

Binary Frequency Shift Keying for Continuous Waveform Radar

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A new binary frequency-shift keying (BFSK) waveform is suggested for continuous wave radar. It provides ideal periodic autocorrelation (PAC) when processed by a matched filter, and perfect periodic cross-correlation (PCC) when processed by a mismatched filter. Ideal PAC implies a uniform sidelobe level, whose ratio to the PAC peak is equal to the inverse of the code length. Perfect PCC implies zero sidelobes. BFSK is relatively spectrum efficient. Design details and processing issues are discussed.

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I. INTRODUCTION

An experimental bistatic continuous wave (CW) radar, described recently [1], employed a two-valued, periodic, phase shift keying (PSK), in which nonantipodal phases [2] and proper families of binary sequences (e.g., Legendre or m-sequences [3]), resulted in perfect periodic autocorrelation (PPAC). When processed by a matched filter, PPAC implies periodic range response with zero sidelobes (SL), namely, the PAC of the sequence $s[n]$

$$R_s[n] = \sum_{k=0}^{N-1} s[n+k]s^*[k \bmod N] \quad (1)$$

is of the form:

$$R_s[n] = E \delta[n \bmod N] \quad (2)$$

namely, periodic peaks of height E . The main drawback of the two-valued PSK waveform is its poor spectral efficiency—broad spectrum and slowly decaying spectral SLs. Spectral SLs interfere with spectral neighbors and require higher sampling rate in order not to alter the waveform's correlation properties. To reduce spectral SLs, the radar described in [1] replaced the rectangular bit shape by a Gaussian windowed sinc (GWS) shape [4]. However, using GWS bit representation resulted in variable amplitude waveform, which required a transmitter with linear power amplifier. The radar described in [1] used a low power (one Watt) linear amplifier, which is readily available.

The problems with PSK prompted a search for a constant amplitude and spectrally clean CW waveform that will yield ideal or perfect periodic range response. The issue of spectrally clean *pulse* waveforms is well referenced [4]–[6]. Some of the approaches used for *pulses* can be adopted to CW, but modifications are needed. The approach suggested in the present paper results in a waveform similar to the one used in [6], which is basically binary frequency shift keying (BFSK). Since [6] deals with a pulse waveform it needed a special treatment of the pulse's rise and fall time, in order to reduce their contribution to spectral SL. The periodic CW case has the advantage of no rise-time and no fall-time (see Fig. 1). This and other differences are listed in Table I.

The concept of a coded periodic CW waveform and its processing is presented in Fig. 1. The top subplot represents the transmitted waveform containing an “endless” number of periods; each period is BPSK or BFSK coded by a common binary sequence of N elements (bits). The second subplot describes a reference waveform, digitally stored in the receiver. It contains P periods of a matched or mismatched reference. That reference has uniform interperiod weight. In the third subplot, the reference is smoothly amplitude weighted by a Hamming window constructed by NP elements. In the bottom subplot, the amplitude weighting is a stepwise P elements Hamming (simpler to implement). Amplitude weighting of the reference reduces Doppler SLs. The penalty is signal-to-noise ratio (SNR) loss and a wider Doppler mainlobe [7]. The resulted periodic delay-Doppler response is slightly better when the weight window is smooth rather than stepped.

TABLE I
Differences Between Pulse and CW-Coded Waveforms

Property	Pulse	CW	Effect
Rise time and fall time	Exist	Do not exist	Cause higher spectral sidelobes
Mismatched filter	Can be longer than the pulse	Of equal length to the signal's period	A longer mismatched filter yields lower delay sidelobes
Perfect delay response (= Zero delay sidelobes)	Unattainable (unless a complementary pair)	Attainable	Requires specific coding of signal and reference

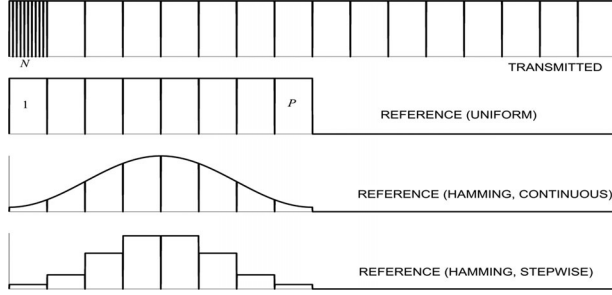


Fig. 1. Indefinite periodic CW-coded waveform (top) and three finite P -period references with different amplitude weightings.

