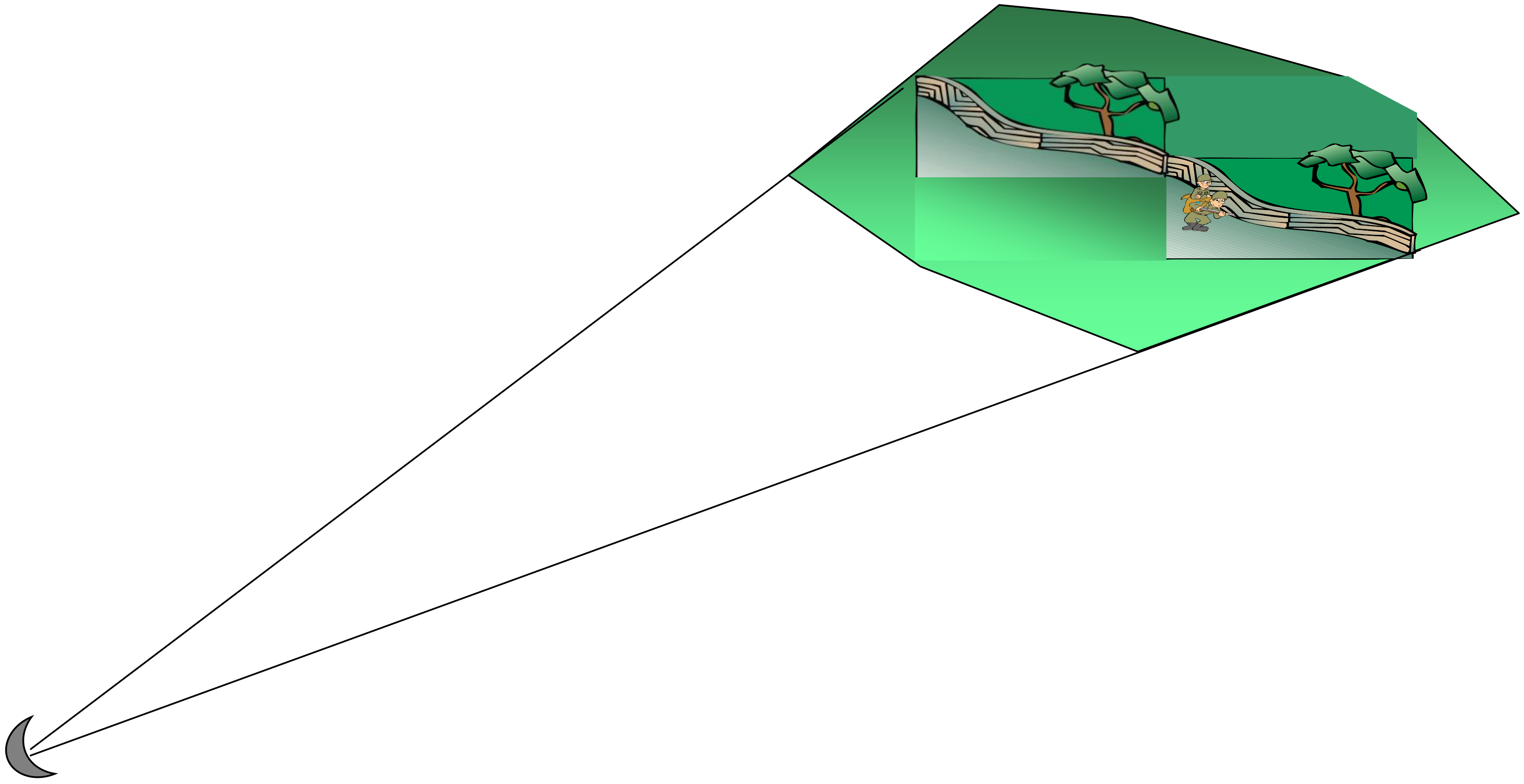
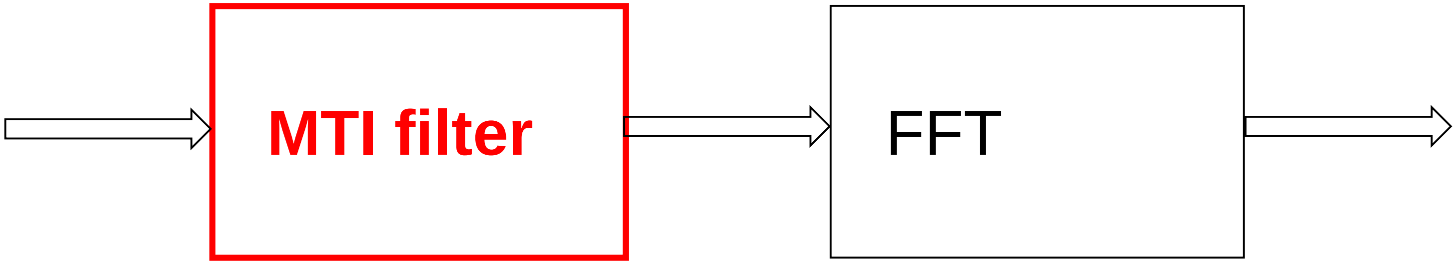
MOVING TARGET INDICATOR (MTI) and/or DOPPLER PROCESSING



MOVING TARGET INDICATOR (MTI) and/or DOPPLER PROCESSING

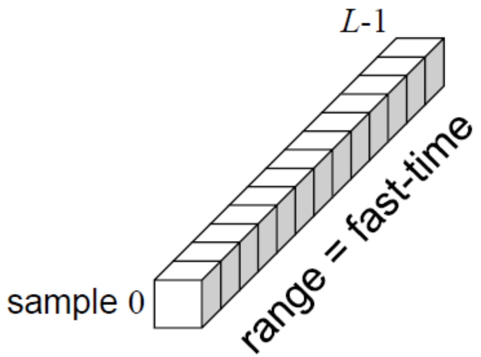
- Involves coherent processing of a pulse train (plain or compressed)
- Purposes: Separate weak returns of moving targets from large clutter returns
- Two basic approaches:
  - Bank of filters (usually DFT, or FFT)
  - Pulse cancellers
- The two approaches are usually combined

Typical radar Doppler processing stages



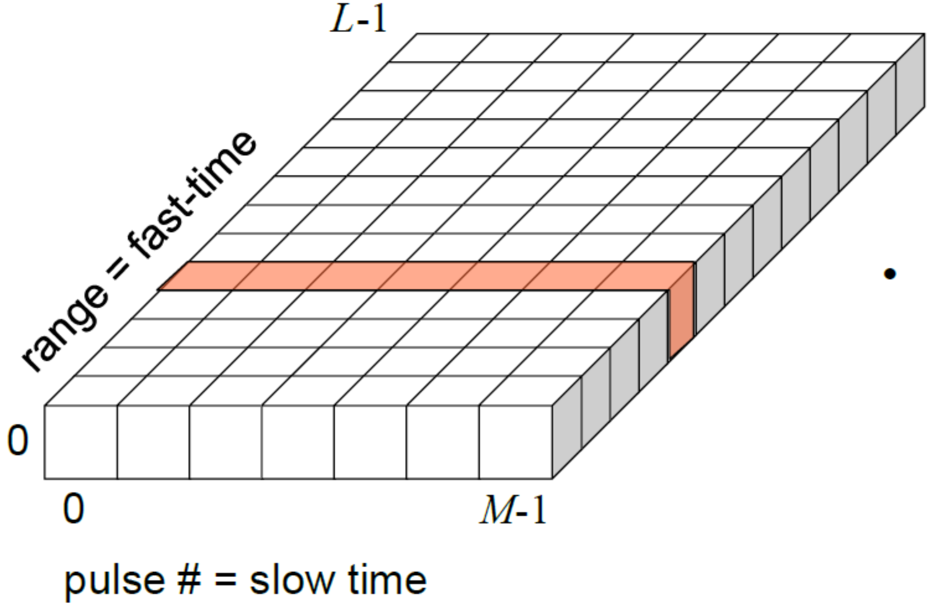
Collecting Pulsed Radar Data, 2: 1 Pulse, Multiple Range Samples

