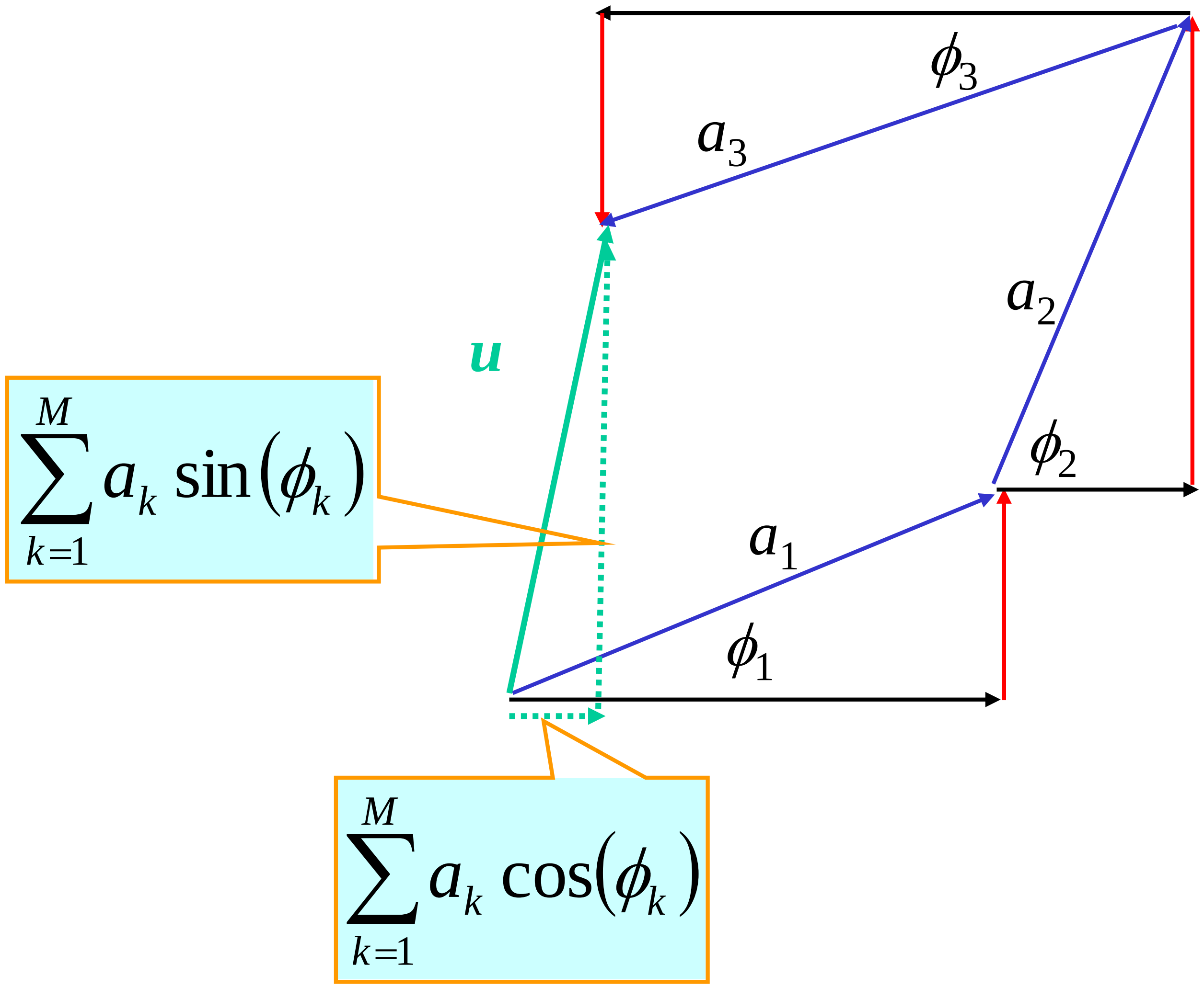
DETECTION - 2: Integration of M pulses

$$u=\sum_{k=1}^{M} a_k \cos(\phi_k) + j\sum_{k=1}^{M} a_k \sin(\phi_k)$$

Here we added reflections from several scatterers



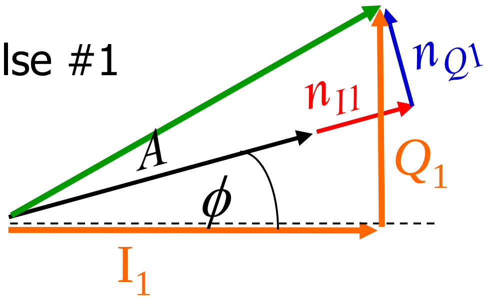
Now we add reflections from several pulses

Reflections of a coherent pulse train from a coherent target

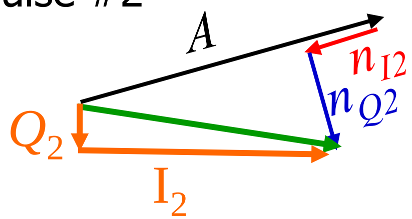
(Signal phasor angle ϕ is fixed but unknown to the receiver)

No motion is assumed (i.e., no Doppler)

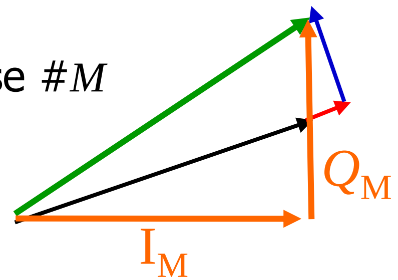
Pulse #1



Pulse #2



Pulse #M



$$\left(\sum_{k=1}^{M} I_k\right)^2 + \left(\sum_{k=1}^{M} Q_k\right)^2$$

$$\sum_{k=1}^{M}\left(I_k^2 + Q_k^2\right)$$

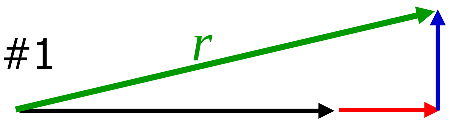
Either coherent (optimal) or non-coherent (entails loss) integration can be performed

If the coherent reflection may contain Doppler shift (causing the target phasor to rotate from pulse to pulse), then the summation of the coherent integration must involve an FFT (which compensates for many different phase rotation rates).

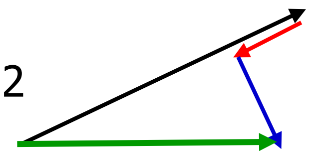
Reflection of a coherent (or non-coherent) transmission from a non-coherent target

(variable and unknown signal phasor angle)

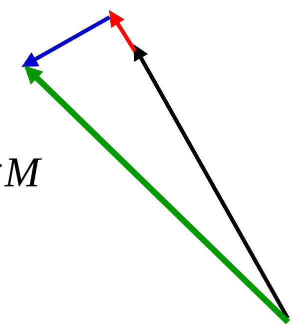
Pulse #1



Pulse #2



Pulse #M



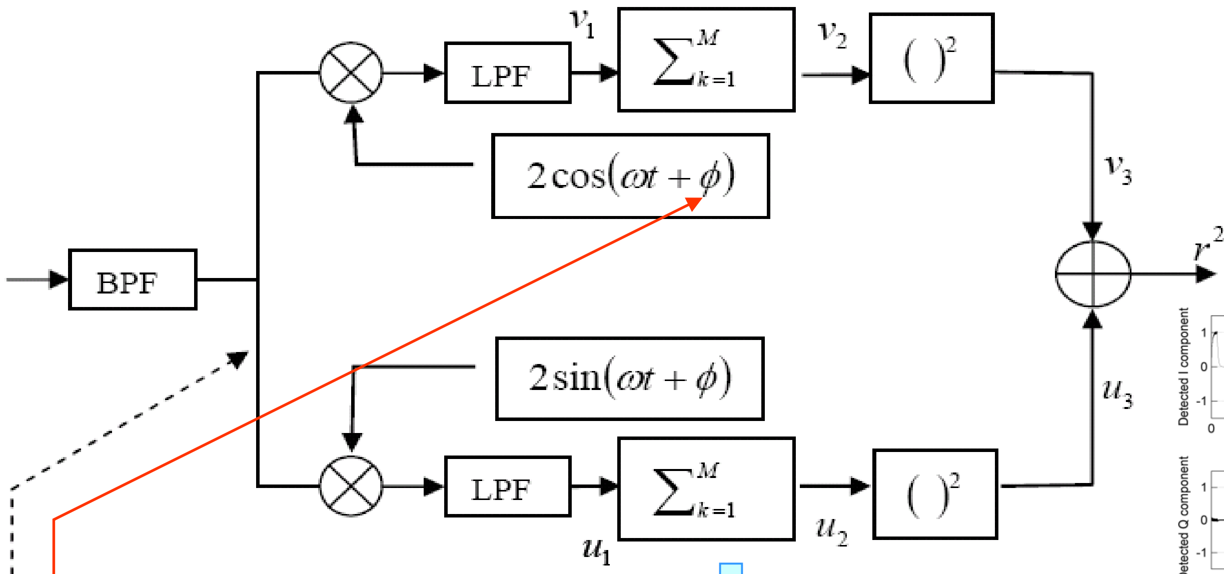
Calls for non-coherent (video) integration after each pulse is envelope detected.

$$\sum_{k=1}^{M}\left(I_k^2+Q_k^2\right)$$

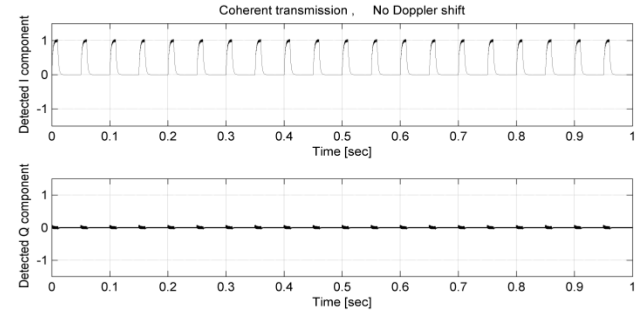
Envelope detection of an individual pulse does not require synchronous detector.
It can be implemented by a simpler circuit (e.g., a diode or diodes bridge).

In this situation, if we perform coherent integration, the results may be <u>much worse</u> than the results of non-coherent integration.

Synchronous detection and coherent integration

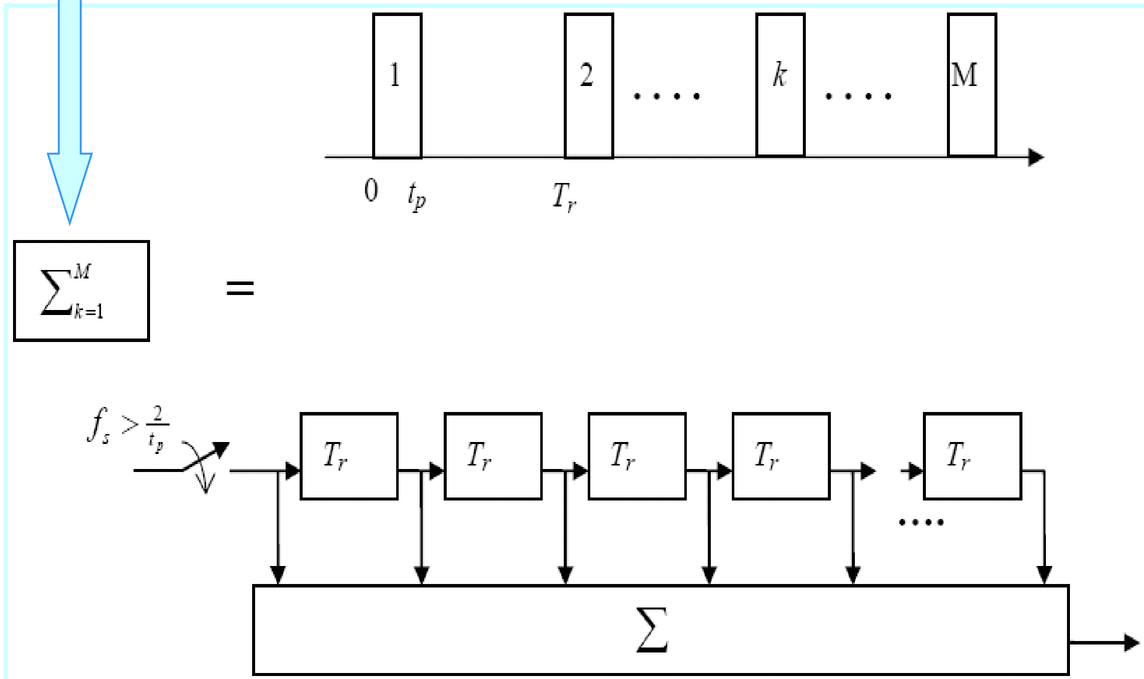


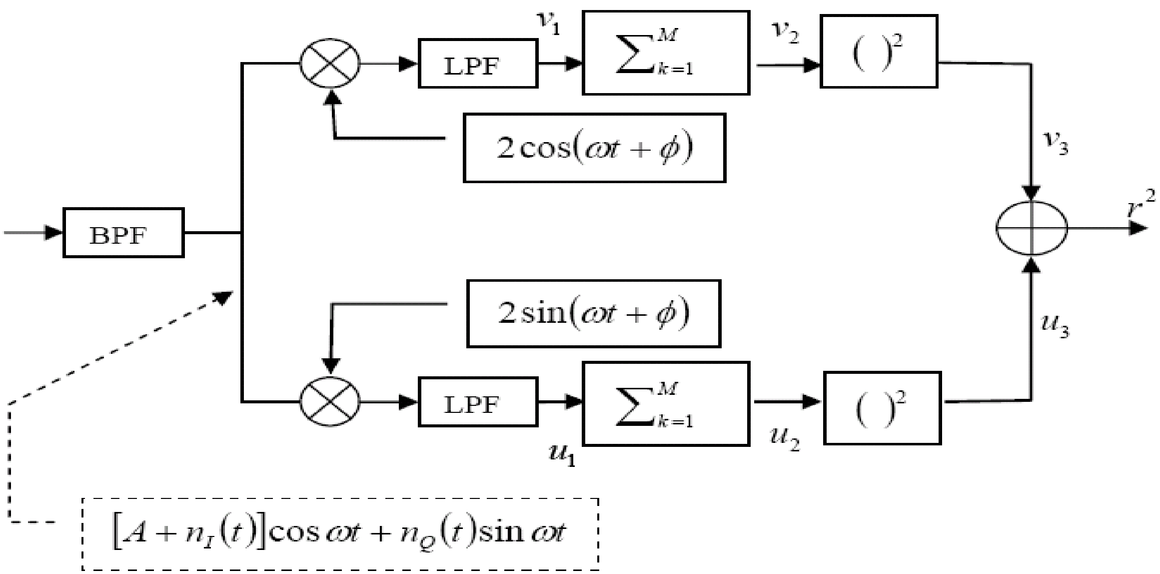
$$[A+n_I(t)]\cos\omega t+n_Q(t)\sin\omega t$$



The unknown but fixed* phase difference between the reflected carrier and the reference carrier.

*when there is no Doppler.





$$\cos\alpha\cos\beta=\tfrac{1}{2}\cos(\alpha-\beta)+\tfrac{1}{2}\cos(\alpha+\beta)$$
$$\sin\alpha\sin\beta=\tfrac{1}{2}\cos(\alpha-\beta)-\tfrac{1}{2}\cos(\alpha+\beta)$$
$$\sin\alpha\cos\beta=\tfrac{1}{2}\sin(\alpha-\beta)+\tfrac{1}{2}\sin(\alpha+\beta)$$

$$v_1=\left\{2\cos(\omega t+\phi)\left[(A+n_I)\cos\omega t+n_Q\sin\omega t\right]\right\}_{\mathrm{LPF}}=(A+n_I)\cos\phi-n_Q\sin\phi$$
$$u_1=\left\{2\sin(\omega t+\phi)\left[(A+n_I)\cos\omega t+n_Q\sin\omega t\right]\right\}_{\mathrm{LPF}}=(A+n_I)\sin\phi+n_Q\cos\phi$$

$$v_2=\left(MA+\sum_{k=1}^{M}n_{Ik}\right)\cos\phi-\left(\sum_{k=1}^{M}n_{Qk}\right)\sin\phi$$
$$u_2=\left(MA+\sum_{k=1}^{M}n_{Ik}\right)\sin\phi+\left(\sum_{k=1}^{M}n_{Qk}\right)\cos\phi$$

$$v_3=\left(MA+\sum_{k=1}^{M}n_{Ik}\right)^2\cos^2\phi+\left(\sum_{k=1}^{M}n_{Qk}\right)^2\sin^2\phi-2\left(MA+\sum_{k=1}^{M}n_{Ik}\right)\left(\sum_{k=1}^{M}n_{Qk}\right)\cos\phi\sin\phi$$
$$u_3=\left(MA+\sum_{k=1}^{M}n_{Ik}\right)^2\sin^2\phi+\left(\sum_{k=1}^{M}n_{Qk}\right)^2\cos^2\phi+2\left(MA+\sum_{k=1}^{M}n_{Ik}\right)\left(\sum_{k=1}^{M}n_{Qk}\right)\cos\phi\sin\phi$$

$$r^2=v_3+u_3=\left(MA+\sum_{k=1}^{M}n_{Ik}\right)^2+\left(\sum_{k=1}^{M}n_{Qk}\right)^2$$

$$r^2 = \left(MA + \sum_{k=1}^{M} n_{Ik}\right)^2 + \left(\sum_{k=1}^{M} n_{Qk}\right)^2$$

$$r^2\Big|_{\text{signal only}} = M^2A^2 \quad , \quad r^2\Big|_{\text{signal only}, M=1} = A^2$$

$$\overline{r^2}\Big|_{\text{noise only}} = \overline{\left(\sum_{k=1}^{M} n_{Ik}\right)^2 + \left(\sum_{k=1}^{M} n_{Qk}\right)^2}$$

$$= \overline{\left(\sum_{k=1}^{M} n_{Ik}\right)^2} + \overline{\left(\sum_{k=1}^{M} n_{Qk}\right)^2} = M\overline{n_I^2} + M\overline{n_Q^2} = 2M\overline{n_I^2} = 2M\beta^2 = 2M\, N_0 f_B$$

$$\overline{r^2}\Big|_{\text{noise only}, M=1} = \overline{n_I^2 + n_Q^2} = \overline{n_I^2} + \overline{n_Q^2} = 2\overline{n_I^2} = 2\beta^2 = 2N_0 f_B$$

$$\frac{r^2\Big|_{\text{signal only}, M=1}}{\overline{r^2}\Big|_{\text{noise only}, M=1}} = \frac{A^2}{2N_0 f_B}$$

$$\frac{r^2\Big|_{\text{signal only}}}{\overline{r^2}\Big|_{\text{noise only}}} = \frac{M^2A^2}{2MN_0 f_B} = M\frac{A^2}{2N_0 f_B} = M\left(\frac{r^2\Big|_{\text{signal only}, M=1}}{\overline{r^2}\Big|_{\text{noise only}, M=1}}\right)$$

Coherent integration

$$\text{SNR}_M = \frac{1}{M}\text{SNR}$$

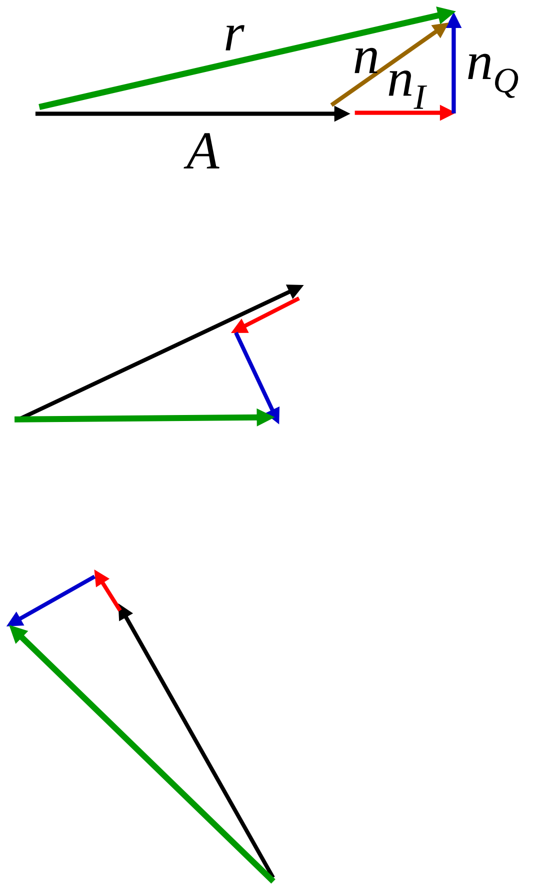
Coherent integration
(*A* is constant during the CPI
and the signal phasor angle does not change)

The target must be Swerling 0, 1 or 3

The detector must be a coherent detector
(e.g., I and Q detector)

Pulse-Doppler radar can operate only on SW 0,1 or 3 targets. The fact that such radars are common place indicates that the targets belong to these SW cases. The most likely case is SW 1.

Non-coherent integration
(unknown signal phasor angle)

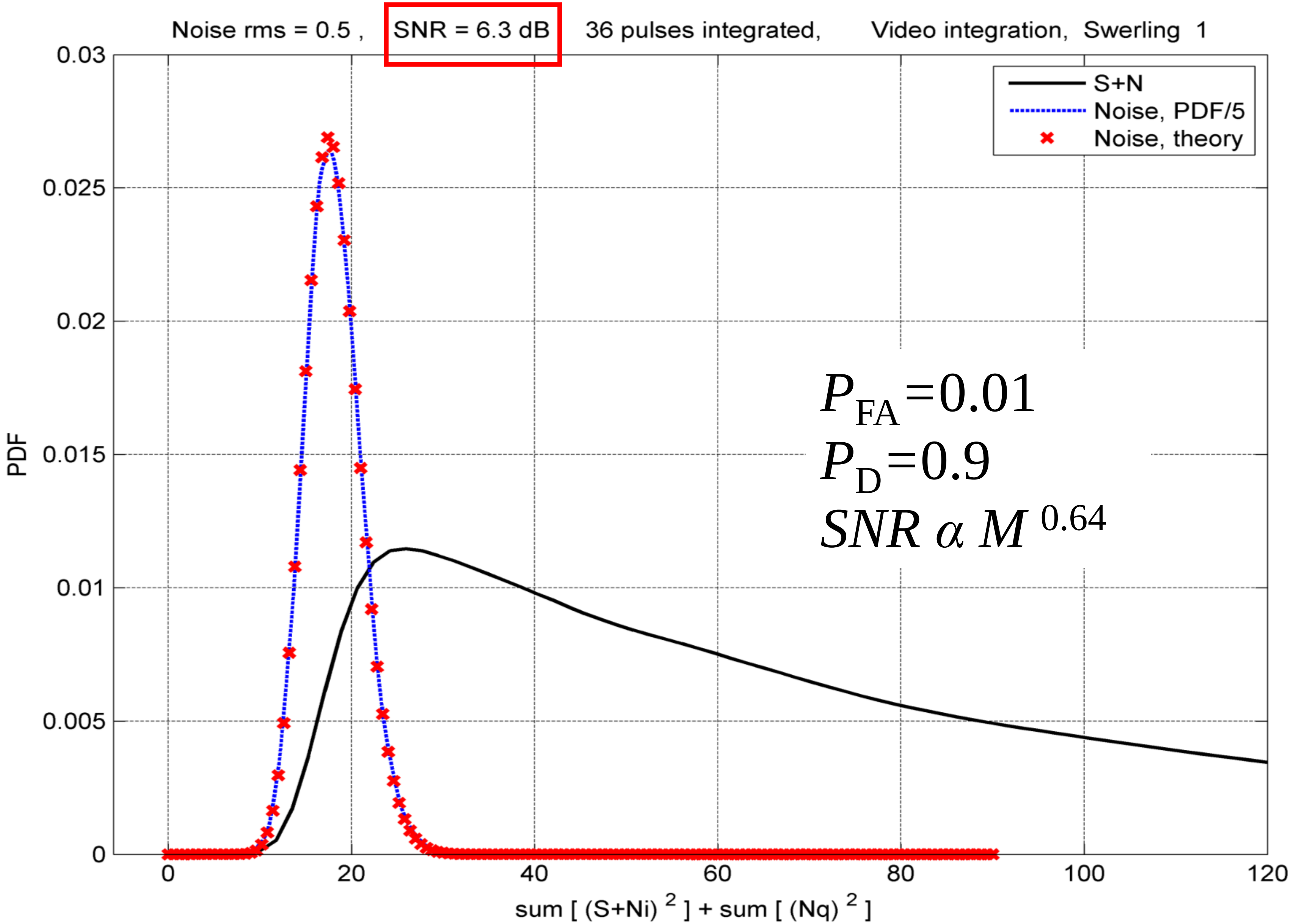


$$r_m = \left|\vec{A} + \vec{n}\right| \quad \text{or} \quad \left|\vec{A} + \vec{n}\right|^2$$

$$v_{\text{out}} = \sum_{m=1}^{M} r_m$$

$$SNR_M \;\; \alpha \;\; \frac{1}{\sqrt{M}} SNR$$

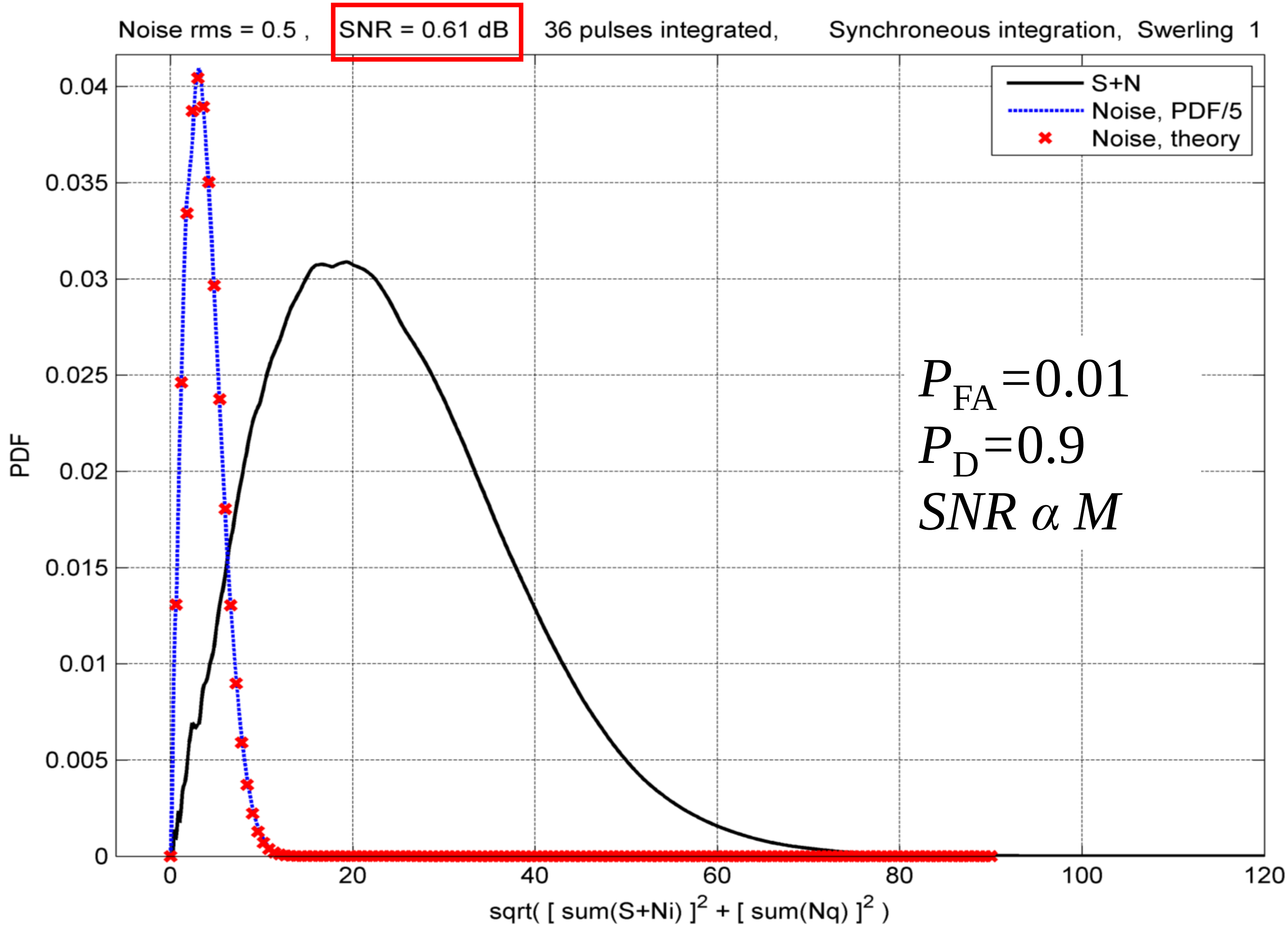
The exact relation depends on if and how A fluctuates

