

Instant Active Positioning with One LEO Satellite

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ABSTRACT: *Autonomous position determination in the Globalstar satellite communication system is discussed. The two-way communication between a user terminal on the earth's surface and a single low earth orbit (LEO) satellite makes it possible to derive range and range-rate and determine the user terminal position instantly. Expected accuracy is presented, and a simple direct solution is given.*

INTRODUCTION

Globalstar [1] is a satellite communication system designed to provide voice and low-rate data communication to user terminals (UTs) on the earth. The satellite constellation includes 48 low earth orbit (LEO) satellites at a height of 1400 km, arranged in eight orbits with an inclination of 52 deg. The Globalstar satellite acts as a bent-pipe relay between a regional gateway (GW) and the UT. The orbit of the satellite is known accurately.

Primarily for operational reasons, before a UT phone call is connected, the system needs to know the UT position with a horizontal accuracy of better than 10 km. Connection of the call cannot be delayed for more than about 3 s, and a call should go through even if the UT sees only one Globalstar satellite. Hence there is a need for instant coarse positioning using the system's own satellite and signals [2]. While two-satellite, two-dimensional (2-D), active positioning is found in other operational systems [3, 4], single-satellite instant positioning is unique. This paper therefore concentrates on this aspect of Globalstar positioning.

Single-satellite positioning is based on measurements of range and range rate. The range between the satellite and the UT is derived from a round-trip delay measurement, from which the known GW-to-satellite leg is removed. Globalstar uses a code division multiple access (CDMA) concept, which utilizes a wideband spread-spectrum signal, as in GPS. Hence the delay measurement resolution is relatively high. Range rate is derived from Doppler measurements. In a *passive* satellite Doppler navigation

system, such as TRANSIT [5], the true Doppler cannot be separated from the UT oscillator frequency offset; hence only Doppler differences (rather than absolute Doppler) can be measured. When the UT is active (receives and transmits) and uses the same master oscillator in its receiver and transmitter, two Doppler measurements—one at the satellite (in practice at the GW) and one at the UT (and communicated to the GW)—provide enough information to separate the true Doppler from the UT frequency offset (see Appendix A).

This paper derives the expected accuracy of 2-D positioning based on one range and one range-rate measurement to a single satellite. In the third dimension, the UT is assumed to be on the earth's surface. The horizontal error resulting from an elevation error is also derived. Finally, a simple direct solution is presented that can be used by itself or to provide a first estimate for an iterative positioning algorithm.

POSITIONING BASED ON RANGE AND RANGE RATE

The positioned UT is located at one of the two intersections between three surfaces (see Figure 1): the range sphere, the range-rate (Doppler) cone, and the earth's surface. The range sphere is centered at the satellite antenna, which is also where the apex of the cone is located. The cone axis of symmetry is the velocity vector. Delay (range) and Doppler (range-rate) errors cause an error in the determined UT position. The cross-track positioning error is magnified dramatically when the intersection is near the satellite subtrack on the earth's surface. An error in the UT assumed height above

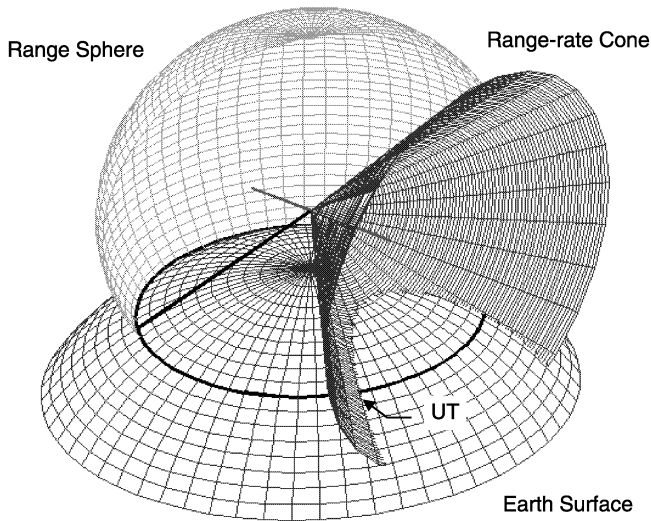


Fig. 1 - Three Surfaces Defining the UT Position

the reference earth surface causes further cross-track positioning error, which also increases as the UT gets closer to the subtrack.

The performances of such a positioning system can be analytically derived for a simplified model that assumes a flat earth and a straight-line orbit. The validity of this assumption is demonstrated by comparing analytic results using the flat earth model with numerical results using a spherical earth model.

