Ambiguity Function of Quadriphase Coded Radar Pulse

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Taylor's quadriphase coding is investigated for nonzero Doppler shifts. While the zero-Doppler cut of the ambiguity function (i.e., the autocorrelation) strongly resembles that of the corresponding biphase code, the remaining ambiguity function differs considerably. The ambiguity function of quadriphase code 13 is typified by a diagonal ridge as found in linear FM signals. The ambiguity function of quadriphase code 28A resembles the three parallel ridges of Frank code 16.

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I. INTRODUCTION

Radar pulse compression using quadriphase coding was introduced by J. W. Taylor, Jr. [1, 2]. The advantages of the signal are in the spectrum falloff rate and the relative ease in which it lends itself to digital processing. Comparison with conventional phase coding, in particular Barker's and Turin's biphase codes (which serve as a basis for the quadriphase codes), show [1] that the pulse compression achieved by biphase codes is maintained in quadriphase codes, and that both have very similar autocorrelation functions.

The similarity of the autocorrelation, which is the zero-Doppler cut of the ambiguity function, indicates similar performances of the two signals against stationary (or known velocity) targets. With regard to targets of unknown velocity, [1] recommends limiting the use of both signals to cases with a maximum Doppler frequency of $0.3/(NT)$, where T is the duration of the subpulse and N is the number of subpulses.

A more detailed comparison of the two signals in the presence of Doppler shift can be obtained from their corresponding ambiguity functions. Ambiguity functions which are quite different are presented here. The ambiguity function of a Barker signal has a major single peak at the origin and smaller sidelobes distributed symmetrically with respect to both the delay and the Doppler axes [3, 4]. The ambiguity function of Taylor's quadriphase code 13 is dominated by a diagonal ridge, whose slope in the Doppler-delay plane is $\approx 0.7/(NT^2)$ Hz/s.

A diagonal ridge with such a slope resembles the diagonal ridge in the ambiguity function of a linear FM signal with a total frequency deviation $\Delta f = 0.7/T$. As a matter of fact, the quadriphase signal effectively switches between two frequencies separated $0.5/T$ apart [2]. This can serve as an intuitive explanation for the ridge.

The diagonal ridge is not a general feature of quadriphase codes. Code 28A [1] exhibits a more uniform pedestal. However, traces of three parallel ridges, typical of Frank code 16, are evident.

II. QUADRIPHASE CODES

The subset of quadriphase codes suggested by Taylor [1], are generated by passing a carrier subpulse with an envelope of a single one half cosine cycle, through a tapped delay line whose weights V_k are related to the corresponding Barker weights W_k as

$$V_k = j^{s(k-1)} W_k \tag{1}$$

where s is fixed, either $+1$ or -1. The one half cycle cosine is of duration $2T$, where T is the tap separation. This scheme creates a narrowband signal whose

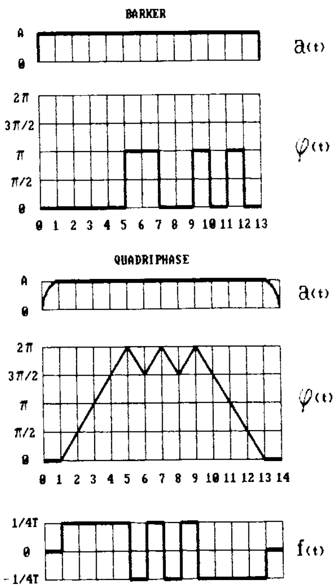


Fig. 1. Magnitude and phase of complex envelope of length 13 Barker coded signal, and corresponding quadriphase signal (length 13).

complex envelope $u(t)$

$$u(t) = a(t)e^{j\phi(t)} \tag{2}$$

has a continuous phase $\phi(t)$, which is obtained from the corresponding biphase code by the following transformation. If the original sequence of phases is θ_k, $k = 1,\ldots,N$, then the transformed phases at multiples of the subpulse duration T, are given by

$$\phi(kT) = \begin{cases} 0, & k = 0 \\ s(k-1)\frac{\pi}{2} + \theta_k, & k = 1,\ldots,N \\ 0, & k = N+1 \end{cases} . \tag{3}$$

In between multiples of T, the phase is given by straight-line segments, connecting the values at multiples of T.

