}\right)^{C}\Gamma(C)},\quad 0 < A$$

Gamma pdf

Mark Richards: *Fundamentals of radar signal processing.*

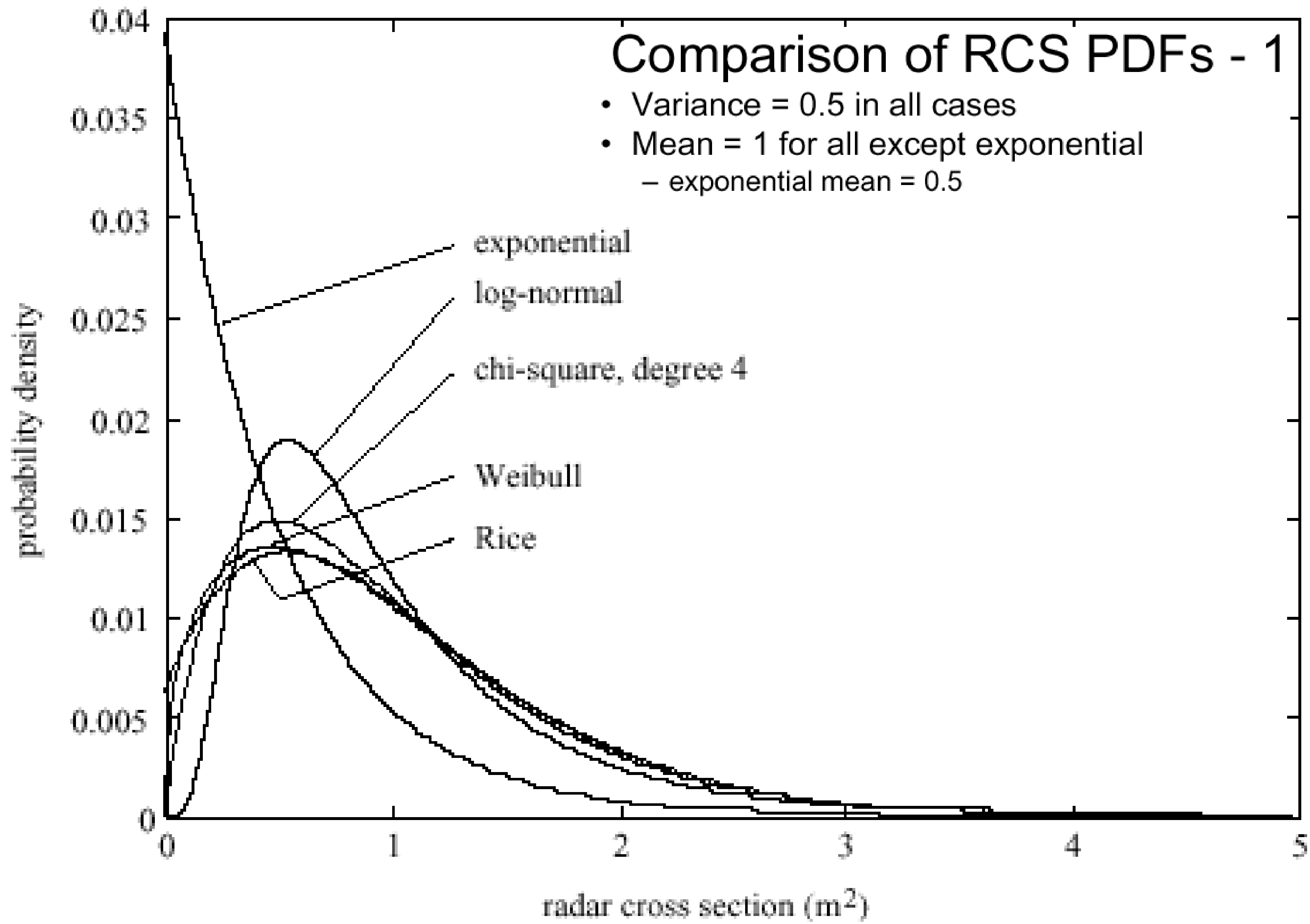
TABLE 2.3 Common Statistical Models for Radar Cross Section

Model name	pdf for RCS σ	Comment
Nonfluctuating, Marcum, Swerling 0, or Swerling 5	$p_\sigma(\sigma) = \delta_D(\sigma - \bar{\sigma})$ $\text{var}(\sigma) = 0$	Constant echo power, e.g., calibration sphere or perfectly stationary reflector with no radar or target motion.
Rayleigh/exponential, chi-square of degree 2	$p_\sigma(\sigma) = \frac{1}{\bar{\sigma}} \exp\left[\frac{-\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma) = \bar{\sigma}^2$	Many scatterers, randomly distributed, none dominant. Used in Swerling case 1 and 2 models.
Chi-square of degree 4	$p_\sigma(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp\left[\frac{-2\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma) = \bar{\sigma}^2/2$	Approximation to case of many small scatterers + one dominant, with RCS of dominant equal to $1+\sqrt{2}$ times the sum of RCS of others. Used in Swerling case 3 and 4 models.
Chi-square of degree $2m$, Weinstock	$p_\sigma(\sigma) = \frac{m}{\Gamma(m)\bar{\sigma}} \left[\frac{m\sigma}{\bar{\sigma}}\right]^{m-1} \exp\left[\frac{-m\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma) = \bar{\sigma}^2/m$	Generalization of the two preceding cases. Weinstock cases correspond to $0.6 \le 2m \le 4$. Higher degrees correspond to presence of a more dominant single scatterer.
Rice or Rician, noncentral chi-square of degree 2	$p_\sigma(\sigma) = \frac{1}{\bar{\sigma}}(1+a^2) \exp\left[-a^2 - \frac{\sigma}{\bar{\sigma}}(1+a^2)\right]$ $\times I_0\left[2a\sqrt{(1+a^2)(\sigma/\bar{\sigma})}\right]$ $\text{var}(\sigma) = \frac{(1+2a^2)}{(1+a^2)^2}\bar{\sigma}^2$	Exact solution for one dominant scatterer plus many small ones. Ratio of dominant RCS to sum of small RCS is a^2.
Weibull	$p_\sigma(\sigma) = CB\sigma^{C-1}\exp[-B\sigma^C]$ $\bar{\sigma} = \Gamma(1+1/C)B^{-1/C}$ $\text{var}(\sigma) = B^{-2/C}[\Gamma(1+2/C) - \Gamma^2(1+1/C)]$	Empirical fit to many measured target and clutter distributions. Can have longer "tail" than previous cases.
Log-normal	$p_\sigma(\sigma) = \frac{1}{\sqrt{2\pi}\, s\sigma} \exp\left[-\ln^2(\sigma/\sigma_m)/2s^2\right]$ $\bar{\sigma} = \sigma_m \exp(s^2/2)$ $\text{var}(\sigma) = \sigma_m^2 \exp(s^2)[\exp(s^2) - 1]$	Empirical fit to many measured target and clutter distributions. "Tail" is longest of previous cases. σ_m is the median value of σ.