The satellite-UT geometry in a flat earth model is depicted in Figure 2. At $t = 0$, the satellite is at the $(0, 0, H)$ coordinate and moving in the y direction with a velocity v . Both H and v are known accurately. The stationary UT is at (x, y, h) . A flat reference earth surface is assumed, represented by the $z = 0$ plane. The range to the UT is given by

$$R(t) = \sqrt{x^2 + (y - vt)^2 + (h - H)^2} \quad (1)$$

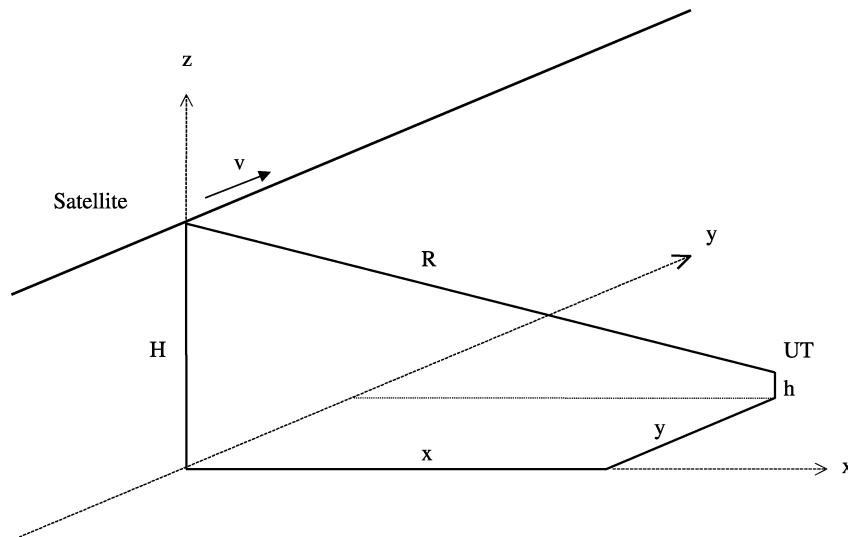


Fig. 2 - Satellite-UT Geometry

and the range rate by

$$\dot{R}(t) = \frac{-v(y - vt)}{\sqrt{x^2 + (y - vt)^2 + (h - H)^2}} \quad (2)$$

Our simple analysis assumes that the range and range rate, derived from the delay and Doppler measurements, are free from bias errors, but suffer from relatively small, independent, random errors with zero mean and standard deviations of σ_R and $\sigma_{\dot{R}}$, respectively. In reality, bias errors do exist. In Globalstar's single-satellite positioning, random measurement errors dominate the position error. The bias errors become a dominant factor in two-satellite positioning. Yet it should be noted that even though the error contour maps in this paper were derived assuming random, zero-mean measurement errors, areas with large sensitivity to random measurement errors are also sensitive to bias measurement errors.

The UT location (x, y) is determined on the reference surface ($z = 0$ plane), assuming erroneously that $h = 0$. Later it is shown that a deviation of the true elevation from the assumed elevation (zero) results in a bias error in the estimated cross-track (x) coordinate of the UT.

LOCATION ERROR AS FUNCTION OF MEASUREMENT ERROR

The random errors in range and range rate cause a UT position error. The position error is separated into cross-track and along-track components, with standard deviations of σ_x and σ_y , respectively. The transformation of errors from the measurement domain to the geometrical domain is done for $t = 0$, and requires the partial derivatives of the measurements with respect to the location parameters. At

$t = 0$, the partial derivatives are given by

$$\frac{\partial R}{\partial x} = \frac{x}{R} \quad (3)$$

$$\frac{\partial R}{\partial y} = \frac{y}{R} \quad (4)$$

$$\frac{\partial \dot{R}}{\partial x} = \frac{-xyv}{R^3} \quad (5)$$

$$\frac{\partial \dot{R}}{\partial y} = \frac{v}{R} \left(1 - \frac{y^2}{R^2} \right) \quad (6)$$

They are arranged as a matrix of partial derivatives:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial R}{\partial x} & \frac{\partial R}{\partial y} \\ \frac{\partial \dot{R}}{\partial x} & \frac{\partial \dot{R}}{\partial y} \end{bmatrix} \quad (7)$$

The errors in range and range rate are assumed to be independent. This assumption is well justified in Globalstar, where the two measurements—delay and Doppler—are nearly uncorrelated. The error covariance matrix is therefore diagonal. (Correla-

tion would result in nonzero off-diagonal elements.) Its inverse, also a diagonal matrix, is termed the weight matrix:

$$\mathbf{W} = \begin{bmatrix} \frac{1}{\sigma_R^2} & 0 \\ 0 & \frac{1}{\sigma_{\dot{R}}^2} \end{bmatrix} \quad (8)$$