- If using a coherent receiver, each range sample comprises one "I" sample and one "Q" sample, forming one complex number $I+jQ$
  - Equivalently, one amplitude and one phase
- So each range cell contains 1 complex number
  - Stored in a single processor memory location
- Each range cell represents echo from a different range interval
- Also called range bins, range gates, range cells, fast time samples, …
- Sample rate determined by data bandwidth …
- Which in turn is determined by waveform bandwidth
  - Typically 1s to 100s of MHz, sometimes higher



Collecting Pulsed Radar Data, 3: Multiple Pulses

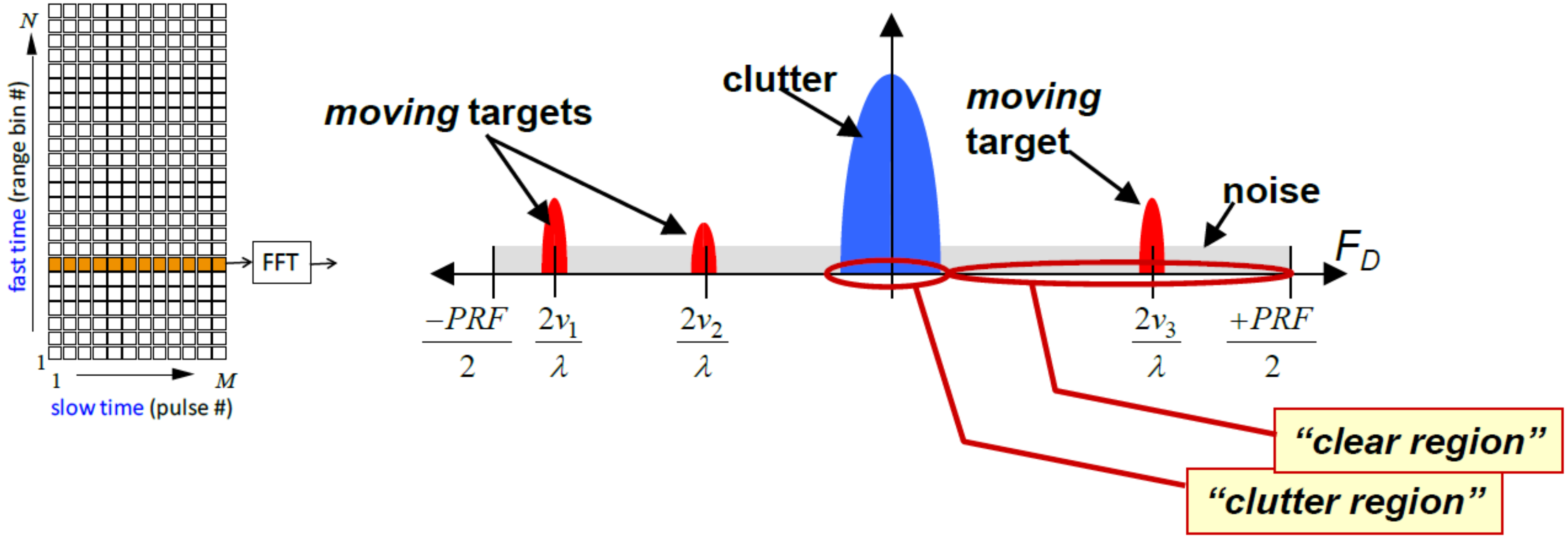
- **Repeat for multiple pulses in a "coherent processing interval" (CPI) or "dwell"**
- **New axis is called pulse number, or slow time**
- **Sample rate is the PRF**
- **Minimum PRF determined by Doppler bandwidth of data**



- Sequence of samples for a fixed range bin represents echoes from same range interval over a period of time
  - if antenna doesn't move significantly

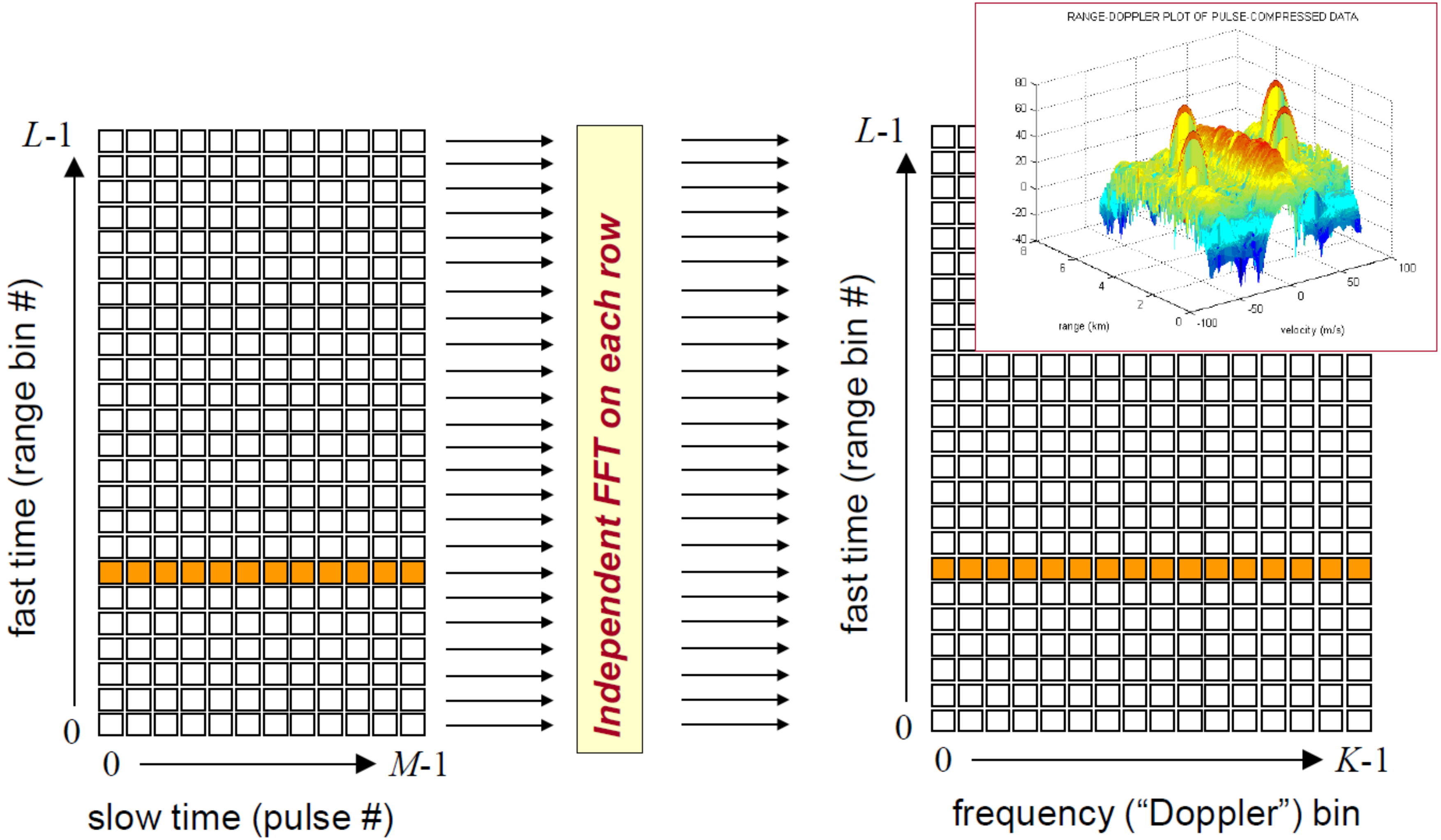
Generic Doppler Spectrum Viewed by a Stationary Radar for 1 Range Bin