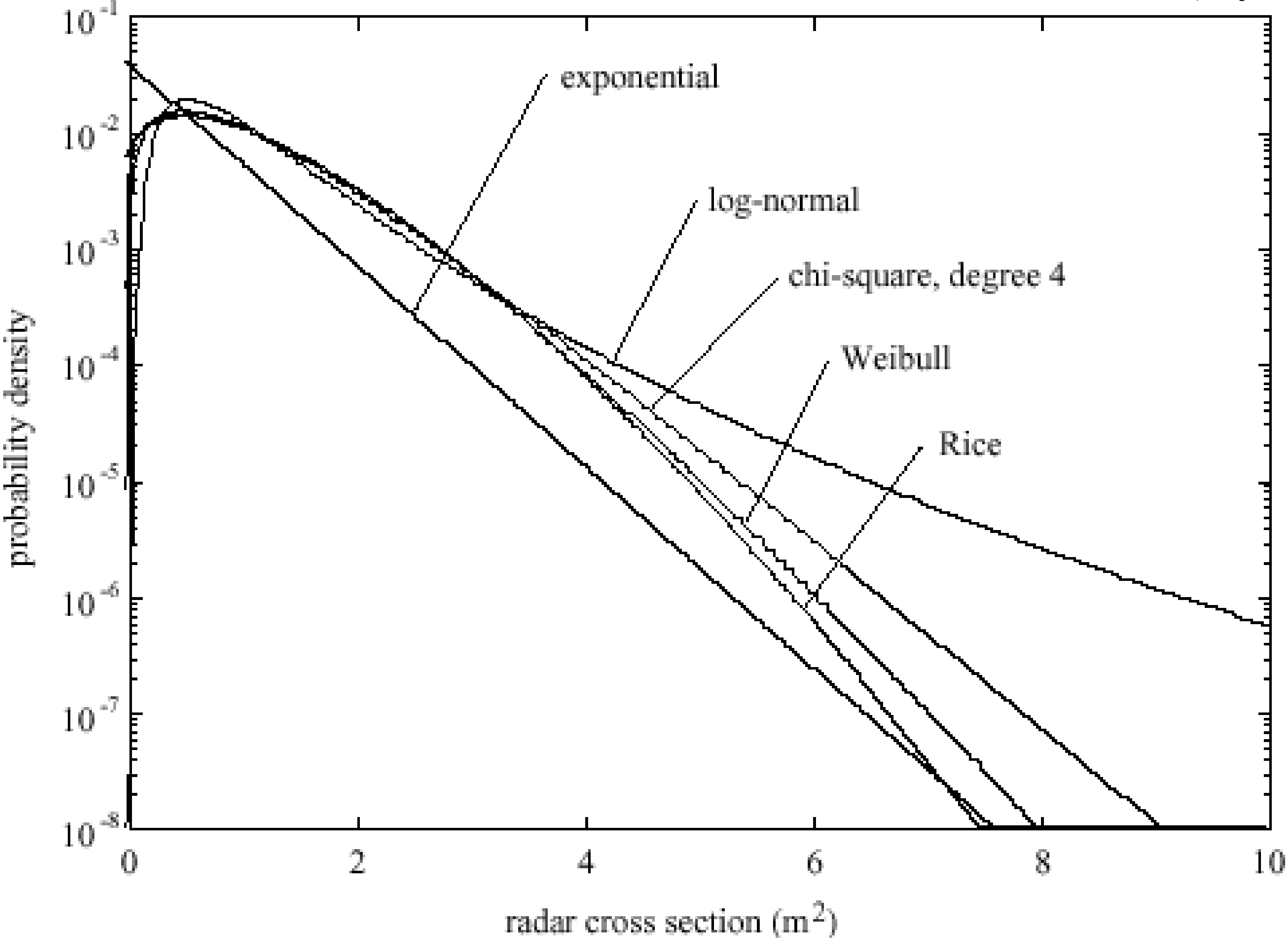
One- *vs.* Two-Parameter PDFs

- One-parameter (mean) *vs.* two-parameter (mean, shape (variance))
  - two-parameter allows separate specification of mean and variance
  - fits wider range of observations
- One-parameter: nonfluctuating, exponential, chi-square of fixed degree
- Two-parameter: Rician, Weibull, log-normal

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech



Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech



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From M.A. Richards, Georgia Tech

- In addition to pdf, we also need to know the correlation properties of RCS or voltage in angle, time, and frequency
- For a rigid target (building, vehicle):
  - if we illuminate from the same aspect and at same frequency, we expect the same measured response
  - if we change frequency or aspect, the relative phase of the individual echoes changes ⇒ measured response changes
- For natural terrain, response typically changes over time even if radar does not move or change frequency
  - in trees, leaves move due to wind
  - waves on ocean surface move
- This "internal motion" causes clutter to decorrelate in time
  - function of type of terrain, wind speed, frequency

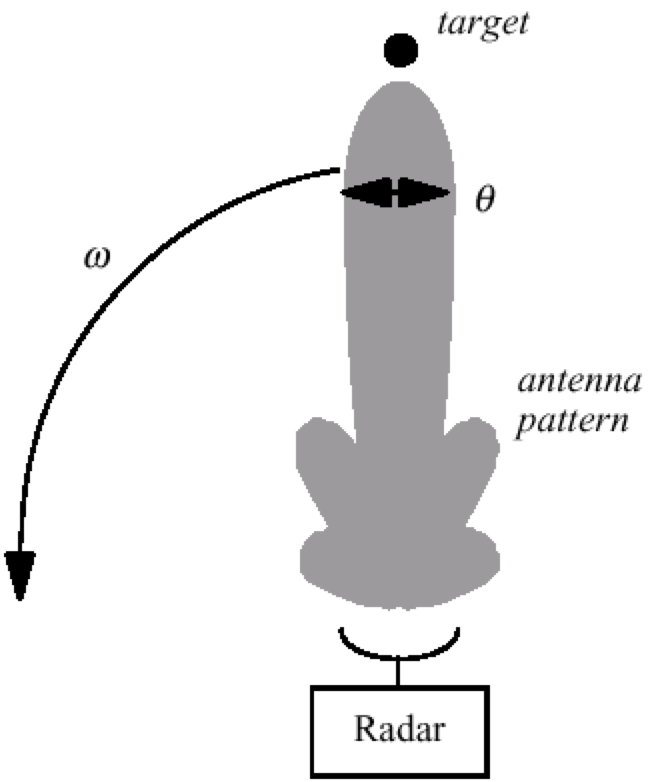
Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