The error variance of the estimated UT coordinates, x and y , can be obtained from \mathbf{H} and \mathbf{W} [6]:

$$\sigma_x^2 = (\mathbf{H}^T \mathbf{W} \mathbf{H})_{1,1}^{-1} \quad (9)$$

$$\sigma_y^2 = (\mathbf{H}^T \mathbf{W} \mathbf{H})_{2,2}^{-1} \quad (10)$$

Symbolic solutions of equations (9) and (10) yield

$$\sigma_x^2 = \frac{R^2}{x^2} \left[\frac{y^2}{v^2} \sigma_R^2 + \left(1 - \frac{y^2}{R^2} \right)^2 \sigma_{\dot{R}}^2 \right] \quad (11)$$

$$\sigma_y^2 = \frac{R^2}{v^2} \sigma_R^2 + \frac{y^2}{R^2} \sigma_{\dot{R}}^2 \quad (12)$$

Figures 3 and 4 are contour maps of the cross-track error, σ_x , and the along-track error, σ_y , re-

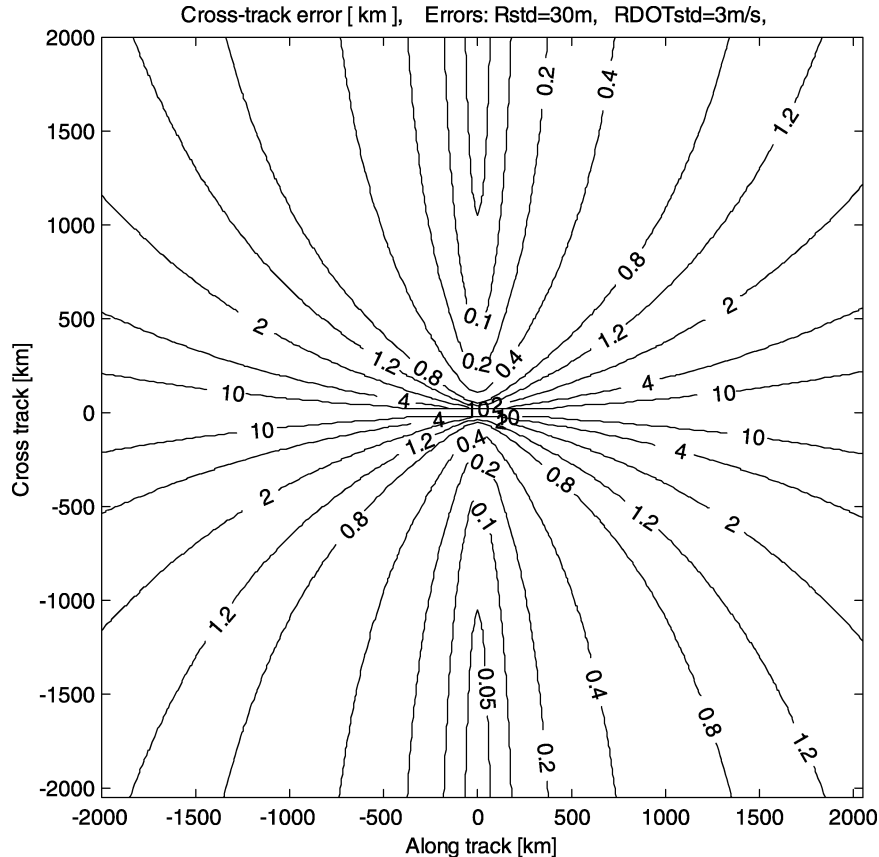


Fig. 3—Cross-Track Location Error (in kilometers)