- "Principal period" of the spectrum of a radar echo signal from one range bin (spectrum repeats periodically):



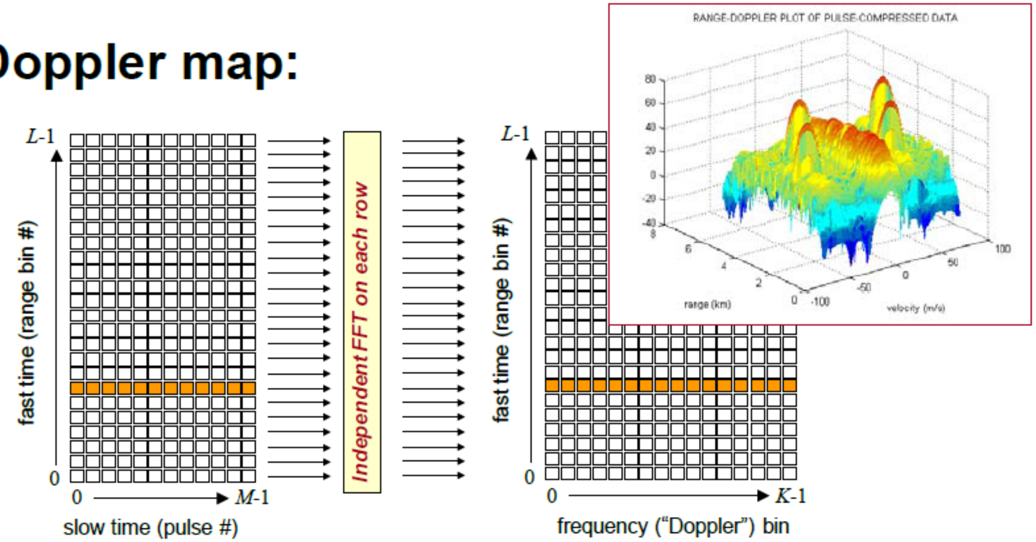
- Noise uniform across spectrum
- Clutter concentrated around zero Doppler
  - But clutter width depends on phenomenology and platform motion
- Target(s) appear at appropriate Doppler based on relative radial velocity