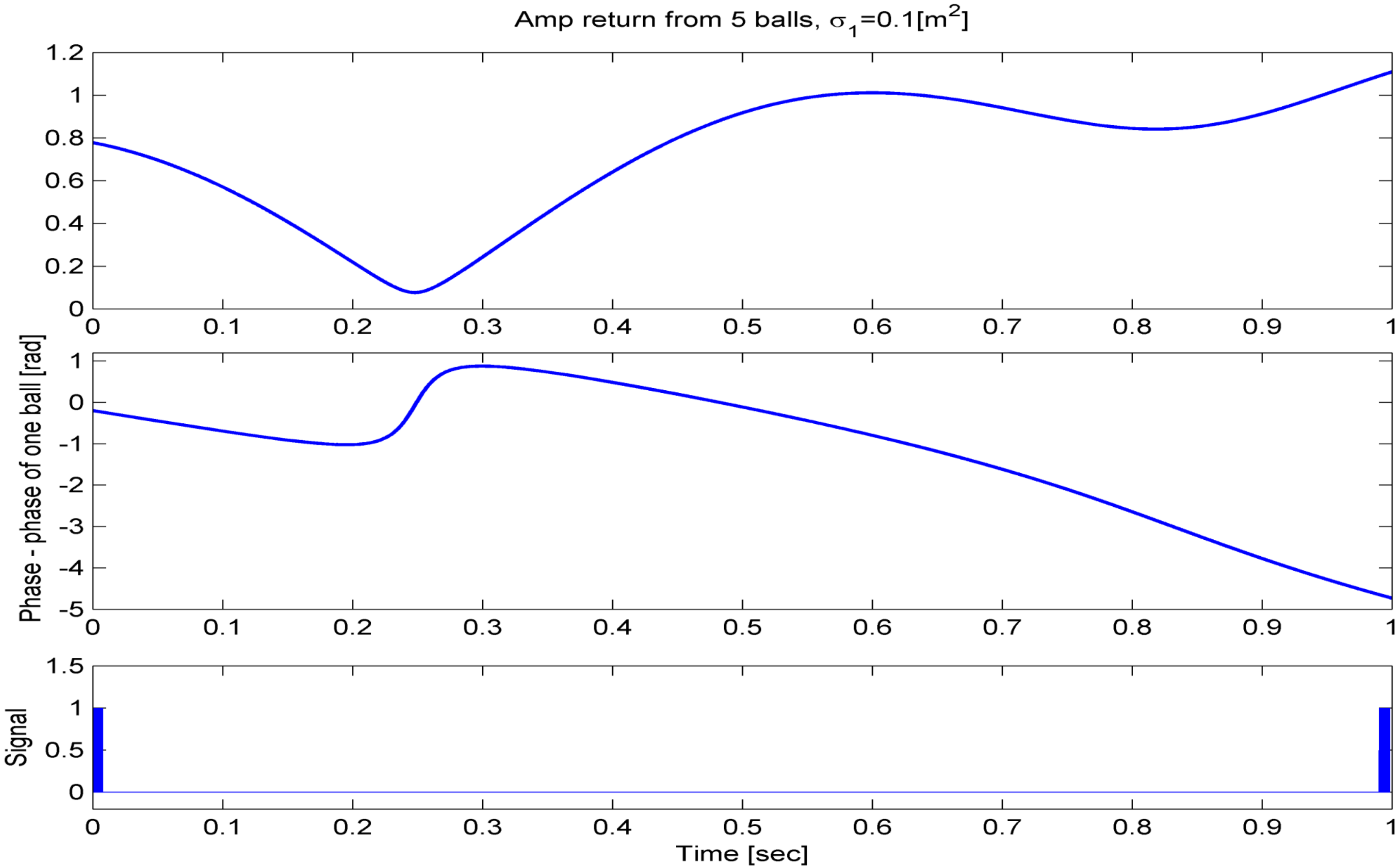
Pulse-to-Pulse Decorrelation

- Many detection algorithms operate on a group of N repeated measurements from N different pulses
  - pulses will be integrated to improve SNR
- An important question is the pulse-to-pulse decorrelation
  - noise aside, do we get the same measurement on successive pulses?
  - or does it change with time?

Rotating Antenna Model

- On each rotation ("scan"), N pulses are received from the target
- How are the N measured echoes at a given range related?
- And how do they relate to the N new measurements from the next scan?





From pulse-train to pulse-train (scan-to-scan) the amplitude and phase change and are practically independent.
The amplitudes are drawn from a Rayleigh PDF. The phases are drawn from a uniform PDF.

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Decorrelation Models for N-Pulse Bursts

- Scan-to-scan: assume the measured pulse amplitudes on one scan are independent of those on the first scan
  - but drawn from same pdf
- Intra-scan: Two limiting assumptions are commonly used:
  - scan-to-scan: all N measurements are identical
  - pulse-to-pulse: all N measurements are independent of one another, but from same pdf

Within a scan

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Swerling Models

- A significant body of detection theory has been developed using the four Swerling models of target RCS and Gaussian noise
  - *relatively* tractable
- Applicable to envelope detection based on N pulses
- Swerling proposed four different target models formed by choosing one each of
  - two different target RCS pdfs
  - two different intra-pulse decorrelation models

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From M.A. Richards, Georgia Tech

Swerling PDFs

- Rayleigh (exponential power)
  - good for complex targets with no dominant scatterer
- Chi-square of degree 4
  - approximation to case of one dominant scatterer and many smaller scatterers
    - total RCS of small scatterers = RCS of dominant scatterer /3 ←
  - Rician pdf is exact for this case
    - but harder to work with

Swerling Decorrelation Models

- Scan-to-scan and pulse-to-pulse models discussed earlier

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Swerling Models

Probability Density Function of RCS	*Decorrelation*	
	scan-to-scan	*pulse-to-pulse*
Rayleigh/exponential	Case 1	Case 2
chi-square, degree 4	Case 3	Case 4

- Nonfluctuating target sometimes called Swerling case 0 or case 5

Peter Swerling
1929 - 2000

P. Swerling "Probability of detection for fluctuating targets", RAND RM-1217, March 17, 1954. Reprinted *IRE Trans. Information Theory*, vol. 6, no. 5, 1960, pp. 269-308.

P. Swerling "Radar probability of detection for some additional fluctuating target cases", *IEEE Trans. Aerospace and Electronic Systems*, vol. 33, no. 2, 1997, pp. 698-709.

*Personal note: **Peter Swerling** was a research assistant to **Jess Ira Marcum** at the Rand Corp. in the 50's. I had the privilege to meet him at a dinner at the home of Solomon Golomb in Pasadena, CA. That evening I learned that the assignment to extend Marcum's work to fluctuating targets took Swerling a whole year to complete, The following related MATLAB example can be completed in few hours.*

בדיקת המודלים של Swerling

המטרה תתואר ע״י 9 מחזירים בעלי שח״מ זהה $n = 1,2,\ldots,9$ $\sigma_n = 0.1\text{m}^2$, ועוד מחזיר עשירי שלו שח״מ הנתון ע״י:

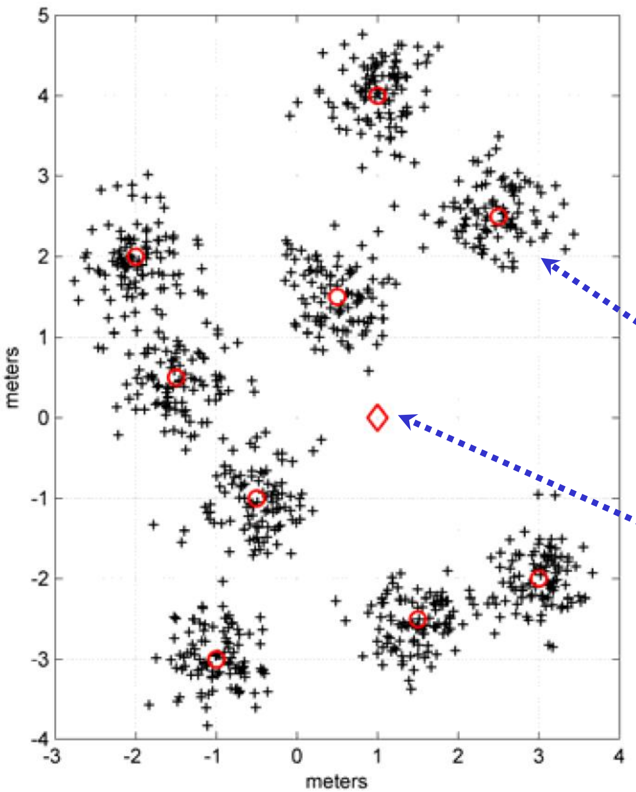
$$\sigma_{10} = \begin{cases} \sigma_1 \;, & \text{SW1,SW2} \\ K\sum_{n=1}^{9}\sigma_n \;, & \text{SW3,SW4} \end{cases} \qquad (4)$$

המיקום הממוצע (במטרים) של תשעת המחזירים הקטנים נתון ע״י הקואורדינטות הקרטזיות שבטבלה:
כדי להשיג אקראיות, המיקום האמיתי בהרצה מס. q נתון ע״י:

$$\begin{aligned} x_{n,q} &= \bar{x}_n + \Delta x_{n,q} \;, \; n = 1,2,\ldots 9 \\ y_{n,q} &= \bar{y}_n + \Delta y_{n,q} \;, \; n = 1,2,\ldots 9 \end{aligned} \;, \quad q = 1,2,\ldots,Q \qquad (5)$$

כאשר $\Delta x_{n,q}$ ו $\Delta y_{n,q}$ לקוחים מפילוג גאוסי עם ממוצא אפס וסטיית תקן 0.35 מטר. המיקום של מחזיר מס. 10 לא ישתנה מהרצה להרצה. דוגמא למיקומים של 10 המחזירים, ב $Q = 100$ הרצות, מופיעה בציור 1.

$\lambda = 0.25\text{m}$



n	$\bar{x}_n$	$\bar{y}_n$
1	3.0	-2.0
2	1.0	4.0
3	-2.0	2.0
4	-1.0	-3.0
5	0.5	1.5
6	-0.5	-1.0
7	2.5	2.5
8	-1.5	0.5
9	1.5	-2.5
10	1.0	0.0

ההרצות

יש לבצע $Q = 100$ הרצות. בכל הרצה מקיף המכ"מ את המטרה בת 10 המחזירים במעגל שרדיוסו 10 ק"מ ומרכזו בנקודה $(x = 0, y = 0)$. המשרעת המתקבלת מהשח"מ השקול (משוואה 1) נמדדת כל 1/4 מעלה. כלומר בכל הרצה מתקבלות 1440 מדידות, ובמאה הרצות מתקבלות $M = 144000$ מדידות.

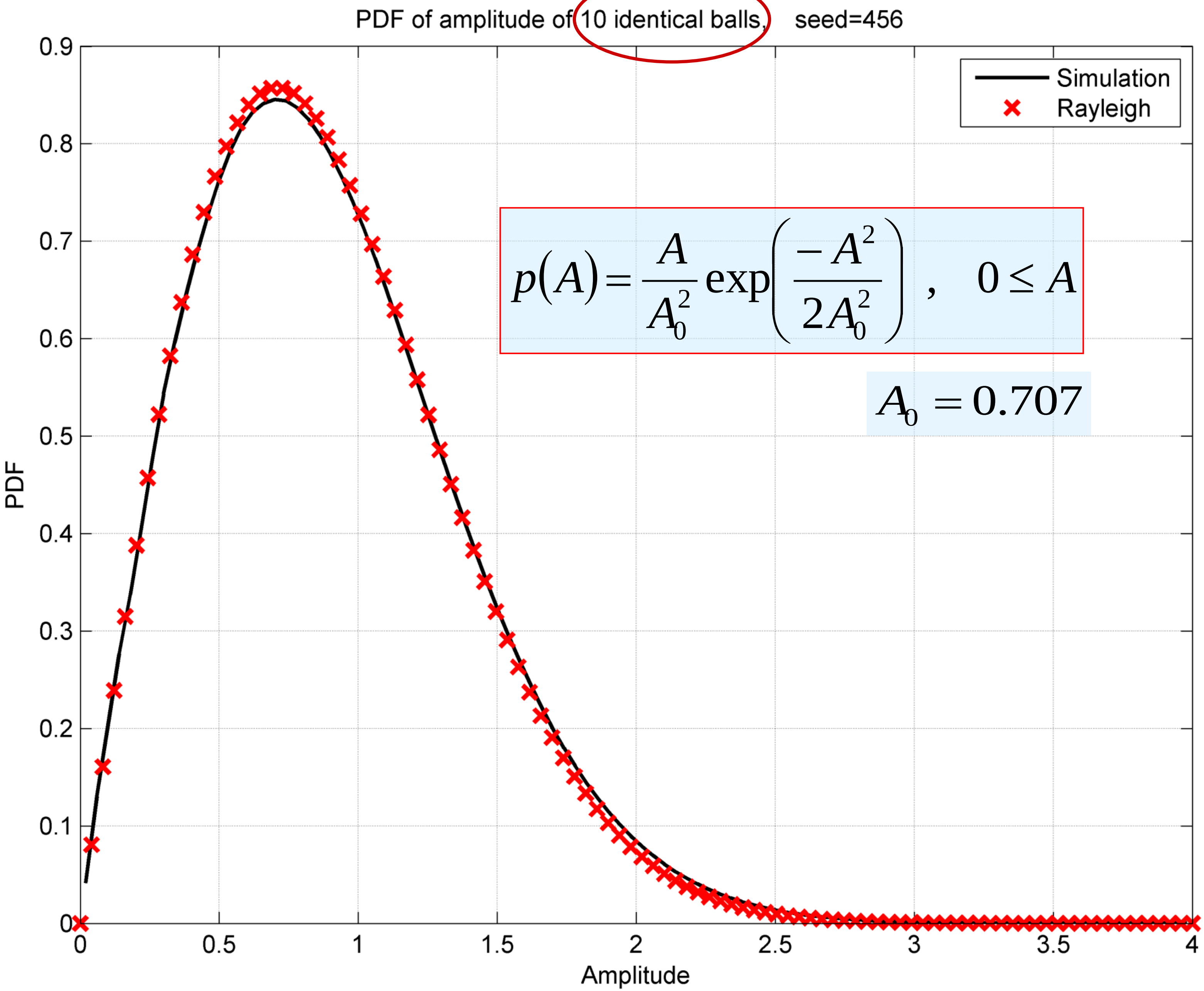
מ M המדידות של A יש לחשב PDF . ממומלץ להשתמש בפקודות MATLB מסוג:

```
vv=linspace(0.02, 3.98);
[qdf(q,:)]=ksdensity(A(q,:),vv,'kernel','epanechnikov','support','positive');
pdf=1/Q*sum(qdf);
```

את ה PDF שהתקבל מהסימולציה יש לצייר יחד עם ה PDF התאורטי לפי נוסחאות (2) או (3) בהתאמה. יש כמובן למצוא את הפרמטר A_0 של ה PDF התאורטי לקבלת ההתאמה הטובה ביותר.