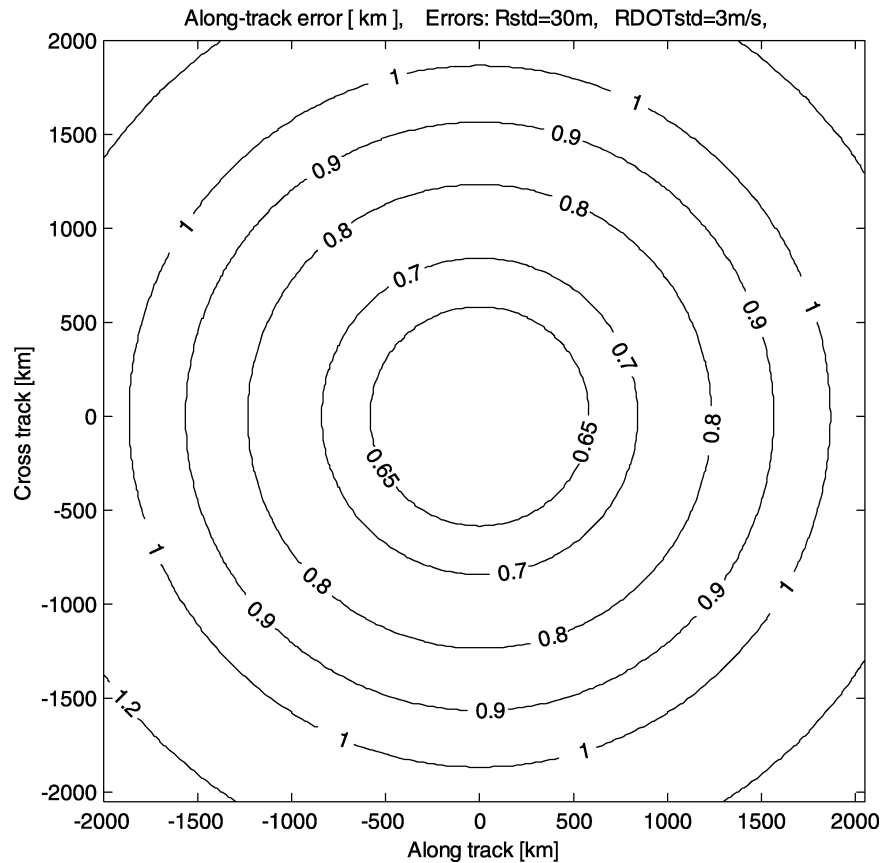


Fig. 4—Along-Track Location Error (in kilometers)

spectively, in kilometers, for the following typical scenario: $H = 1400$ km, $v = 7$ km/s, $\sigma_R = 30$ m, and $\sigma_{\dot{R}} = 3$ m/s. The most prominent feature is the large cross-track error in positioning UTs located near the satellite subtrack (cross-track = 0). This feature implies large geometric dilution of precision (GDOP) near the subtrack. The poor GDOP stems from the fact that on the subtrack, the circle drawn on the earth's surface by the intersection with the range sphere (Figure 1) is tangential to the hyperbola drawn on the earth's surface by the intersection with the range-rate cone. Any small error in range or range rate will cause a large cross-track error in the calculated location.

As the UT location gets farther away from the subtrack, the GDOP singularity is replaced by another problem—ambiguity. Single-satellite positioning based on one pair of range and range-rate measurements always yields two symmetrical solutions on the two sides of the subtrack (Figure 1). When the true UT location is far from the subtrack, the two solutions are far from each other. In that case, the true solution can be identified using information from several satellite antenna beams. When the true UT location is close to the subtrack, the

two solutions are close to each other and may both fall within the illumination area of the same antenna beam. In that case, the ambiguity cannot be resolved.

Figures 3 and 4 are plots of the analytic results of equations (11) and (12), respectively, which were derived using a flat earth model. To demonstrate their validity for the real earth, a numerical error analysis was performed using a spherical earth model and a circular orbit. The result for the cross-track error is presented in Figure 5. The excellent agreement between Figures 3 and 5 (and similar agreement, not plotted, with respect to the along-track error) proves that the use of a flat earth model is justified, and that equations (11) and (12) apply to more realistic earth models.

ELEVATION-INDUCED ERROR

Equations (1) and (2) indicate that in both the range and the range-rate measurement, the UT's elevation, h , and cross-track coordinate, x , are coupled together in the expression $x^2 + (h - H)^2$. Hence an error, Δh , in the assumed elevation must create an error, Δx , in the cross-track coordinate.