Range-Doppler Map

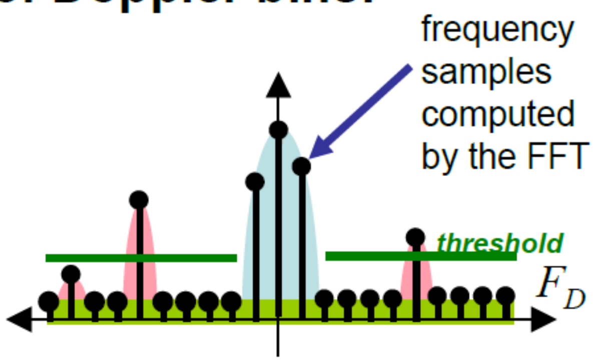


Basic Pulse Doppler Detection Process

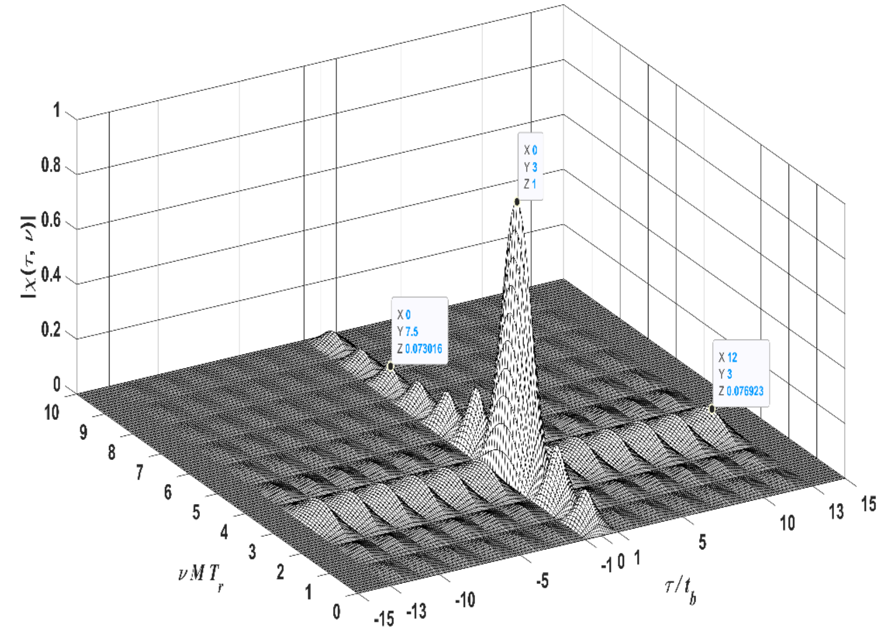
- **Form range-Doppler map:**

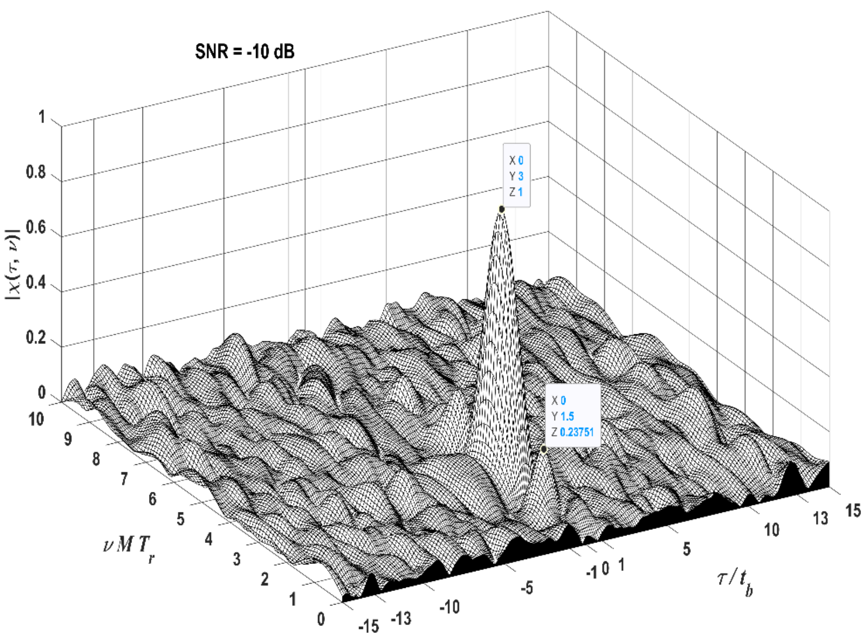


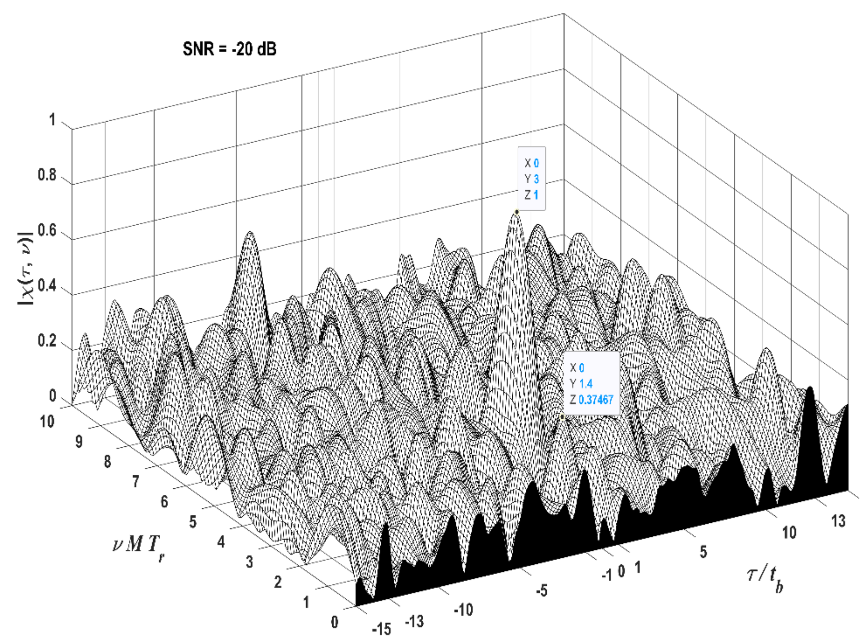
- **Threshold testing of Doppler bins:**



- **So we need to maximize SNR in each Doppler bin …**


$|\chi(\tau,\nu)|$
$\nu M T_r$
τ / t_b
X 0
Y 3
Z 1
X 0
Y 7.5
Z 0.073016
X 12
Y 3
Z 0.076923

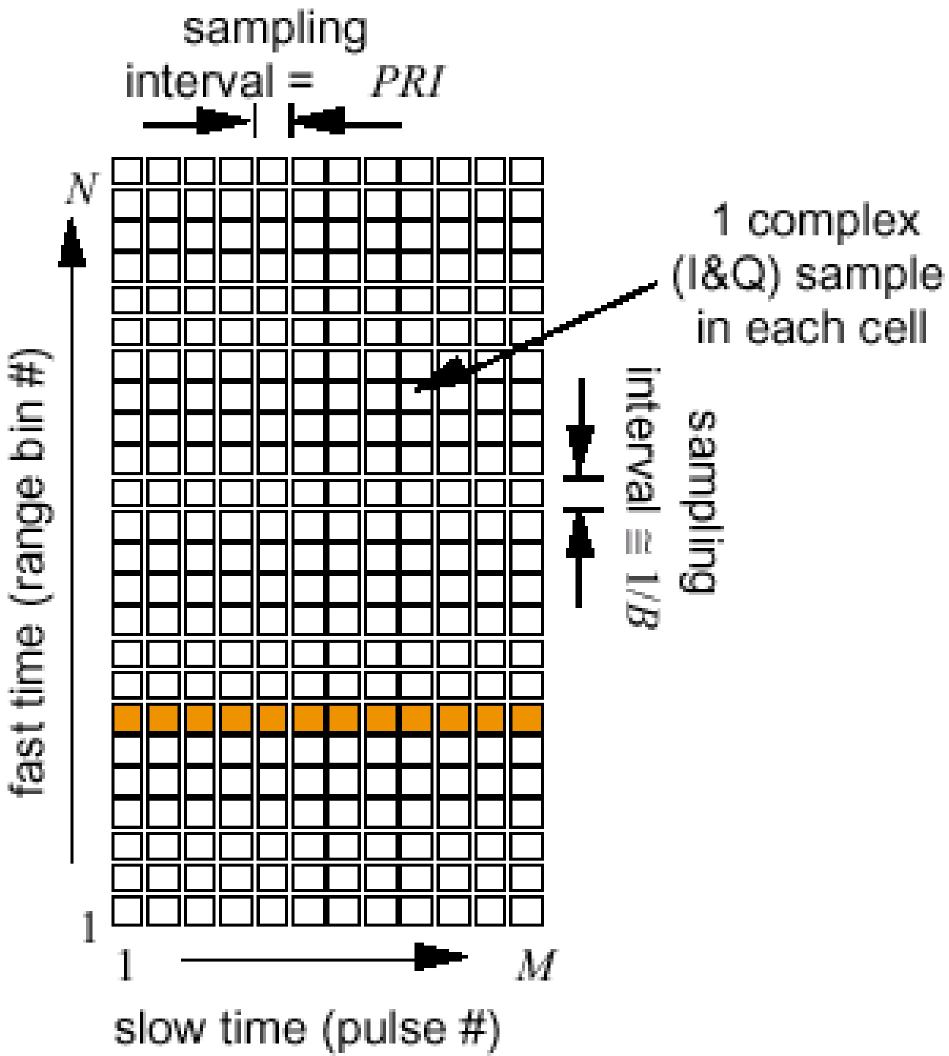

SNR = -10 dB
$|\chi(\tau,\nu)|$
$\nu M T_r$
τ / t_b
X 0
Y 3
Z 1
X 0
Y 1.5
Z 0.23751


SNR = -20 dB
$|\chi(\tau,\nu)|$
$\nu M T_r$
τ / t_b
X 0
Y 3
Z 1
X 0
Y 1.4
Z 0.37467