The waveform proposed here is BFSK. It was selected because of four properties:

- 1) Constant amplitude;
- 2) Simple two-valued modulation;
- 3) Cleaner spectrum than BPSK; and
- 4) With proper coding it can produce *ideal* or *perfect* periodic delay response.

Ideal periodic cross-correlation (IPCC) can be obtained with a matched reference, meaning that the PAC of the waveform $s[n]$ is two-valued:

$$R_s[n] = \begin{cases} E, & n \bmod N = 0 \\ F, & \text{elsewhere} \end{cases} \quad (3)$$

where typically $|F| = 1$. *Perfect* periodic cross-correlation (PPCC) requires a unique mismatched reference waveform, where the PCC of the waveform $s[n]$ with the mismatched filter $h[n]$ would have zero SLs:

$$\begin{aligned} R_{s,h}[n] &= \sum_{k=0}^{N-1} s[n+k]h^*[k \bmod N] \\ &= \tilde{E} \delta[n \bmod N]. \end{aligned} \quad (4)$$

Using a mismatched filter would entail SNR loss.

In the mismatched periodic case (4), s and h are periodic and have the same period N . The different correlations (a-periodic and periodic) can be further demonstrated by noting the corresponding MATLAB scripts: The a-periodic correlation is given by $R = \text{xcorr}(s,h)$, while the periodic correlation is given by $R = \text{ifft}(\text{fft}(s) \cdot \text{conj}(\text{fft}(h)))$, recalling that in the periodic case, the s and h sequences must be of the same length. The last two lines of the MATLAB script in the Appendix demonstrate IPCCs and PPCCs.

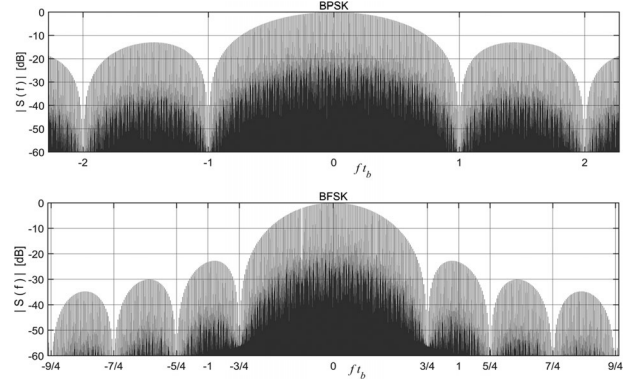


Fig. 2. Spectrum comparison of BPSK and BFSK coding.

A waveform has three parameters that can be modulated or keyed (individually or combined): amplitude, phase, and frequency. Periodic amplitude “ON–OFF” keying for a CW laser range finder was field tested and described in [8]. Periodic BPSK for a bistatic CW radar was field tested and described in [1]. The present paper completes the trio by describing CW-BFSK waveforms.

It should be made clear at the beginning that in the proposed BFSK coding, the delay resolution is a function of the duration of the frequency step, namely, the width of the code element. There is no similarity what so ever between our BFSK coding and the relatively slow BFSK switching used to implement the so called “two frequency measurement” [9], [10]. In the latter, the purpose of the FSK is to repeatedly transmit two close frequencies next to each other and the duration of the frequency step has no bearing on the range resolution. It is also important to clarify that the proposed BFSK waveform is not viable to stretch processing [11], [12]; hence, the sampling rate should match the waveform bandwidth, and will be independent of the expected targets’ delays.

The following sections will describe the construction of the BFSK waveform; the mismatched reference needed to move from ideal to perfect periodic delay response; the resulted correlations and spectra; and discuss issues of parameter tolerances and their effect on the range response. Before going into the details of the waveform, Fig. 2 demonstrates the spectra of BPSK and BFSK waveforms, both with a rectangular bit shape. The frequency scale is normalized by multiplying the frequency variable by the code element duration t_b . The waveforms used code length of 103 elements and the CPI contained 32 code periods. In the

BPSK-coded waveform, the first null appears at $f t_b = 1$ and the remaining nulls appear at the following integers. The delay resolution (first null of the delay response) of the BPSK waveform is at $\tau = t_b$, as shown in [1, Fig. 5].