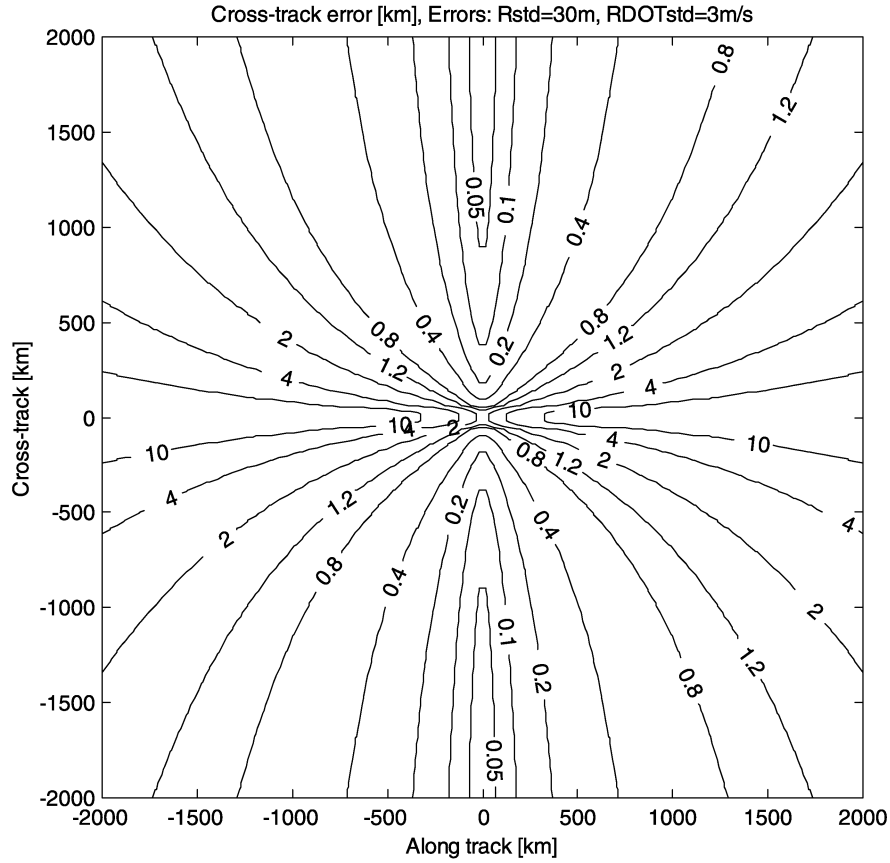


Fig. 5 – Cross-Track Location Error (in kilometers) Derived Numerically Using Spherical Earth Model

To keep the expression unchanged, the following relationship must hold

$$\Delta x = \frac{H - h}{x} \Delta h \approx \frac{H}{x} \Delta h \quad (13)$$

According to equation (13), when the cross-track distance is approximately equal to the satellite height, the elevation-induced cross-track error is equal to the elevation error. Clearly the error increases as the UT gets closer to the subtrack. In the actual Globalstar positioning algorithm, the horizontal position is first solved using sea-level elevation. Then the terrain elevation for the solved position is obtained from a topographic map and used instead of sea level, and an improved position is obtained. Unless the terrain is very rough, convergence is expected after two or three iterations.

The elevation-induced horizontal positioning error can be incorporated in the error contour maps by declaring the elevation obtained from the topo-

graphic map to be a ‘measurement’ with error standard deviation σ_E . The partial derivative and weight matrix are increased in size to 3×3 :

$$\mathbf{H} = \begin{bmatrix} \frac{\partial R}{\partial x} & \frac{\partial R}{\partial y} & \frac{\partial R}{\partial h} \\ \frac{\partial \dot{R}}{\partial x} & \frac{\partial \dot{R}}{\partial y} & \frac{\partial \dot{R}}{\partial h} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial h} \end{bmatrix} \quad (14)$$

$$\mathbf{W} = \begin{bmatrix} \frac{1}{\sigma_R^2} & 0 & 0 \\ 0 & \frac{1}{\sigma_{\dot{R}}^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_E^2} \end{bmatrix} \quad (15)$$

where the additional elements are

$$\frac{\partial \mathbf{R}}{\partial \mathbf{h}} = \frac{\mathbf{h} - \mathbf{H}}{\mathbf{R}} \quad (16)$$

$$\frac{\partial \dot{\mathbf{R}}}{\partial \mathbf{h}} = \frac{-(\mathbf{h} - \mathbf{H})\mathbf{y}\mathbf{v}}{\mathbf{R}^3} \quad (17)$$

$$\frac{\partial \mathbf{h}}{\partial \mathbf{x}} = 0, \quad \frac{\partial \mathbf{h}}{\partial \mathbf{y}} = 0, \quad \frac{\partial \mathbf{h}}{\partial \mathbf{h}} = 1 \quad (18)$$

The cross-track, along-track, and height error variances are derived from the three diagonal elements of the matrix $(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1}$, yielding

$$\sigma_x^2 = \frac{\mathbf{R}^2}{\mathbf{x}^2} \left[\frac{\mathbf{y}^2}{\mathbf{v}^2} \sigma_R^2 + \left(1 - \frac{\mathbf{y}^2}{\mathbf{R}^2} \right)^2 \sigma_R^2 \right] + \frac{(\mathbf{h} - \mathbf{H})^2}{\mathbf{x}^2} \sigma_E^2 \quad (19)$$

$$\sigma_y^2 = \frac{\mathbf{R}^2}{\mathbf{v}^2} \sigma_R^2 + \frac{\mathbf{y}^2}{\mathbf{R}^2} \sigma_R^2 \quad (20)$$

$$\sigma_h^2 = \sigma_E^2 \quad (21)$$

Note that the along-track error expression (equation (20)) is identical to the 2-D case (equation (12));

hence Figure 4 holds for the 3-D case as well. The cross-track error (equation (19)) differs from equation (11) in the additional term predicted by equation (13). The slightly degraded cross-track error contour map assuming an elevation measurement error standard deviation of $\sigma_E = 200$ m is presented in Figure 6. There is no need to plot the standard deviation of the error in the estimated height since it must everywhere be equal to σ_E , as indicated by equation (21).