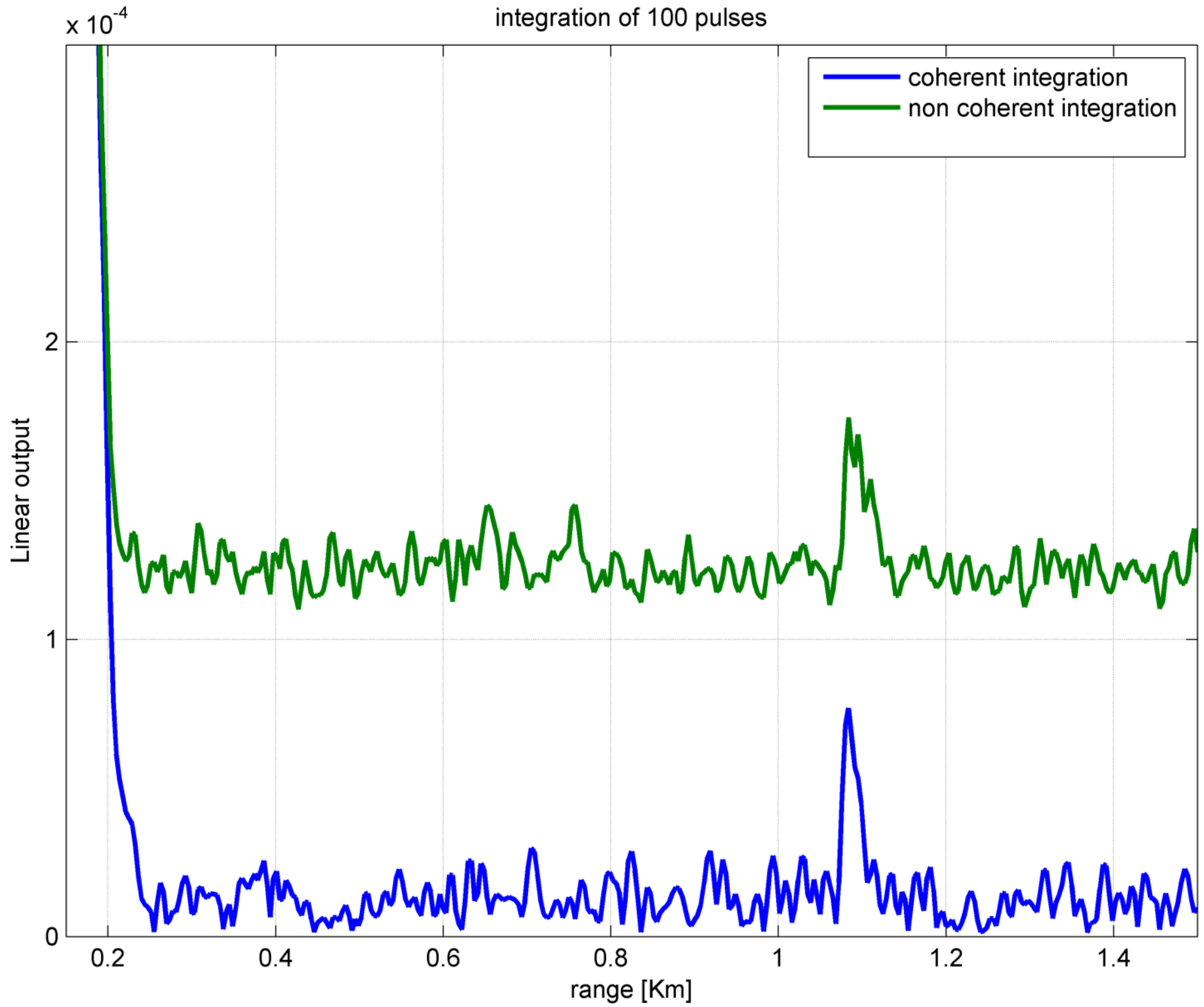


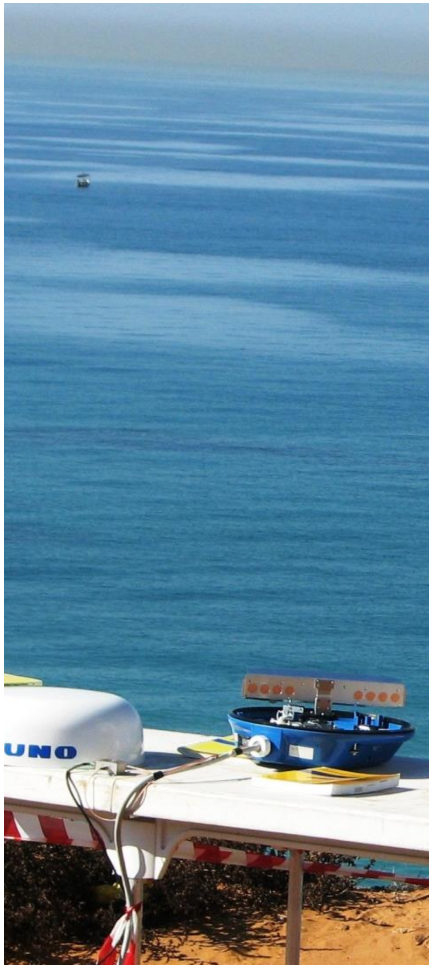
$P_{FA}=0.01$

$P_D=0.9$

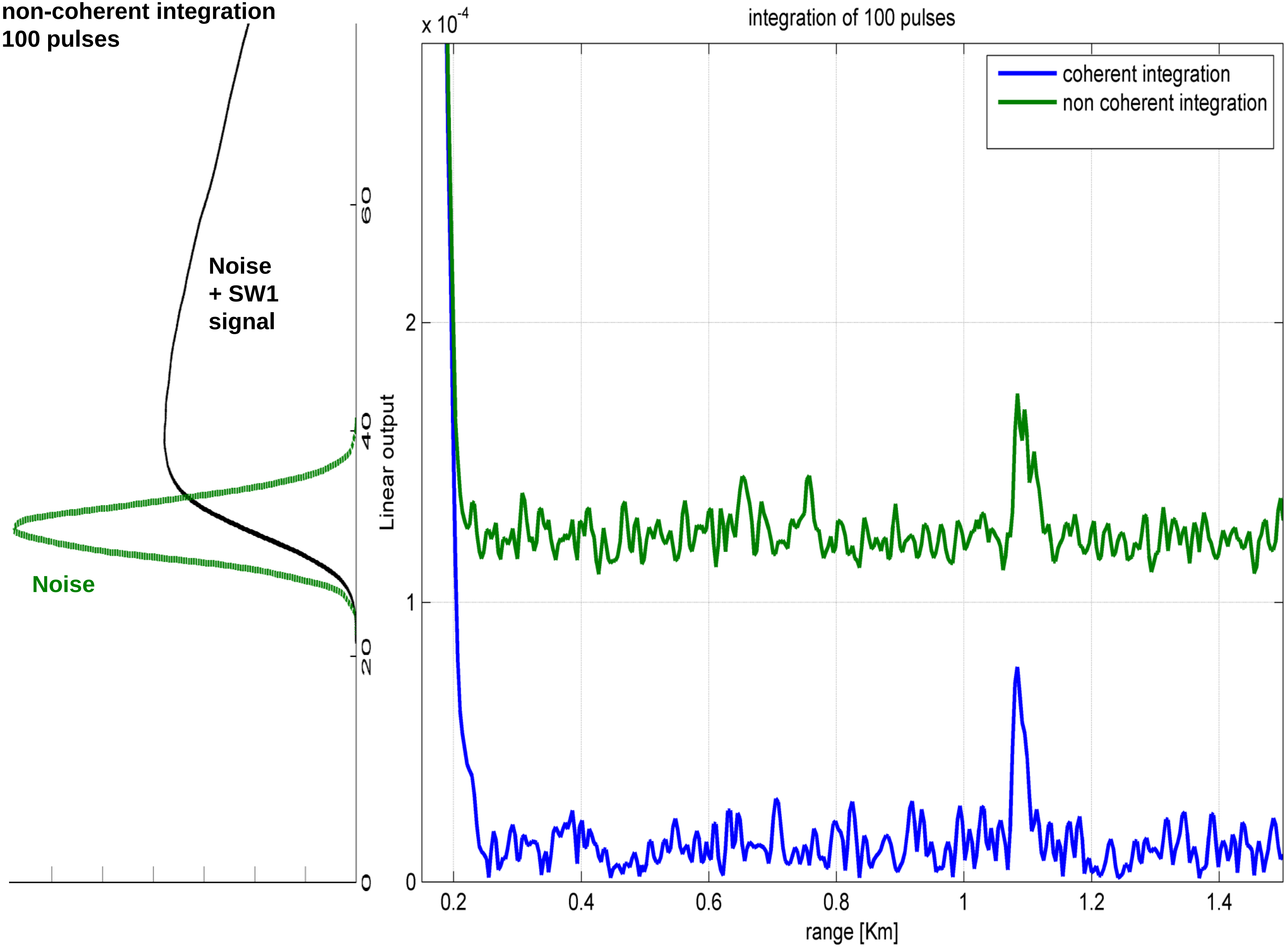
$SNR\ \alpha\ M^{0.64}$


Noise rms = 0.5 , SNR = 0.61 dB 36 pulses integrated, Synchroneous integration, Swerling 1
S+N
Noise, PDF/5
Noise, theory
PDF
sqrt([sum(S+Ni)]^2 + [sum(Nq)]^2)
P_{FA}=0.01
P_D=0.9
SNR α M

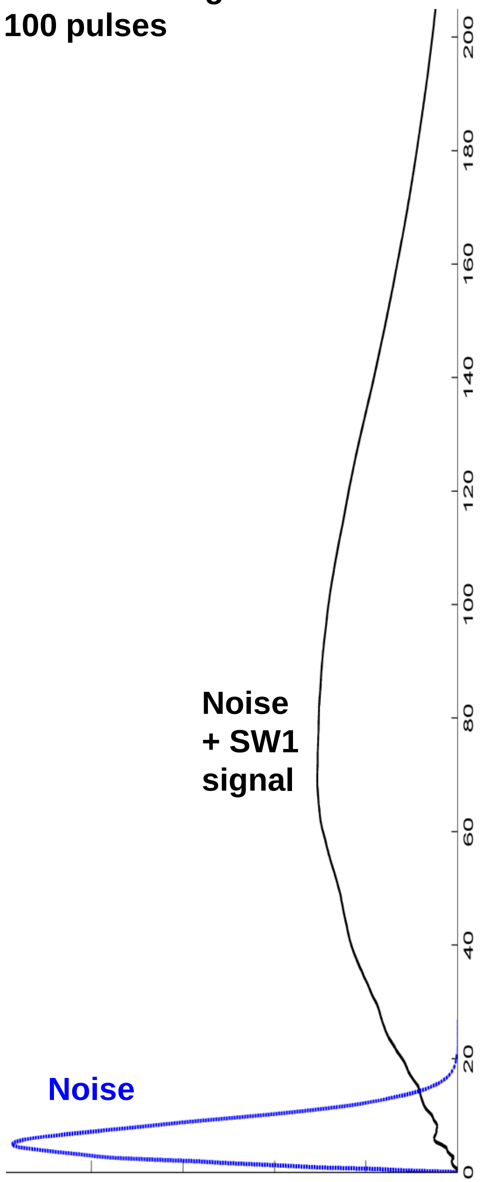

integration of 100 pulses
x 10^-4
coherent integration
non coherent integration
Linear output
range [Km]
0
1
2
0.2
0.4
0.6
0.8
1
1.2
1.4

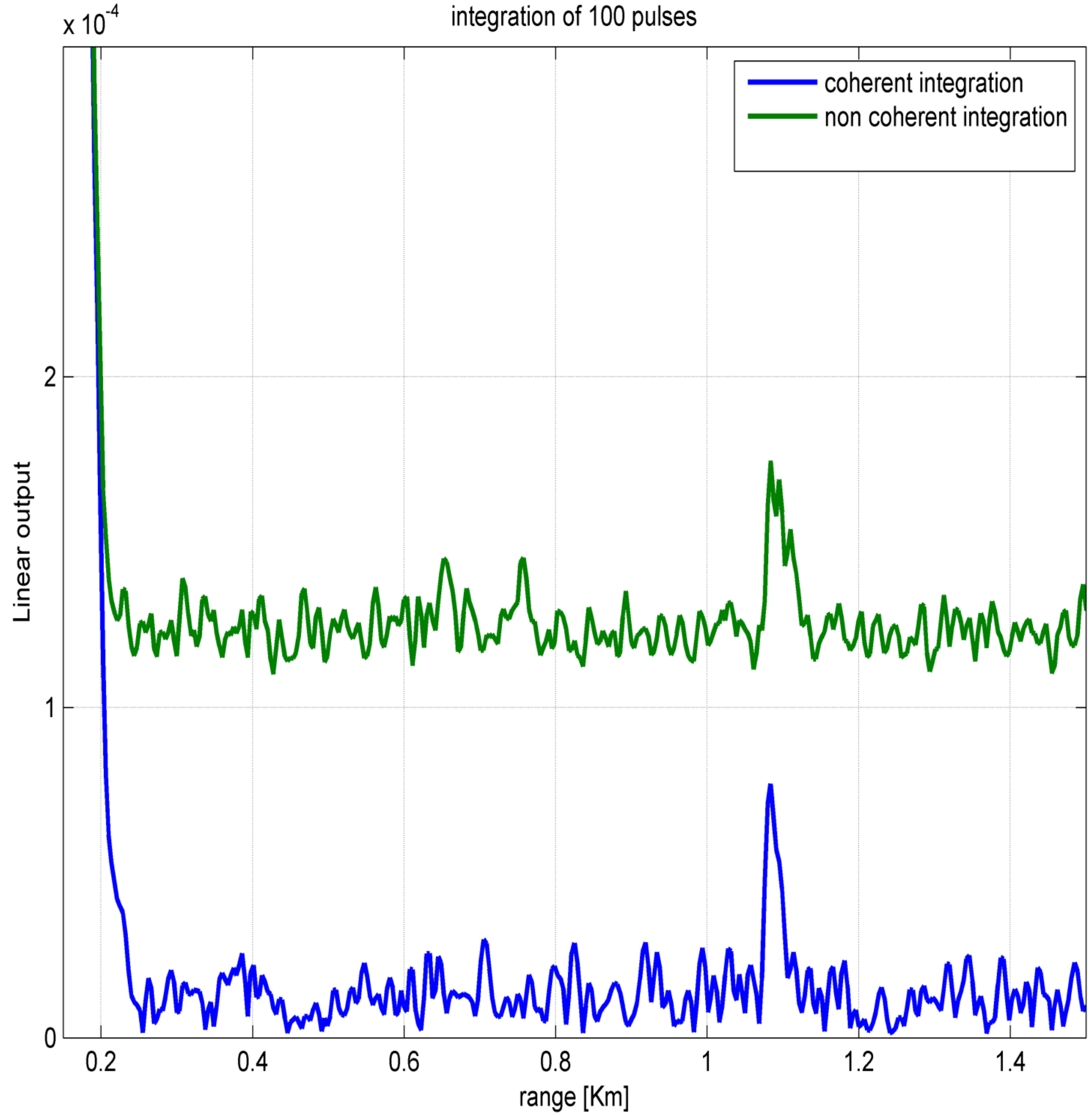


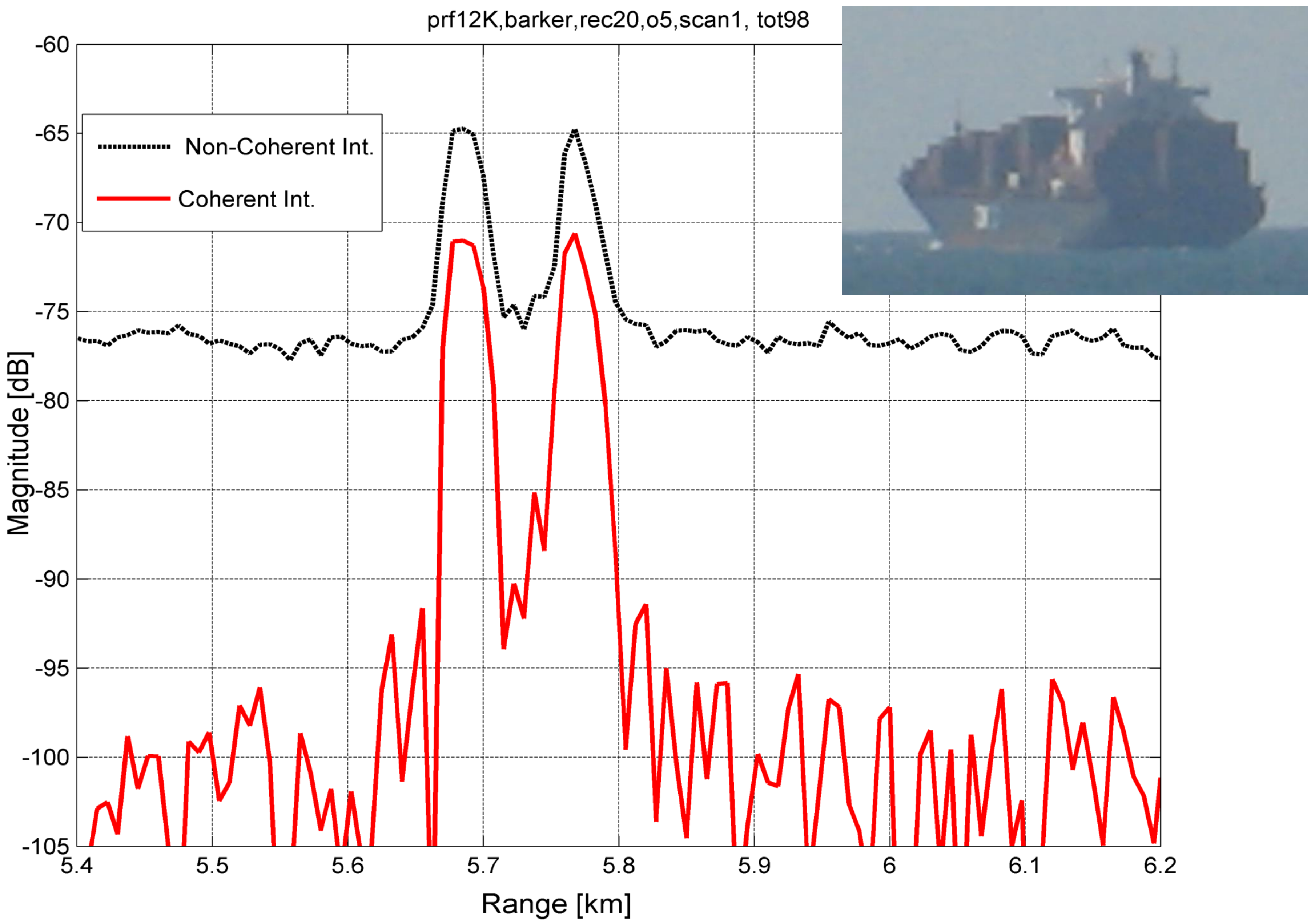
non-coherent integration 100 pulses



coherent integration
100 pulses






prf12K,barker,rec20,o5,scan1, tot98
Non-Coherent Int.
Coherent Int.
Magnitude [dB]
Range [km]
-60
-65
-70
-75
-80
-85
-90
-95
-100
-105
5.4
5.5
5.6
5.7
5.8
5.9
6
6.1
6.2

Since the SNR gain due to coherent integration is so much better, is there any motivation to implement non-coherent integration?

Coherent integration:

- Requires the transmitted pulses to be coherent (pulse-to-pulse).
- Requires the target to maintain coherence during the CPI.
- Requires the receiver to maintain local oscillator stability and perform coherent processing, which is more complex.

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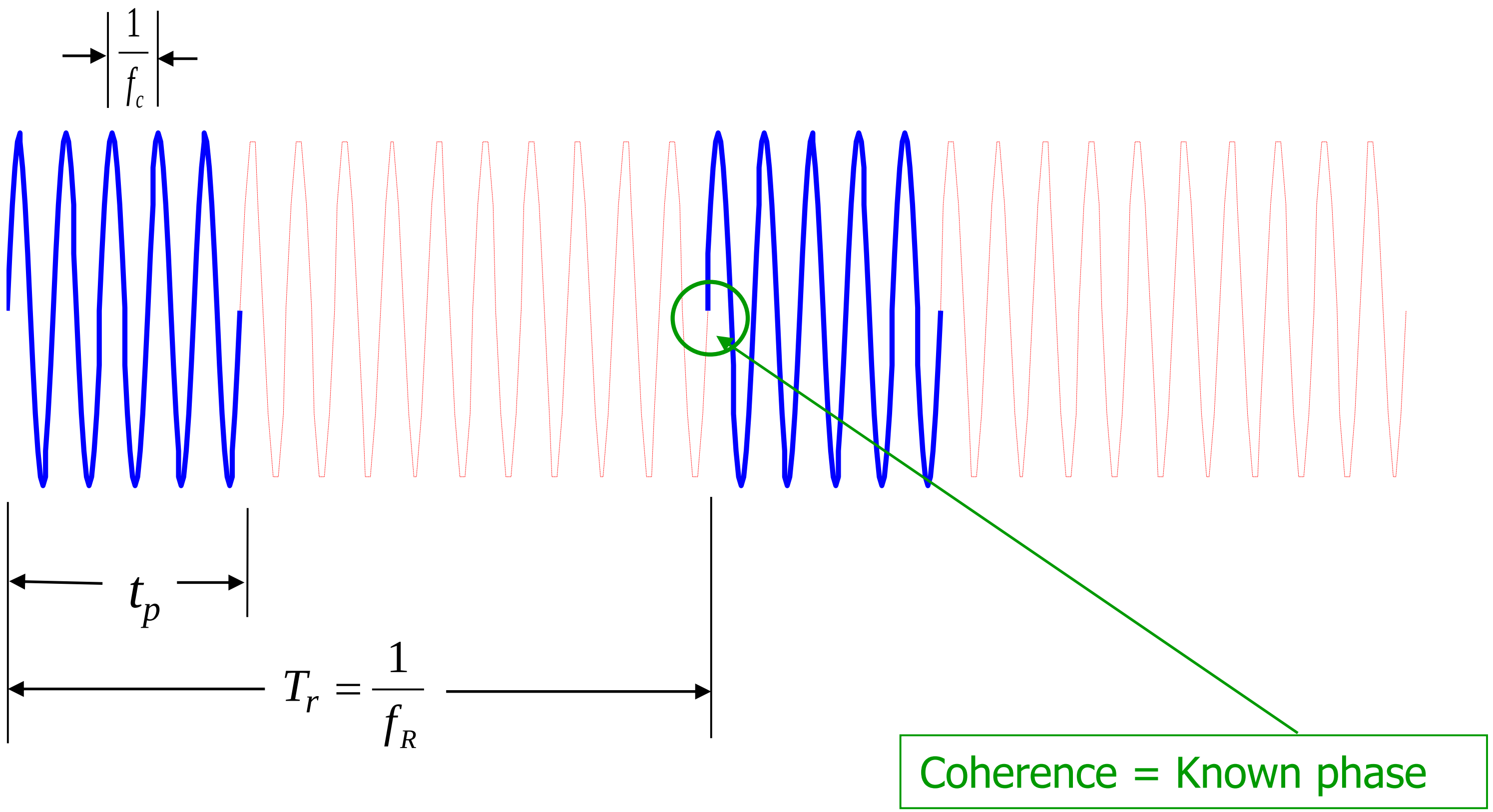
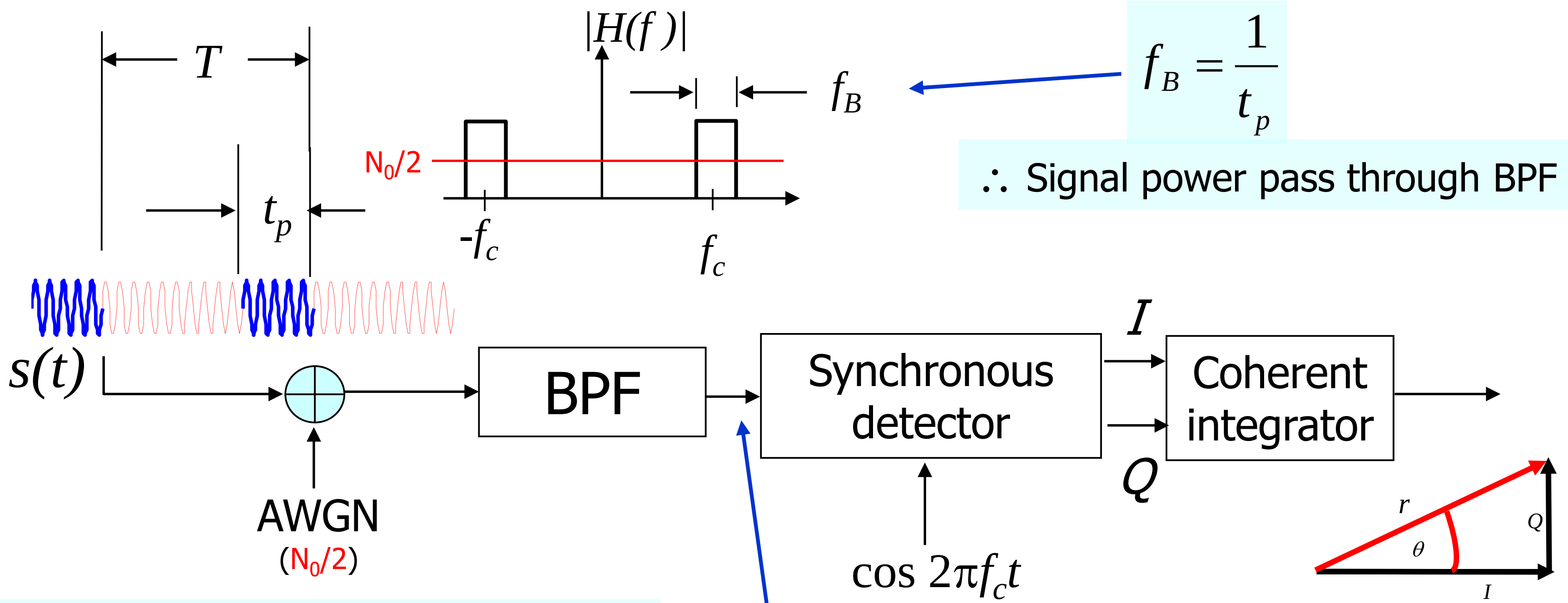
SNR version of the radar equation

(Will be demonstrated on a coherent pulse train, but true for all signals)