יש לבצע את הסימולציה פעם אחת עבור SW1,SW2, כאשר מתקיימת השורה העליונה של משוואה (4).

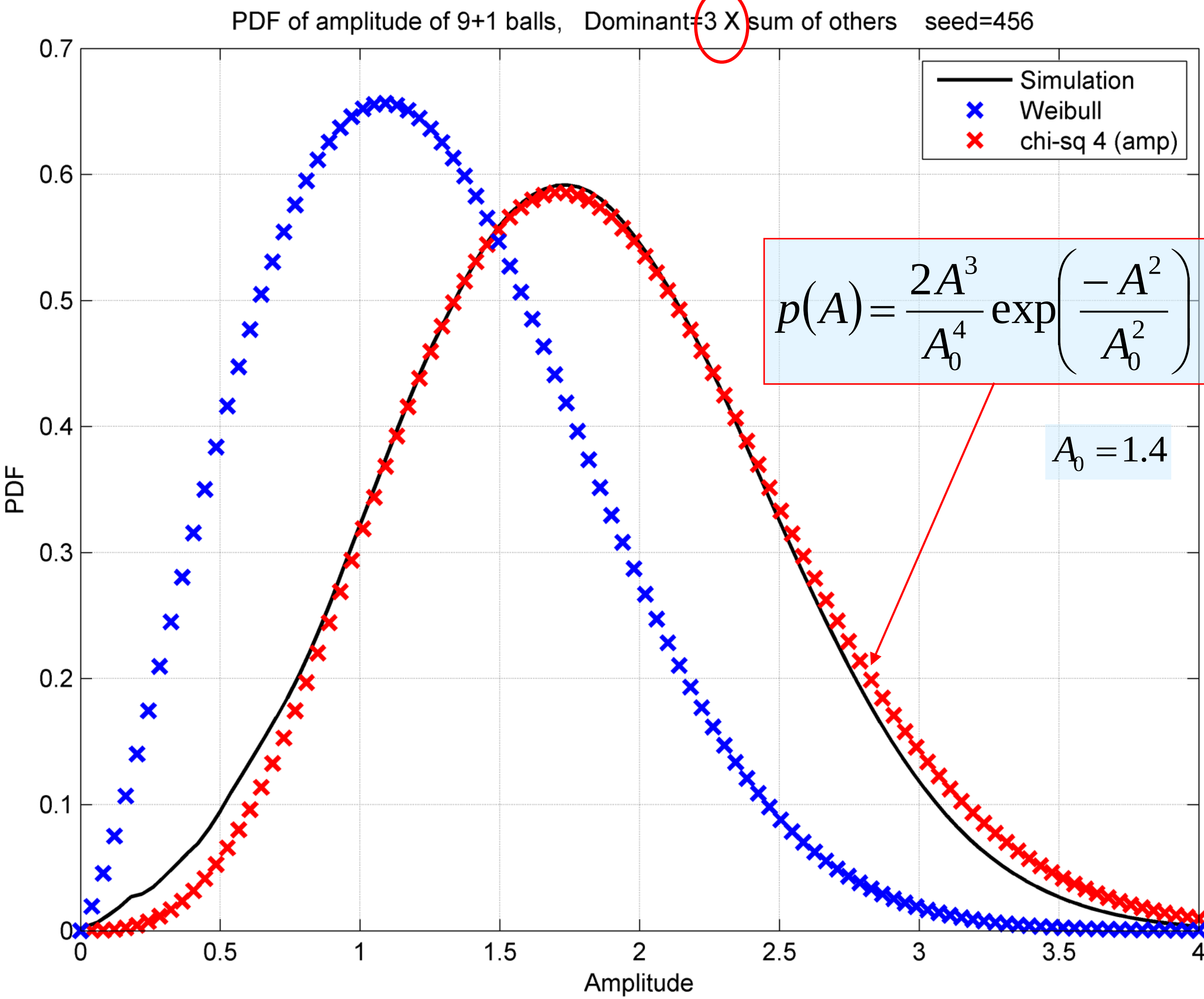
ופעם שנייה עבור SW3,SW4, להם מתקיימת השורה התחתונה של משוואה (4).