OTHER ERRORS

As the various error contour maps indicate, with the expected random measurement error, typical positioning errors are between 0.5 and 10 km (excluding the area near the satellite subtrack). This performance level can be considered *coarse* positioning, but it meets the system's needs. Other error sources, such as ionospheric effects and ephemeris errors, that are considered in more accurate positioning systems can be neglected in our error analysis. The main remaining source of error is UT velocity. UT velocity generates additional Doppler, which effectively increases the range-rate error. Because we are using a single Doppler measurement, the fact that it is a bias rather than a random error makes no difference, and the effect of

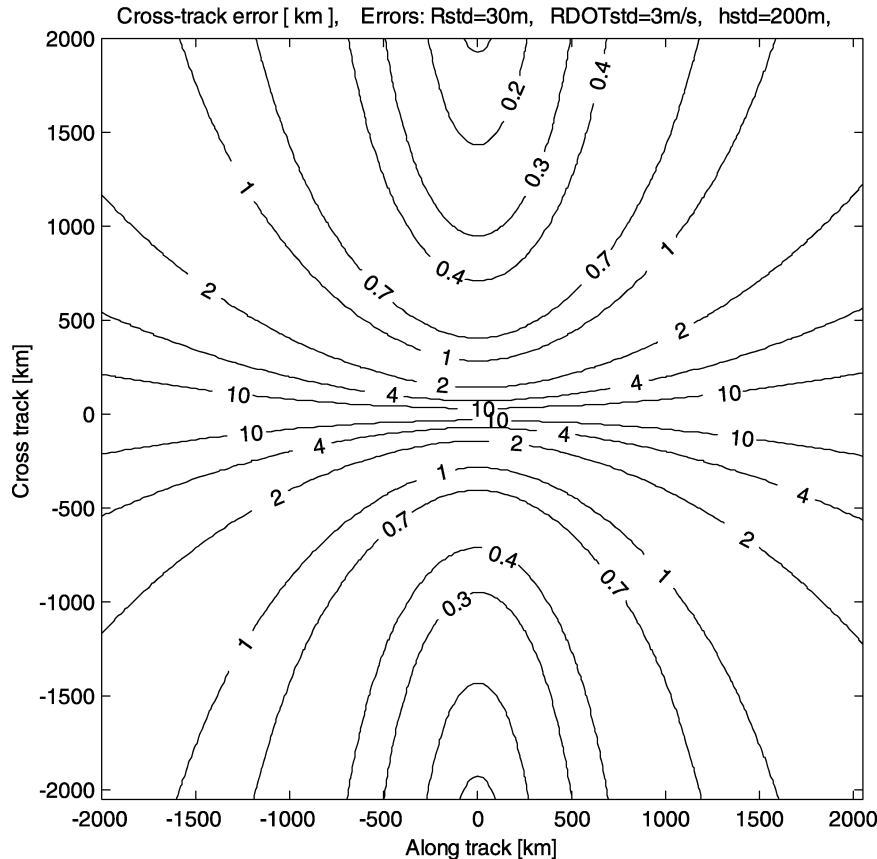


Fig. 6 – Cross-Track Location Error with Contribution from Elevation Error (in kilometers)

UT velocity can be incorporated into the error analysis by increasing the range-rate random error.

POSITIONING ALGORITHM

The operational positioning algorithm applies to all possible scenarios, including the two- and three-satellite cases. The latter are overdetermined systems, which invites a least-squares solution. The general algorithm thus uses an iterative weighted least-squares solution. Such an algorithm requires an initial estimate of the UT location. A simple approach to obtaining a good first estimate is to solve explicitly the minimal set of two equations (measurements): the range and range rate to a single satellite. One explicit solution is described in Appendix B. In a single-satellite situation, the explicit solution can replace the iterative solution.

CONCLUSIONS

Autonomous positioning in Globalstar may necessitate determining the UT position when only one satellite is available. The random error analysis presented in this paper demonstrates that with the expected measurement accuracy, coarse position can be determined instantly for UTs everywhere except near the satellite subtrack.