SNR_p = for a single pulse

SNR = for integration time

T_I = Coherent integration interval

M = number of pulses in T_I

$$SNR_p = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 N_0 f_B}$$

$$M = \frac{T_I}{T} = T_I f_R$$

$$SNR = M\, SNR_p$$

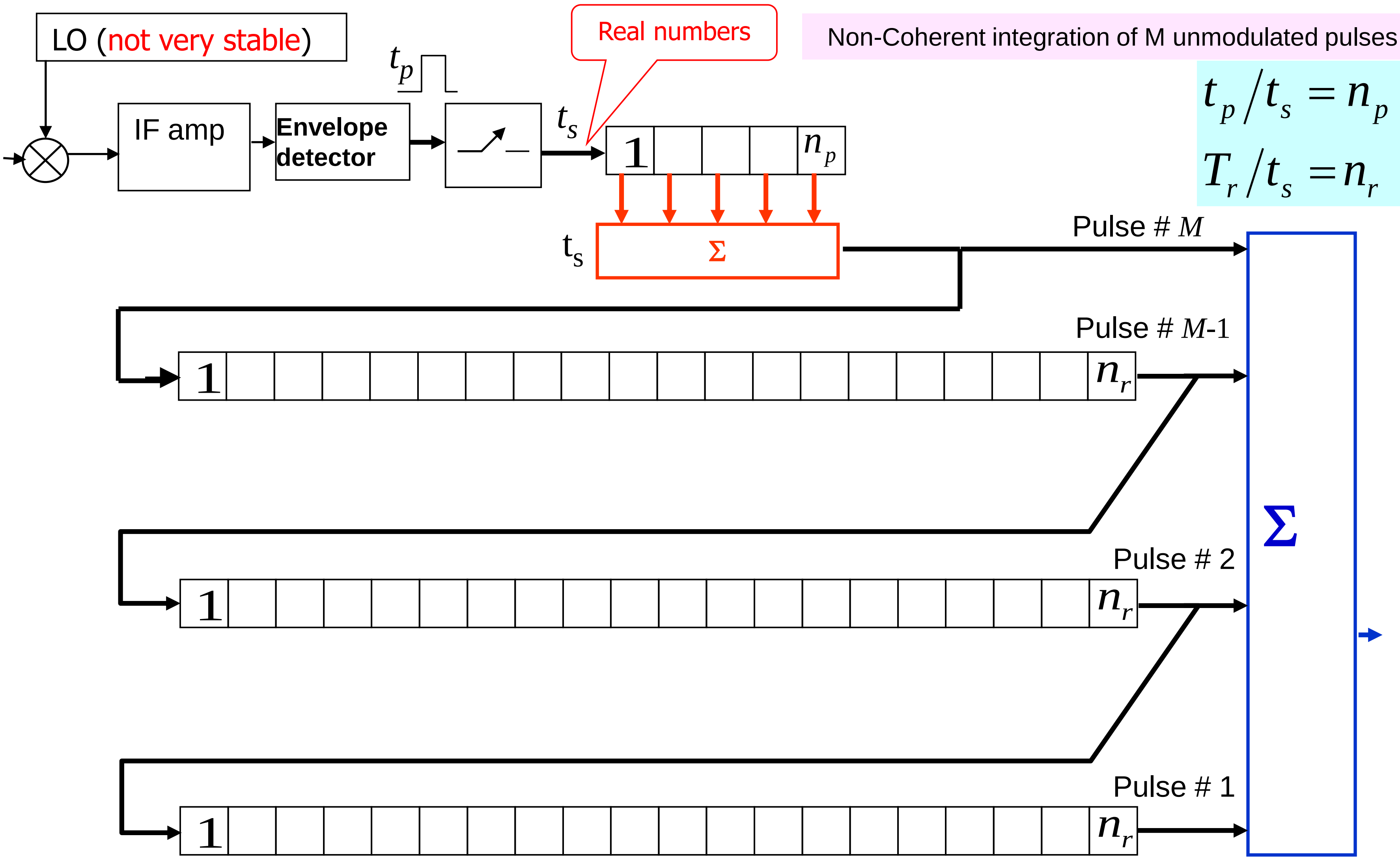
$$P_{ave} = \frac{P_T t_p}{T} = \frac{P_T M}{f_B T_I}$$

$$SNR = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \frac{P_{ave} T_I}{N_0}$$

Non-Coherent integration of M unmodulated pulses

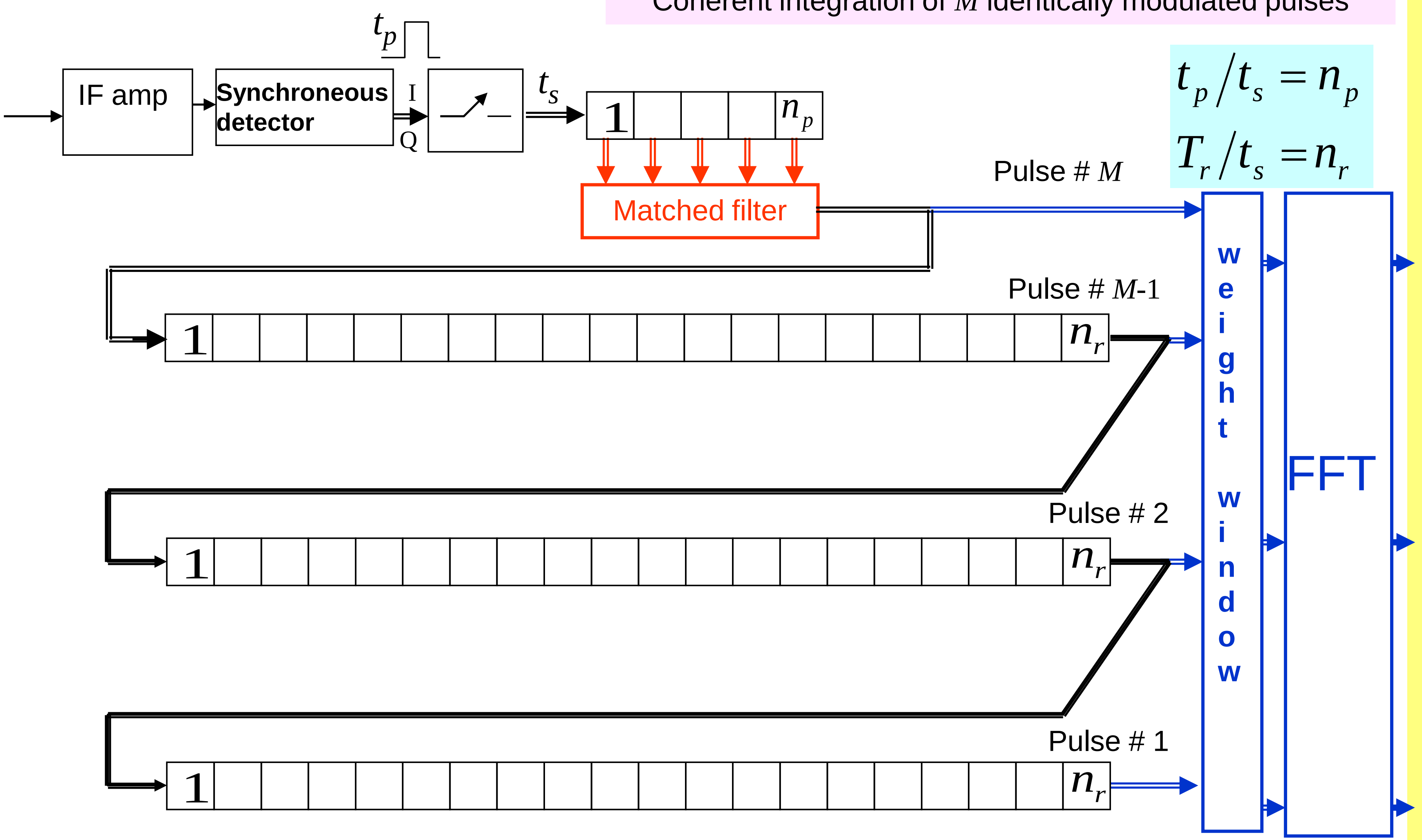
$$t_p / t_s = n_p$$

$$T_r / t_s = n_r$$



Integration of M pulses

Coherent integration of M identically modulated pulses

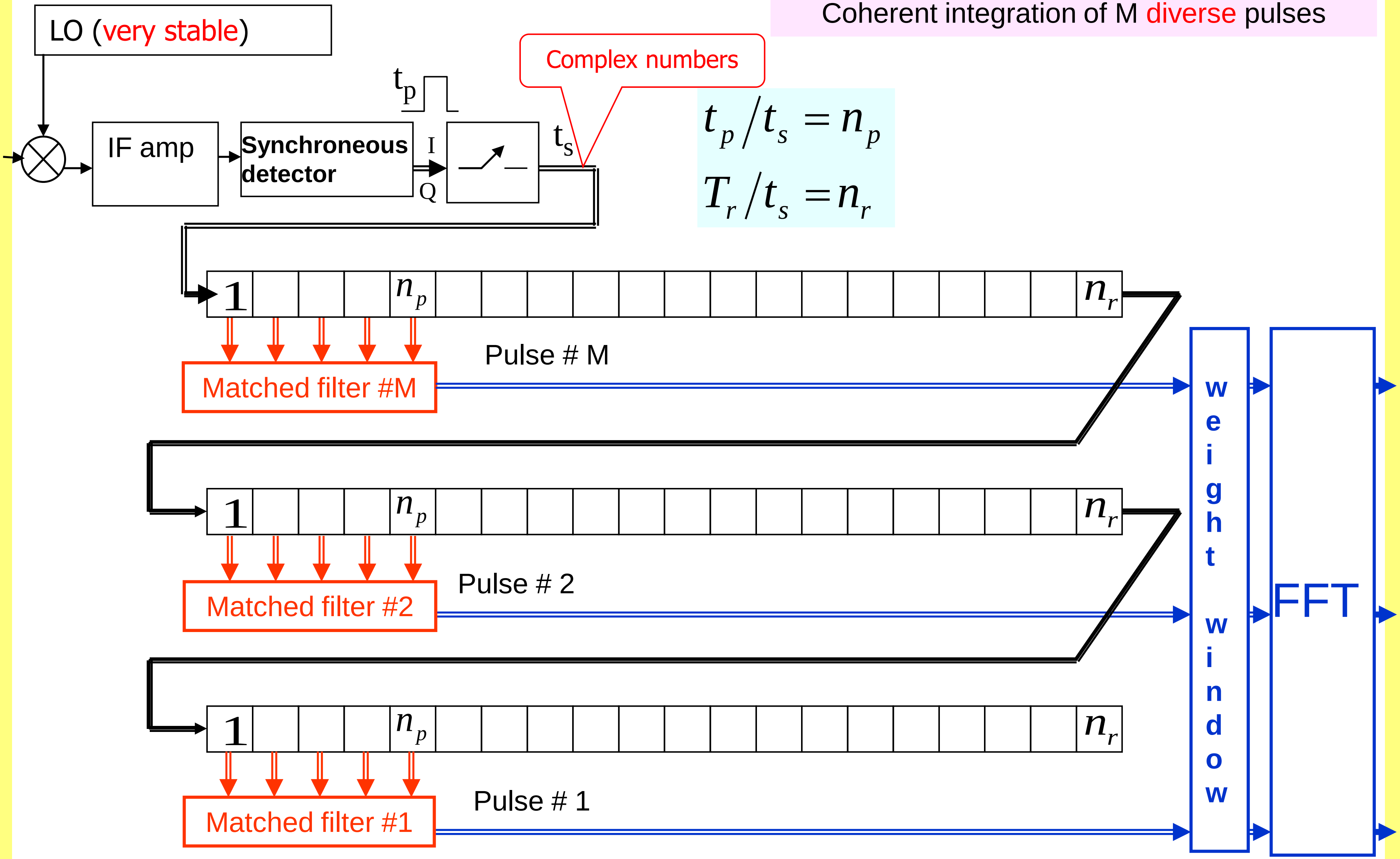


Integration of M pulses

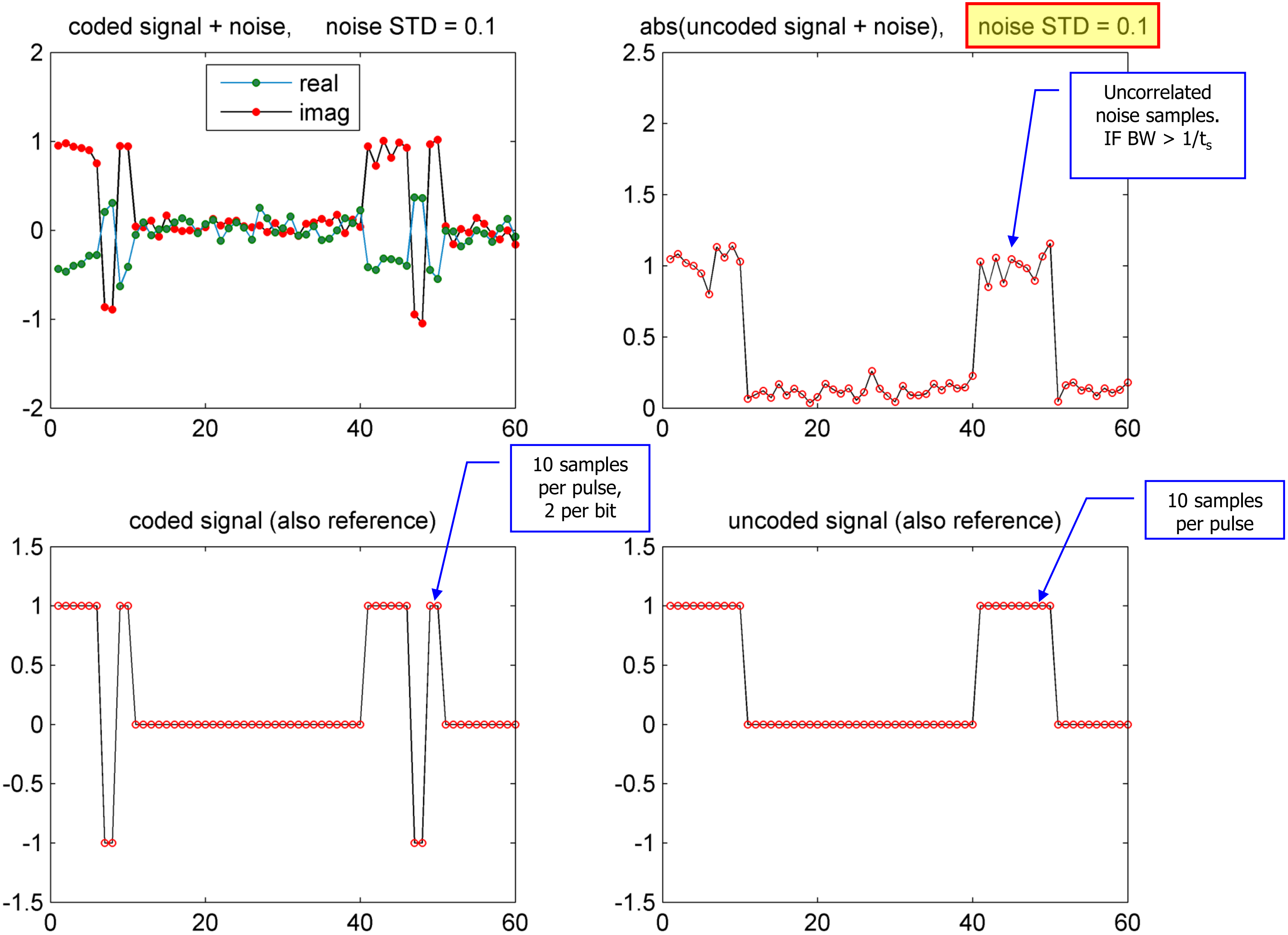
Coherent integration of M diverse pulses

$$t_p / t_s = n_p$$

$$T_r / t_s = n_r$$



Integration of M pulses


coded signal + noise, noise STD = 0.1
real
imag
abs(uncoded signal + noise), noise STD = 0.1
Uncorrelated noise samples. IF BW > 1/t$_s$
coded signal (also reference)
10 samples per pulse, 2 per bit
uncoded signal (also reference)
10 samples per pulse

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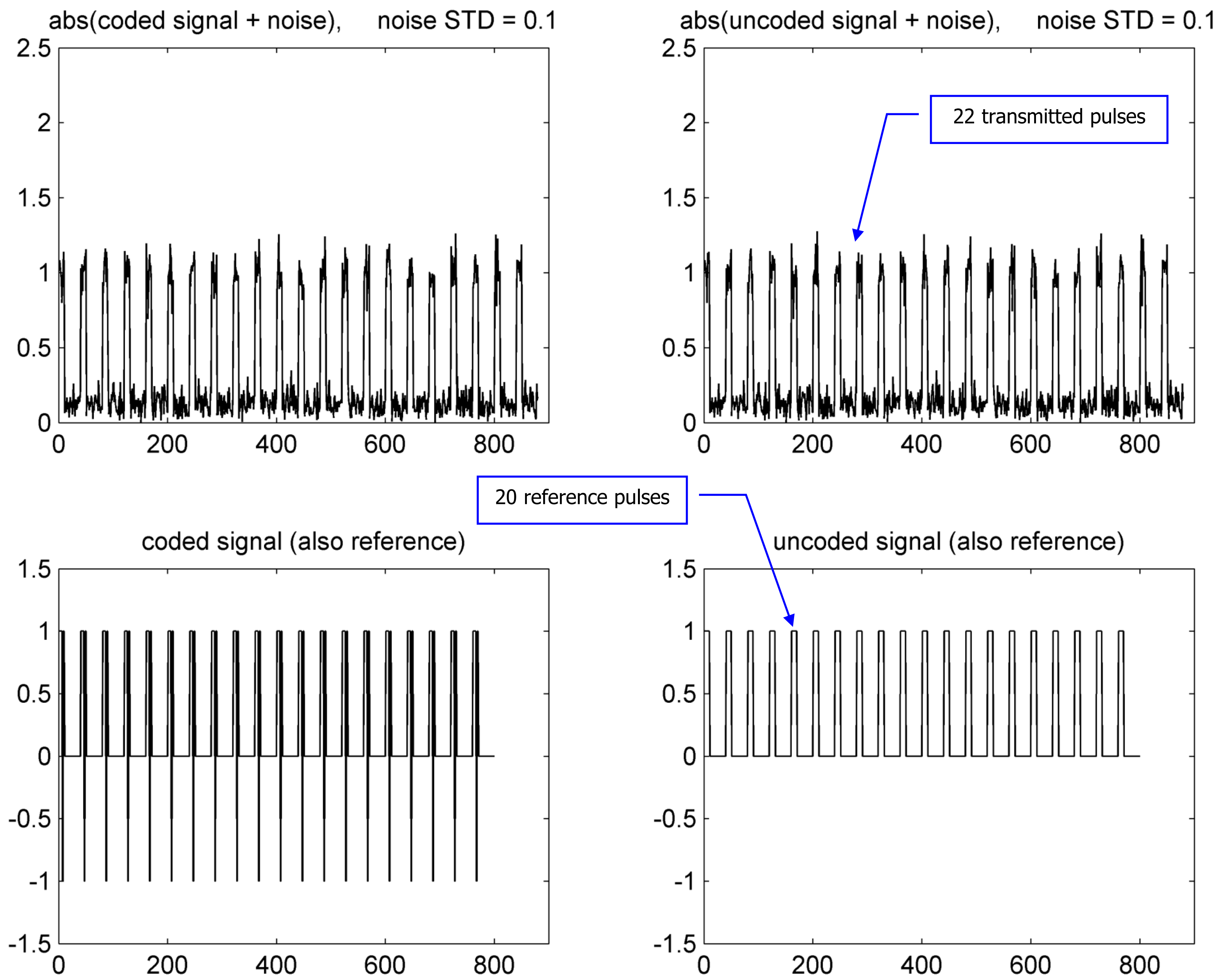
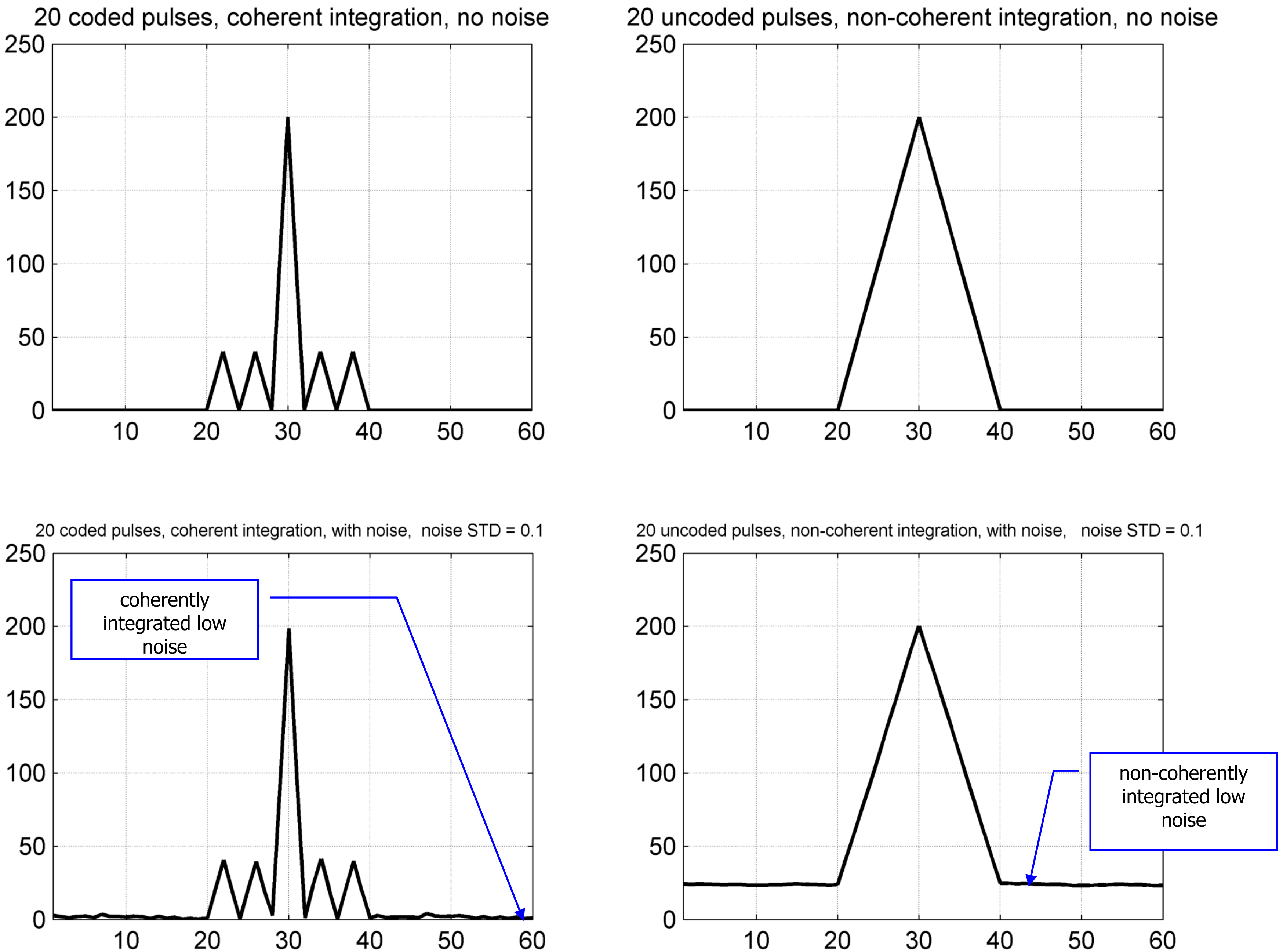
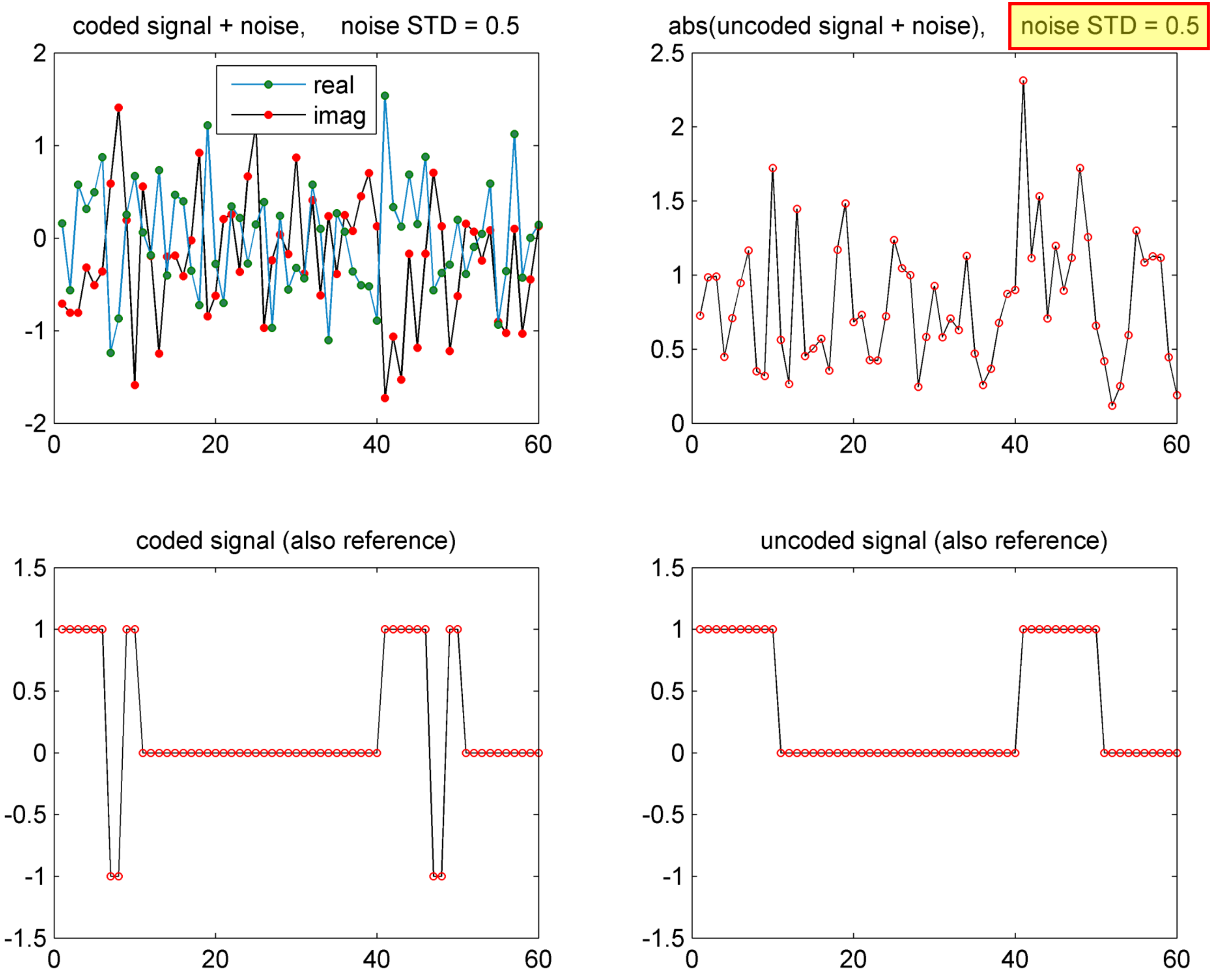
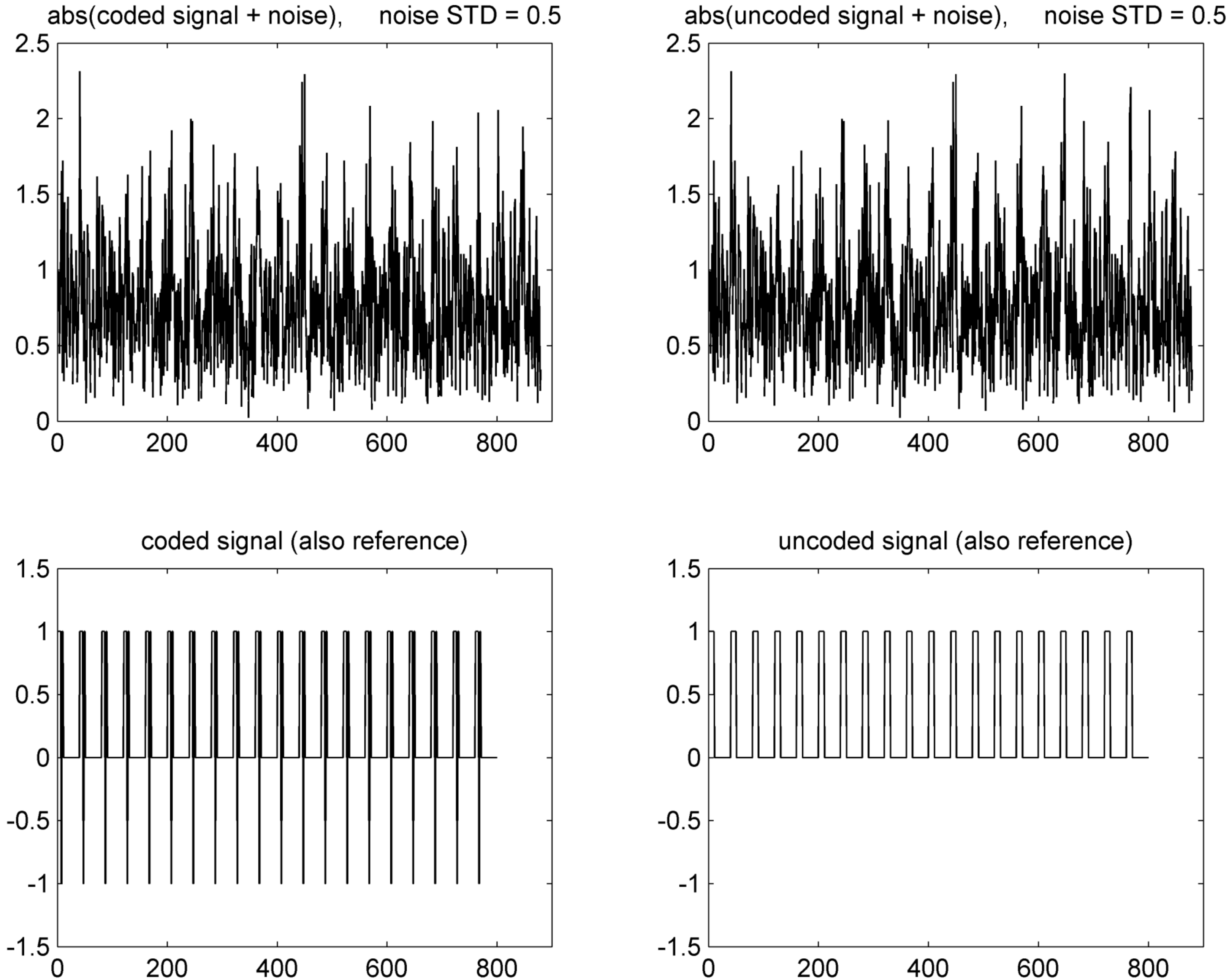