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

2-D Pulse Doppler Data Matrix

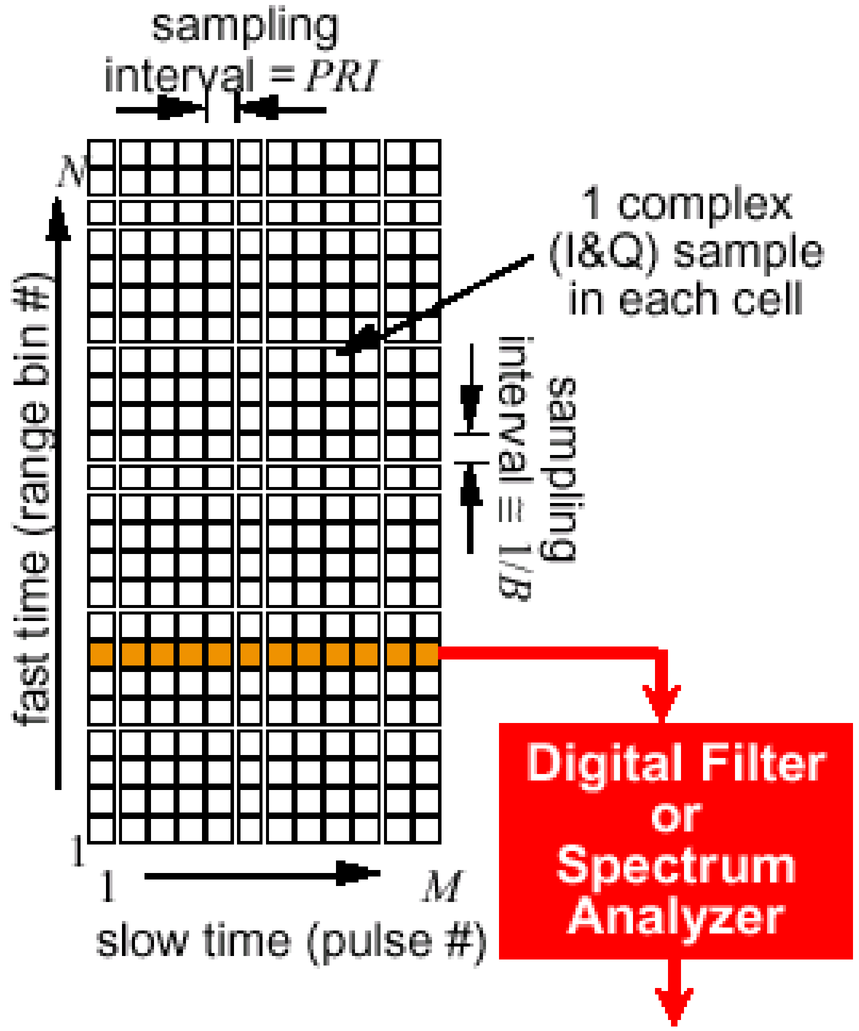
- 2-D data matrix of coherent, baseband returns for M pulses
  - fast time sample rate is 10^5 to 10^8 samp/s
  - slow time sample rate is the PRF, 10^3 to 10^5 samp/s
  - collected using the pulse burst waveform



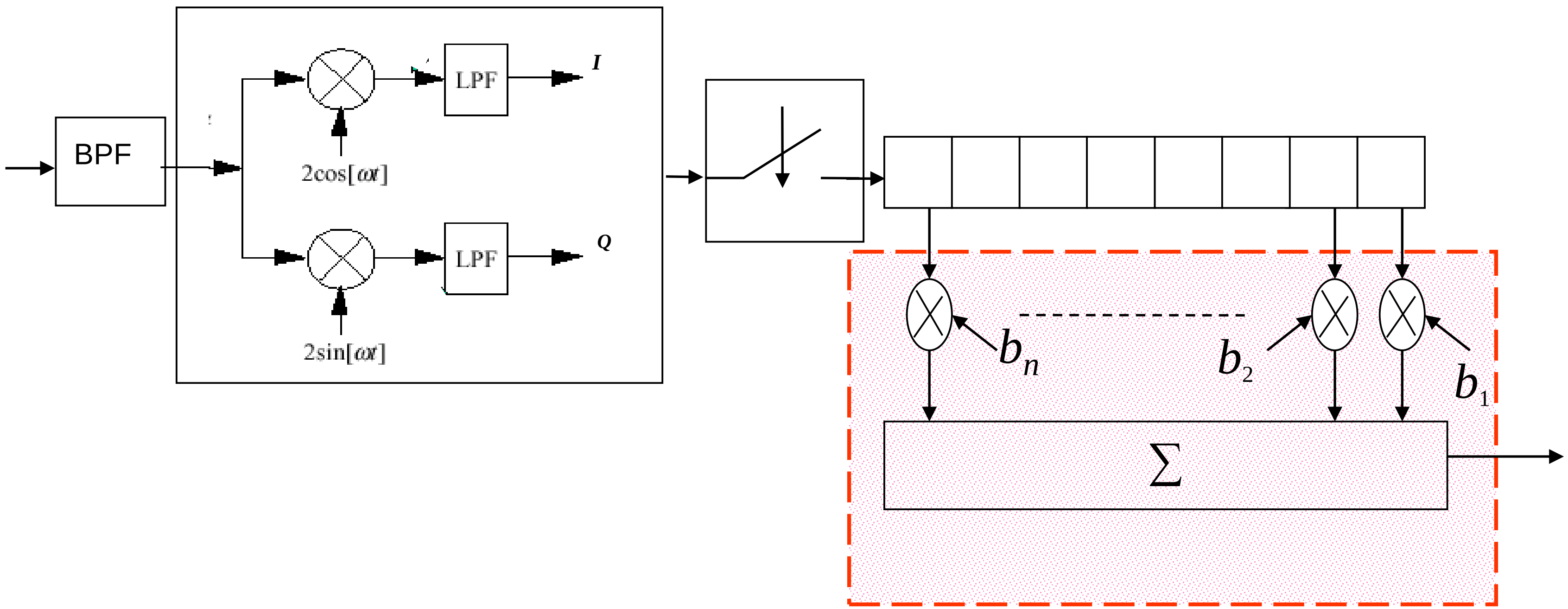
Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

2-D Pulse Doppler Data Matrix

- Doppler processing operates on a row of this matrix
- Two major classes of processing:
  - *MTI* applies a linear filter to the row
  - *Pulse Doppler* applies a spectrum estimator to the row