In the BFSK-coded waveform, described in the next section, the spectrum mainlobe is narrower and the first spectral null appears at $3/4$. The remaining nulls are spaced by $1/2$. The narrower BFSK spectral mainlobe hints that the correlation mainlobe of the BFSK will be wider than that of the BPSK, which is indeed the case. In general, Fig. 2 demonstrates the much faster decay of spectral SLs in the BFSK waveform case.

II. DESIGN OF THE BFSK WAVEFORM

The waveform design is based on binary m-sequences [3], available at lengths $N = 2^k - 1$, $k = 2, 3, 4, \dots$, or on binary Legendre sequences available at all lengths Q if Q is an odd prime that satisfies $Q = 4k - 1$, $k = 1, 2, 3, \dots$. The Appendix suggests a MATLAB function that generates binary Legendre sequences. Note that Q contains about half the prime numbers, those that cannot be described as the sum of two squares. For example, the primes 103 and 107 are suitable, while the primes $109(= 10^2 + 3^2)$ or $113(= 8^2 + 7^2)$ are not.

The transformation from the original binary phase elements $\phi_n\{0, \pi\}$, $n = 1, 2, \dots, N$ to the normalized BFSK elements $f_n\{0, (1/t_b)\}$, $n = 1, 2, \dots, N$ is given [13] by the following expression (\oplus is XOR or sum modulo 2):

$$2t_b f_n = \left(\frac{1}{\pi} \phi_n\right) \oplus \left(\frac{1}{\pi} \phi_{(n+1) \bmod N}\right). \quad (5)$$

The transformation will be demonstrated using a 23 element Legendre-based binary phase code

$$\phi_n/\pi = \{1\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1\}. \quad (6)$$

Applying the transformation in (5) to the sequence in (6) yields

$$2t_b f_n = \{1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\}. \quad (7)$$

When processed by a matched filter, a constant amplitude waveform, frequency coded by (7), produces a two-level ideal PAC, where the SL level is $1/N$ relative to the mainlobe peak (see Fig. 3). However, the same signal will produce PPCC with a reference signal having the same frequency coding, but also amplitude “ON-OFF” coding, identical to the sequence in (7) (top subplot of Fig. 4).

That result is related to the fact that the cross-correlation between a unipolar version $\{1, 0\}$ of a Legendre or m-sequence code and its binary version $\{+1, -1\}$ is perfect with a peak equal to the number of ones in the code and uniform off-peak SL level which is identically zero [14], [8].

Since the number of zeroes in a Legendre sequence of length N is $(N + 1)/2$ or $(N - 1)/2$, turning the amplitude of the reference “OFF” during each “0” code element implies using only half the received signal’s energy, namely, an SNR loss of approximately 3 dB.

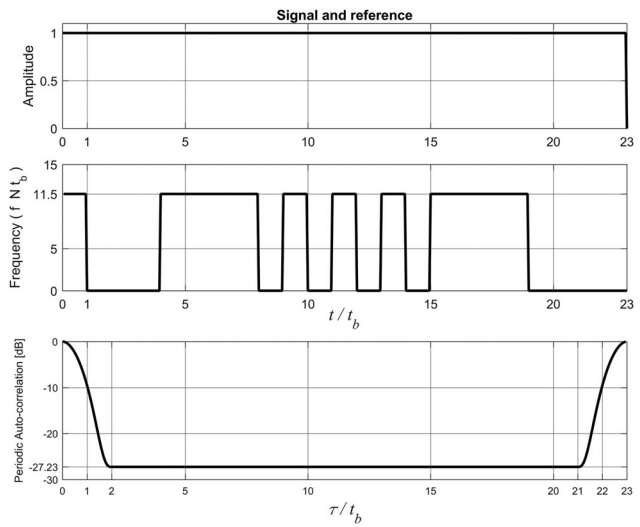


Fig. 3. One period of the BFSK code, Legendre 23: Amplitude (top). Frequency (middle). Periodic autocorrelation (bottom).