20 coded pulses, coherent integration, no noise
20 uncoded pulses, non-coherent integration, no noise
20 coded pulses, coherent integration, with noise, noise STD = 0.1
coherently integrated low noise
20 uncoded pulses, non-coherent integration, with noise, noise STD = 0.1
non-coherently integrated low noise
0
50
100
150
200
250
10
20
30
40
50
60


coded signal + noise, noise STD = 0.5
real
imag
abs(uncoded signal + noise), noise STD = 0.5
coded signal (also reference)
uncoded signal (also reference)


abs(coded signal + noise), noise STD = 0.5
abs(uncoded signal + noise), noise STD = 0.5
2.5
2
1.5
1
0.5
0
0
200
400
600
800
coded signal (also reference)
uncoded signal (also reference)
1.5
1
0.5
0
-0.5
-1
-1.5

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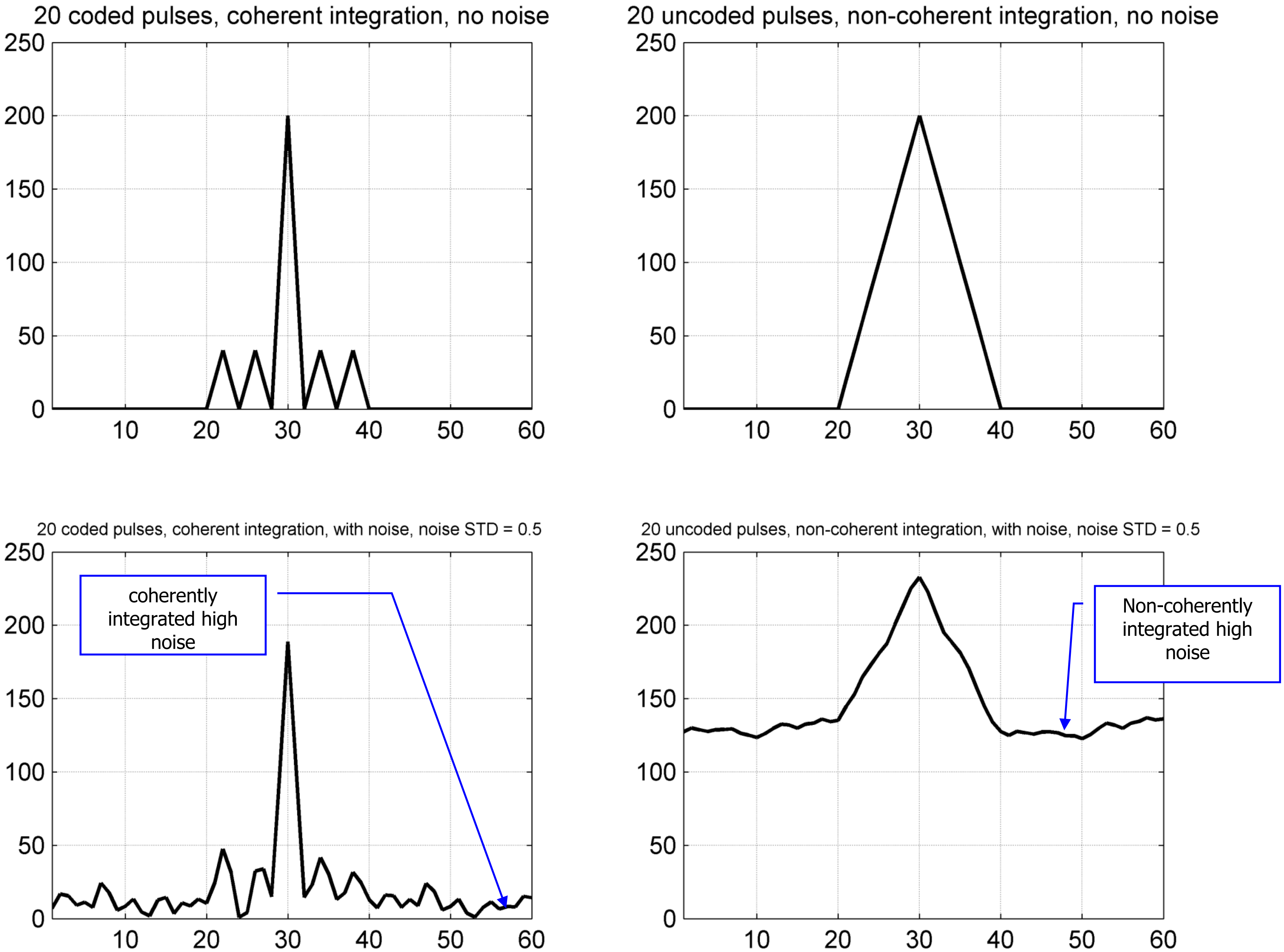
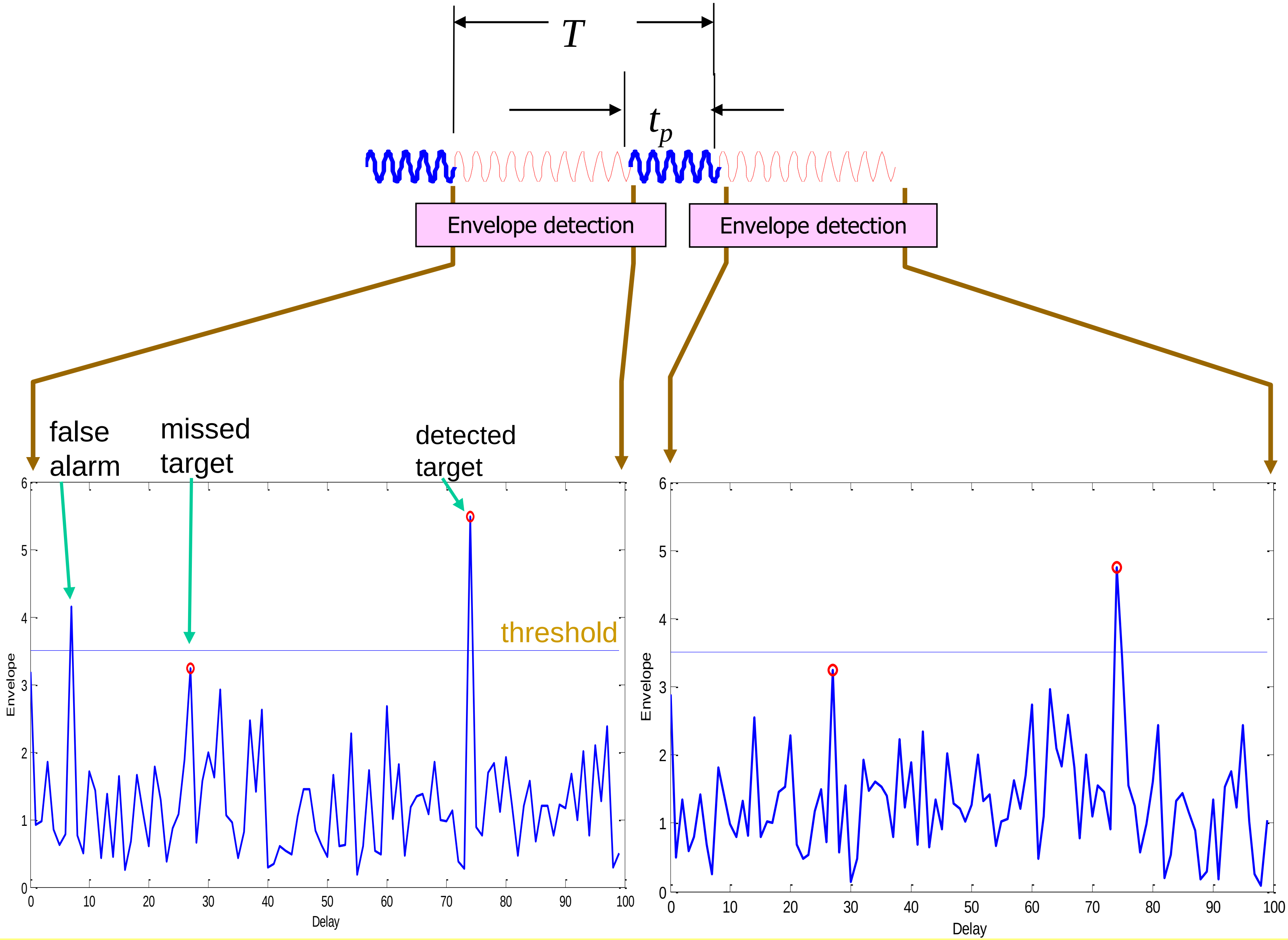
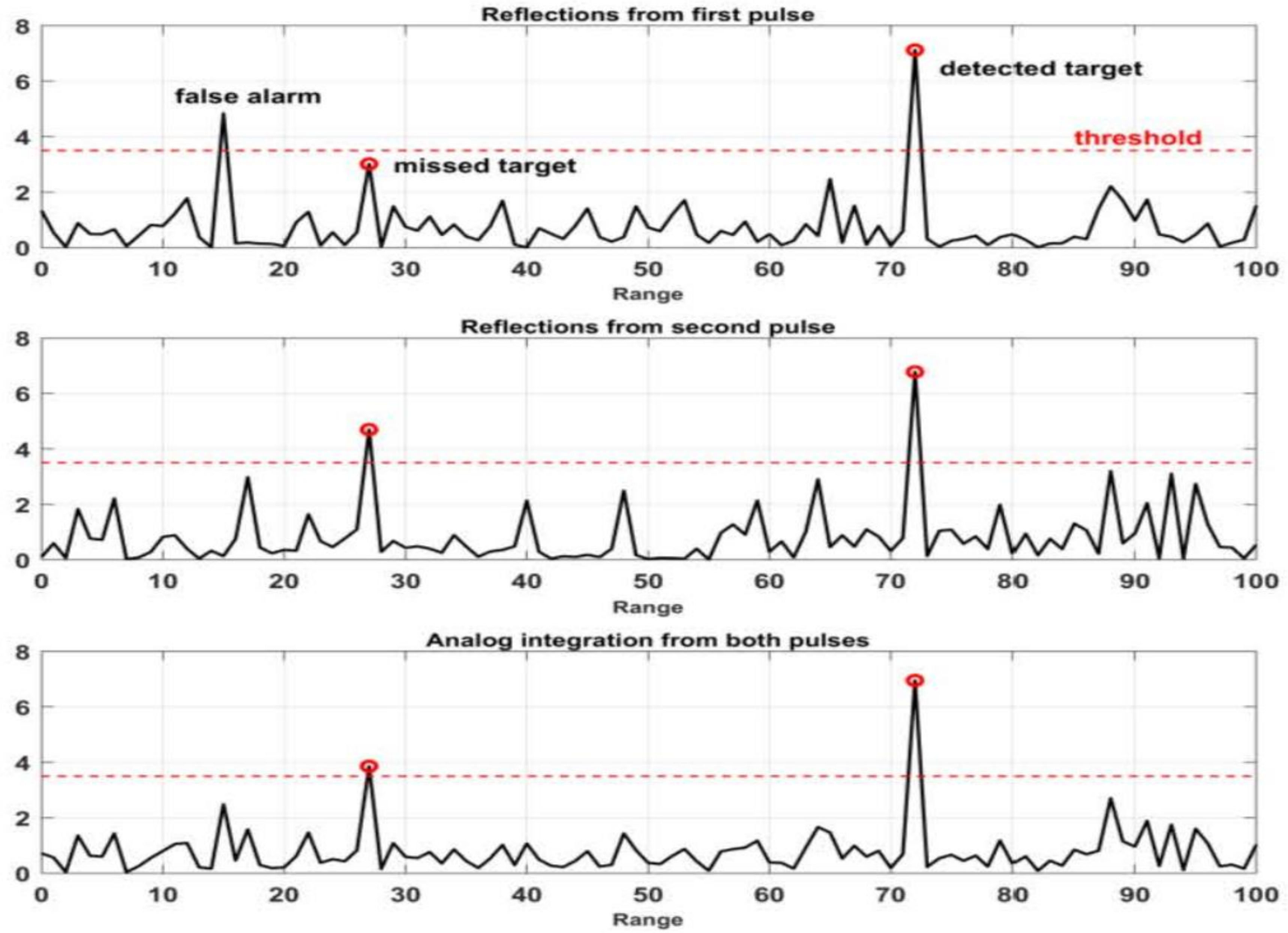


Non-coherent integration of *M* pulses




Reflections from first pulse
false alarm
missed target
detected target
threshold
Range
Reflections from second pulse
Range
Analog integration from both pulses
Range

Non-coherent integration of *M* pulses

First case: Swerling 2 fluctuating target, square-law detector

We start from detection of a single pulse from a fluctuating target (square-law detector)

Single pulse – Rayleigh fluctuating, square law detector

$$p(r\,|\,A)=\frac{r}{\beta^2}\exp\frac{-\left(r^2+A^2\right)}{2\beta^2}\,\mathrm{I}_0\!\left(\frac{rA}{\beta^2}\right)$$

Rician distribution

$$\frac{A^2}{2\beta^2}=SNR$$

No signal, noise only , $A=0$

Rayleigh distribution

$$p_{\text{noise}}(r)=p(r)\Big|_{A=0}=\frac{r}{\beta^2}\exp\frac{-r^2}{2\beta^2}$$

$$z=\frac{r^2}{2\beta^2}$$

$$p(z\,|\,A)=\exp\left[-\left(z+\frac{A^2}{2\beta^2}\right)\right]\mathrm{I}_0\!\left(\sqrt{\frac{2zA^2}{\beta^2}}\right)$$

In SW1 or SW2 the signal intensity is drawn from a Rayleigh distribution:

$$p(A)=\frac{A}{A_0^2}\exp\left(\frac{-A^2}{2A_0^2}\right),\quad \begin{matrix} A>0 \\ A_0^2=\tfrac{1}{2}\overline{A^2}\end{matrix}$$

$$\overline{SNR}=\frac{A_0^2}{\beta^2}$$

$$p(z)=\int_0^\infty p(z\,|\,A)\,p(A)\,dA$$

$$\text{hint}:\quad \int_0^\infty \exp(-ax)\,\mathrm{I}_0\!\left(b\sqrt{x}\right)dx=\frac{1}{a}\exp\left(\frac{b^2}{4a}\right)$$

$$p(z)=D\exp(-Dz)$$

$$D=\frac{1}{1+\dfrac{A_0^2}{\beta^2}}=\frac{1}{1+\overline{SNR}}$$

$$P_D = \int_T^\infty p(z)dz = \int_T^\infty D\exp(-Dz)dz = \left(e^{-T}\right)^D$$

$$D = \frac{1}{1+\frac{A_0^2}{\beta^2}} = \frac{1}{1+\overline{SNR}}$$

$$P_{FA} = P_D\Big|_{A_0=0} \qquad A_0 = 0 \Rightarrow D = 1 \qquad P_{FA} = e^{-T}$$

$$P_D = \left(P_{FA}\right)^D$$

For a single pulse from a Rayleigh fluctuating target we get ⇒

$$P_D = P_{FA}^{\frac{1}{1+\overline{SNR}}}$$

Because it is a single pulse, the correlation between pulses is of no concern, hence the single pulse result applies to both SW1 and SW2

This single-pulse result is also very important for <u>coherent</u> integration of M pulses, returning from a SW1 target !!!
In that case (if SNR is that of a single pulse):

$$P_D = P_{FA}^{\frac{1}{1+M\,\overline{SNR}}}$$

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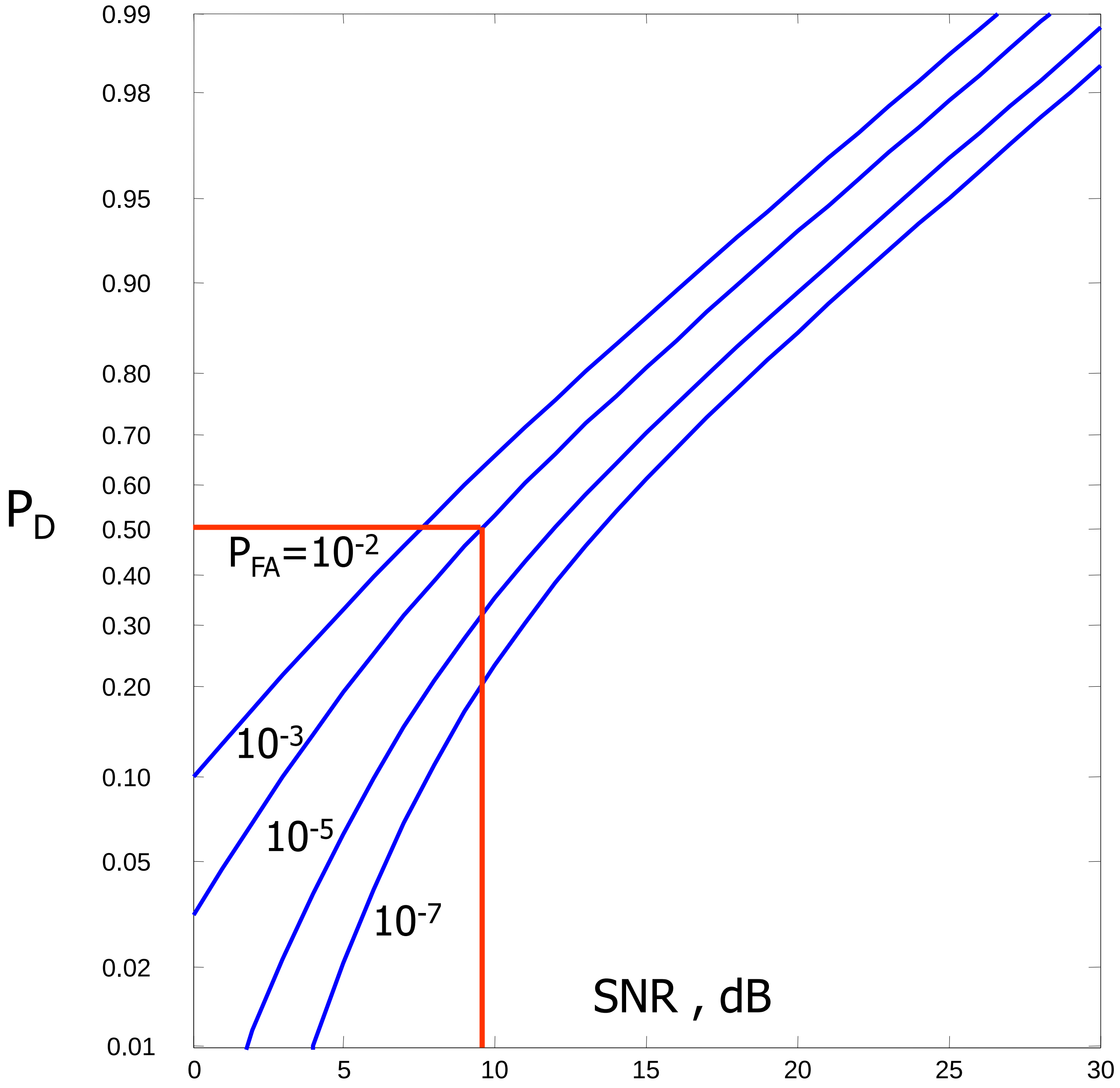
Single pulse SW1 or SW2 fluctuating

$$P_D = P_{FA}^{\frac{1}{1+\overline{SNR}}}$$



Single pulse – Swerling 3 fluctuating, square law detector

$$p(r\,|\,A)=\frac{r}{\beta^2}\exp\frac{-\left(r^2+A^2\right)}{2\beta^2}\,\mathrm{I}_0\left(\frac{rA}{\beta^2}\right)$$

Rician distribution

$$\frac{A^2}{2\beta^2}=SNR$$

No signal, noise only , $A=0$

Rayleigh distribution

$$p_{\text{noise}}(r)=p(r)\Big|_{A=0}=\frac{r}{\beta^2}\exp\frac{-r^2}{2\beta^2}$$

$$z=\frac{r^2}{2\beta^2}$$

$$p(z\,|\,A)=\exp\left[-\left(z+\frac{A^2}{2\beta^2}\right)\right]\mathrm{I}_0\left(\sqrt{\frac{2zA^2}{\beta^2}}\right)$$

In SW3 or SW4 the signal intensity is drawn from the distribution: (Developed by Nitzan Raybi)

$$p(A)=\frac{2A^3}{A_0^4}\exp\left(\frac{-A^2}{A_0^2}\right)\ ,\quad A>0$$

$$\overline{SNR}=\frac{A_0^2}{\beta^2}$$

$$p(z)=\int_0^\infty p(z\,|\,A)\,p(A)\,dA$$

$$p(z)=e^{-z}\frac{2}{A_0^4}\int_0^\infty A^3e^{-aA^2}I_0(b|A|)\,dA\,,\ a=\frac{1}{2\beta^2}+\frac{1}{A_0^2},\ b=\sqrt{\frac{2z}{\beta^2}}$$

The integral boundaries are positive, hence, within the integral we can replace $|A|=A$

From the book “Integrals and Series”, Volume 2 by A. P. Prudnikov, Yu. A. Brychkov, O. I. Marichev, we will use the equation:

$$\int_0^\infty x^{\alpha-1}e^{-ax^2}I_\nu(bx)\,dx = 2^{-\nu-1}b^\nu a^{-\frac{\alpha+\nu}{2}}\frac{\Gamma\left(\frac{\alpha+\nu}{2}\right)}{\Gamma(\nu+1)}\,{}_1F_1\left(\frac{\alpha+\nu}{2},\nu+1,\frac{b^2}{4a}\right), \qquad \mathcal{R}e(p)>0, \mathcal{R}e(\alpha+\nu)>0,\ -\pi<\arg b<\pi$$

Where ${}_1F_1$ is the *Confluent Hypergeometric function of the first kind.*

We will simplify the equation from the book by setting: $\nu = 0,\ \alpha = 2n,\ n\in\mathbb{Z}$

$$\int_0^\infty x^{2n-1}e^{-ax^2}I_0(bx)dx = \frac{(n-1)!}{2a^n}e^{\frac{b^2}{4a}}L_{n-1}\left(-\frac{b^2}{4a}\right)$$

Where L_n are *Laguerre polynomials of order n*. For our case we will set n=2 and get:

$$p(z) = e^{-z}\frac{2}{A_0^4}\int_0^\infty A^3e^{-aA^2}I_0(bA)dA = e^{-z}\frac{2}{A_0^4}\cdot\frac{1}{2a^2}e^{\frac{b^2}{4a}}\left(1+\frac{b^2}{4a}\right) = D^2[1+z(1-D)]e^{-Dz},$$

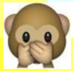
$$D = \frac{1}{1+\frac{A_0^2}{2\beta^2}} = \frac{1}{1+\frac{\overline{SNR}}{2}}$$

$$\overline{SNR} = \frac{A_0^2}{\beta^2}$$

We will now calculate P_D assuming a threshold T :

$$P_D = \int_T^\infty p(z)dz = \int_T^\infty D^2[1+z(1-D)]e^{-Dz}dz = [1+(1-D)DT]e^{-DT}$$

$$P_{FA} = P_D\big|_{A_0=0}\ ,\ A_0=0 \Rightarrow D=1,\ P_{FA} = [1+(1-D)DT]e^{(-DT)}\big|_{D=1} = e^{-T}\ ,\ P_{FA}=e^{-T} \Rightarrow T=-\ln P_{FA}$$



$$P_D = [1+(1-D)DT]e^{(-DT)}\Big|_{D=\frac{1}{1+\frac{\overline{SNR}}{2}},\, T=-\ln P_{FA}} \Rightarrow$$