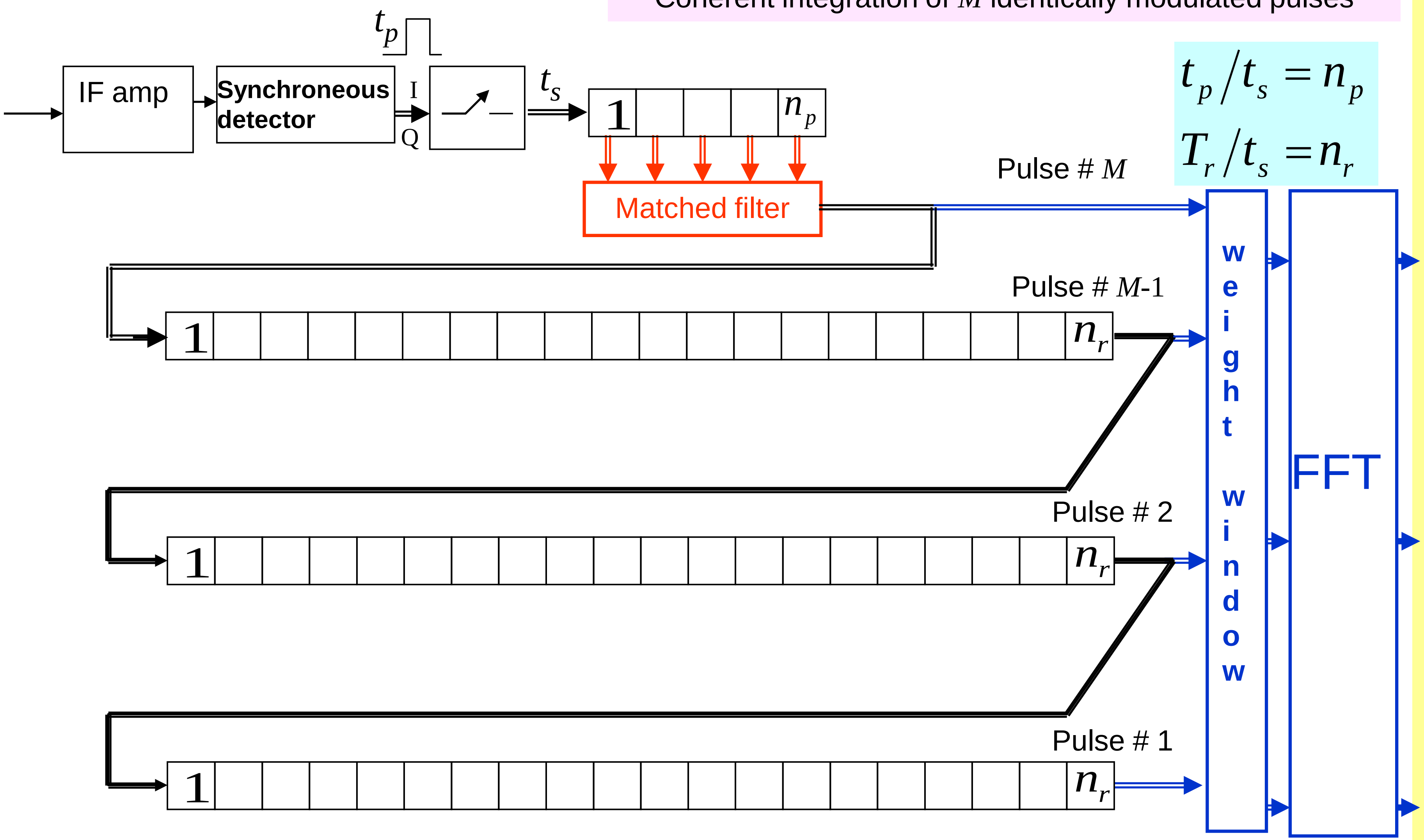


Synchronous detection and matched filtering



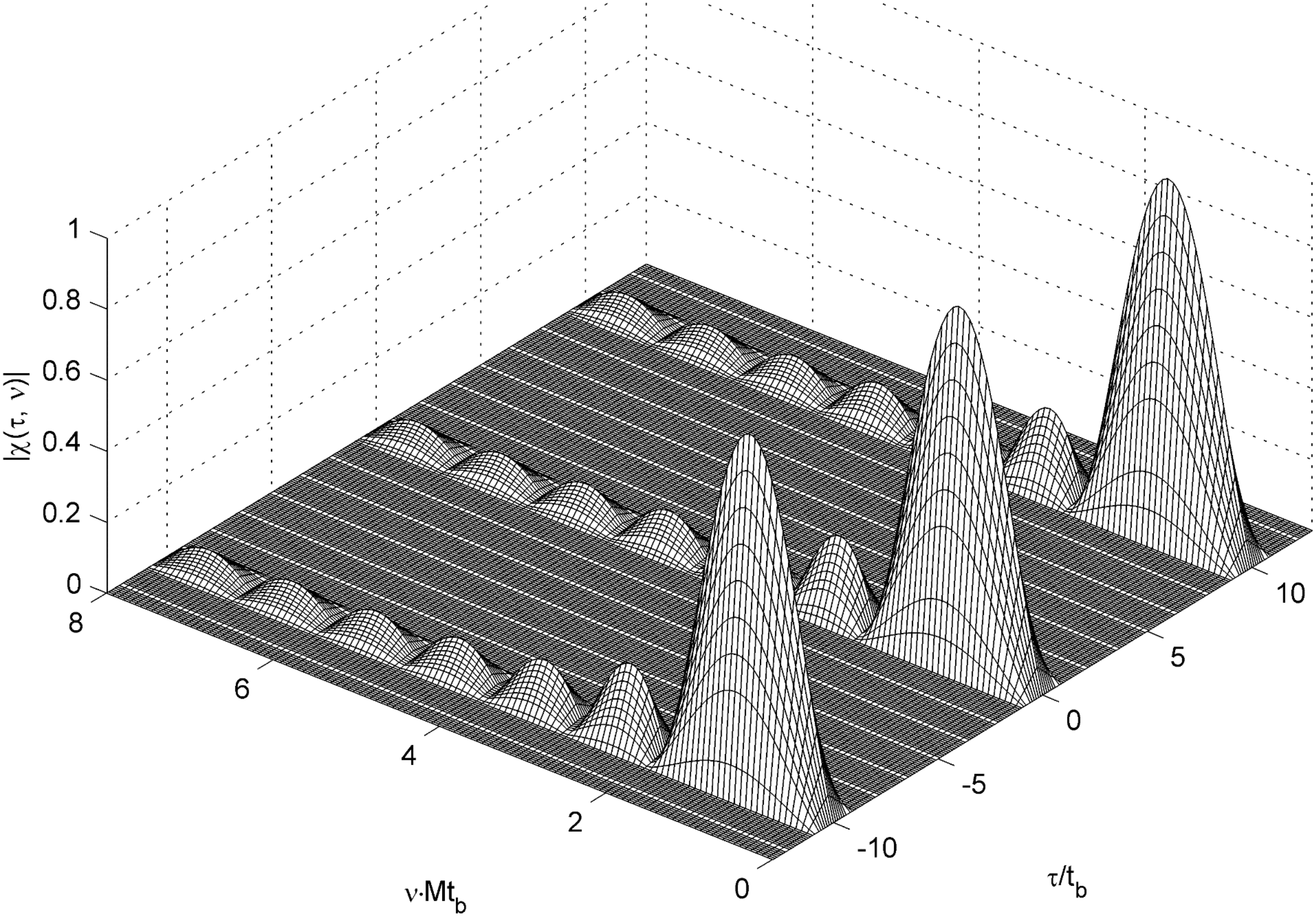
Nadav Levanon, Tel-Aviv University

Coherent integration of M identically modulated pulses

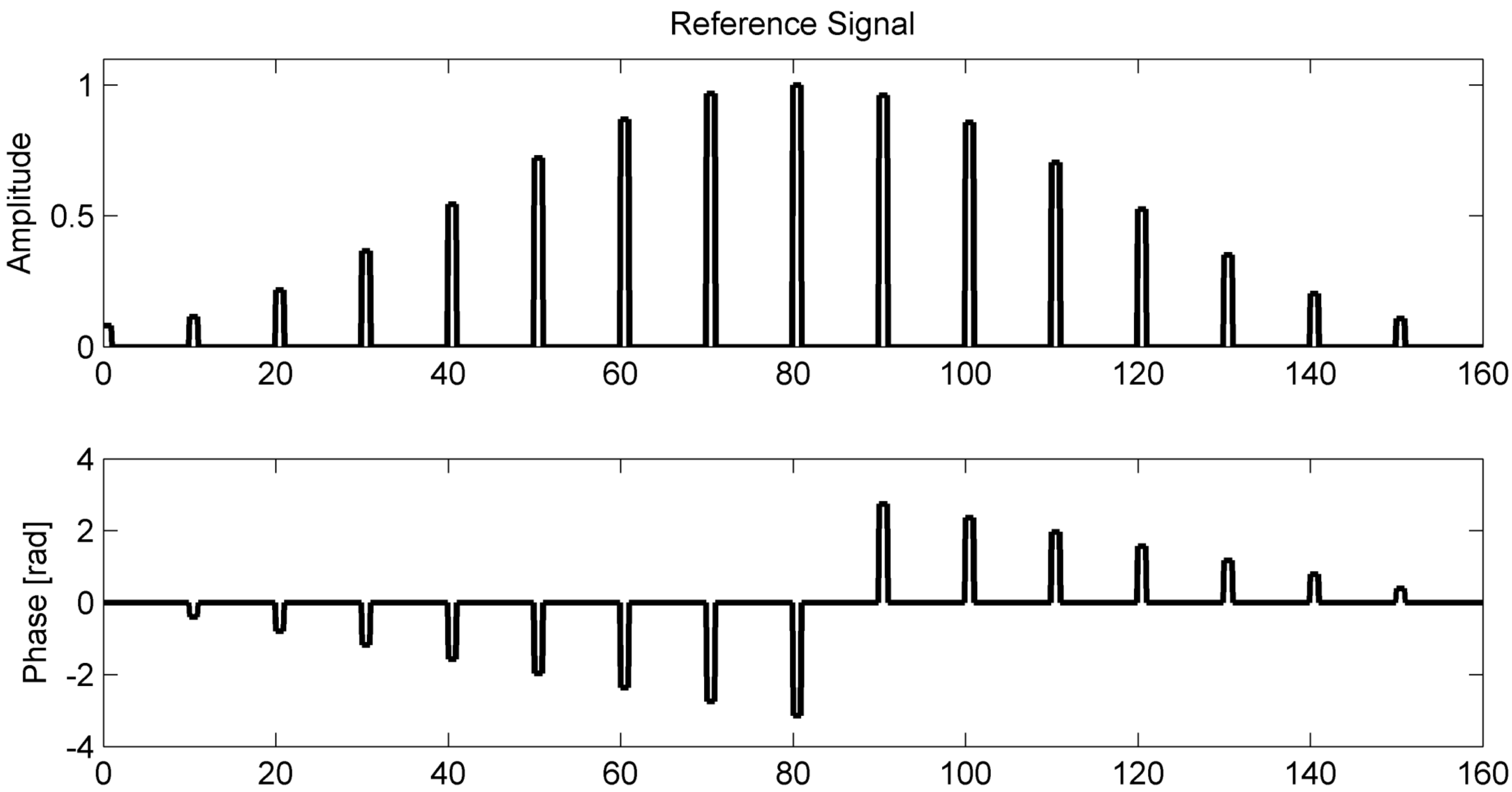


Integration of M pulses