The magnitude $a(t)$, which is a rectangular in the biphase case, is given in the quadriphase case as

$$a(t) = \begin{cases} A\sin(2\pi t/4T), & 0 \le t \le T \\ A, & T \le t \le NT \\ A\cos[2\pi(t-NT)/4T], & NT \le t \le (N+1)T \end{cases} \tag{4}$$

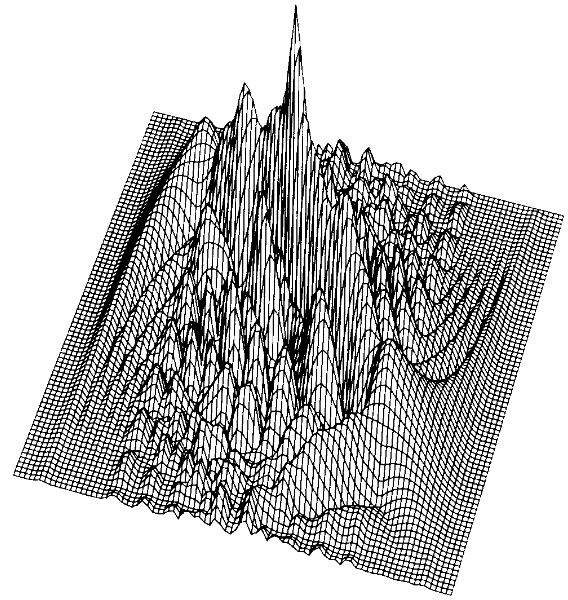
The magnitude and phase of a length 13 Barker code, and the corresponding quadriphase code, are shown in Fig. 1. For the quadriphase code, the frequency is also plotted.

III. AMBIGUITY FUNCTION

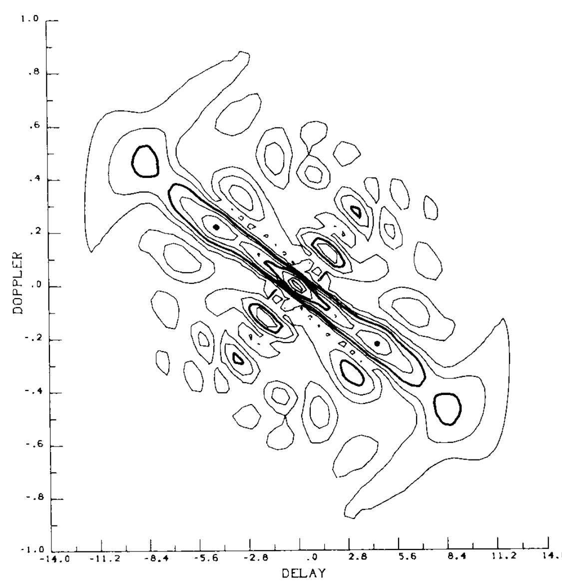
Fig. 2 presents the ambiguity function (with an inverted delay axis) of the quadriphase code of length 13, (the signal described in Fig. 1). The plotted function is

$$|\chi(\tau,\nu)| = \left| \int_{-\infty}^{\infty} u(t)u^*(t+\tau)e^{j2\pi\nu t}\,dt \right| . \tag{5}$$

Fig. 2(a) is a 3-D view of the ambiguity function (linear scale), and Fig. 2(b) is a contour plot of the same function. Both parts of Fig. 2 cover the delay axis