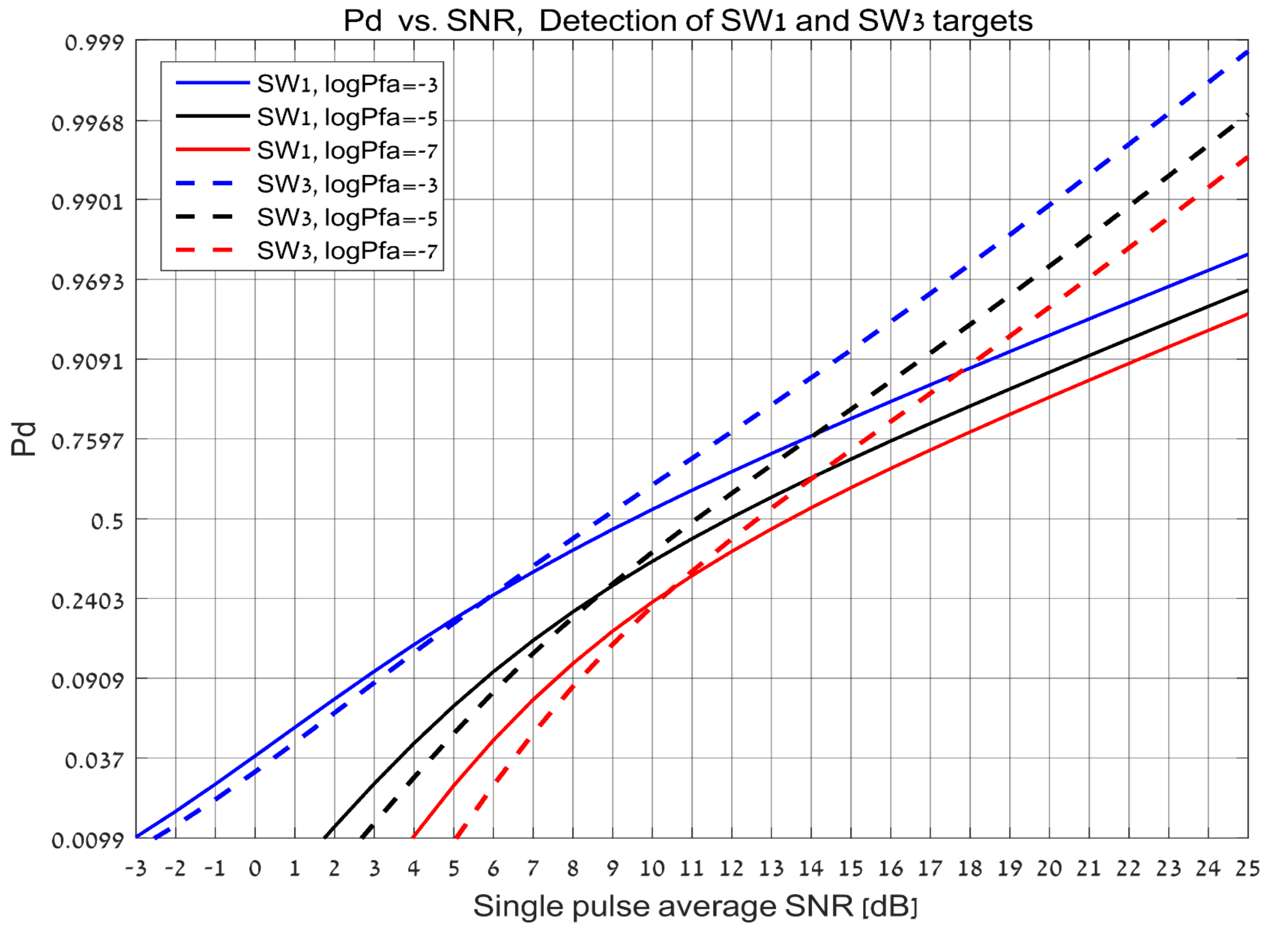
$$P_D = \left[1 - \frac{2\overline{SNR}\ln P_{FA}}{(2+\overline{SNR})^2}\right] P_{FA}^{\frac{1}{1+\frac{\overline{SNR}}{2}}} = \left[1 - \frac{2\overline{SNR}\ln P_{FA}}{(2+\overline{SNR})^2}\right] e^{\frac{2\ln P_{FA}}{2+\overline{SNR}}}$$

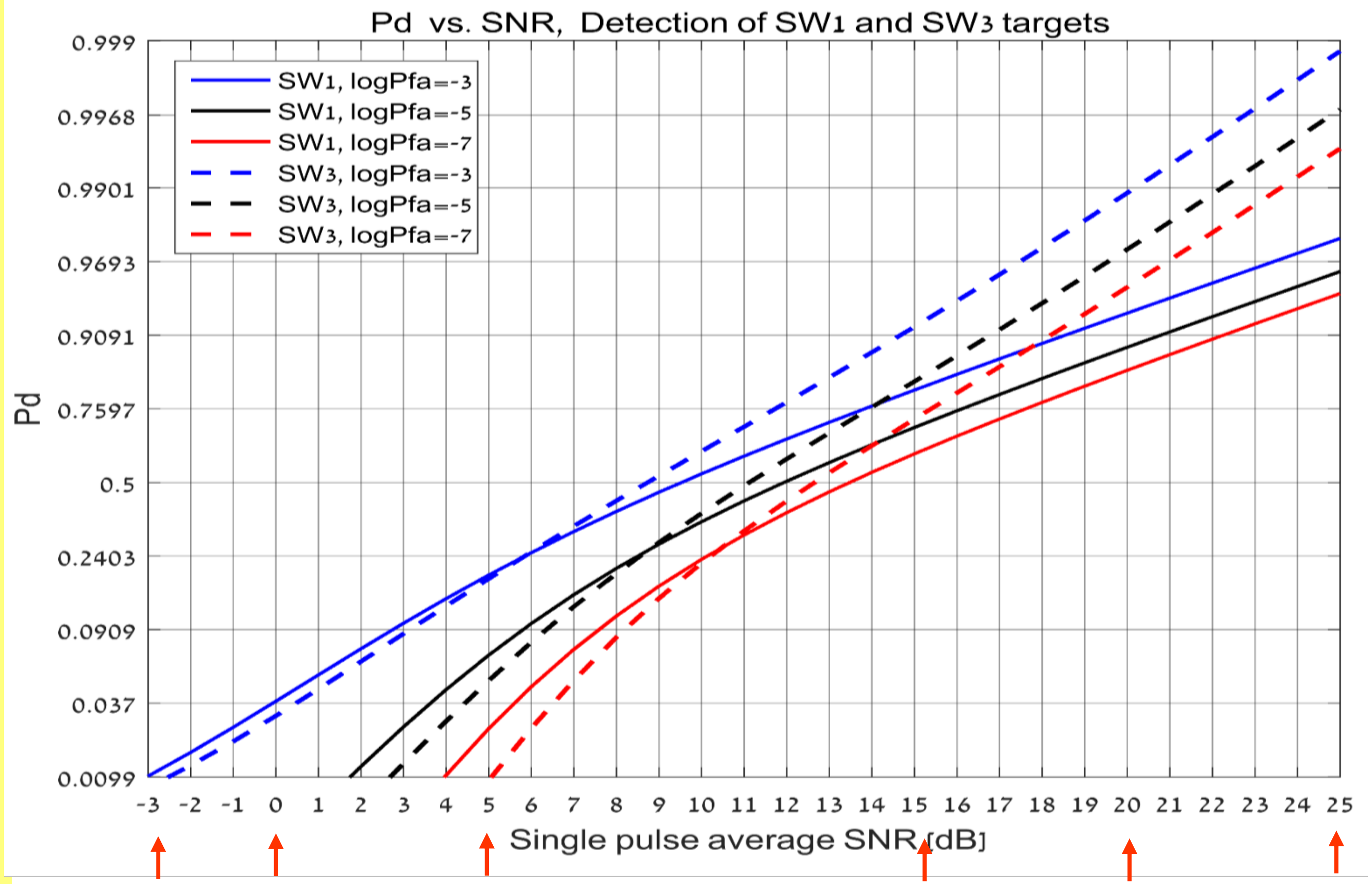
See same result in Eq. 6.127 in “***Basic Radar Analysis***”, by Marvin C. Budge Jr. and Shaw R. German, Artech House, 2015

This result is also very important for coherent integration of M pulses, returning from a SW3 target !!!
In that case (if $\overline{SNR}$ is that of a single pulse):

$$P_D = \left[1 - \frac{2M\overline{SNR}\ln P_{FA}}{(2+M\overline{SNR})^2}\right] e^{\frac{2\ln P_{FA}}{2+M\overline{SNR}}}$$

There are no similar results for SW2 or SW4 targets. These targets fluctuate pulse-to pulse, not allowing coherent integration.


Pd vs. SNR, Detection of SW1 and SW3 targets
SW1, logPfa=-3
SW1, logPfa=-5
SW1, logPfa=-7
SW3, logPfa=-3
SW3, logPfa=-5
SW3, logPfa=-7
Pd
0.999
0.9968
0.9901
0.9693
0.9091
0.7597
0.5
0.2403
0.0909
0.037
0.0099
-3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Single pulse average SNR [dB]



Single pulse average SNR [dB] **when** ***M* =10 SW1 or SW3 pulses are integrated coherently**

$10*\log_{10}(10)=10$

This single-pulse result is also very important for <u>coherent</u> integration of *M* pulses, returning from a SW1 target !!!
In that case (if *SNR* is that of a single pulse):

$$P_D = P_{FA}^{\frac{1}{1+M\,\overline{SNR}}}$$

(Video) Inegration = algebraic sum of M envelope values at each cell

$$z_m = \frac{r_m^2}{2\beta^2} \qquad y = \sum_{m=1}^{M} z_m$$

M pulses, SW2 fluctuating

$$p(y) = p(z_1) * p(z_2) * \ldots * p(z_M) \ , \quad p(z_1) = p(z_2) = \ldots = p(z_M) = p(z)$$

In SW2 $\quad p(z) = De^{-Dz}$

$$D = \frac{1}{1 + \overline{\overline{SNR}}}$$

Use Laplace transform

(Moment generating function)

$$\mathrm{L}[p(z)] = \int_0^\infty e^{-uz} p(z)\, dz = h(u)$$

$$h(u) = \int_0^\infty e^{-uz} De^{-Dz}\, dz = \frac{D}{D+u}$$

$$\mathrm{L}[p(y)] = \{\mathrm{L}[p(z)]\}^M = \left(\frac{D}{D+u}\right)^M$$

$$p(y) = \frac{D^M}{(M-1)!} y^{M-1} e^{-Dy}$$

P_{FA} , P_{D} and SNR

y_T = Normalized threshold

$$D = \frac{1}{1+\overline{SNR}}$$

M pulses, SW2 fluctuating

$$P_{\mathrm{D}} = \int_{y_T}^{\infty} p(y)\,dy$$

$$p(y) = \frac{D^M}{(M-1)!} y^{M-1} e^{-Dy}$$

$$P_{\mathrm{D}} = e^{-y_T D} \sum_{r=0}^{M-1} \frac{(y_T D)^r}{r\,!}$$

$$P_{\mathrm{FA}} = P_{\mathrm{D}}\Big|_{D=1} = e^{-y_T} \sum_{r=0}^{M-1} \frac{y_T^r}{r\,!}$$

$M, P_{\mathrm{FA}} \Rightarrow y_T$ (iterative process)

$M, D, y_T \Rightarrow P_{\mathrm{D}}$

or

$y_{TD} = 0.5*\mathrm{chi2inv}(1-P_D, 2*M)$

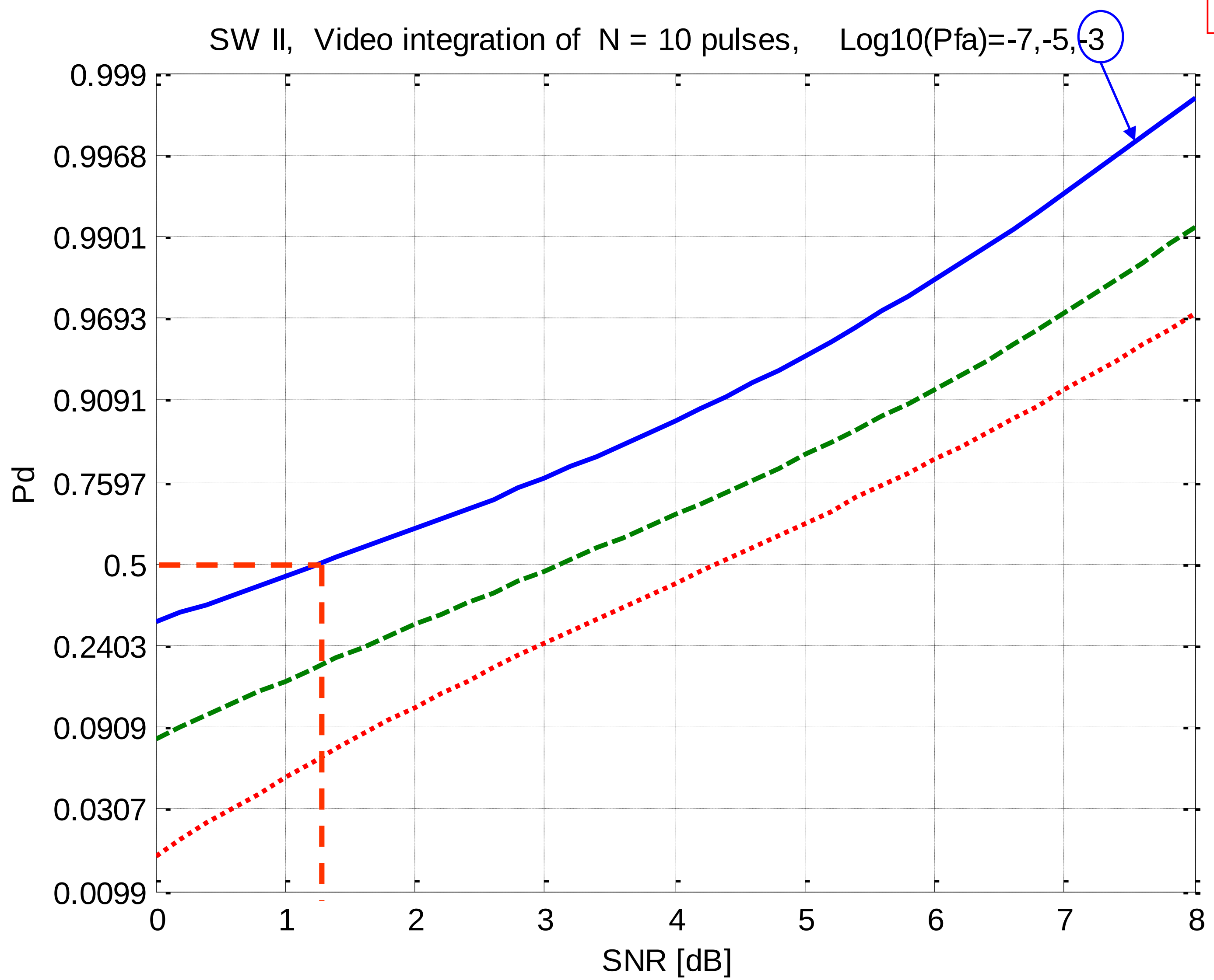
$SNR = y_T / y_{TD} - 1$

or use MATLAB instruction

$y_T = 0.5*\mathrm{chi2inv}(1-P_{FA}, 2*M)$

Victor Chernyak "On a closed form for the noncoherent gain factor", IEEE TAES Vol. 48, No. 2, Apr 2012, pp. 1770-1771

M pulses,
SW2 fluctuating



Noncoin2.m

```
% "noncoherent_integration_sw2.m"
% creates pd, pfa, snr curves
% written by Nadav Levanon on 17 April 2012

mm=10; % number of pulses integrated

pf=[1e-3 1e-5 1e-7];
pd=[0.0099 0.0307 0.0909 0.2403 0.5  0.7597 0.9091 0.9693 0.9901  0.9968 0.999];
pdscale=log(pd./(1-pd));

for q=1:3
    pfa=pf(q);
    yt=1/2*chi2inv(1-pfa,2*mm);
        for p=1:11
        pd1=pd(p);
        ytd=1/2*chi2inv(1-pd1,2*mm);
        snr(q,p)=yt/ytd-1;
    end
end
snrdb=10*log10(snr);

figure(1), clf, hold off
plot(snrdb(1,:),pdscale, 'b', 'linewidth',1.5)
hold on
plot(snrdb(2,:),pdscale, 'g', 'linewidth',1.5)
plot(snrdb(3,:),pdscale, 'r', 'linewidth',1.5)
axis([0 8 -4.61 6.91])
grid on
title(' SW 2,  noncoherent integration of 10 pulses, log10(Pfa)=-7,-5,-3 ')
xlabel(' SNR [dB]')
ylabel('Pd')

set(gca, 'YTickLabel',{'0.0099', '0.037', '0.0909' , '0.2403','0.5', '0.7597', '0.9091', '0.9693', …

'0.9901' , '0.9968' ,'0.999'}, 'YTick', pdscale);
```

M pulses, SW2 fluctuating, **with** normalization

$$z_m = \frac{r_m^2}{2\beta^2}$$

$$y = \sum_{m=1}^{M} z_m$$

$$p(y) = \frac{D^M}{(M-1)!} y^{M-1} e^{-Dy}$$

$$D = \frac{1}{1+\overline{\overline{SNR}}}$$

$$\overline{SNR} = \frac{A_0^2}{\beta^2}$$

Noise only $\Rightarrow D=1$

$$p\big|_{A=0}(y) = \frac{1}{(M-1)!} y^{M-1} e^{-y}$$

M pulses, SW2 fluctuating, **without** normalization

$$r_s = \sum_{m=1}^{M} r_m^2$$

Unnormalized sum at the output of square-law envelope detector.

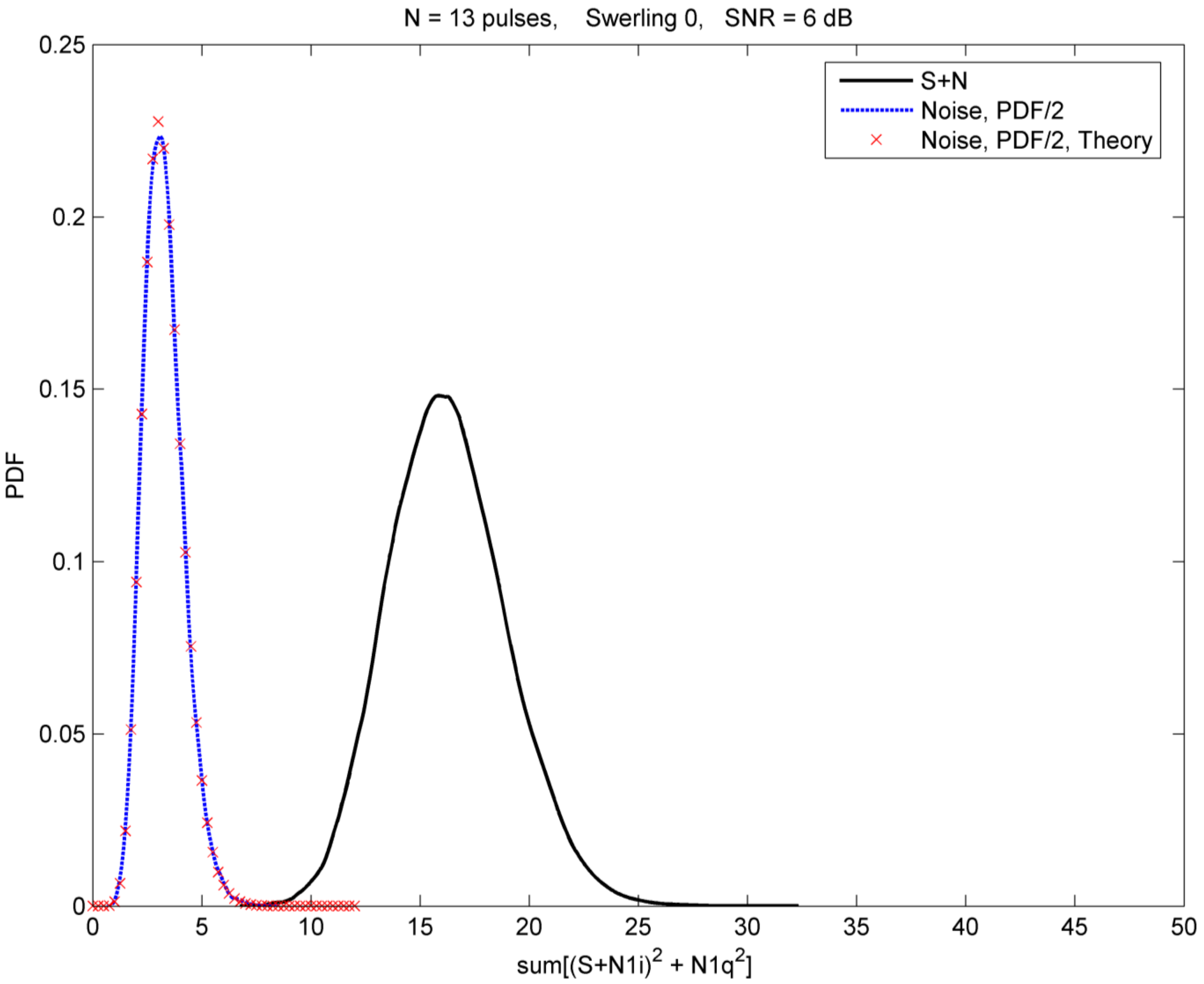
$$p\big|_{A=0}(r_s) = \frac{1}{2\beta^2 (M-1)!}\left(\frac{r_s}{2\beta^2}\right)^{M-1} \exp\left(\frac{-r_s}{2\beta^2}\right) \approx \frac{1}{2\beta^2\sqrt{2\pi(M-1)}}\left[\frac{e\, r_s}{(M-1)2\beta^2}\right]^{M-1} \exp\left(\frac{-r_s}{2\beta^2}\right)$$

When the input is noise-only the target model is of no concern.

```
if swerling==0
   signal=A*ones(1,N);
end

if swerling==2
   signal=raylrnd(Ao,1,N);
end

if swerling==1
   signal=raylrnd(Ao,1,1)*ones(1,N);
end
```



Swerling 0

Non-coherent integration

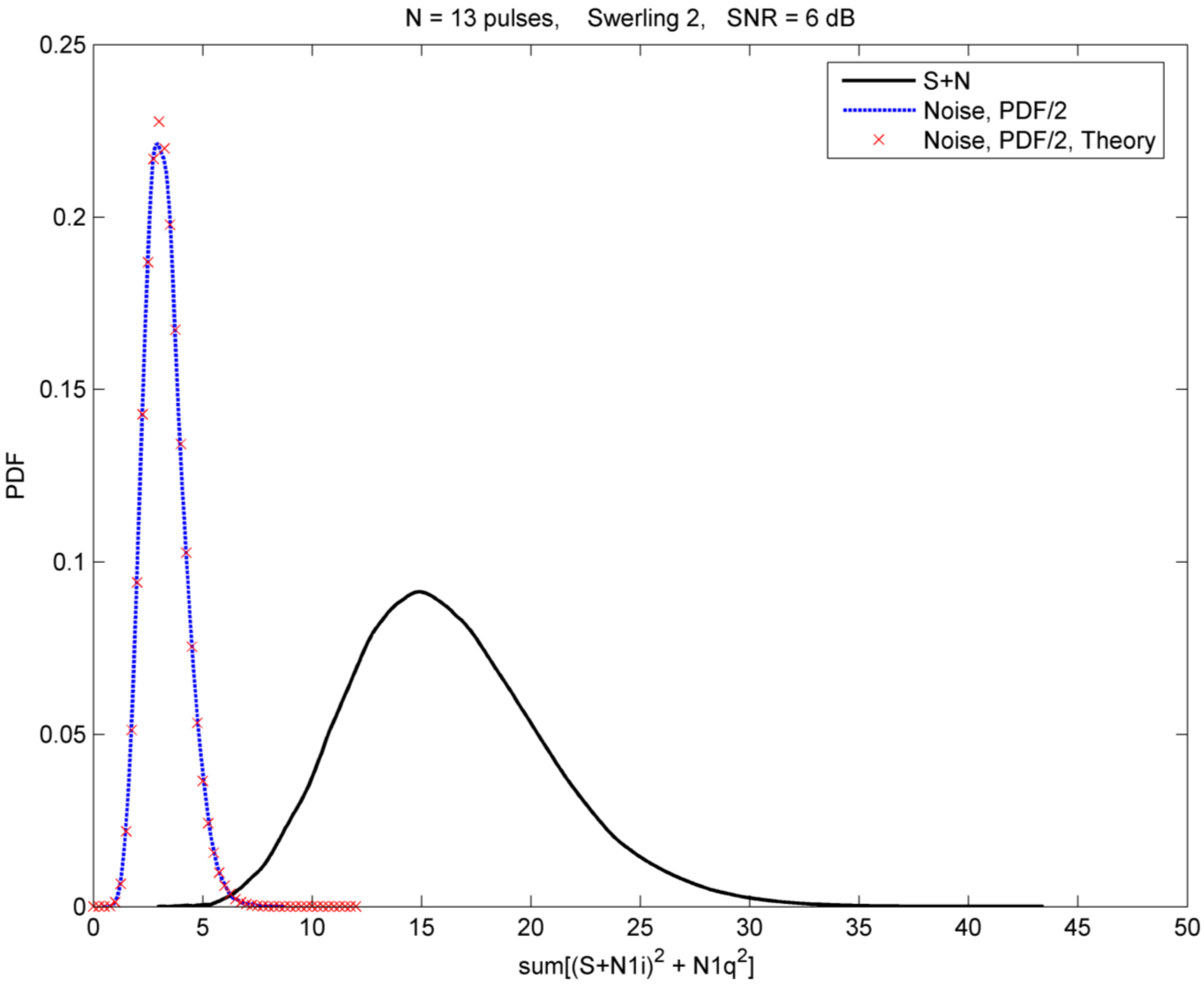
A=1
β=0.3544
N=13 pulses
runs=40000

$$SNR = \frac{A^2}{2\beta^2}$$

$$r_s = \sum_{m=1}^{M} r_m^2$$

$$p(r_s) = \frac{1}{2\beta^2 (N-1)!}\left(\frac{r_s}{2\beta^2}\right)^{N-1} \exp\left(\frac{-r_s}{2\beta^2}\right)$$

```
[pdf,r]=ksdensity(sum_sn,'kernel','epanechnikov')
```



Swerling 2

Non-coherent integration

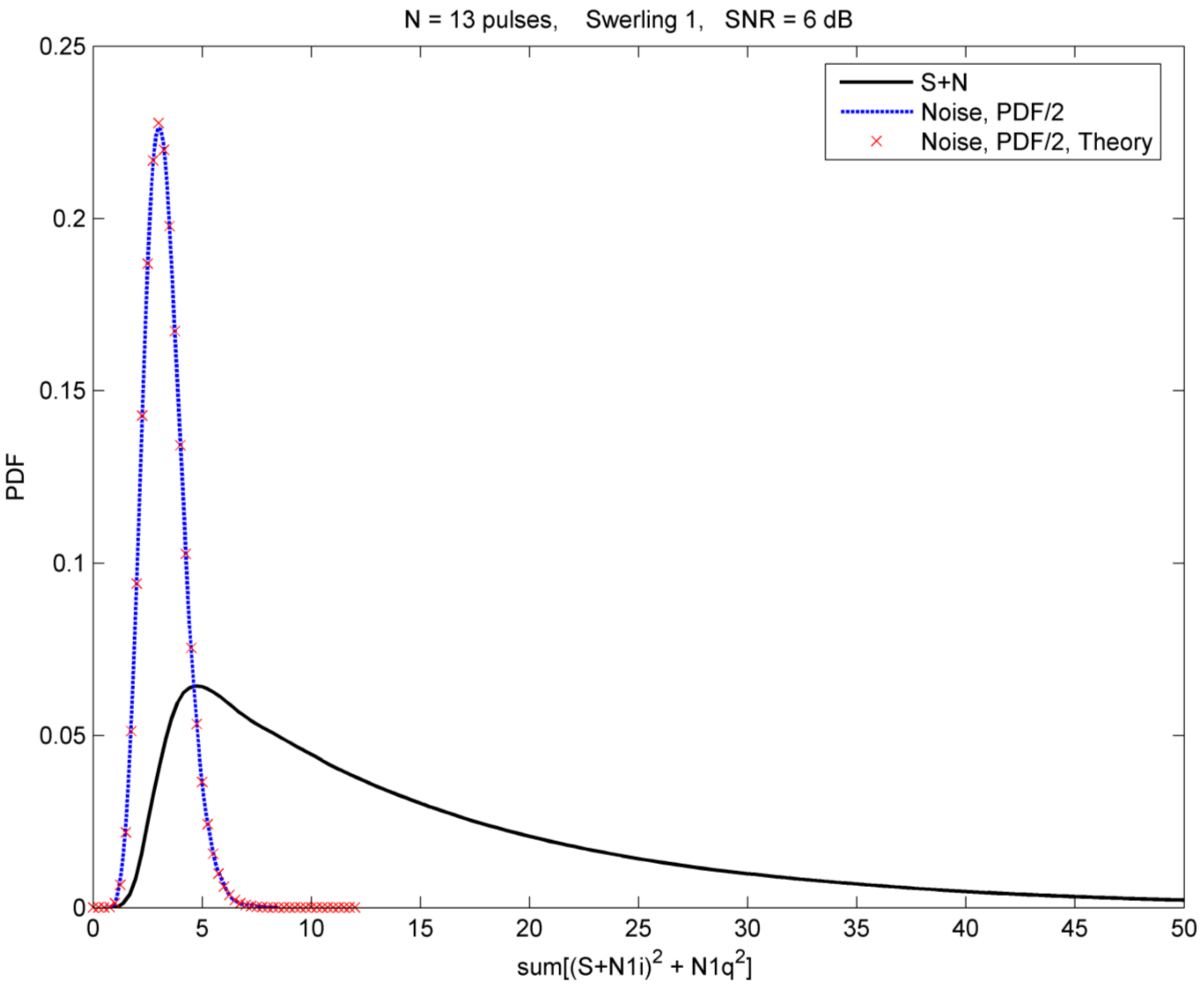
A_0=0.707
β=0.3544
N=13 pulses
runs=40000

$$p(A) = \frac{A}{A_0} \exp\left(\frac{-A^2}{2A_0^2} \right)$$

$$\overline{SNR} = \frac{A_0^2}{\beta^2}$$

$$r_s = \sum_{m=1}^{M} r_m^2$$

$$p(r) = \frac{1}{2\beta^2 (N-1)!} \left(\frac{r}{2\beta^2} \right)^{N-1} \exp\left(\frac{-r}{2\beta^2} \right)$$