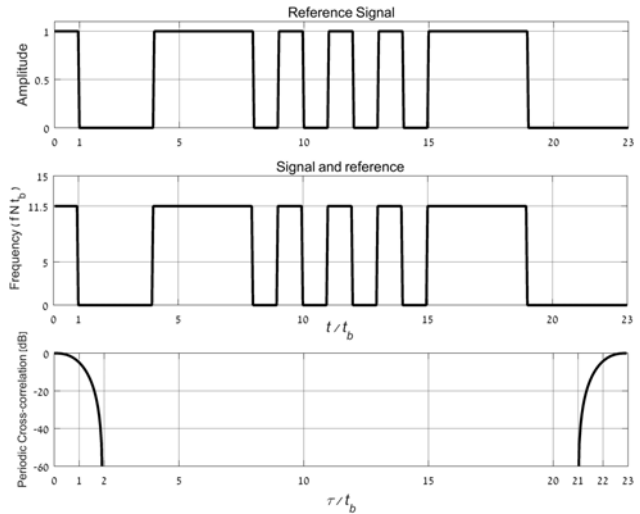


Fig. 4. One period of the BFSK code, Legendre 23: Amplitude of the BFSK reference (top). Frequency of signal and reference (middle). Periodic cross-correlation (bottom).

From (7), we learn that the FSK’s frequency step is $\delta f = 1/(2t_b)$. The physical meaning of that relationship says that during a “0” bit the intermediate frequency (IF) f_c , on which the receiver performs the synchronous sampling, completes $n_c(0)$ cycles per bit, where $n_c(0) = t_b f_c$, while during a “1” bit it should complete exactly one half-cycle more, shown as follows:

$$\begin{aligned} n_c(1) &= t_b (f_c + \delta f) = t_b \left(f_c + \frac{1}{2t_b}\right) = t_b f_c + \frac{1}{2} \\ &= n_c(0) + \frac{1}{2}. \end{aligned} \quad (8)$$

The result in (8) is independent of the bit duration or the delay resolution.

From the PCC (see Fig. 4, bottom subplot), we see that the null is at a delay $\Delta\tau = 2t_b$, which can be referred to

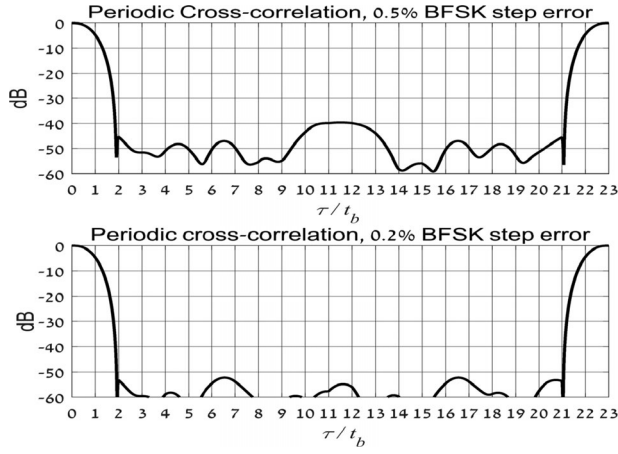


Fig. 5. Periodic cross-correlation with frequency step errors of 0.2% and 0.5%.

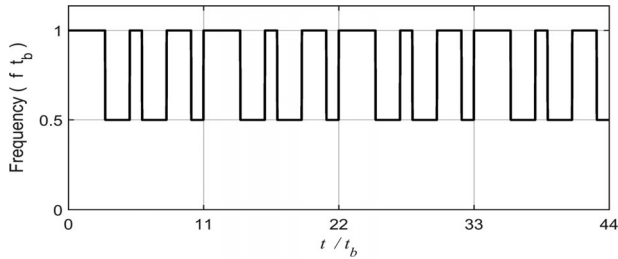


Fig. 6. Frequency evolution of four periods of FSK 11 (total of 44 bits), $f_{\text{LOW}} = 0.5/t_b$.

