relatively small values of T_A [see (4)], the accuracy associated with the phase principle is essentially equal to the accuracy associated with the maximum likelihood principle.

In Fig. 1 we have considered the case where $T_L \ll T_A$. The case where $T_A \doteq T_L$ has been studied by Blum [5, Fig. 10]. In this latter case the measurement is independent of system gain fluctuations but the sensitivity, using the normalization as in (8), is only 2. Thus the performance is worse than that of the balanced Dicke radiometers.

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A. J. RAINAL
Bell Telephone Laboratories, Inc.
Whippany, N. J. 07981

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Accurate Pulse-Radar Altimeter for Meteorological Balloons

Abstract—A simple, light-weight pulse-radar altimeter package has been designed and tested to accurately measure altitudes of meteorological balloons. This altimeter uses a superregenerative RF stage which is controlled by an adaptive feedback system. Altitudes to 20 km have been measured accurately over Lake Michigan using a peak power of one watt.

There is a growing need for an accurate radio altimeter which is simple and light enough to be carried aloft by standard sounding balloons. Some basic ideas for implementing such an altimeter are discussed briefly in this letter.

The altimeter described here makes use of a superregenerative stage serving as both the transmitter and the receiver. This stage is part of a feedback system such that the period between transmitted pulses is a measure of the altitude. The relatively slow rate of change of altitude permits the averaging of many successive returns with a corresponding improvement in the signal-to-noise ratio.

The superregenerative stage when gated on by the quench pulse produces an RF pulse whose envelope area depends on the RF input signal present when the leading edge of the quench pulse is applied. This operation is centered on the growth of oscillations in a highly oscillatory circuit. This action continues until the trailing edge of the quench pulse stops the oscillations and is described in more detail in the literature.[1] The superregenerative circuit has the advantages that 1) a high receiver gain can be achieved using a single stage, and 2) a single stage can double as transmitter and receiver. It is for both reasons that the superregenerative circuit was chosen for the balloon altimeter package.

The principles of operation of the feedback-controlled superregenerative stage can be described best by referring to Fig. 1. When the period of the quench pulses is equal to the delay of the return pulse, the superregenerative stage detects its own pulses. As the period (or repetition rate) of the quench pulses is varied, the output of the superregenerative detector reaches a maximum near the repetition rate f_q as given by

$$f_q = \frac{c}{2h}$$

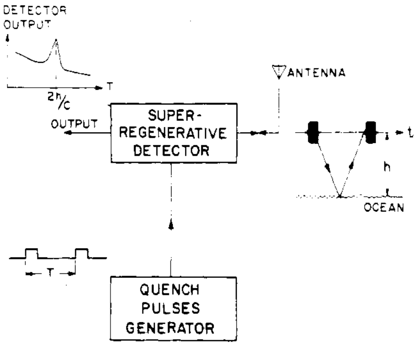

Fig. 1. The output of the superregenerative detector peaks when the quench pulse period equals the delay of the return.

where h is the altitude and c is the velocity of light. Varying the repetition rate about this value is equivalent to scanning the pulse for its time of arrival. The remainder of the feedback loop is designed to adjust the repetition rate to the peak of the returned pulse. Before discussing the feedback, however, we shall look at the returned pulse characteristics.

The expected returns at near-vertical incidence are largely due to area scatter[2] and the terrain serves as an imperfect integrator on the pulse envelope.[3] Thus returns from the vertical (i.e., minimum path length) are, on the average, reinforced by returns arriving from small angular displacements from the vertical. The net effect is that if the illuminated area is limited by the pulse length rather than by the antenna beamwidth, the peak or maximum of the returned pulse envelope occurs at the end of the transmitted pulse shape (delayed by the round trip to the altimeter subpoint). Such a condition guarantees that the fixed additional delay between the beginning of the superregenerative quench pulse and the peak of the return is exactly the length of the quench pulse. This can be controlled accurately in the design of the altimeter.

The principle of the feedback system is to always maintain the condition that the leading edge of the quench pulse occurs at the peak of the returned pulse. To detect the error, a small sinusoidal perturbation is introduced in the estimate of the pulse delay, which is the pulse repetition rate. The net overall effect of the perturbation is to take the derivative of the averaged returned pulse envelope. Thus a null condition, corresponding to the peak of the returned pulse, results. This can be used as an error signal in a feedback loop to drive the repetition rate to the correct value. Measuring the arrival time of the peak of the returned pulse is in contrast to conventional pulse–radar altimeters which use the leading edge of the returned pulse for an indication of height.

A block diagram of the pulse–radar altimeter system is shown in Fig. 2. The voltage-controlled oscillator (VCO) determines the repetition rate of the quench pulses. This rate is linearly related to the voltage at the output of the integrator and is sinusoidally perturbed at a preset rate f_p Hz. The dependence of the output of the superregenerative detector with quench frequency serves to frequency demodulate the perturbation signal. If the quench frequency is too high, the demodulated perturbation signal (at the frequency f_p) is out of phase compared to the modulating signal. On the other hand, if the quench frequency is too low, the detected signal is in phase, and when it is at the correct frequency (or rate) only higher harmonics of f_p are present.

A synchronous detector is then used to compare the phase of the demodulated perturbation signal with the injected perturbation signal to yield both the magnitude and sign of the error signal. The integrator smooths this voltage and locks the VCO to that frequency which gives the peak output of the superregenerative stage. The optimization of this feedback loop was based on data available for the optimization of phase-locked loops which are very similar.