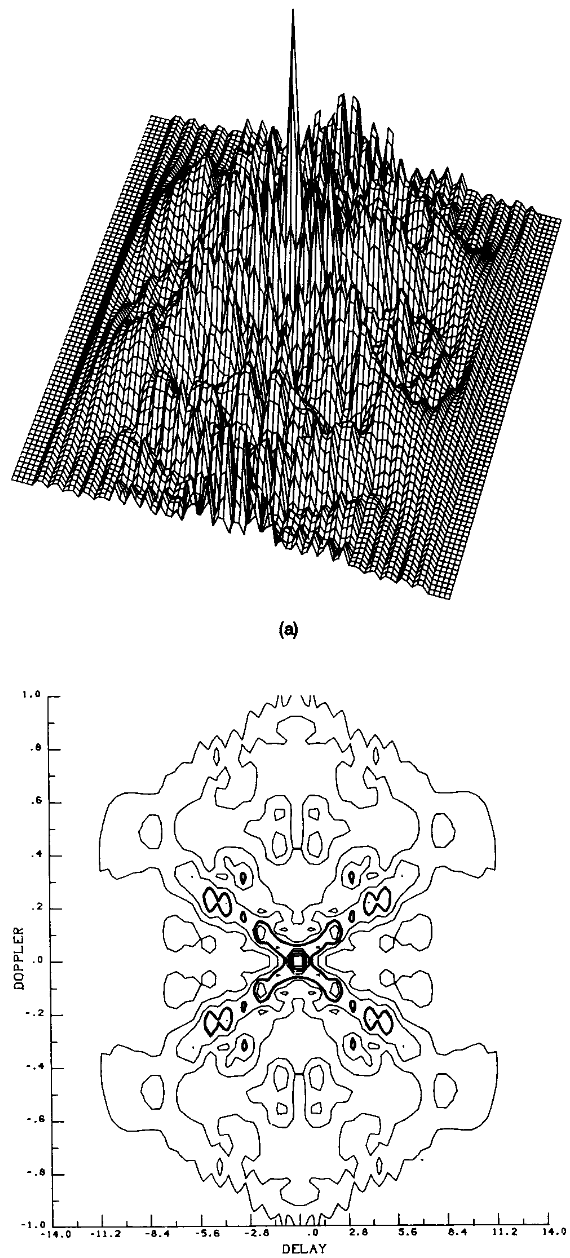


(a)



(b)

Fig. 2. Ambiguity function of length 13 quadriphase pulse, (a) 3-D view. (b) Contour plot (contours 0.3 and 0.6 emphasized).



(a)

(b)

Fig. 3. Ambiguity function of length 13 Barker coded pulse, (a) 3-D view. (b) Contour plot (contours 0.3 and 0.6 emphasized).

from $-14T$ to $+14T$, and the Doppler axis from $-1/T$ to $+1/T$. The contour lines begin at the level of 0.1 and the spacing between lines is also 0.1. Contours 0.3 and 0.6 are emphasized.

The dominant feature of the ambiguity function is the diagonal ridge, which is evident in both the contour plot and the 3-D view. This ridge exists as part of an X-shaped ridge in the ambiguity function

TABLE I
Ambiguity Function Cuts At Three Doppler Shifts

Doppler shift, ν	0	$0.02/T$	$0.05/T$
A. Main Lobe (dB)			
Quadriphase	0	−0.7	−2.7
Barker	0	−1.7	−6
Linear FM	0	−0.4	−0.8
B. Peak Sidelobe (dB)			
Quadriphase	−22	−15.5	−10.5
Barker	−22	−21.5	−16*
Linear FM	−14.5	−14.2	−14
C. Delay Of Main Lobe			
Quadriphase	0	$0.4T$	T
Barker	0	0	0
Linear FM	0	$0.4T$	T

Note: Middle column ($\nu = 0.02/T = 0.28/14T$) corresponds approximately to the $\nu_{max} = 0.3/NT$, Doppler limit suggested in [1].