as the delay resolution. We can, therefore, relate δf and t_b to the range resolution ΔR as follows:

$$\delta f = \frac{1}{\Delta \tau} = \frac{C}{2 \Delta R}, \quad t_b = \frac{\Delta R}{C} \quad (9)$$

where C is the velocity of propagation. For example, range resolution of $\Delta R = 3$ m will require frequency step $\delta f = 50$ MHz and bit duration $t_b = 10$ ns. If the IF is 200 MHz, the number of IF cycles during a “0” bit is $n_c(0) = t_b f_c = 2$. If the next bit is “1” then the number of cycles in it will be $n_c(1) = t_b(f_c + \delta f) = 2.5$, a difference of half a cycle, as predicted in (6). Returning to the example mentioned above, the specification requires that for the next 10 ns, the IF should be exactly 250 MHz.

The physical meaning of this specification is interesting and rather harsh. It says, for example, that in order to get range resolution of 3 m the bit duration should be $t_b = 10$ ns. It also says that at the transmitted frequency, or at the receiver’s IF, the frequency jump between different code bits must be exactly 50 MHz; Fig. 5 shows how the PCC deviates from a perfect one (bottom subplot of Fig. 4), if the frequency step is 0.2% off the nominal value (e.g., 50.1 MHz instead of 50 MHz), or 0.5% (e.g., 50.25 MHz instead of 50 MHz).

A demonstration of the expected real signal is shown in Figs. 6 and 7. The IF where the received signal is synchronously sampled was selected in such a way that the low

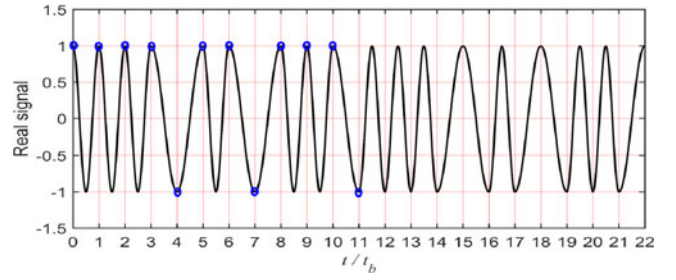


Fig. 7. Two periods (22 bits) of the real signal, whose frequency is shown in Fig. 6.

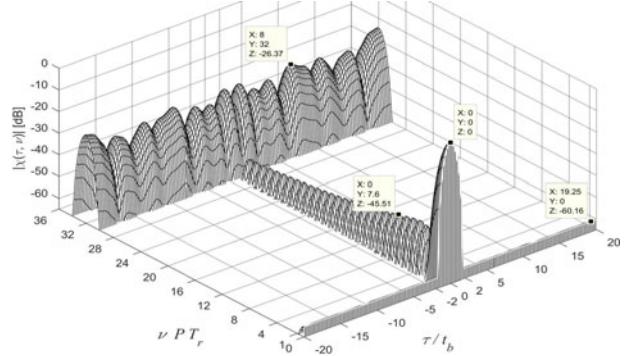


Fig. 8. Normalized periodic delay-Doppler response. Matched FSK, Legendre 1019, $P = 32$. Hamming weighted reference.

and high BFSK frequencies are (see Fig. 6)

$$f_{\text{LOW}} = \frac{f t_b}{t_b} = \frac{0.5}{10^{-8}} = 50 \text{ MHz}$$

$$f_{\text{HIGH}} = \frac{f t_b}{t_b} = \frac{1}{10^{-8}} = 100 \text{ MHz}.$$

$f_{\text{LOW}} = 50$ MHz is an arbitrary choice used to enable simple demonstration of the half-cycle difference between “1” and “0”. As a result of this selection, the real signal intersects the bits boundaries at two fixed values (see Fig. 7). Because of this specific selection, during bits # 1, 2, and 3, the signal completes one cycle per bit, while during bits # 4 and 5 it completes $\frac{1}{2}$ cycle per bit.

The strict requirement on the accuracy of the frequency step raises concern regarding implementation of the frequency modulation through voltage input to a voltage-controlled oscillator (VCO).