$$p(A) = \frac{A}{A_0^2} \exp\left(\frac{-A^2}{2A_0^2} \right), \quad 0 \le A$$

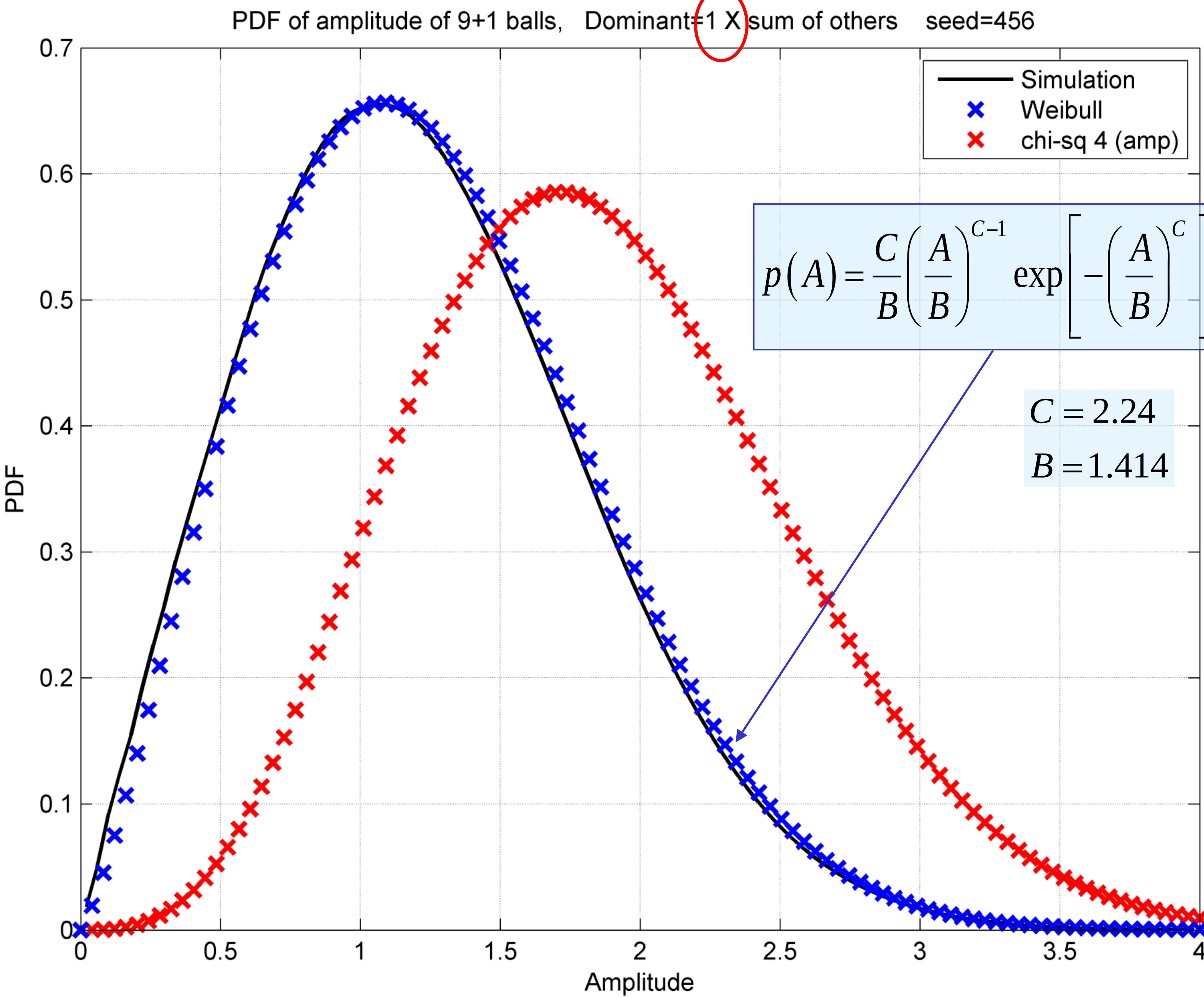
$$A_0 = 0.707$$

Nadav Levanon, Tel-Aviv University



$$p(A) = \frac{2A^3}{A_0^4} \exp\left(\frac{-A^2}{A_0^2}\right), \quad 0 \le A$$

$A_0 = 1.4$

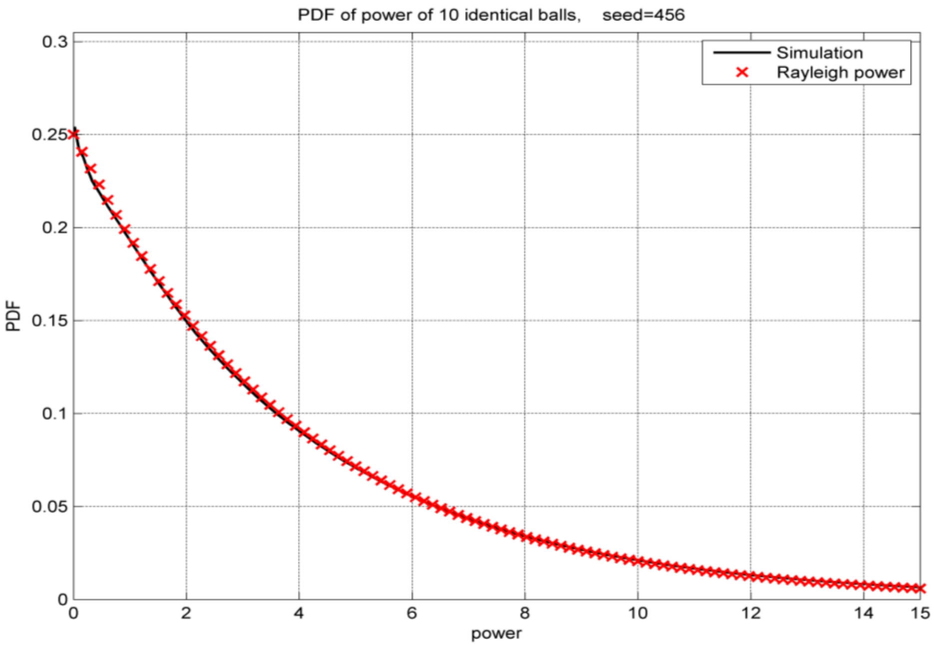


$$p(A) = \frac{C}{B}\left(\frac{A}{B}\right)^{C-1} \exp\left[-\left(\frac{A}{B}\right)^{C}\right], \quad 0 \le A$$

$C = 2.24$

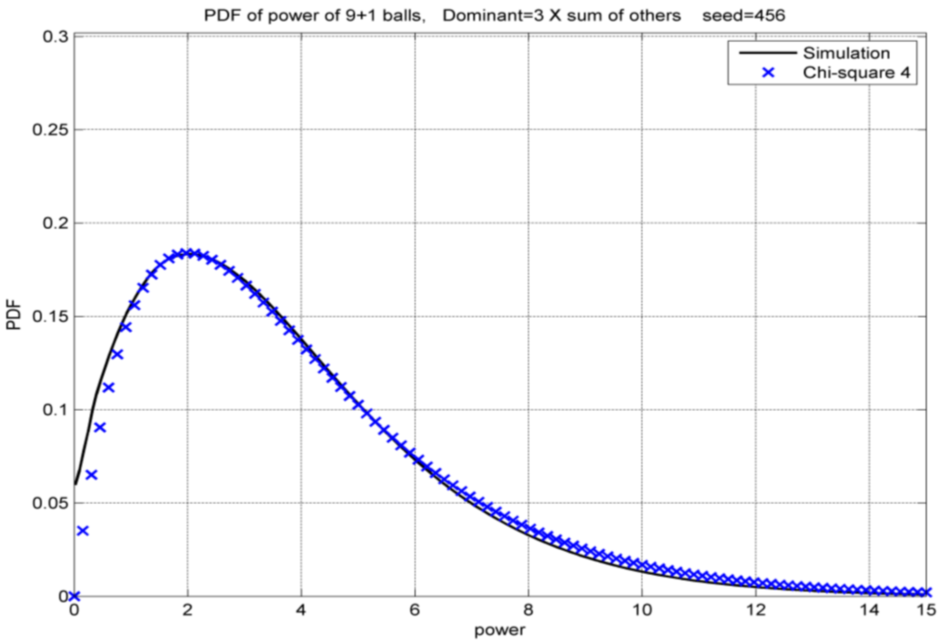
$B = 1.414$

Model Name	pdf for RCS σ	Comment
Nonfluctuating, Marcum, Swerling 0, or Swerling 5	$p(\sigma)=\delta_D(\sigma-\bar{\sigma})$ $\text{var}(\sigma)=0$	Constant echo power, *e.g.* calibration sphere or perfectly stationary reflector with no radar or target motion.
Rayleigh / Exponential, Chi-square of degree 2	$p(\sigma)=\frac{1}{\bar{\sigma}}\exp\left[\frac{-\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma)=\bar{\sigma}^2$	Many scatterers, randomly distributed, none dominant. Used in Swerling case 1 and 2 models.
Chi-square of degree 4	$p(\sigma)=\frac{4\sigma}{\bar{\sigma}^2}\exp\left[\frac{-2\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma)=\bar{\sigma}^2/2$	Approximation to case of many small scatterers + one dominant, with RCS of dominant equal **3x** sum of RCS of others. Used in Swerling case 3 and 4 models.
Chi-square of degree *M*	$p_\sigma(\sigma)=\chi^2\{\sigma;M,\bar{\sigma}\}=\frac{\sigma^{\frac{M}{2}-1}}{\left(\frac{\bar{\sigma}}{\frac{M}{2}}\right)^{\frac{M}{2}}\Gamma\left(\frac{M}{2}\right)}\exp\left(\frac{-M}{2}\frac{\sigma}{\bar{\sigma}}\right)$	Moment generating function $C_{\chi^2}(s)=\left(1-s\frac{\bar{\sigma}}{\frac{M}{2}}\right)^{\frac{M}{2}}$