Note: * The −6 dB sidelobe is considered part of the main lobe.

of the corresponding length 13 Barker signal. That ambiguity function is given in Figs. 3(a) and 3(b). The ambiguity function of the Barker signal resembles more of a thumbtack shape with a narrow peak in the origin and a pedestal which has a shape of the figure "8". The phase discontinuities and the sharp rise and fall of the magnitude of the envelope of the Barker signal are reflected in the much sharper features of the ambiguity function.

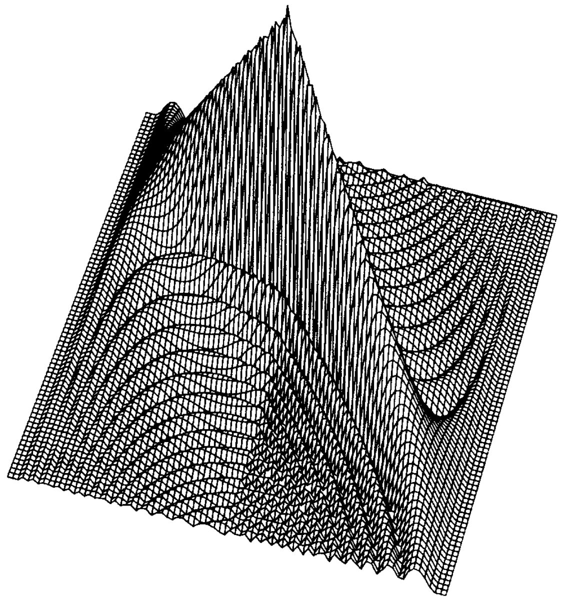
The diagonal ridge of the length 13 quadriphase signal resembles the ridge of a linear FM signal. Fig. 4 contains the 3-D and contour plot of the ambiguity function of a linear FM signal of duration $14T$, and a total frequency deviation, during that duration, of $\Delta f = 0.7/T$. We note that the ridge in the linear FM case has the same slope as in the quadriphase case. The sidelobes spread, however, is considerably less.

IV. CUTS ALONG SMALL DOPPLER SHIFTS

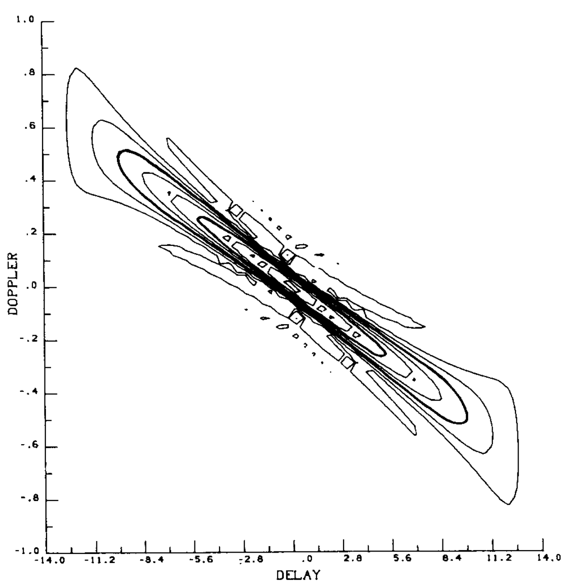
Cuts of the ambiguity functions at zero Doppler (the autocorrelation function) of both the quadriphase and the Barker signals appear in [1]. Here we present cuts at intervals of $0.01/T$ Hz (from 0 to $-0.05/T$) using a dB scale. Figs. 5–7 present the results for the three signals, quadriphase, Barker, and linear FM, respectively. The main results are summarized in Table I.

V. QUADRIPHASE CODE 28A

To check if the diagonal ridge observed in the ambiguity function of quadriphase code 13 is typical of other quadriphase codes, we have plotted the ambiguity function of code 28A [1]. That ambiguity function turned out to have more of a thumbtack



(a)



(b)

Fig. 4. Ambiguity function of linear FM pulse of length $14T$, with total frequency deviation $\Delta f = 0.7/T$. (a) 3-D view. (b) Contour plot (contours 0.3 and 0.6 emphasized).

shape, with a relatively uniform pedestal. However, a blown-up contour plot shown in Fig. 8 (covering about half the total delay span of the ambiguity function) reveals three parallel ridges, broken into peaks. A similar pattern of three ridges is found in the ambiguity function of code Frank of length 16 [4].