III. DELAY-DOPPLER RESPONSE

Ideal periodic delay-Doppler response of the BFSK waveform (see Fig. 8) is demonstrated using a long code (Legendre, 1019 elements). The number of code periods coherently processed is $P = 32$. Ideal response is obtained when the processor uses a reference waveform identical to the transmitted signal, except for Hamming weighting. The expected pedestal at zero Doppler should have a level of $20 \log(1/1019) = -60.16$ dB. Fig. 8 zooms on a limited delay axis $|\tau| \leq 20 t_b$. The floor of the dB scale was set to -65 dB to enable observation of the SL pedestal at zero Doppler.

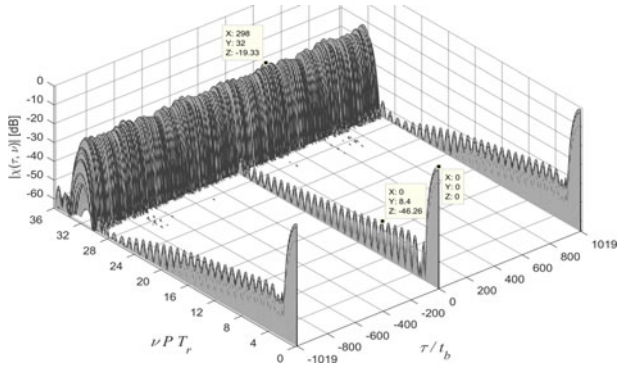


Fig. 9. Normalized periodic delay-Doppler response. Mismatched BFSK, Legendre 1019, $P = 32$. Hamming weighted reference.

The main features of Fig. 8 are

- 1) Mainlobe that reaches a null at $\tau = 2t_b$;
- 2) Typical Hamming SLs (< -40 dB) along the Doppler axis;
- 3) A recurrent Doppler SL, with height of -26 dB, at $\nu = 1/T_r$, namely, at $\nu P T_r = 32$, where $T_r = N t_b$ is the code repetition period;
- 4) A range SL pedestal of -60 dB at zero-Doppler, which is acceptable for most applications.

Increasing the Legendre code length to 2011 reduced the SL pedestal at zero-Doppler to -66 dB ($\approx 20 \log(2011)$), while the recurrent Doppler ridge at $\nu = 1/T_r$ dropped to a level of -30 dB. That trend is expected to continue as the code length increases.

The normalized perfect periodic delay-Doppler response (see Fig. 9) is demonstrated using the same long code (1019 element), and the same number of code periods coherently processed ($P = 32$). The mismatched reference signal has “ON-OFF” amplitude modulation. It was also Hamming-weighted in order to reduce Doppler SLs. In Fig. 9, the delay axis extends to a full period, namely, $|\tau| \leq 1019 t_b$.

The normalization is reflected by a peak value of 0 dB. Without normalization the entire function, including the peak at the origin, will be lower by 4.5 dB, reflecting the loss due to the “ON-OFF” modulation and the Hamming weighting of the mismatched reference waveform. The main features of Fig. 9 are: zero SLs on the delay axis; typical Hamming SLs (< -40 dB) on the Doppler axis; Doppler recurrent ridge at $\nu P T_r = 32 \Rightarrow \nu = 1/T_r$, with a typical level ≈ -20 dB.

IV. DELAY-DOPPLER PROCESSING CONSIDERATIONS

The following two facts have important implications on the delay-Doppler processing of the BFSK-coded signal: a) The BFSK’s frequency step of $\delta f = 1/(2t_b)$ implies that during a bit the signal’s phase accumulates (linearly) $+\pi/2$ or $-\pi/2$ radians; b) In an m-sequence or Legendre sequence of length N , the number of ones is $(N + 1)/2$ or

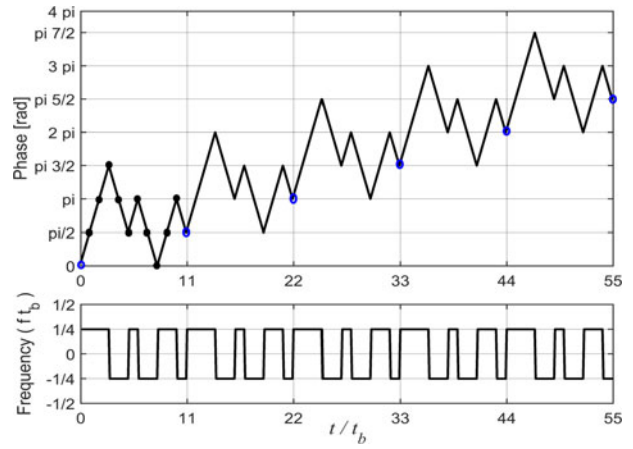


Fig. 10. Frequency (bottom) and phase (top) evolution during five periods of BFSK 11 (total of 55 bits). Centered reference frequency.