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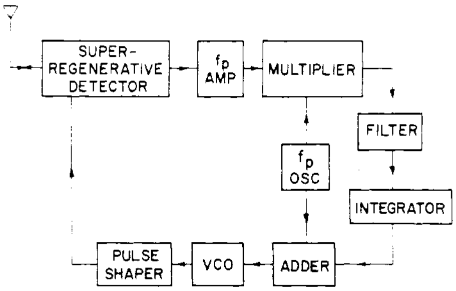

Fig. 2. Block diagram of the radio altimeter.

Two prototypes of the altimeter described have been constructed and flown in two balloon flights over Lake Michigan.[4] Both versions made accurate measurements of balloon altitude from 2 km up to 20 km using a peak power of one watt. Antenna gain was 3.2 dB over that of a dipole and the wavelength used was 70 cm. The RF pulse width (not including the rise time) was 0.5 μs.

The integration time used in the altimeter was one second. The pulse repetition rate varied between 5×10^4 to 10^5 pps, which implies that at least 5×10^4 pulses were averaged for each reading. Data was made available at the ground station every two seconds by counting the repetition rate of the arriving pulses over a period of one second. At an altitude of 10 km, the rms random error in the altitude measurements was less than 0.07 percent. The range ambiguity of the readings was equal to at least 2 km and was easily resolved by the pressure and temperature data or the history of the flight. These flight tests have demonstrated the capability of the design described here. Further work in this area is continuing.

Nadav Levanon
Ferrel G. Stremler
Dept. of Elec. Engrg.
University of Wisconsin
Madison, Wisc. 53706

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Examination of an Error in an Early Work on Interfering Response in Superheterodynes

Abstract—An early work by Morgan describes four types of interfering responses in superheterodynes. In this letter, it is pointed out that all four are examples of the larger class of mixer *p–q* responses. Furthermore, one of Morgan's formulas predicts responses that are not observed in practice.

The four receiver spurious responses described by Morgan[1] are in fact all special cases of what are now commonly known as mixer *p–q* responses,[2] and the formula given by Morgan for group A responses is in error. The *p–q* equation is obtained by considering that mixer generated multiples of an input signal at f_{SP} and a local oscillator at f_{LO} can combine to yield the intermediate frequency f_{IF} in the presence of a mixer nonlinearity of order $p+q$:

$$p(f_{LO}) - q(f_{SP}) = \mp f_{IF} \tag{1}$$

or

$$f_{SP} = \frac{p(f_{LO}) \pm f_{IF}}{q}. \tag{2}$$

Morgan uses s to denote either the desired or the image frequency, o and i to represent the local oscillator and intermediate frequencies, and n for the harmonic orders. Presumably, the harmonic orders for o and s can differ, although Morgan is vague in this regard. The group A formulation

$$f = n_1 o \pm n_2 s \tag{3}$$

is in error, regardless of whether or not $n_1 = n_2$. This is seen by rewriting (3) as

$$f = n_1 o \pm n_2(o \pm i) = (n_1 \pm n_2)o \pm n_2 i \tag{4}$$

where the relationship between s, o, and i has been utilized. This is identical to (2), with $q=1$ and $p=n_1 \pm n_2$, except for the factor n_2 by which i is multiplied. Such responses exist in fact only for $n_2=1$. Morgan gives no experimental evidence to the contrary, and a more recent investigation has failed to disclose any such responses[3] for $n_2 \neq 1$.

The responses in Morgan's groups B, C, and D are special cases of the *p–q* type (generated by mixer nonlinearity), and those of group C can also come about as a result of IF harmonic radiation into the receiver front end. In either case, the group C responses are observed only with the receiver tuned near a multiple of the IF—a restriction that does not apply to *p–q* responses in general. It is because of this restriction that the term *ni* appears in the group C frequency relationship. Since the restriction does not apply to group A responses, the group A frequency relationship, as expressed in (4), should *not* contain an integer multiplier of the intermediate frequency.

The combination of groups A through D is incomplete in that it fails to predict all of the responses given by the *p–q* relationship.

The treatment of whistles in superheterodynes in Henney's radio engineering handbook is based on (1), although Morgan's paper is given as a reference.[4] Terman's treatment of spurious responses in superheterodynes is based directly on Morgan and contains the error discussed above.[5]

I am indebted to the reviewer for pointing out that the example of a type *D* response given by Morgan on pp. 1167–1168[1] is in fact the (9, 10, +) *p–q* response, so that Morgan's assumption of recombination "by plate modulation" on p. 1168[1] is unnecessary.

R. G. Huenemann
IIT Research Institute
Chicago, Ill. 60616

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A Self-Pumped Tunnel Diode Parametric Amplifier

Abstract—A tunnel diode oscillating at 804 MHz is shown to produce an 8 dB self-pumped parametric amplification of an input signal at 7.24 GHz. No external pumping power is required.

Self-pumped parametric amplification has been observed in avalanche diodes [1] and Gunn diodes [2], [3] where the pump frequency is several times that of the signal to be amplified. A tunnel diode self-oscillating converter has been suggested [4]. In this letter, self-pumped parametric amplification in tunnel diodes is examined for the case where the pumping signal frequency is much *less* (one-ninth) than the frequency to be amplified.

The tunnel diode employed was a Sylvania Model D4966 with a maximum observed oscillation frequency of 3.4 GHz. The tunnel diode mount employed was a section of *X*-band tapered waveguide [5]. One side of the waveguide mount was terminated with a matched load. The other side of the mount was connected to a slotted section with a waveguide to coax termination on the end. The signal to be amplified was applied to the coaxial input. The output was taken from a probe in the slotted section of

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