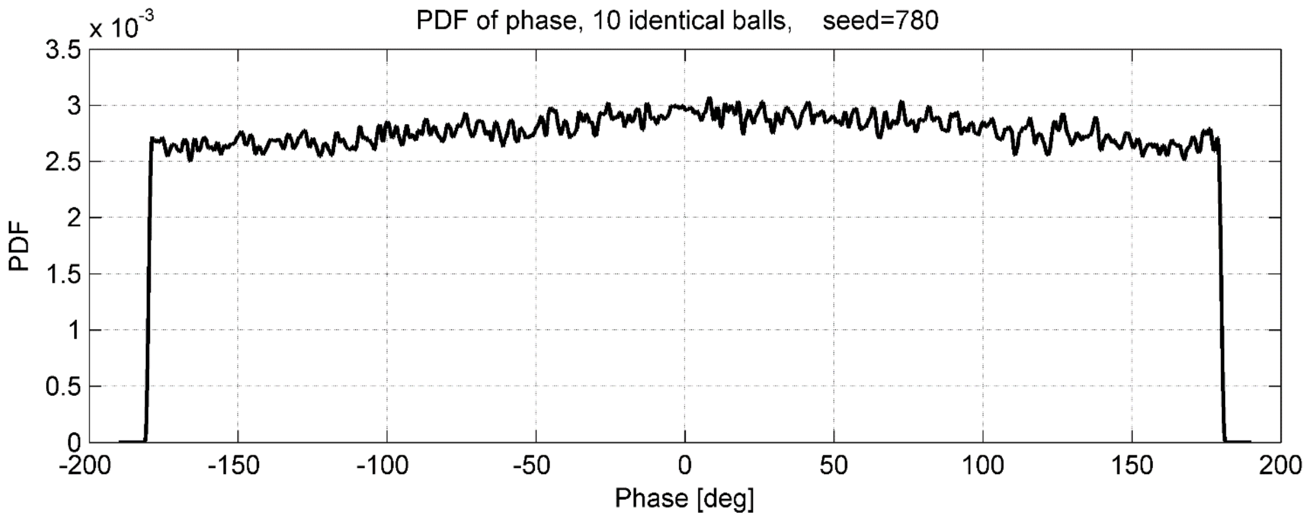
PDF of RCS or Power (SW1, SW2)

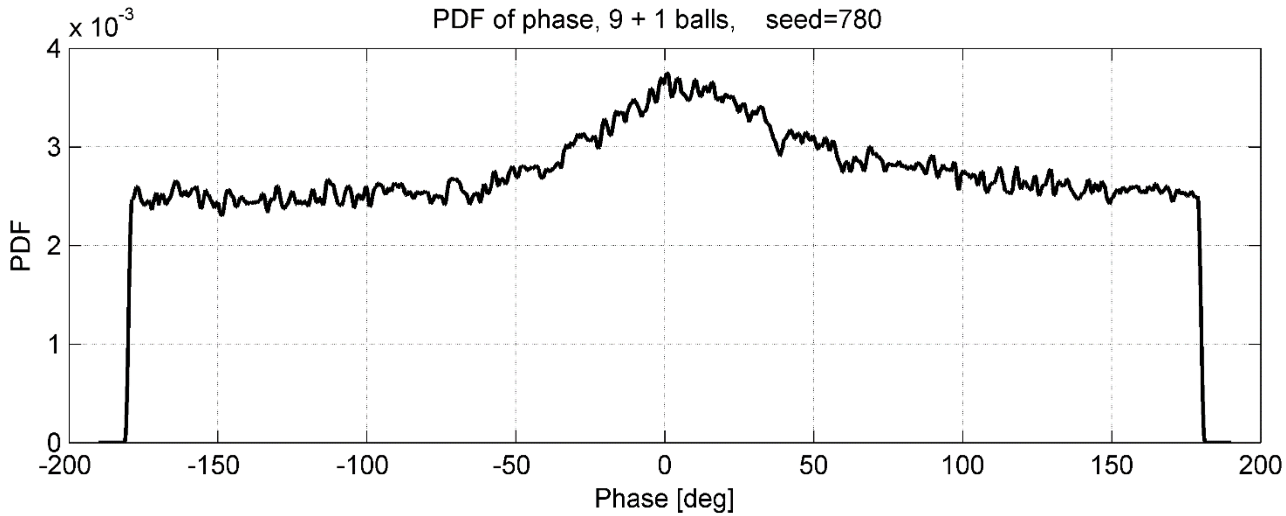
$$p_\sigma(\sigma) = \frac{1}{\bar{\sigma}} \exp\left(\frac{-\sigma}{\bar{\sigma}}\right), \ \bar{\sigma} = 4$$



PDF of RCS or Power (SW3, SW4)

$$p_\sigma(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp\left(\frac{-2\sigma}{\bar{\sigma}}\right), \ \bar{\sigma} = 4$$


PDF of phase, 10 identical balls, seed=780
x 10^-3
3.5
3
2.5
2
1.5
1
0.5
0
PDF
-200
-150
-100
-50
0
50
100
150
200
Phase [deg]


PDF of phase, 9 + 1 balls, seed=780
x 10^-3
4
3
2
1
0
PDF
-200
-150
-100
-50
0
50
100
150
200
Phase [deg]

Feature Article:

DOI. No. 10.1109/MAES.2017.160167

IEEE A&E SYSTEMS MAGAZINE SEPTEMBER 2017

RCS Measurements and ISAR Images of Small UAVs

***Massimiliano Pieraccini, Lapo Miccinesi, Neda Rojhani,* University of Florence, Firenze, Italy**

Pieraccin, M., Miccinesi, L. and Rojhani, N. "RCS measuremenys and ISAR images of small UAVs", *IEEE Aerospace and Electronic Systems Magazine*, vol. 32 (9), pp. 28-32, 2017.