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APPENDIX A SEPARATING DOPPLER FROM UT OSCILLATOR OFFSET

To determine the satellite-UT range rate, it is necessary to separate the UT oscillator offset from the Doppler shift (in the satellite-UT leg). In an unpublished report, Steven A. Kremm suggested a method that applies when the UT uses the same local oscillator for both transmit and receive. The method is explained with the help of Figure A.1 and the index below. It is assumed that the Doppler shifts in the GW-satellite leg are perfectly known and removed.

- \dot{R} = range rate of the satellite-to-UT leg
- C = propagation velocity (speed of light)
- f_F = forward link nominal carrier frequency (2500 MHz)
- f_R = reverse link nominal carrier frequency (1600 MHz)
- $\frac{f_{\text{off}}}{f_0}$ = normalized frequency offset of UT's oscillator

Two measured frequencies are available at the GW: the reported measurement from the UT

$$f_{\text{meas, UT}} = f_F \left(-\frac{\dot{R}}{C} - \frac{f_{\text{off}}}{f_0} \right) \quad (\text{A-1})$$

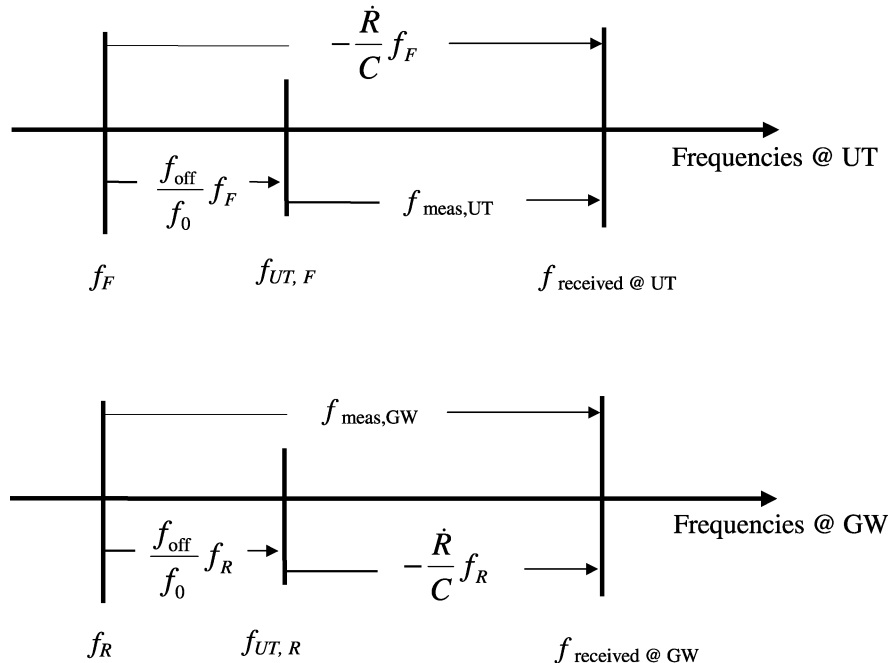


Fig. A.1 – Relationship Between Frequencies Used to Separate Doppler from Frequency Offset

and the measurement performed at the GW itself

$$f_{\text{meas,GW}} = f_R \left(-\frac{\dot{R}}{C} + \frac{f_{\text{off}}}{f_0} \right) \quad (\text{A-2})$$

Adding and subtracting equations (A-1) and (A-2) yields both the UT offset and the range rate:

$$\frac{f_{\text{off}}}{f_0} = \frac{1}{2} \left(\frac{f_{\text{meas,GW}}}{f_R} - \frac{f_{\text{meas,UT}}}{f_F} \right) \quad (\text{A-3})$$

$$\dot{R} = -\frac{C}{2} \left(\frac{f_{\text{meas,GW}}}{f_R} + \frac{f_{\text{meas,UT}}}{f_F} \right) \quad (\text{A-4})$$

APPENDIX B DIRECT SOLUTION

The direct solution assumes a smooth ellipsoid earth model and uses earth-centered, earth-fixed Cartesian coordinates. There are many references to direct solutions for geolocation using time-difference or frequency-difference measurement [7]. However, the direct solution becomes very simple when absolute measurements are available.