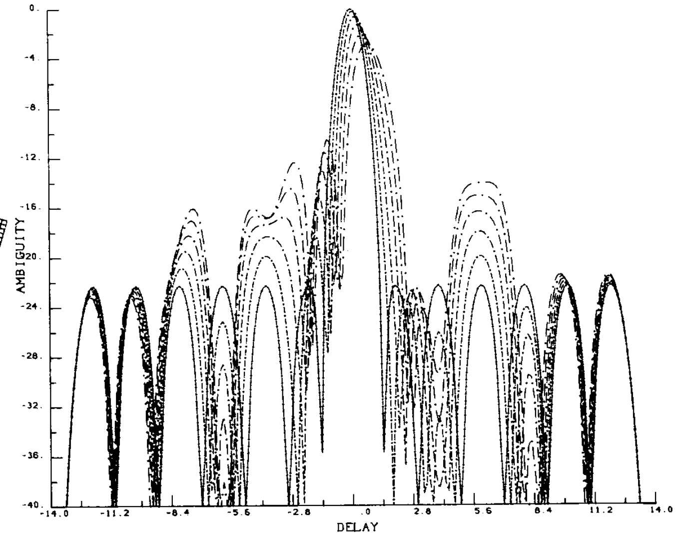


Fig. 5. Quadriphase signal. Cuts of ambiguity function at $\nu = -.01n/T$, $n = 0,1,\ldots,5$. (Most dense line corresponds to $n = 0$ and least dense to $n = 5$. Vertical scale is in dB.)

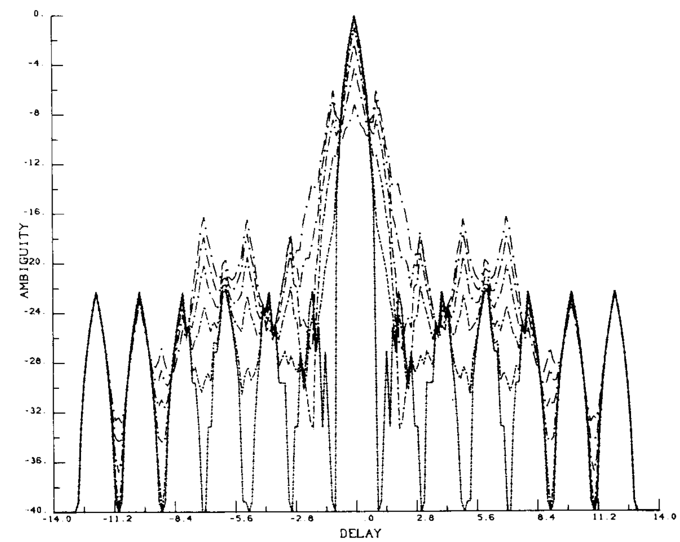


Fig. 6. Barker signal. Cuts of ambiguity function at $\nu = -.01n/T$, $n = 0,1,\ldots,5$. (Most dense line corresponds to $n = 0$ and least dense to $n = 5$. Vertical scale is in dB.)

VI. CONCLUSIONS

Outside the zero Doppler line, the ambiguity function of a quadriphase coded pulse does not necessarily resembles the ambiguity function of the corresponding Barker code. Nor is there a uniform ambiguity pattern for all quadriphase codes.