$(N - 1)/2$, namely, one more or one less than the number of zeroes.

The frequency and phase evolution of the complex envelope of the signal, presented in Fig. 10, is obtained using a third fact: c) The reference unmodulated sinewave, used in the coherent synchronous detection in the receiver, is set exactly at the middle between the two BFSK frequencies. In Fig. 10, the signal is BFSK coded using Legendre 11 and the number of periods displayed is $P = 5$. The fact that the reference frequency is exactly centered is seen in the bottom subplot where the normalized frequency $f t_b$ is switching between $-1/4$ and $+1/4$. The resulted phase evolution is seen in the top subplot of Fig. 10. We note clearly the linear accumulation of $+\pi/2$ or $-\pi/2$ radians during each bit. Because of fact (b) mentioned above, there is a phase accumulation of $+\pi/2$ per each code period lasting 11 bits. With the three settings (a, b, and c), from the phase point of view, the true periodicity of the waveform is four code periods. Only after four periods (44 bits), the phase, modulo 2π , returns to its initial value.

Normally, after matched filtering with one code period, the processor performs fast Fourier transform on P complex samples taken at an equal delay, from each one of the coherently processed P consecutive code periods. Phase-wise periodicity that is different from the code periodicity can create a false fixed Doppler shift [of $1/(4T_r)$ in this case]. To prevent that, the phase-wise periodicity can be made equal to one code period if the frequency switching is not symmetrical about the reference frequency. If the number of ones is larger than the number of zeroes, the normalized frequency around the reference frequency (marked as $f t_b = 0$) should switch between the two values:

$$f t_b = \pm \frac{1}{4} \left(1 \mp \frac{1}{N} \right). \quad (10)$$

Namely, the reference frequency is offset from the center.

In this case, the frequency and phase evolution, for code length 11, are shown in Fig. 11. Equation (10) shows that the frequency offset decreases with the code length N . In a

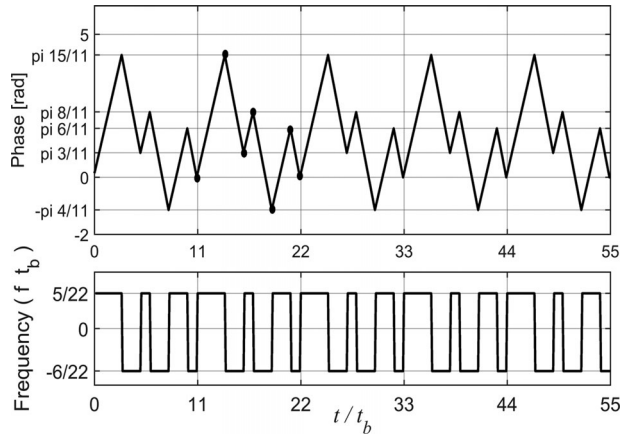


Fig. 11. Frequency (bottom) and phase (top) evolution during five periods of BFSK 11 (total of 55 bits). Offset reference frequency.

monostatic CW radar, where the transmitted signal is also available to the receiver, the two FSK frequencies f_H , f_L , relative to the reference frequency f_C , can be simply preset as indicated in the following equation and the carrier at f_C should be used in the synchronous detection:

$$f_H = f_C + \frac{N-1}{4N t_b}, \quad f_L = f_C - \frac{N+1}{4N t_b}. \quad (11)$$

In a bistatic or multistatic radar system, a synchronization mechanism is required at the receiver that will estimate and lock to f_C , just as it needs to estimate the exact bit width t_b or the code period $T_r (= N t_b)$.