Swerling 1

Non-coherent integration

A_0=0.707
β=0.3544
N=13 pulses
runs=40000

$$p(A)=\frac{A}{A_0}\exp\left(\frac{-A^2}{2A_0^2}\right)$$

$$\overline{SNR}=\frac{A_0^2}{\beta^2}$$

$$r_s=\sum_{m=1}^{M} r_m^2$$

$$p(r_s)=\frac{1}{2\beta^2(N-1)!}\left(\frac{r_s}{2\beta^2}\right)^{N-1}\exp\left(\frac{-r_s}{2\beta^2}\right)$$

Swerling 1

$$z_m = \frac{r_m^2}{2\beta^2} \qquad y = \sum_{m=1}^{M} z_m$$

M pulses,
SW1 fluctuating

$$p(y) = \left(1 + \frac{\beta^2}{MA_0^2}\right)^{M-2} \frac{\beta^2}{MA_0^2} \exp\left(\frac{-y}{1 + \frac{MA_0^2}{\beta^2}}\right) f(u,v)$$

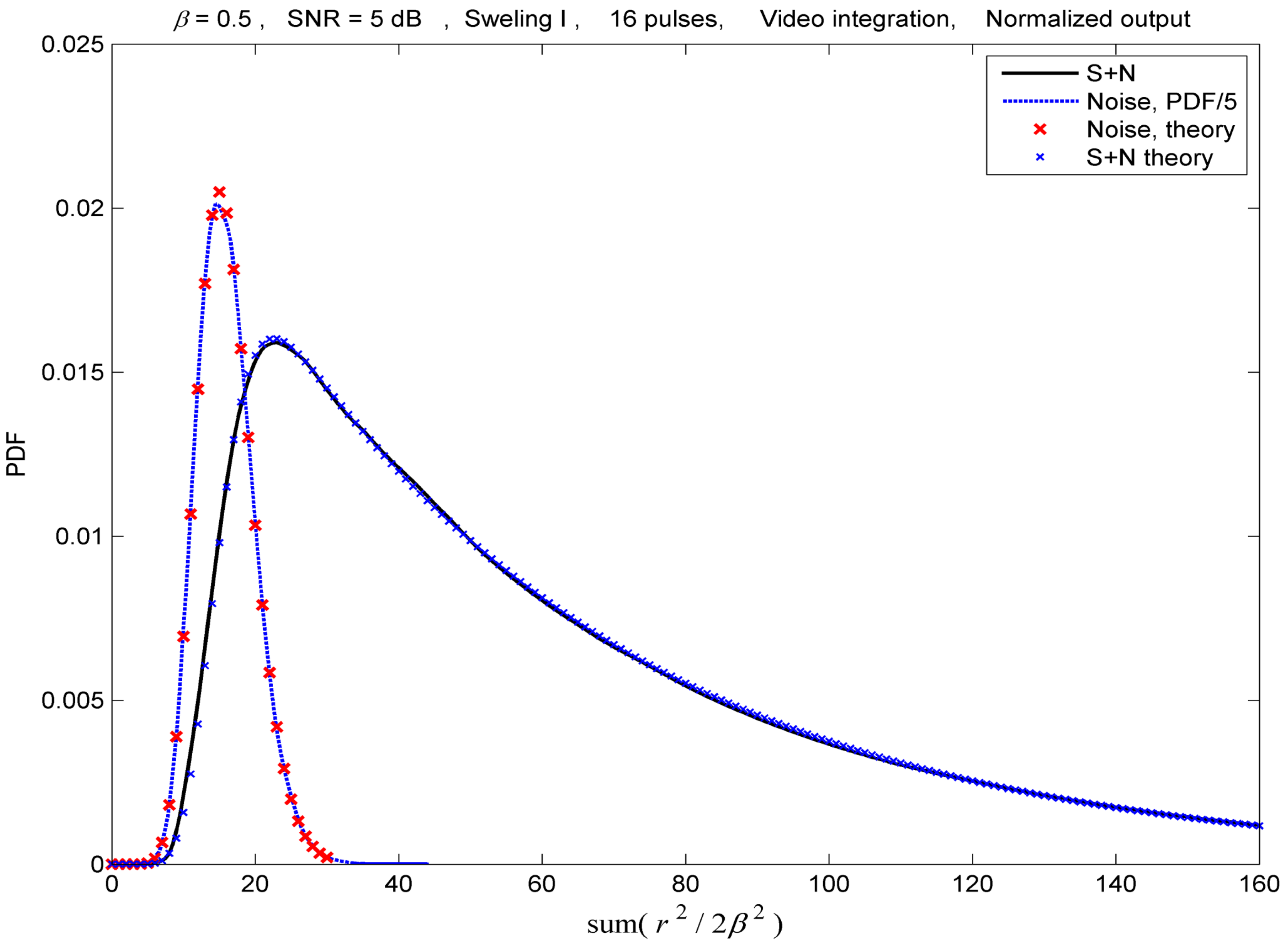
$$u = \frac{y}{1 + \frac{\beta^2}{MA_0^2}} \quad , \quad v = M - 2$$

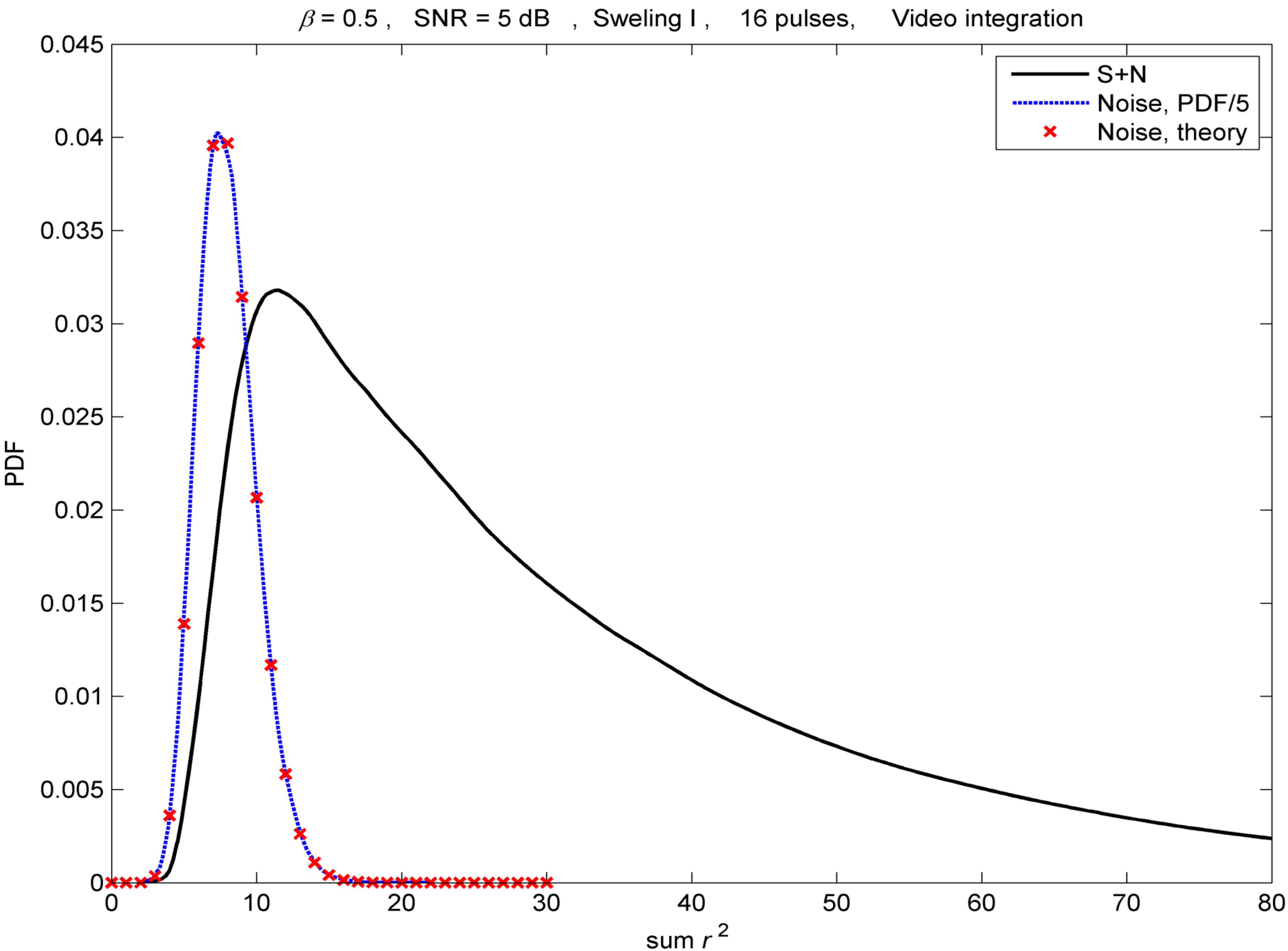
$$f(u,v) = 1 - \sum_{k=0}^{v} \frac{u^k}{k!} e^{-u} \quad , \quad \text{Incomplete Gamma function}$$

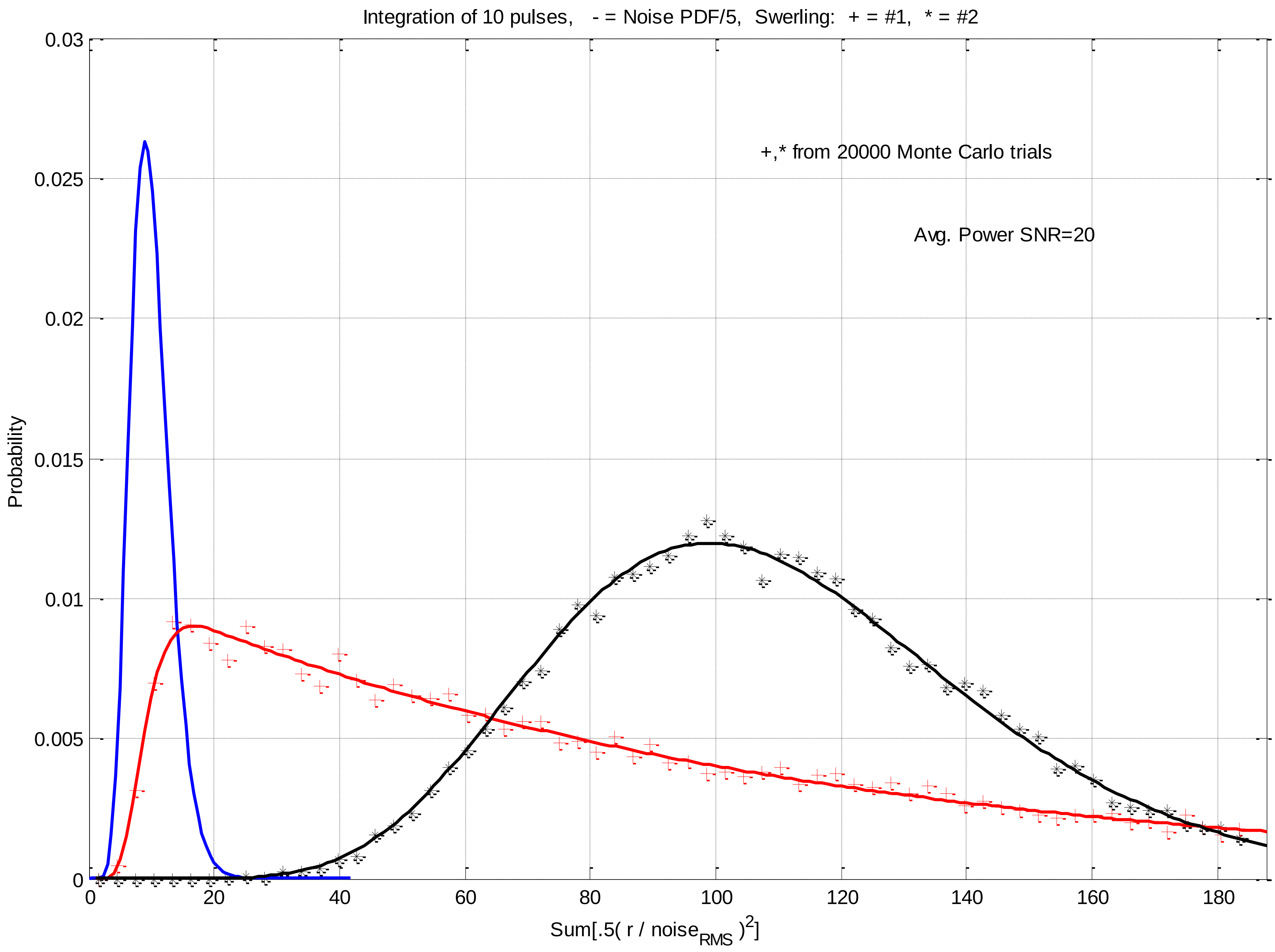
In SW 1 Calculating the relation between SNR, M, P_D and P_{FA} is cumbersome

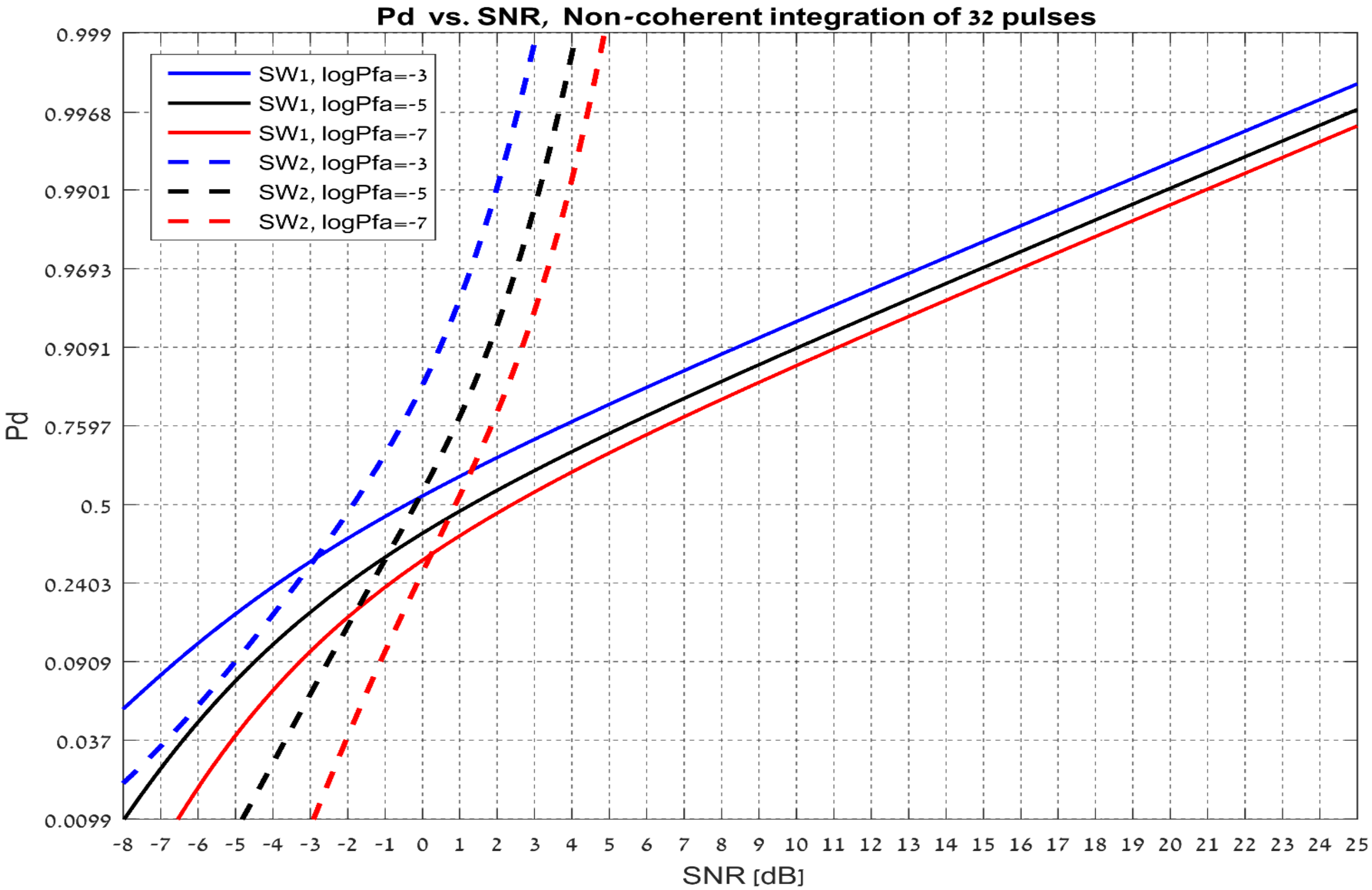
Swerling 2

$$p(y) = \frac{D^M}{(M-1)!} y^{M-1} e^{-Dy}$$


β = 0.5 , SNR = 5 dB , Sweling I , 16 pulses, Video integration, Normalized output
S+N
Noise, PDF/5
Noise, theory
S+N theory
PDF
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
140
160
sum($r^2 / 2\beta^2$)

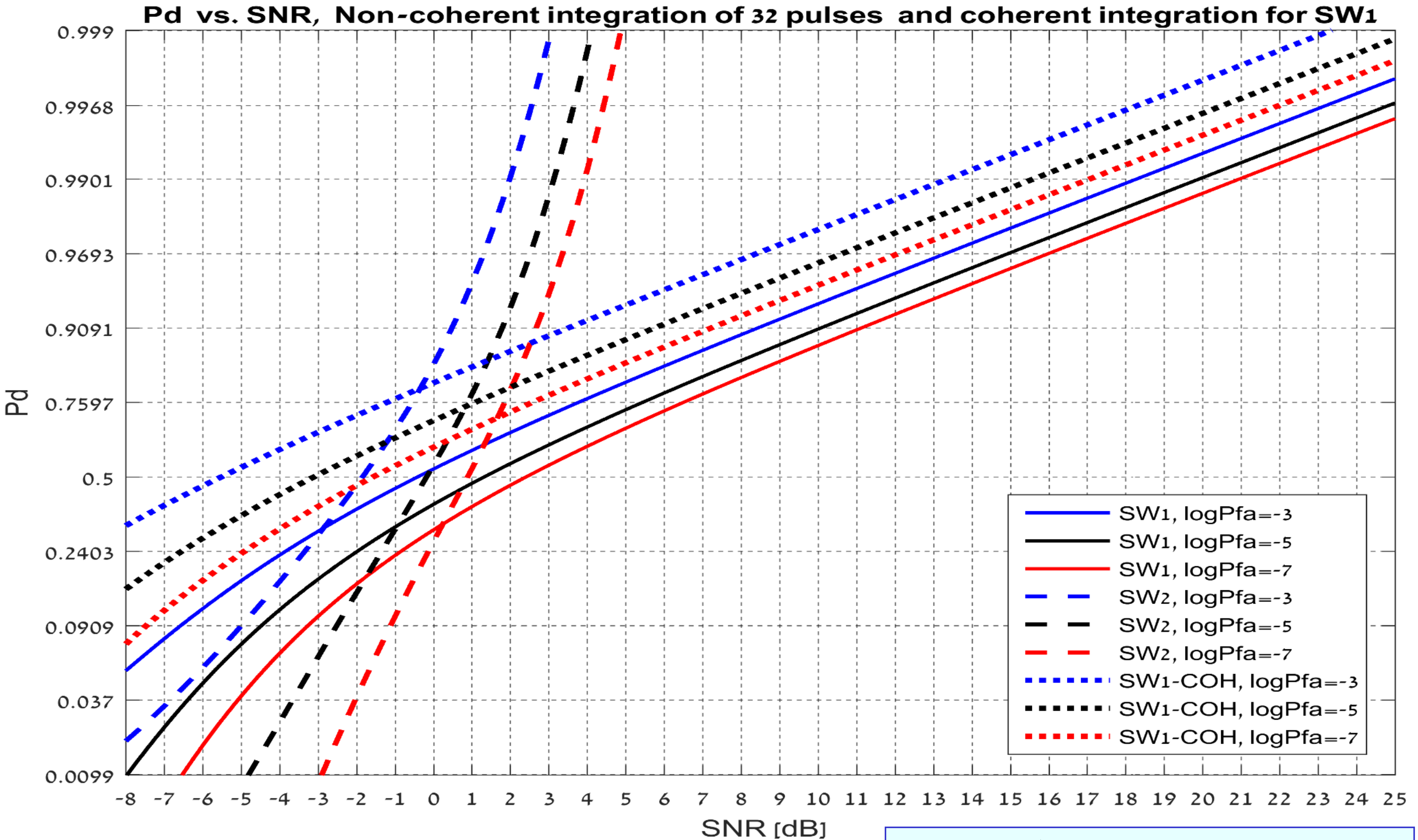

β = 0.5 , SNR = 5 dB , Sweling I , 16 pulses, Video integration
S+N
Noise, PDF/5
Noise, theory
PDF
sum r 2
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50
60
70
80


Integration of 10 pulses, - = Noise PDF/5, Swerling: + = #1, * = #2
+,* from 20000 Monte Carlo trials
Avg. Power SNR=20
Probability
0.03
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
140
160
180
Sum[.5(r / noise$_{RMS}$)^2]



Victor Chernyak's equations

```
pd_sw2=1-chi2cdf(chi2inv(1-pfa,2*M)./(1+(10.^(0.1*snr))),2*M);     % SNR in dB, M = number of pulses
pd_sw1=((1+1./(M*(10.^(0.1*snr)))).^(M-1)).*exp(-((0.5*chi2inv(1-pfa,2*M))/(1+M*(10.^(0.1*snr)))));
```

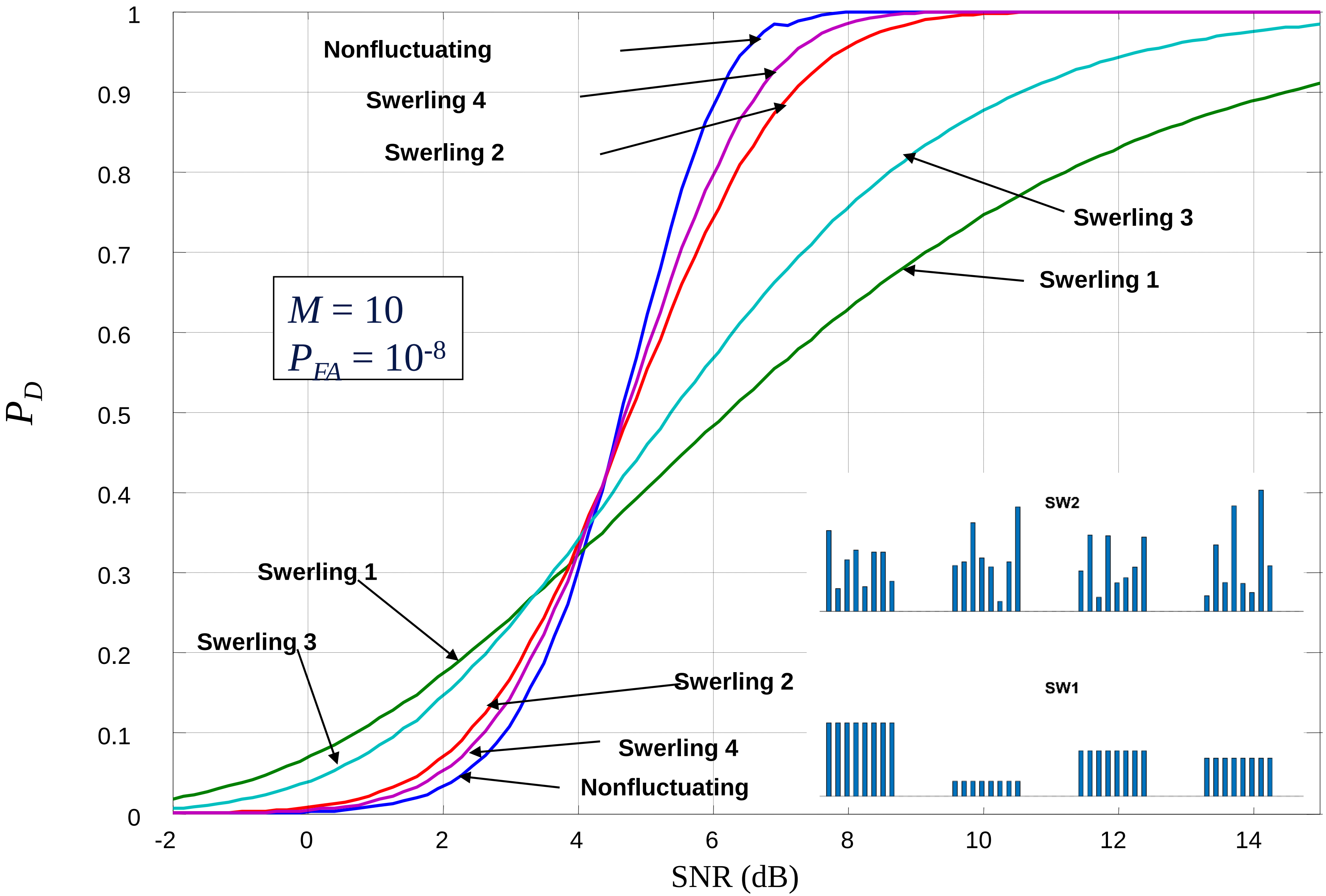


$$P_D = P_{FA}^{\frac{1}{1+M\,\overline{SNR}}}, \quad \text{coherent integration of SW1}$$

Victor Chernyak's equations for non-coherent integration

```
pd_sw2=1-chi2cdf(chi2inv(1-pfa,2*M)./(1+(10.^(0.1*snr))),2*M);    % SNR in dB, M = number of pulses
pd_sw1=((1+1./(M*(10.^(0.1*snr)))).^(M-1)).*exp(-((0.5*chi2inv(1-pfa,2*M))./(1+M*(10.^(0.1*snr)))));
```