The particular ambiguity function of a quadriphase code 13 is a hybrid from the ambiguity function of the corresponding Barker signal and the ambiguity function of a linear FM signal with a total frequency

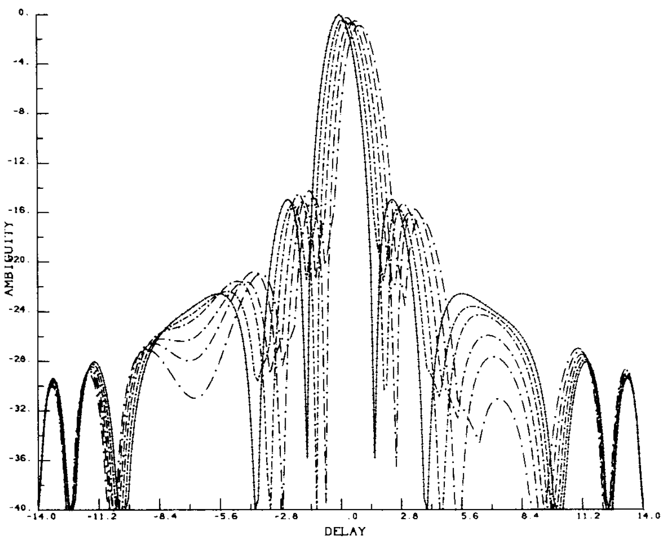


Fig. 7. Linear FM signal. Cuts of ambiguity function at $\nu = -.01n/T$, $n = 0,1,\ldots,5$. (Most dense line corresponds to $n = 0$ and least dense to $n = 5$. Vertical scale is in dB.)

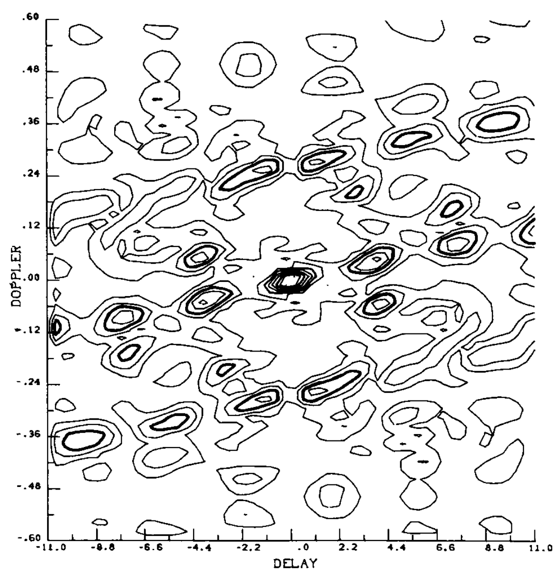


Fig. 8. Central part of ambiguity function of quadriphase code 28A. (Contours 0.3 and 0.6 emphasized.)

deviation $\Delta f = 0.7/T$, where T is the duration of the subpulse. The zero Doppler shift of the ambiguity function is very similar to that of the Barker signal. Cuts at nonzero Doppler exhibit a delay of the peak position, which increases with Doppler shift. Such a coupling between delay and Doppler, is due to the diagonal ridge in the ambiguity function of a linear FM pulse.

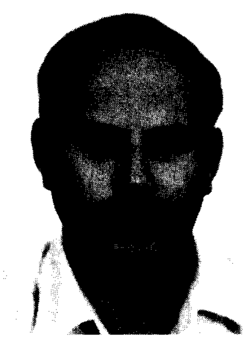
REFERENCES

[1] Taylor, J. W., Jr., and Blinchikoff, H. J. (1988)
Quadriphase code—a radar pulse compression signal with unique characteristics.
IEEE Transactions on Aerospace and Electronic Systems, **AES-23**, 2 (Mar. 1988), 156–170.

[2] Vale, C. R. (1981)
SAW quadraphase code generator.
IEEE Transactions on Microwave Theory and Techniques, **MTT-29** (May 1981), 410–414.

[3] Rihaczek, A. W. (1969)
Principles of High Resolution Radar
New York: McGraw-Hill, 1969, p. 217.

[4] Levanon, N. (1988)
Radar Principles
New York: Wiley, 1988, ch. 8.



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