Definitions

The UT Cartesian coordinates (earth-centered, earth-fixed) are

$$\mathbf{u} = [xyz]^T \quad (\text{B-1})$$

Initially, the UT is assumed to be on a sphere of radius r_c (whose value is arbitrarily selected between the equatorial earth radius r_E and the polar earth radius r_P). That assumption yields the first equation:

$$r_c^2 = \mathbf{u}^T \mathbf{u} = x^2 + y^2 + z^2 \quad (\text{B-2})$$

Six satellite parameters at the measurement epoch

are known: the position coordinates

$$\mathbf{s} = [x_s y_s z_s]^T \quad (\text{B-3})$$

and the velocities

$$\mathbf{v} = [v_x v_y v_z]^T \quad (\text{B-4})$$

The radius of the satellite orbit is

$$r_s^2 = \mathbf{s}^T \mathbf{s} = x_s^2 + y_s^2 + z_s^2 \quad (\text{B-5})$$

The two available measurements are range

$$R = |\mathbf{s} - \mathbf{u}| = \sqrt{(x_s - x)^2 + (y_s - y)^2 + (z_s - z)^2} \quad (\text{B-6})$$

and range rate

$$\begin{aligned} \dot{R} &= \frac{(\mathbf{s} - \mathbf{u})^T \mathbf{v}}{r} \\ &= \frac{1}{r} [(x_s - x)v_x + (y_s - y)v_y + (z_s - z)v_z] \end{aligned} \quad (\text{B-7})$$

Outline

Define

$$\mathbf{A} = \begin{bmatrix} x_s & y_s \\ v_x & v_y \end{bmatrix} \quad (\text{B-8})$$

$$\mathbf{b} = \frac{1}{2} [r_s^2 + r_c^2 - R^2] \quad (\text{B-9})$$

$$\mathbf{c} = \mathbf{s}^T \mathbf{v} - R\dot{R} = x_s v_x + y_s v_y + z_s v_z - R\dot{R} \quad (\text{B-10})$$

Obtain

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{b} & z_s \\ \mathbf{c} & v_z \end{bmatrix} \quad (\text{B-11})$$

The z coordinates of the UT's true and mirror solutions are obtained from

$$\hat{z}_{1,2} = \frac{\alpha\beta + \gamma\delta \pm \sqrt{(\alpha\beta + \gamma\delta)^2 - (1 + \delta^2 + \beta^2)(\alpha^2 + \gamma^2 - r_c^2)}}{1 + \delta^2 + \beta^2} \quad (\text{B-12})$$

and the remaining coordinates of the two solutions are given by

$$\hat{x}_{1,2} = \alpha - \beta \hat{z}_{1,2} \quad (\text{B-13})$$

$$\hat{y}_{1,2} = \gamma - \delta \hat{z}_{1,2} \quad (\text{B-14})$$

Resolving which of the two is the true solution requires additional information. That information can be obtained, for example, from the satellite antenna beam through which the UT signal was received.

Explanation

The range and range-rate equations (B-6) and (B-7) can be rewritten as

$$[\mathbf{s} \quad \mathbf{v}]^T \mathbf{u} = [\mathbf{b} \quad \mathbf{c}]^T \quad (\text{B-15})$$

where \mathbf{b} and \mathbf{c} are as defined in equations (B-9) and (B-10). From equation (B-15), x and y can be expressed as functions of z :

$$x = \alpha - \beta z \quad (\text{B-16})$$

$$y = \gamma - \delta z \quad (\text{B-17})$$

where α , β , γ , and δ are as defined in equation (B-11). Inserting equations (B-16) and (B-17) in equation (B-2) yields a quadratic equation of z

$$(\alpha - \beta z)^2 + (\gamma - \delta z)^2 + z^2 = r_c^2 \quad (\text{B-18})$$

whose two solutions are given by equation (B-12) above. Using the two solutions for z back in equations (B-16) and (B-17) yields the corresponding two solutions for x and y , as given in equations (B-13) and (B-14) above.

Ellipsoid Earth Model

The first improvement to the above simple solution is to assume that the UT is on an oblate ellipsoid that has a polar radius r_p and an equatorial radius r_E . From the Cartesian coordinates derived assuming a spherical earth, we find sine and cosine terms of the approximate geocentric latitude ϕ' :

$$\sin \phi' = \frac{z}{r_c}; \quad \cos \phi' = \frac{\sqrt{x^2 + y^2}}{r_c} \quad (\text{B-19})$$

Using the ellipsoid model and the approximate geocentric latitude yields a more accurate distance to

the center of the earth:

$$r_c^2 = \frac{r_p^2}{\sin^2 \phi' + \left(\frac{r_p}{r_E}\right)^2 \cos^2 \phi'} \quad (\text{B-20})$$

This local earth radius (for the approximate UT location) is used in equation (B-9), and the direct solution is repeated to yield more accurate Cartesian coordinates of the UT location.

When the direct solution is used to obtain a first estimate for the iterative solution, the accuracy achieved thus far is sufficient. If the direct solution is used by itself, an elevation correction is required. To obtain the elevation, the Cartesian coordinates must be converted into longitude and geodetic latitude [8], the surface elevation for that location is obtained from a digital terrain map, the distance to the earth center is adjusted accordingly, and the direct solution is repeated.

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