Non-coherent integration

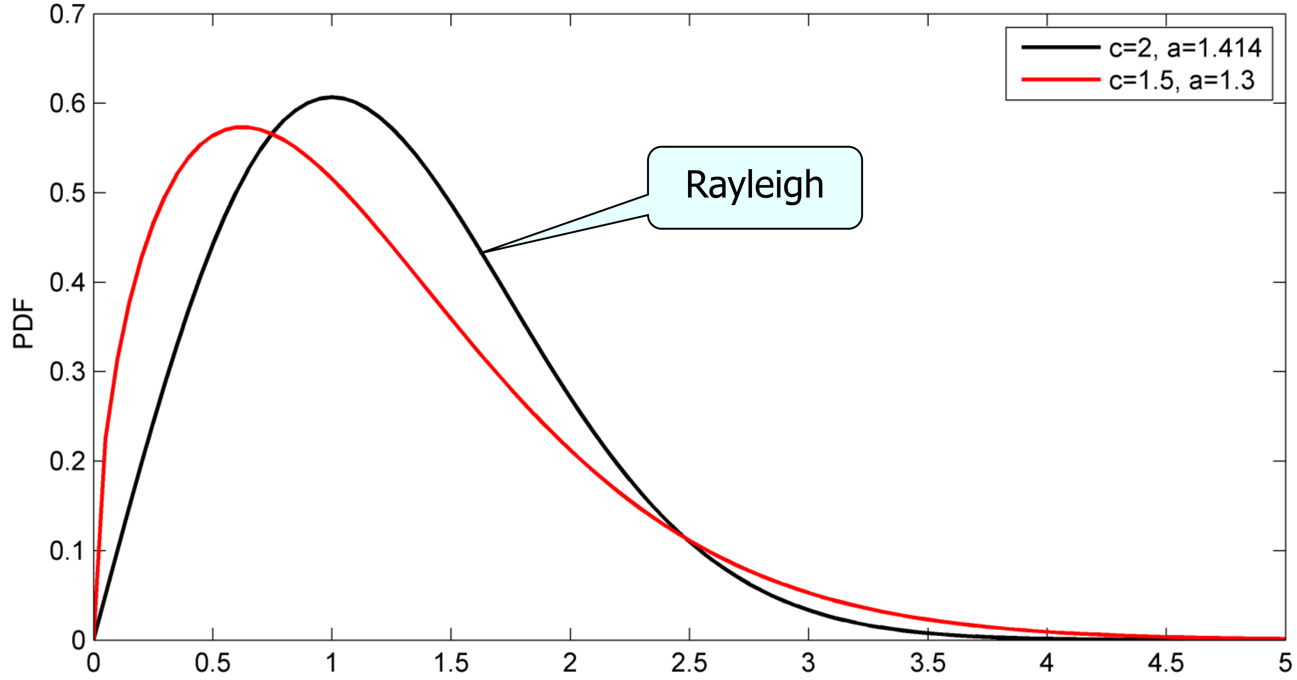


Non-coherent integration of M noise samples (Rayleigh noise envelope)

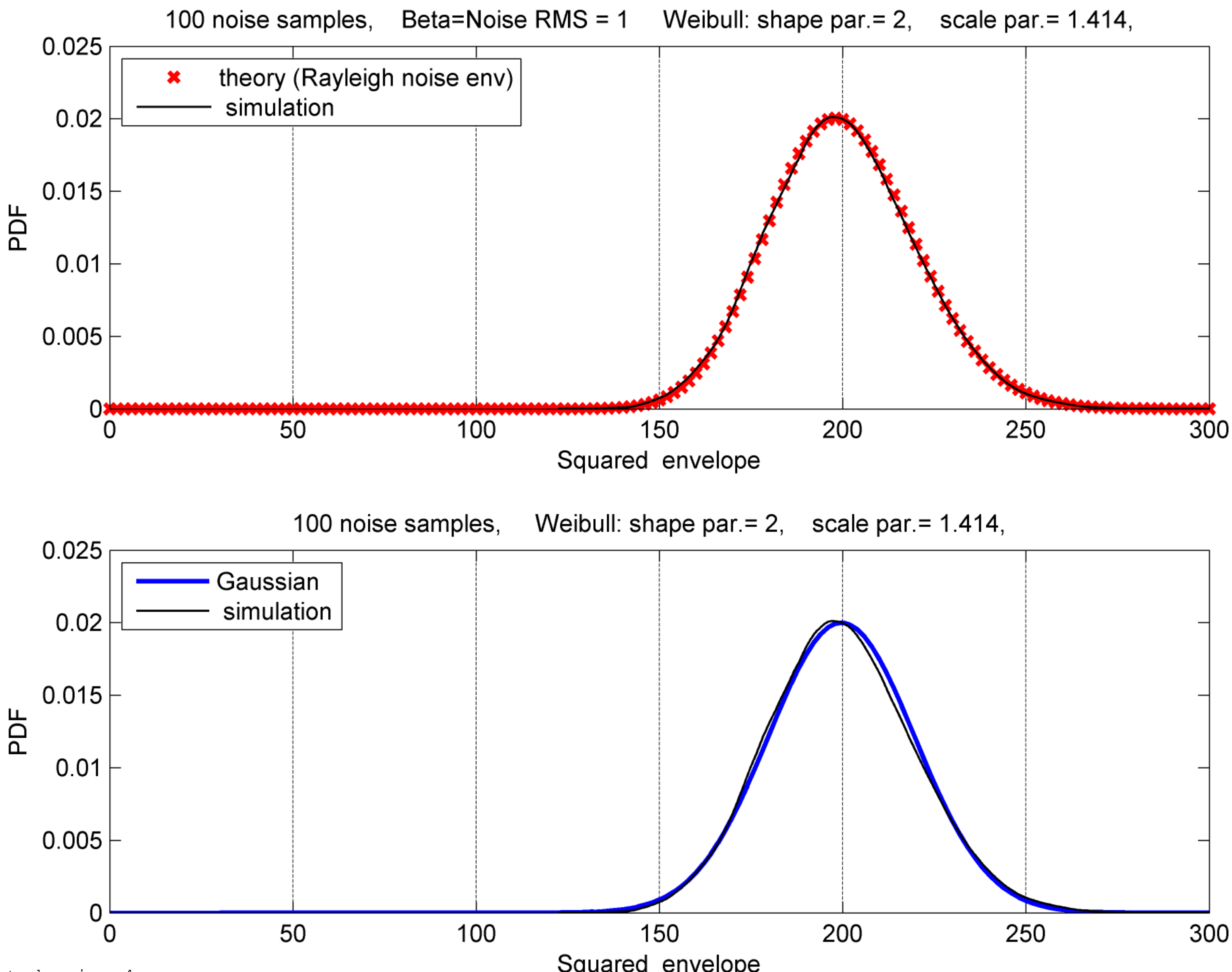
$$r_s = \sum_{m=1}^{M} r_m^2$$

Unnormalized sum at the output of square-law envelope detector.

$$p\big|_{A=0}(r_s) = \frac{1}{2\beta^2 (M-1)!}\left(\frac{r_s}{2\beta^2}\right)^{M-1} \exp\left(\frac{-r_s}{2\beta^2}\right) \approx \frac{1}{2\beta^2\sqrt{2\pi(M-1)}}\left[\frac{r_s e}{(M-1)2\beta^2}\right]^{M-1} \exp\left(\frac{-r_s}{2\beta^2}\right)$$

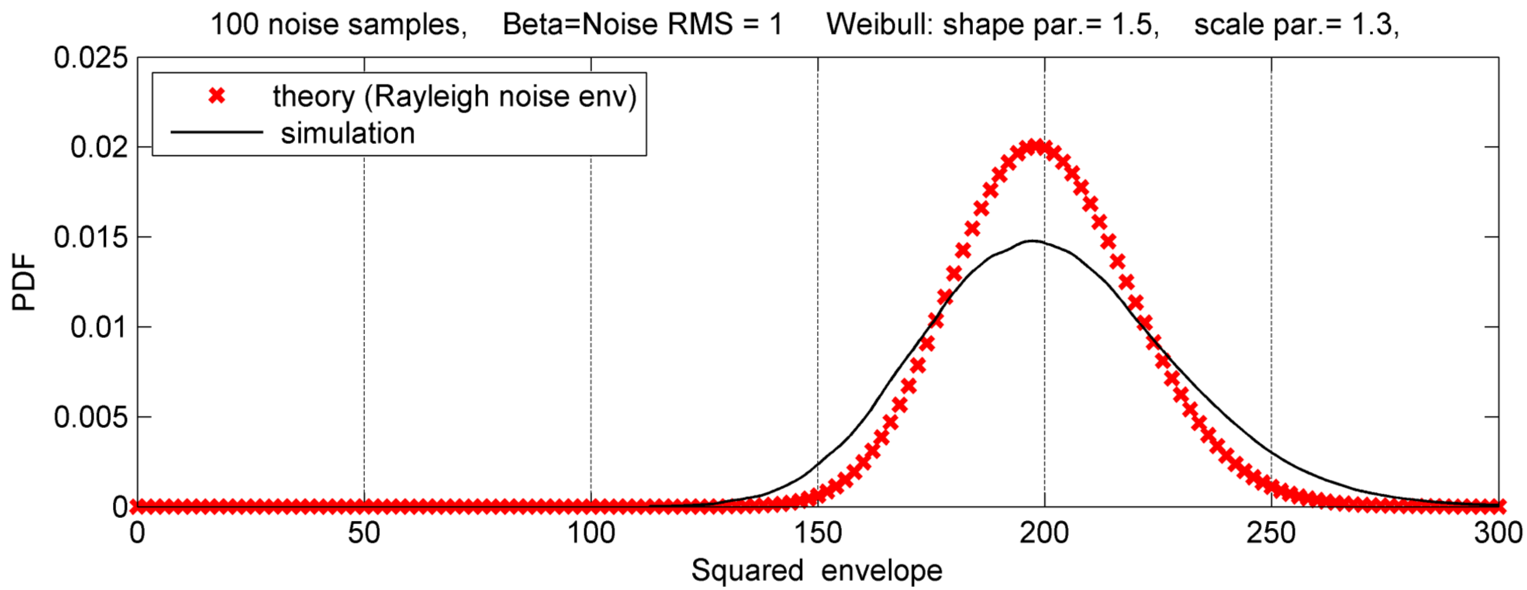


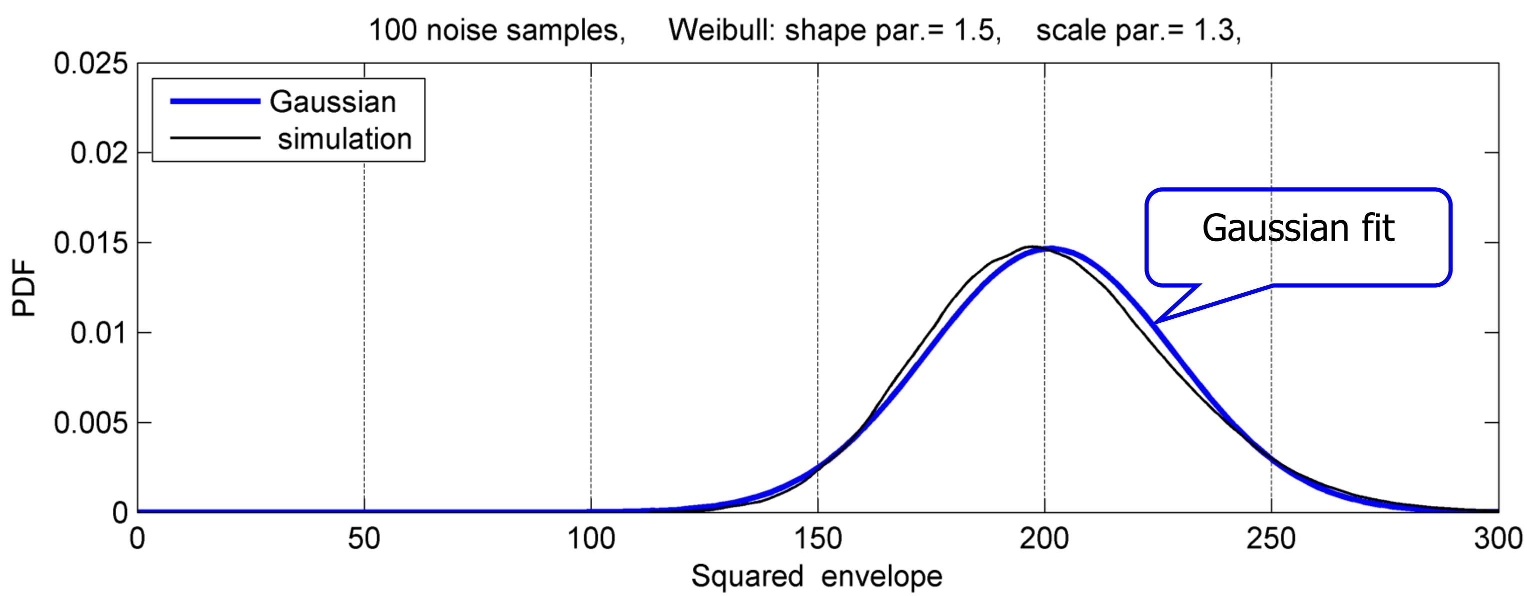
Rayleigh noise envelope



integrated_noise_4.m

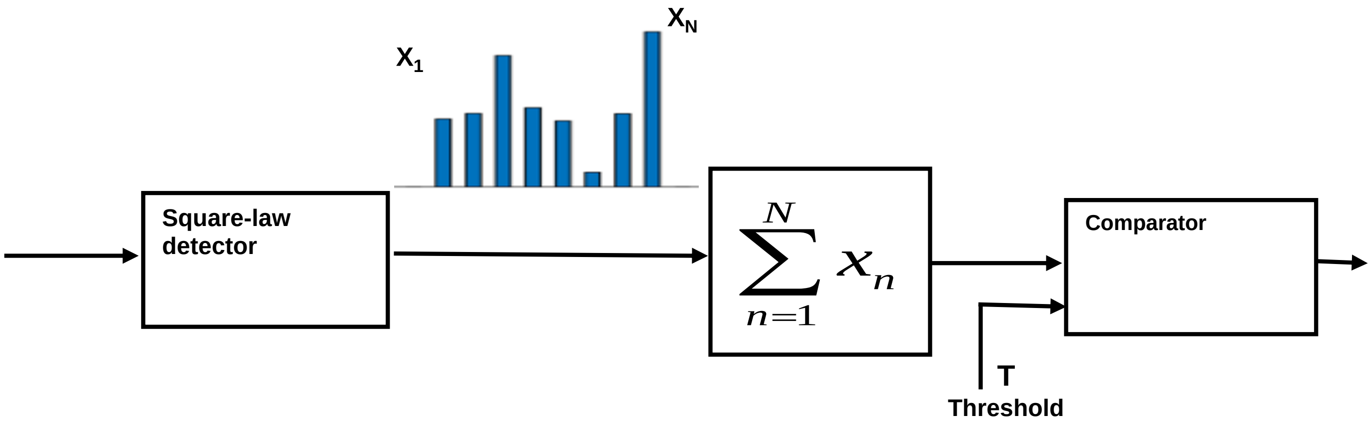
Non-Rayleigh noise envelope





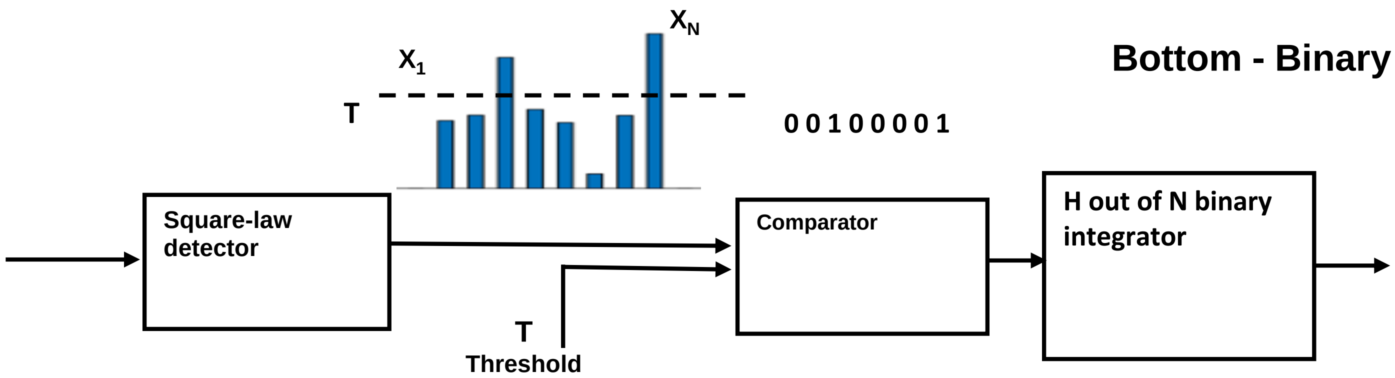
Binary Integration

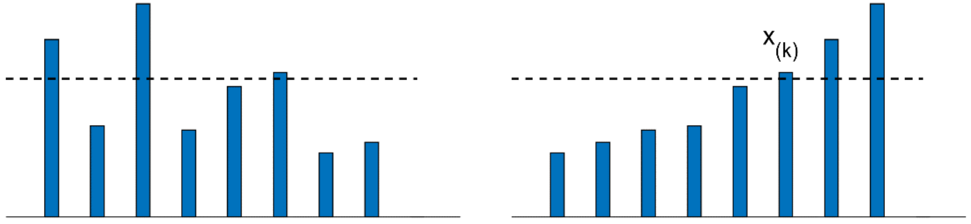
- Data is "binary" after threshold detection
  - 1 = "target present"
  - 0 = "target absent"
- We could combine data from several pulses, scans, or resolution cells *after* threshold detection into a higher-order decision logic
  - *e.g.*, "it's not really a detection unless we see it on 2 out of 3 tries"
- How does this affect P_D and P_{FA}?



Top -Analog (video) integration

Bottom - Binary integration





Time sequence (left) **Sorted sequence (right)**

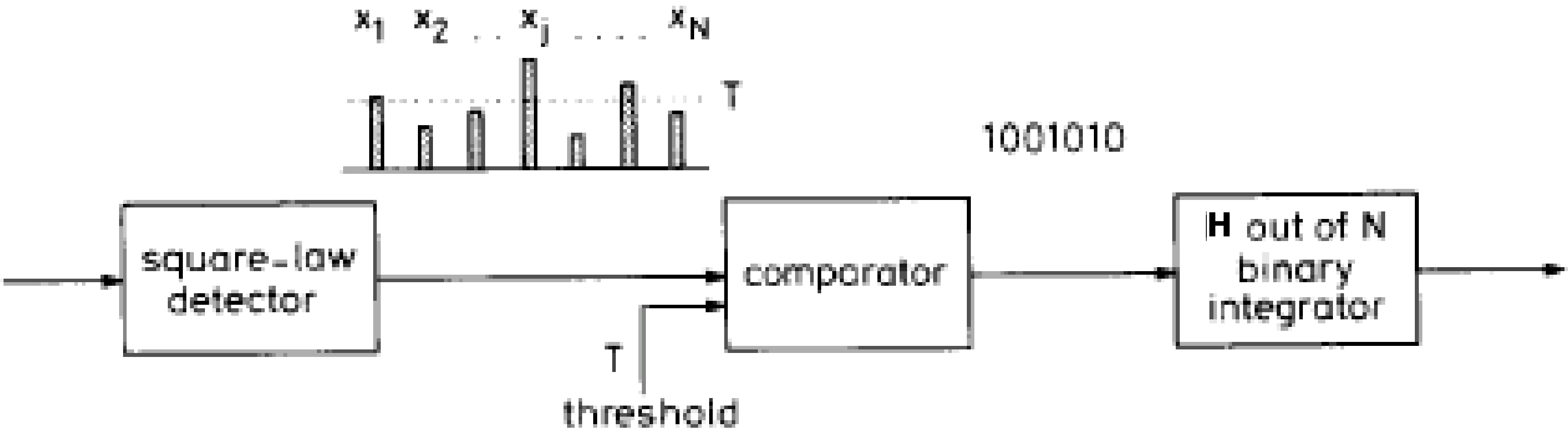


Fig. 2A *Binary integration*

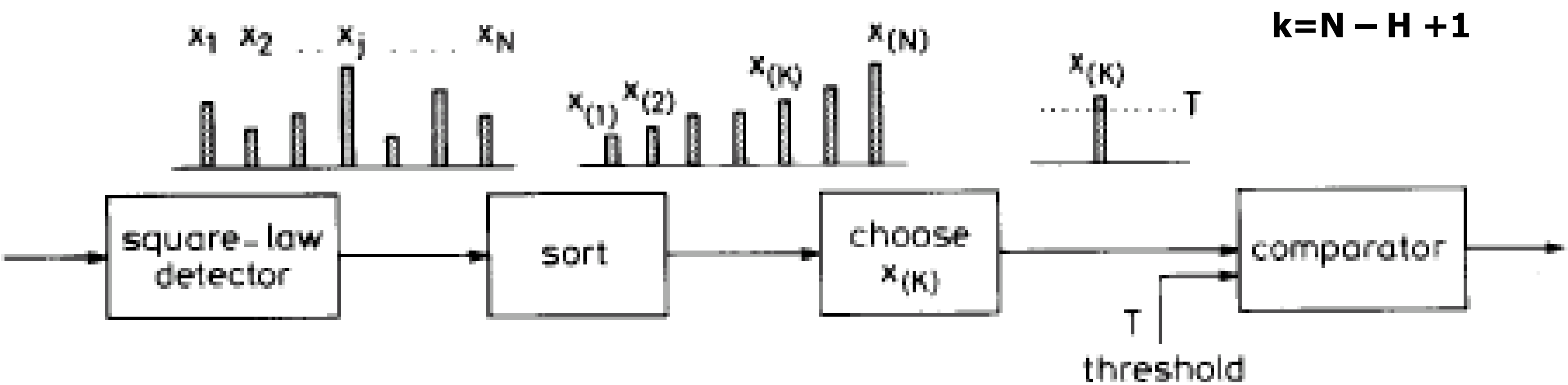


Fig. 2B *Order statistics integration*

If we choose k=N-H+1, then both integration schemes are identical

N pulses,
SW2 fluctuating

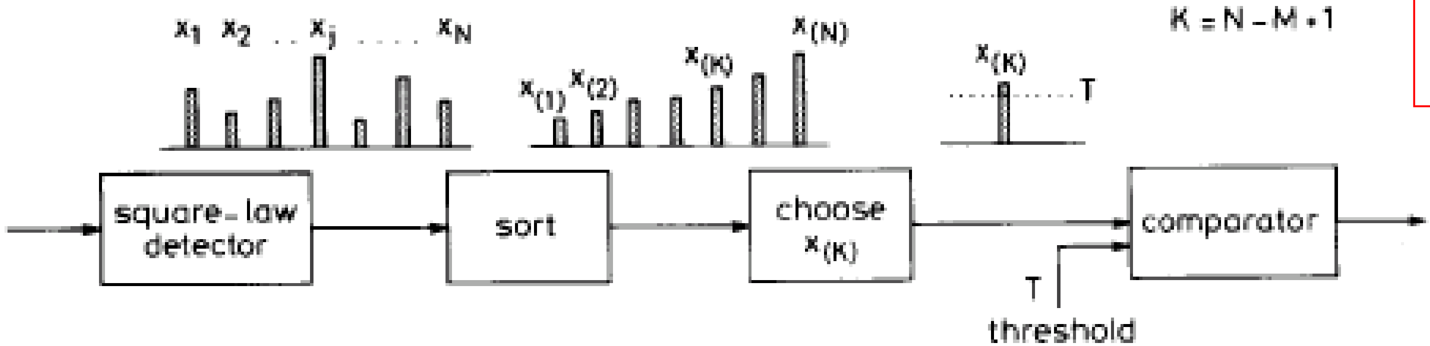


Fig. 2B *Order statistics integration*

The probability distribution function $P_K(z)$ of the K'th ordered sample, out of a total of N samples of i.i.d. r.v. z whose probability distribution function is $P(z)$, is:

$$P_K(z) = \sum_{r=K}^{N} \binom{N}{r} [P(z)]^r [1 - P(z)]^{N-r}$$

In SW 2 $\quad D = \frac{1}{1 + \overline{SNR}}$

$$p(z) = D \exp(-Dz)$$

$$P(z) = 1 - \exp(-Dz)$$

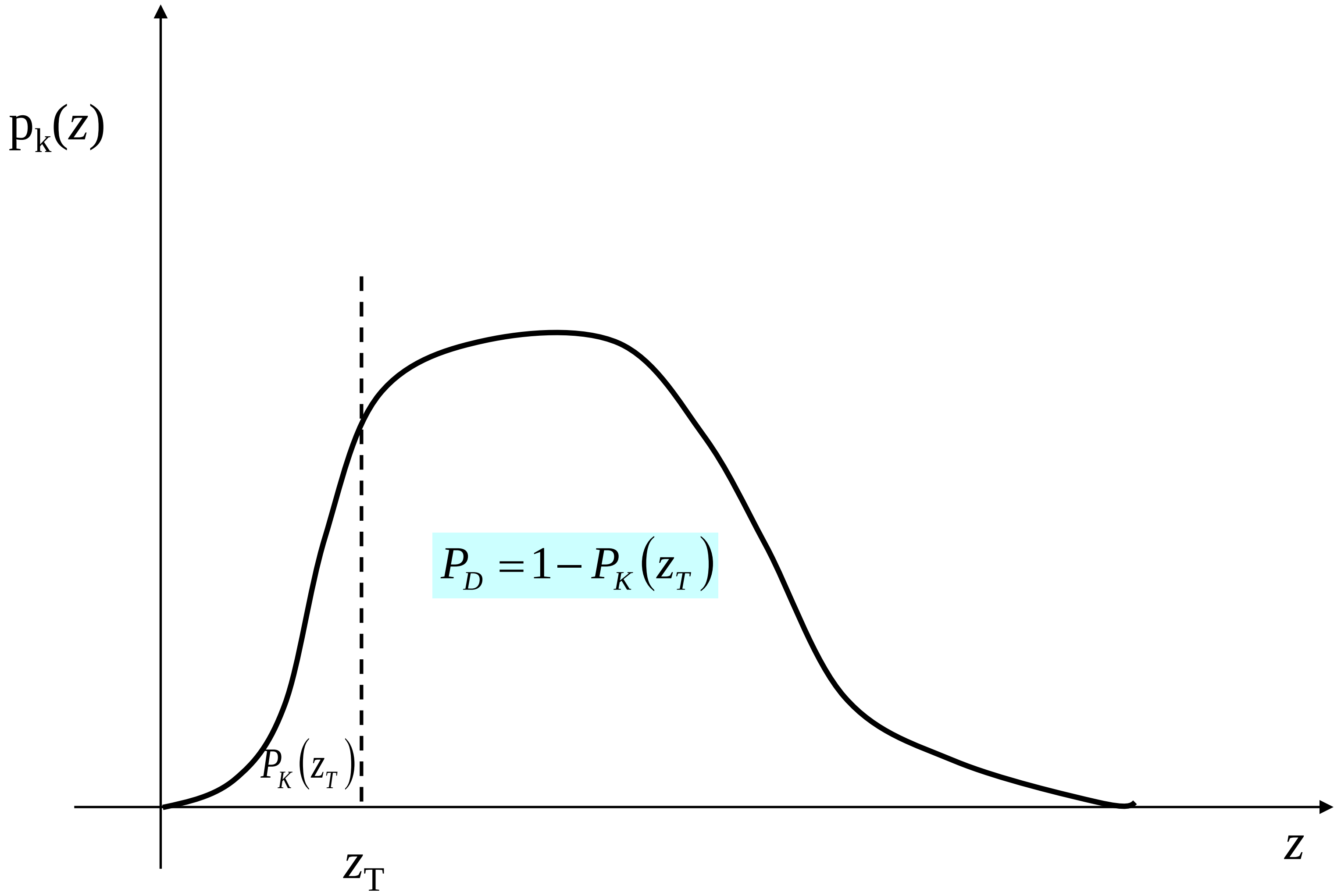
$$P_D = 1 - P_K(z_T)$$

$$P_{FA} = \exp(-z_T N) \sum_{r=0}^{K-1} \binom{N}{r} [\exp(z_T) - 1]^r$$

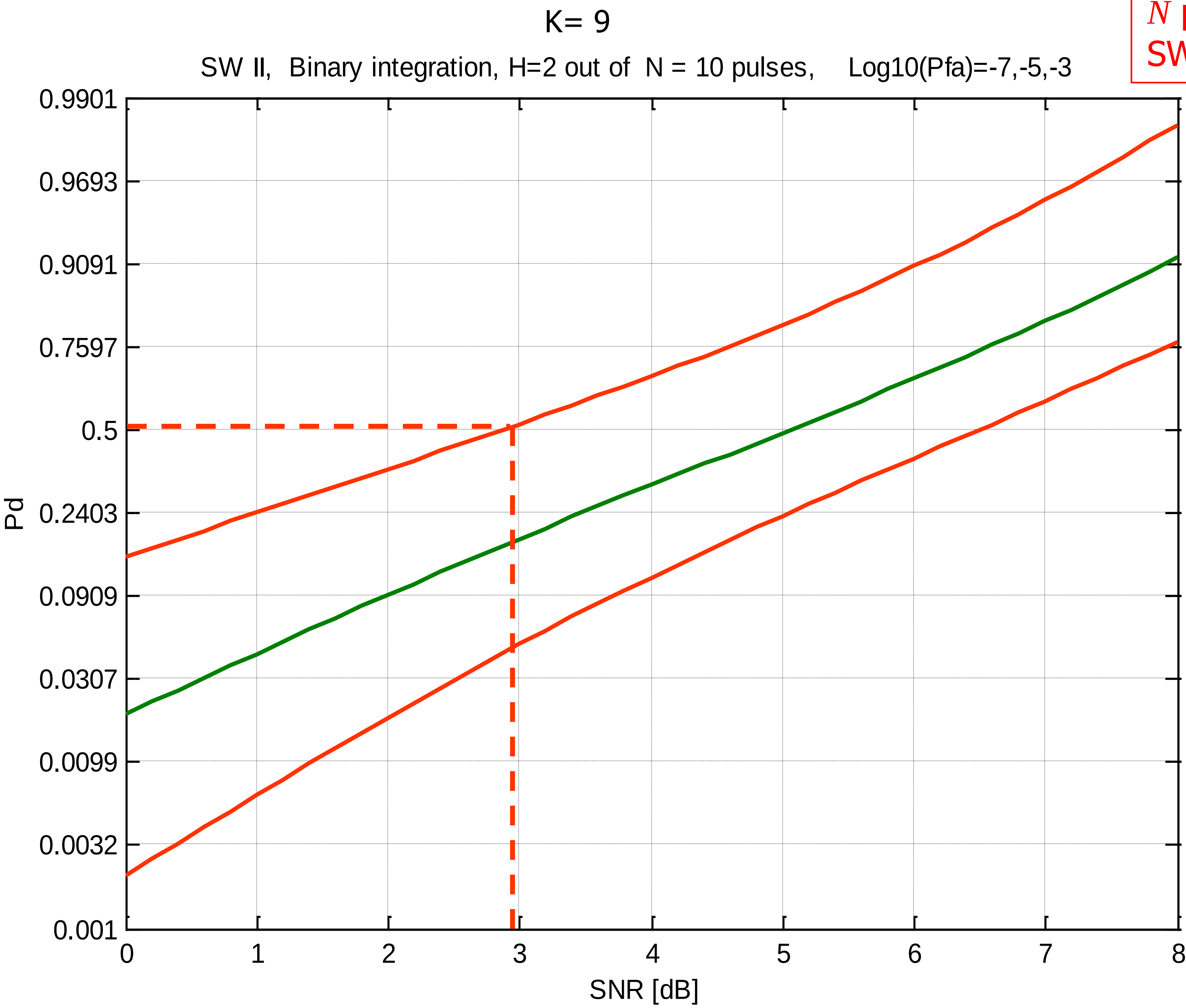
$D = 1$

$$P_D = \exp(-z_T DN) \sum_{r=0}^{K-1} \binom{N}{r} [\exp(z_T D) - 1]^r$$

Calulate Z_T iteratively


$p_k(z)$
$P_D = 1 - P_K(z_T)$
$P_K(z_T)$
z_T
z

N pulses,
SW2 fluctuating



Binary integration:

$\approx$ 1.5dB loss Vs. video integration

Immunity against strong interference (if $K<N$)

Simpler implementation

Optimal choice $K \approx 0.7\,N$, $(K=N-H+1 \Rightarrow H \approx 0.3\,N+1)$

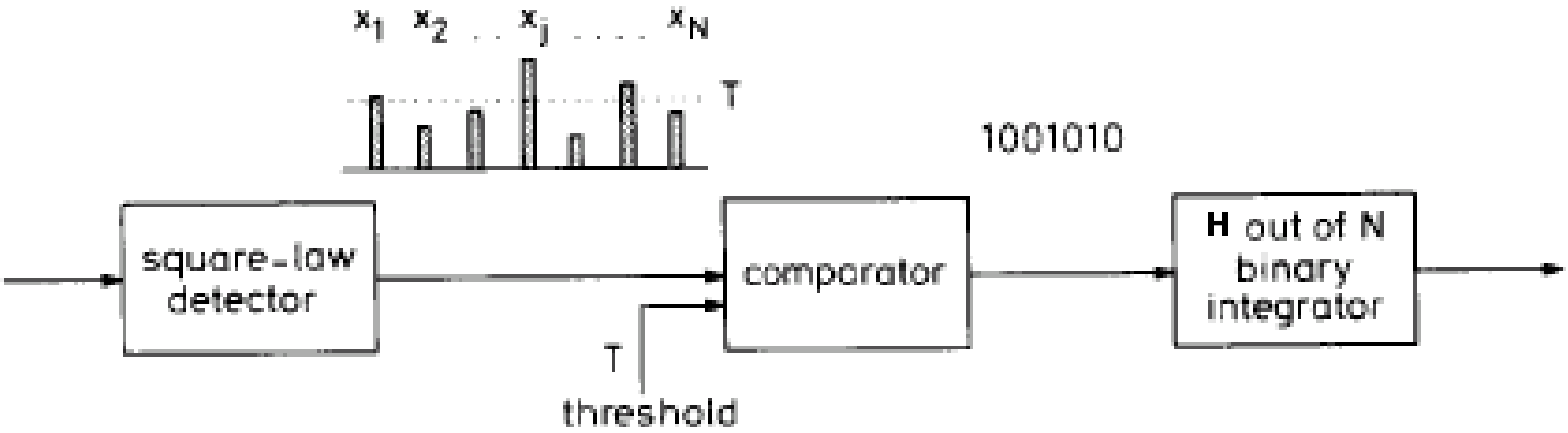


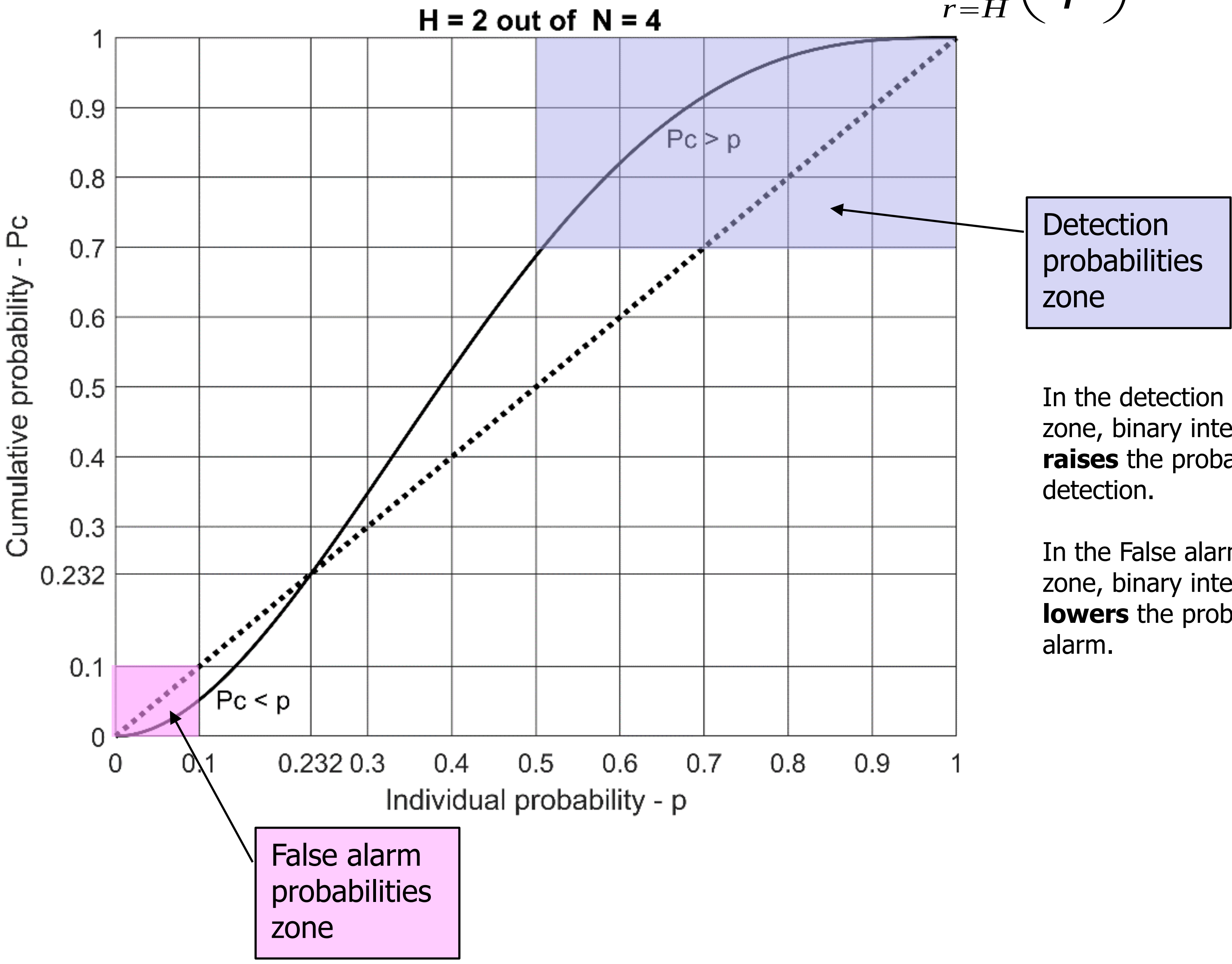
Fig. 2A *Binary integration*

The cumulative probability P_C of H "successes" in N trials, when the probability of a "success" in a single trial is p, is:

$$P_C = \sum_{r=H}^{N} \binom{N}{r} p^r (1-p)^{N-r}$$

"Success" can be both **false alarm** or **detection**

$$P_C = \sum_{r=H}^{N} \binom{N}{r} p^r (1-p)^{N-r}$$



In the detection probabilities zone, binary integration **raises** the probability of detection.

In the False alarm probabilities zone, binary integration **lowers** the probability of false alarm.

NON-COHERENT INTEGRATION

of

NON-FLUCTUATUAING TARGETS (Swerling 0)

Numerical calculations

Albersheim's Equation

- Empirical, easily computable approximation to the Swerling 0/5 case
  - nonfluctuating target in white noise
  - linear detector
  - noncoherent integration
- Error in the estimate of SNR_1 is claimed to be less than 0.2 dB (0.4 dB if used for square law) for
  - $10^{-7} \le P_{FA} \le 10^{-3}$
  - $0.1 \le P_D \le 0.9$
  - $1 \le N \le 8096$

$$A = \ln\left(\frac{0.62}{P_{FA}}\right), \quad B = \ln\left(\frac{P_D}{1-P_D}\right)$$

$$SNR_1 = -5\log_{10} N + \left(6.2 + \left(\frac{4.54}{\sqrt{N+0.44}}\right)\right) \bullet \log_{10}\left(A + 0.12AB + 1.7B\right) \quad \text{dB}$$

Number of pulses integrated

SNR of the individual pulse

Albersheim, W. J. "A closed form approximation to Robertson's detection characteristics", *Proc. IEEE*, July 1981, p.839

The gain achieved by non-coherent integration of *N* pulses from a non-fluctuating target (SW 0)

$$G(N)_{\mathrm{dB}} = SNR_{1[\mathrm{dB}]}(N=1) - SNR_{1[\mathrm{dB}]}(N=N)$$

$$G(N)_{\mathrm{dB}} = 5\log_{10} N + 4.54\left(\frac{1}{\sqrt{1.44}} - \frac{1}{\sqrt{N+0.44}}\right)\log_{10}\left(A + 0.12AB + 1.7B\right)$$

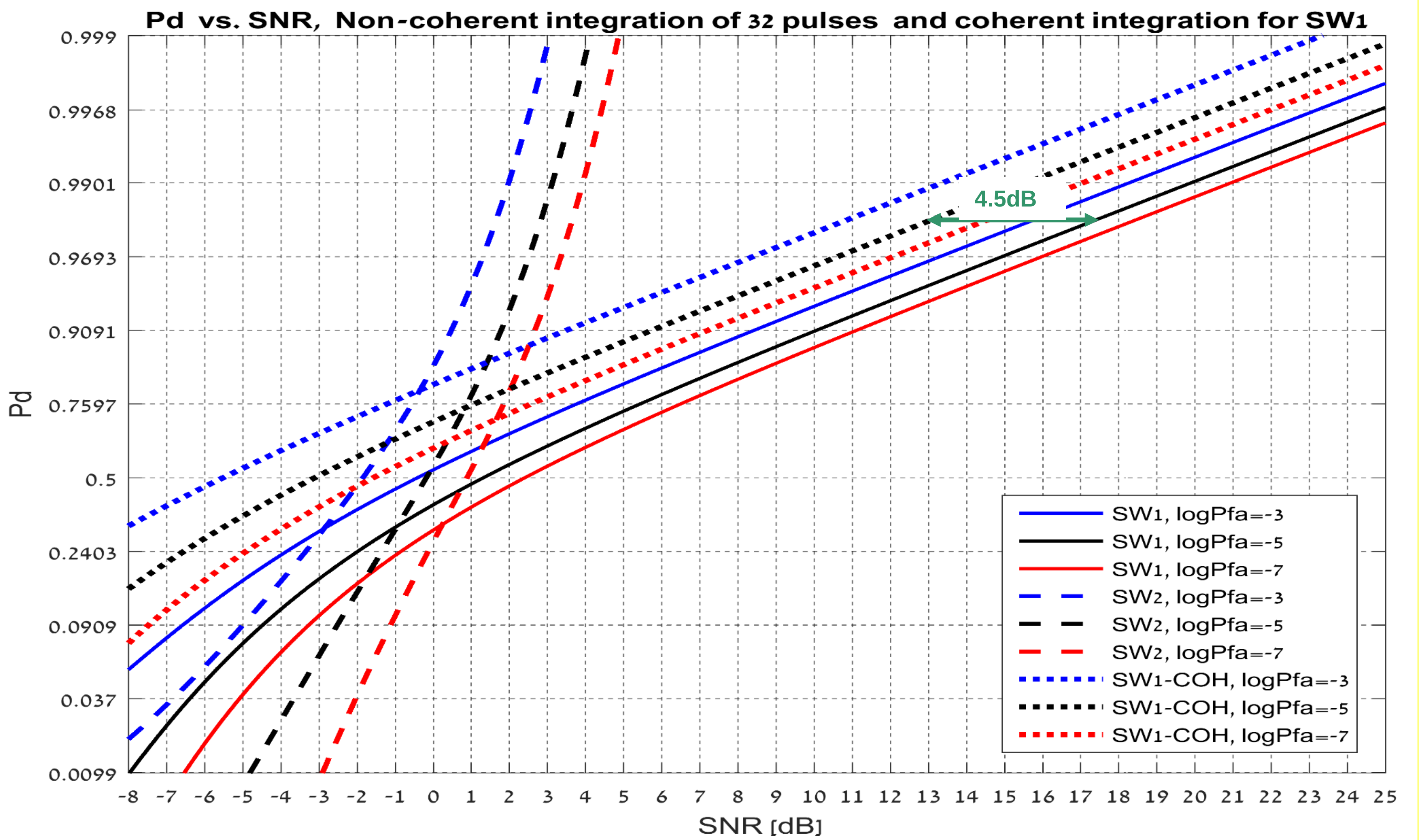
$$A = \ln\left(\frac{0.62}{P_{FA}}\right), \quad B = \ln\left(\frac{P_D}{1-P_D}\right)$$

$$G(N) = N^{\frac{\gamma}{10}}$$

$$G(N)_{\mathrm{dB}} = 10\log_{10}\left(N^{\frac{\gamma}{10}}\right) = \gamma \log_{10} N$$

$$\gamma = 5 + \frac{4.54}{\log_{10} N}\left(\frac{1}{\sqrt{1.44}} - \frac{1}{\sqrt{N+0.44}}\right)\log_{10}\left(A + 0.12AB + 1.7B\right)$$

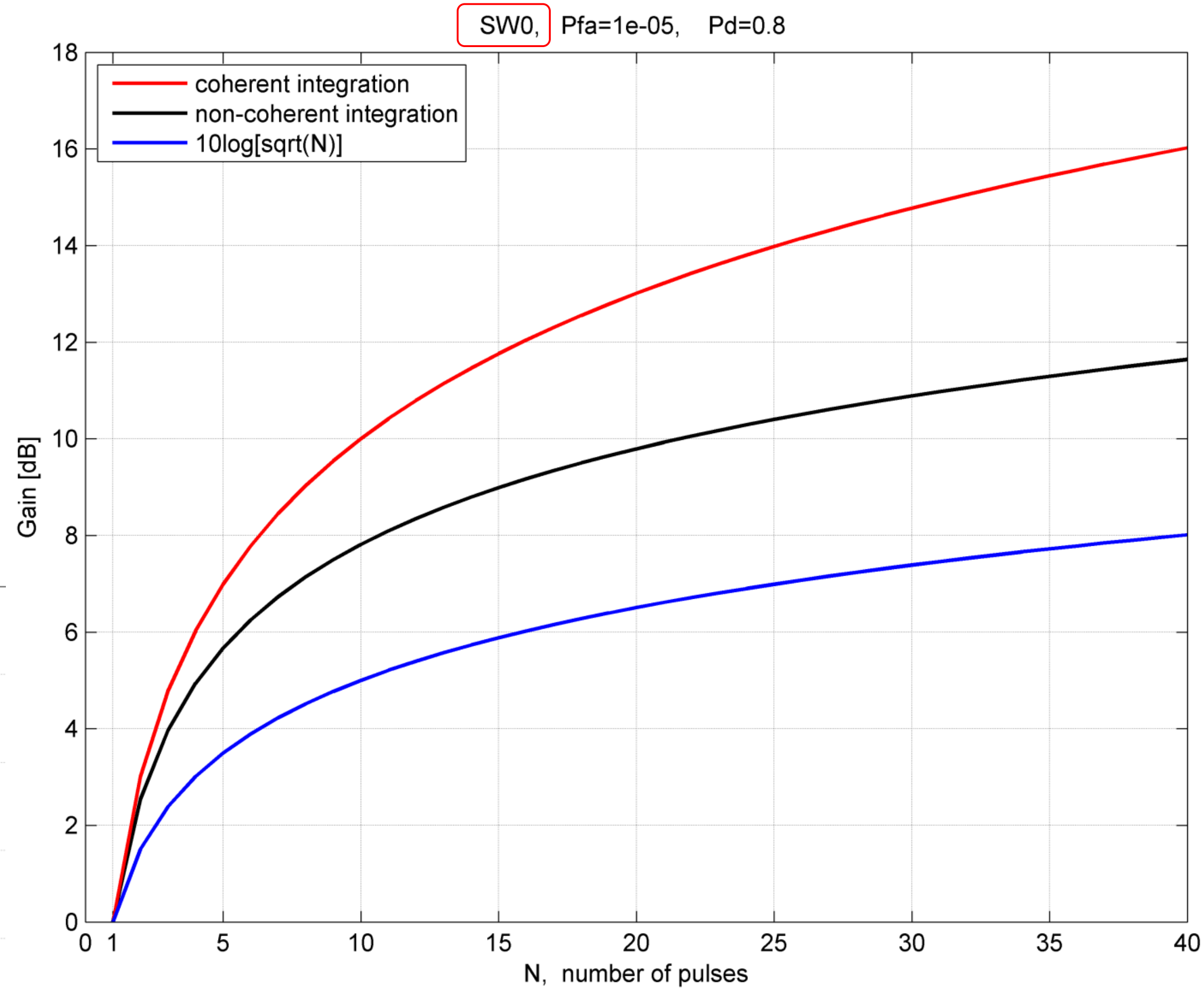
Harry Urkowitz “A closed form for the noncoherent gain factor”, IEEE TAES Vol. 46, No. 2, Apr 2010, pp. 943-944



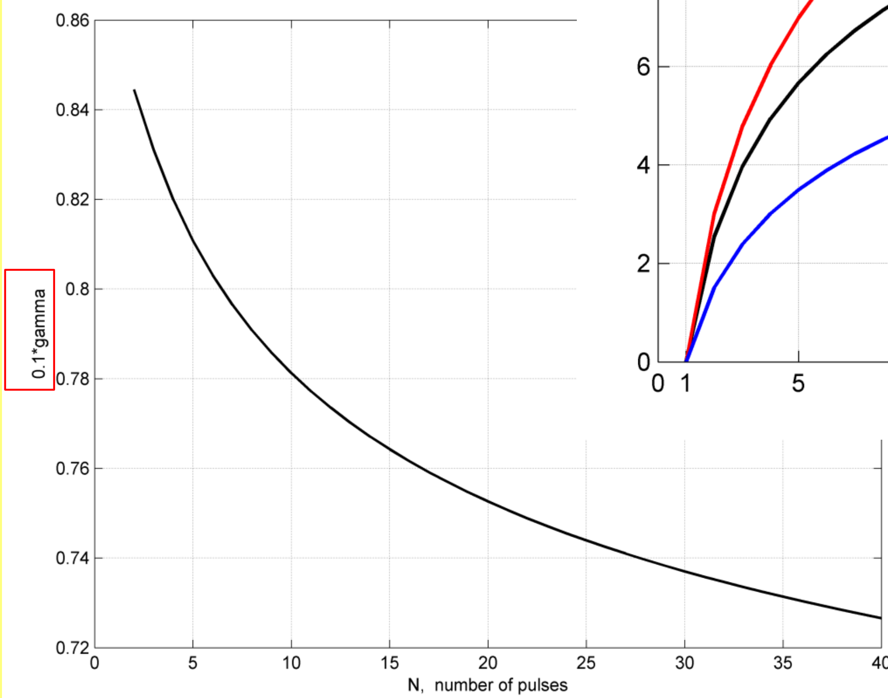
$$P_D = P_{FA}^{\frac{1}{1+M\overline{SNR}}}, \quad \text{coherent integration of SW1}$$

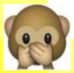
Victor Chernyak's equations for non-coherent integration

```
pd_sw2=1-chi2cdf(chi2inv(1-pfa,2*M)./(1+(10.^(0.1*snr))),2*M);    % SNR in dB, M = number of pulses
pd_sw1=((1+1./(M*(10.^(0.1*snr)))).^(M-1)).*exp(-((0.5*chi2inv(1-pfa,2*M))/(1+M*(10.^(0.1*snr)))));
```

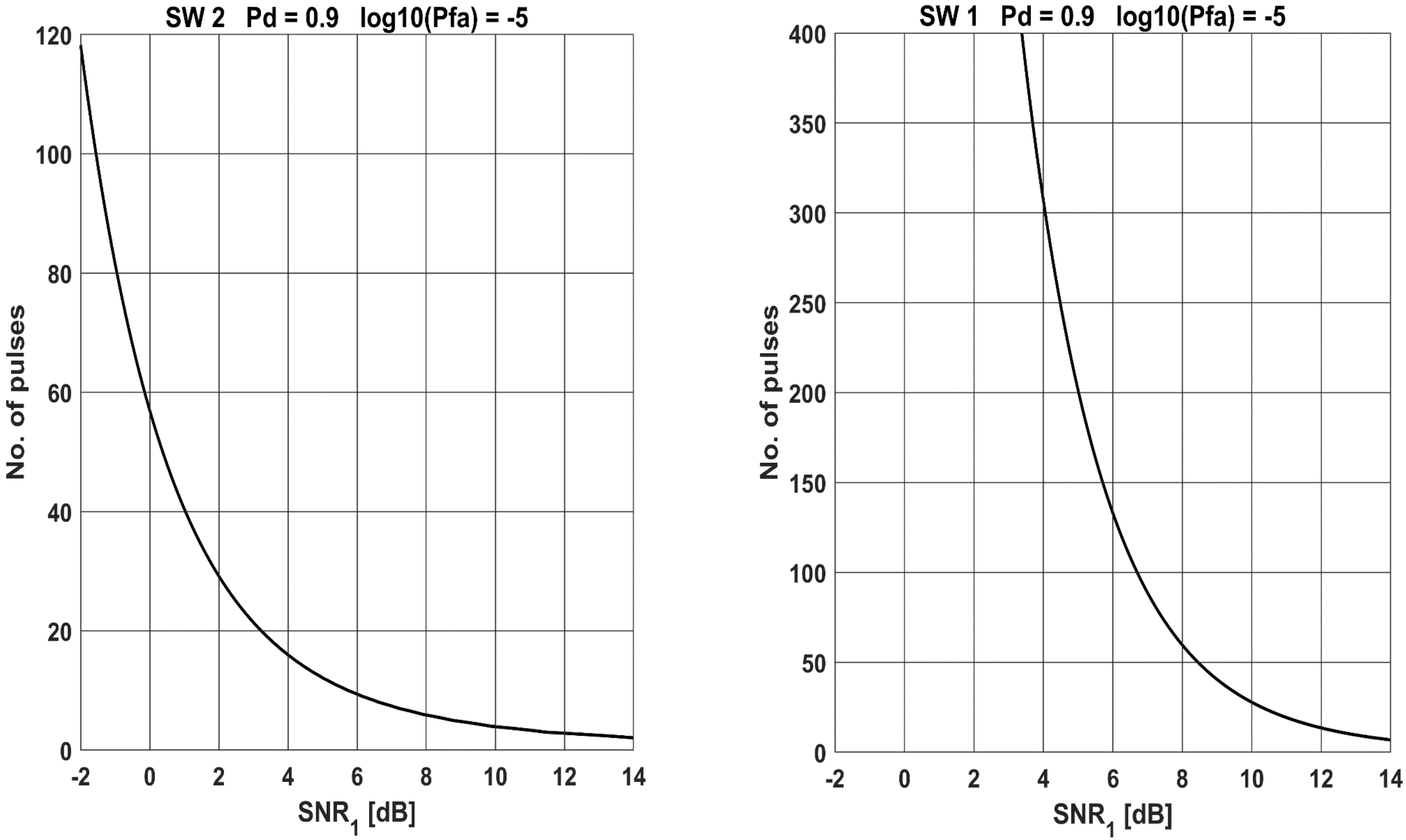


$$G(N) = N^{\frac{\gamma}{10}}$$





The number of video integrated pulses needed to reach a specific Pd (SW1 and SW2)



Victor Chernyak's equations

```
pd_sw2=1-chi2cdf(chi2inv(1-pfa,2*M)./(1+(10.^(0.1*snr))),2*M); % single-pulse SNR in dB, M = number of pulses
pd_sw1=((1+1./(M*(10.^(0.1*snr)))).^(M-1)).*exp(-((0.5*chi2inv(1-pfa,2*M))/(1+M*(10.^(0.1*snr)))));
```