The 2st output of the FFT, with no weighting, produces the delay-Doppler response below. Note that the zero-Doppler clutter is nullified.



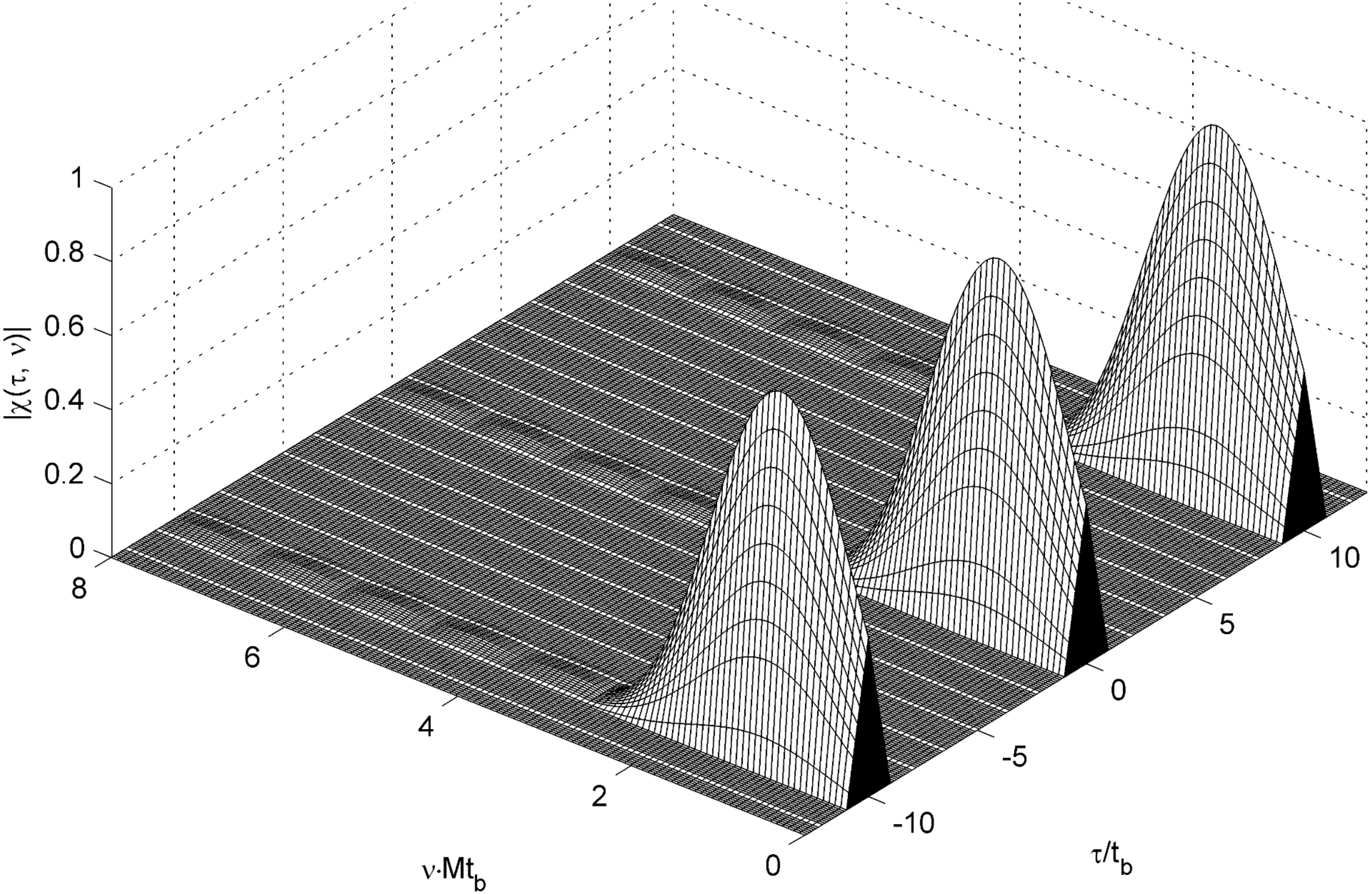
The 2nd output of the FFT, following Hamming weighting, is equivalent to cross correlation with the reference signal below.