V. SEPARABILITY

An important property of coded waveforms is the richness of waveforms belonging to the same family. The variability is mostly due to different code lengths. This property is important in multistatic radar scenes, where several transmitters operate simultaneously, emitting different waveforms. In order to enable a receiver to choose any signal with minimum interference from the others, the signals should be separable. One simple measure of separability is the a-PCC between P periods of BFSK-coded waveform of length N_1 , and P periods of BFSK-coded waveform of length N_2 (same t_b and same intensity).

A-PCC is used because the signals' period durations are different. Such a cross-correlation, for the case $N_1 = 1019$, $N_2 = 1031$, and $P = 128$, is shown in Fig. 12 (pink plot). For comparison, the a-PAC of the signal $N_1 = 1019$, $P = 128$ is also plotted (black). The drawing shows difference of 45 dB between the autocorrelation peak and the highest value of the cross-correlation. That difference increases with an increase of the number of periods processed coherently. Note that because we plotted the *a-periodic* (rather than the *periodic*) autocorrelation of the waveform with length 1019, the SLs are not uniform at a level of -60 dB ($=20 \log(1019)$), but fluctuate around that value.

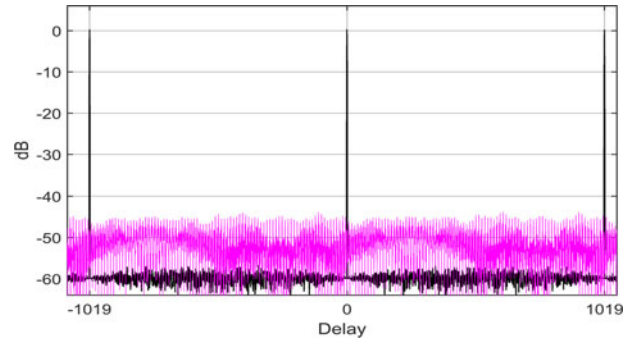


Fig. 12. A-periodic cross-correlation between: (pink) $P(= 128)$ periods of BFSK-code of length 1019 and P periods of BFSK-code of length 1031; (black) a-periodic autocorrelation of P periods of FSK-code of length 1019.

VI. CONCLUSION

A new binary frequency-coded, constant amplitude, periodic waveform was proposed for CW radars. Depending on whether the reference is matched or mismatched, the output exhibits either ideal or perfect periodic range response, respectively. The waveform is mainly based on Legendre binary sequences; hence, the available lengths include about half the prime numbers. When processed by a mismatched reference, the output sustains an SNR loss of about 3 dB. The BFSK waveform exhibits better spectral efficiency than a BPSK waveform. Being frequency-coded waveform, it can be generated by a VCO, but the VCO needs to be of high quality because the signal's parameters allow little tolerance. However, since the VCO characteristic is tapped at only two points, stability of the frequency versus voltage curve is more important than the linearity of that curve. Another useful property of the BFSK-coded waveform is the good separability between two codes of close but different period durations, when a large number of periods are processed coherently.

APPENDIX

A MATLAB function for generating Legendre sequences

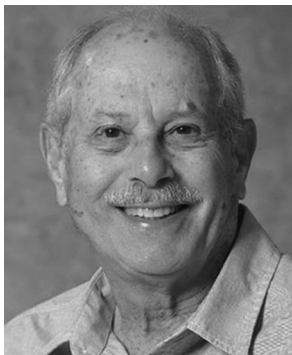
```
function [p_code] = binary_legendre_sequence(nn)
% Creates a binary Legendre sequence
of length nn
% nn must be an odd prime that satisfies
nn = 4k - 1, k is an integer
if isprime(nn) == 0
disp('Not a prime')
return
end
if rem((nn + 3)/4, 1) == 0
disp('Not a suitable prime')
return
else
p_code = ones(1,nn);
p_code(mod((1:nn-1).^2,nn) + 1) = 0;
end
```

A MATLAB Demonstration of ideal and perfect periodic correlations:

```
s = 2*p_code-1; % conversion to bipolar sequence
r_ideal = ifft(fft(s).*conj(fft(s)));
r_perfect = ifft(fft(s).*conj(fft(p_code))).
```

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