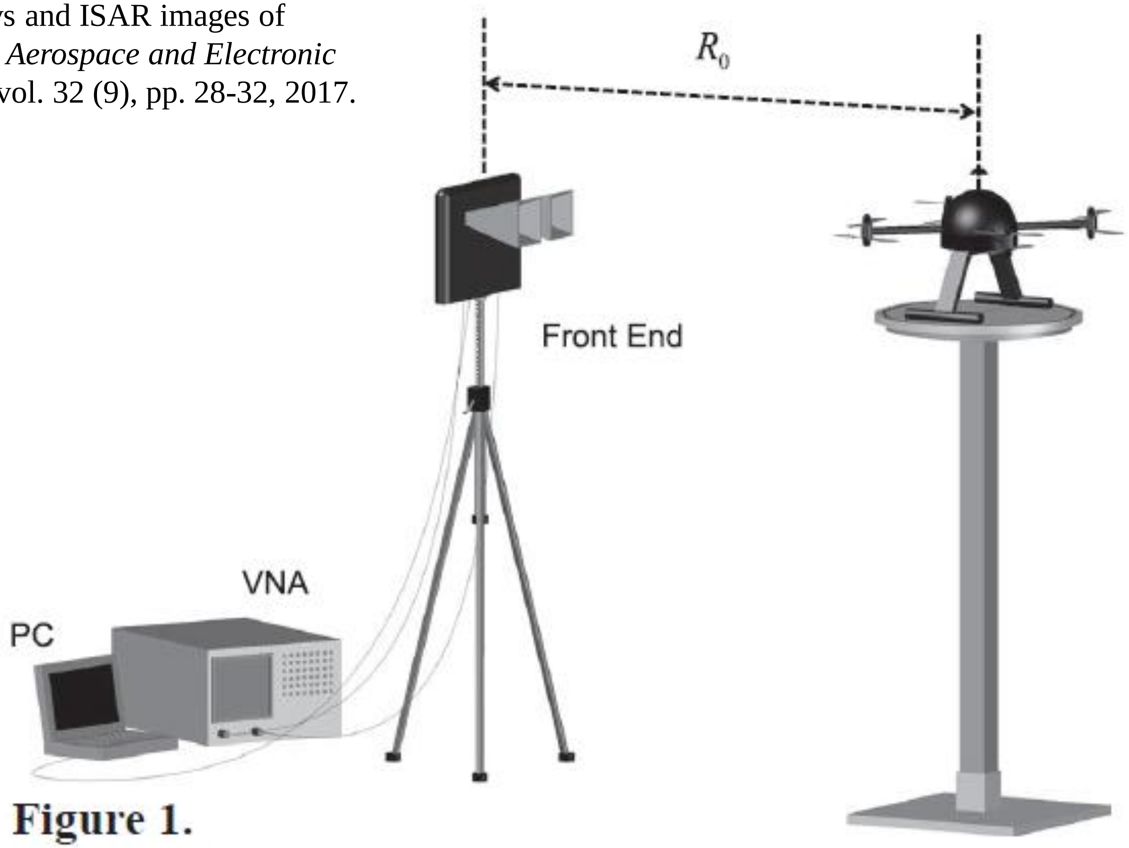


Figure 1.
Sketch of the measurement equipment.

Figure 2.
SYMA quadcopter.

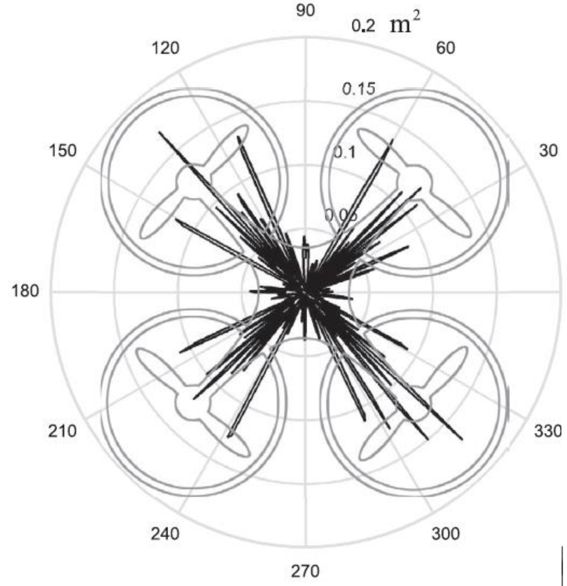


Figure 4.
RCS angular pattern of SYMA quadcopter.

Finally, by using the whole measurement set (9,270 single acquisitions), we calculated the RCS statistical distribution, as shown in Figure 10, that resulted in very good agreement with the well-known Swerling Case I distribution [12]:

$$p(\sigma) = \frac{1}{\bar{\sigma}} e^{-\frac{\sigma}{\bar{\sigma}}} \qquad (4)$$

where $\bar{\sigma}$ is the mean value of RCS. Indeed the χ^2 goodness-of-fit test rejected the null hypothesis at the 1% significance level.

Figure 3.
AirVision quadcopter.

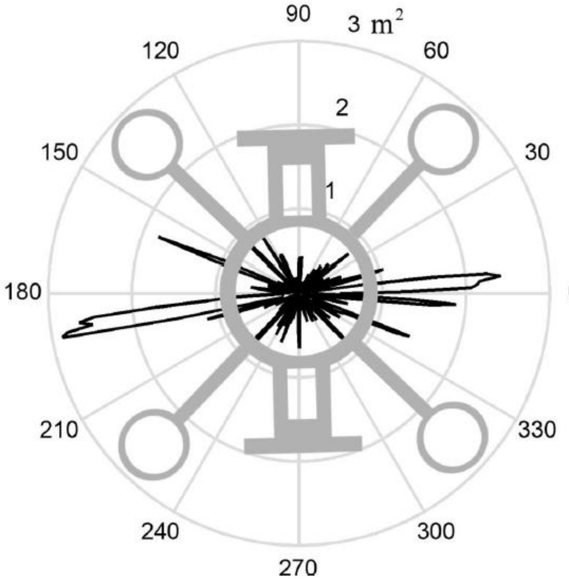


Figure 5.
RCS distribution of AirVision quadcopter.

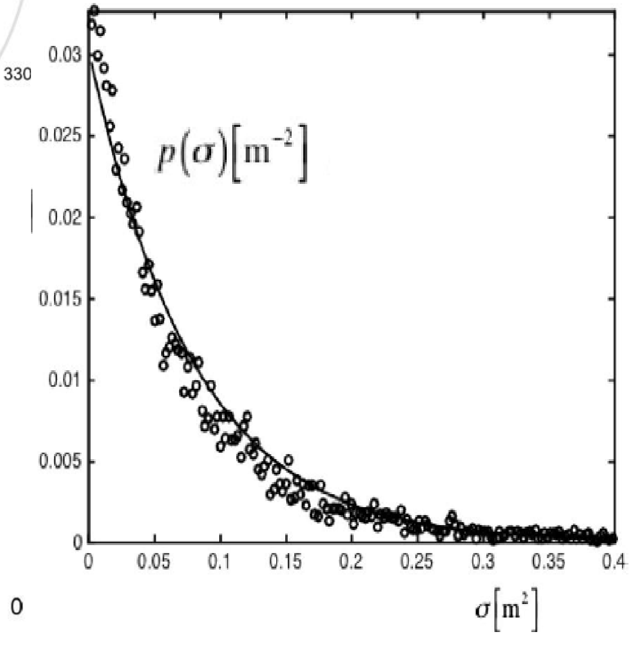


Figure 10.
RCS distribution of AirVision quadcopter using the whole measurement set for 3D ISAR. The full line is the Swerling distribution.