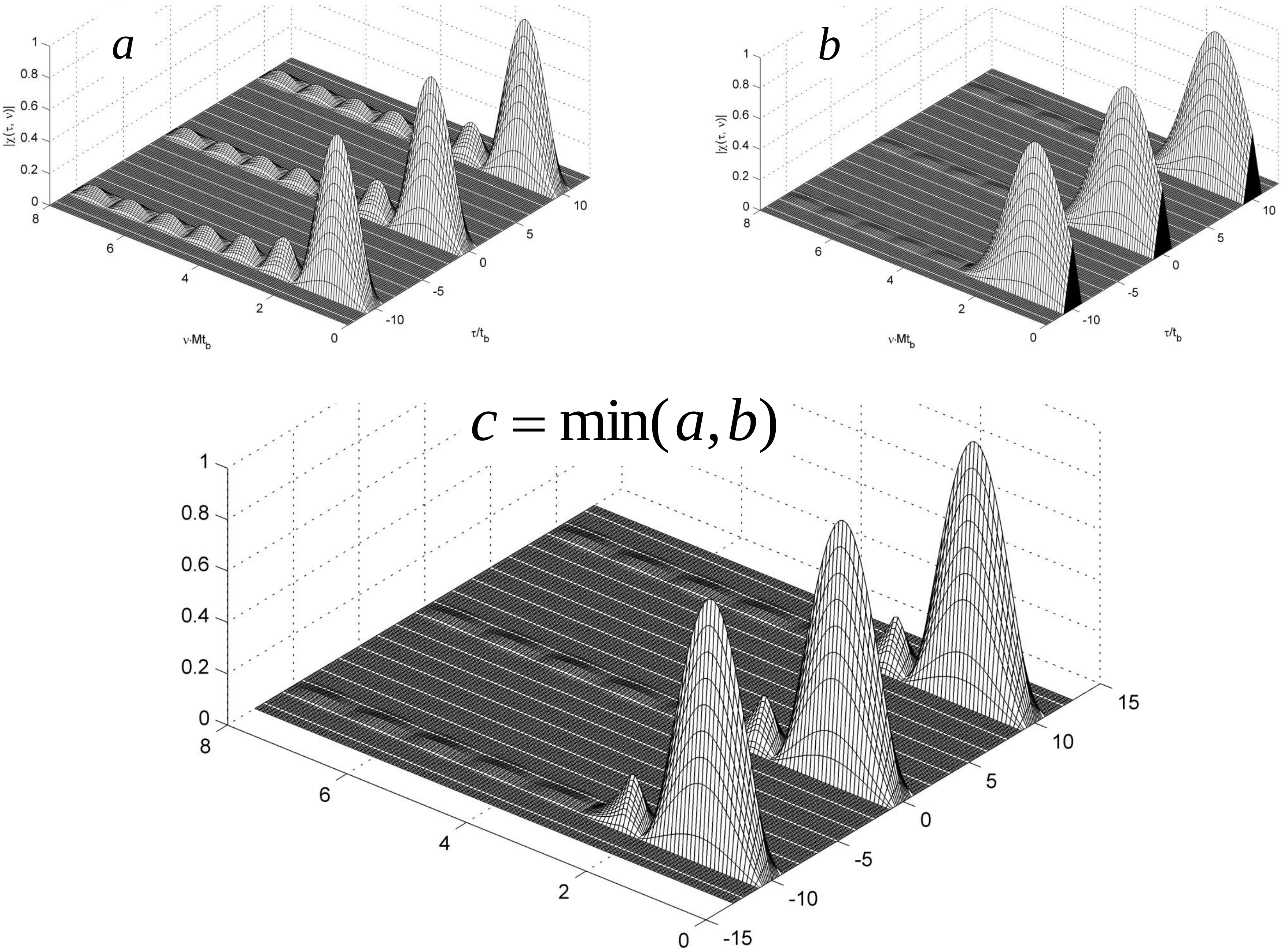


The 2nd output of the FFT, following Hamming weighting, produces the delay-Doppler response below.

Note that the zero-Doppler clutter is not nullified because the main-lobe is widened.

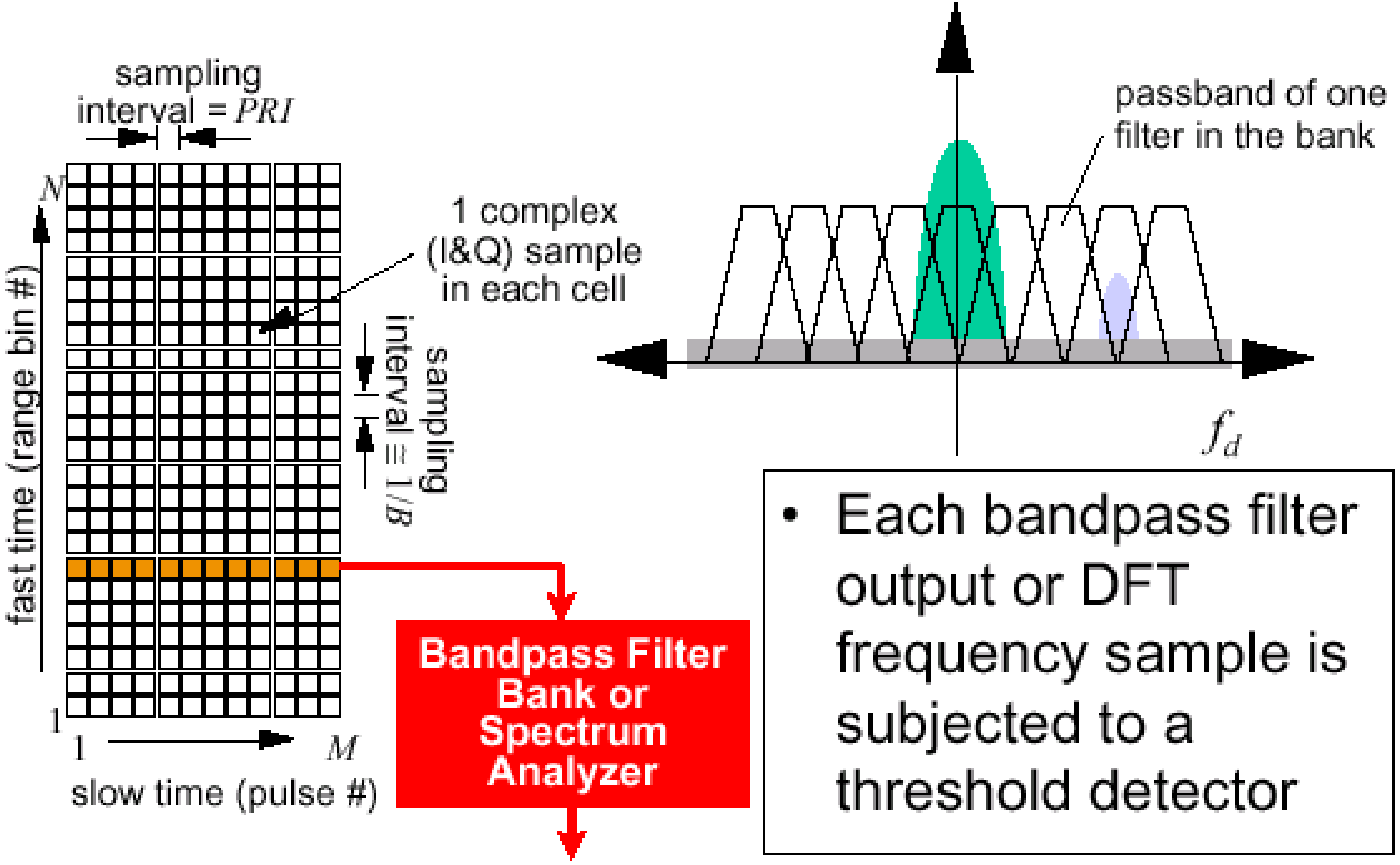
Note the reduced sidelobes.




a
b
$c = \min(a, b)$
|χ(τ, ν)|
ν·Mt_b
τ/t_b
1
0.8
0.6
0.4
0.2
0
8
6
4
2
0
-15
-10
-5
5
10
15

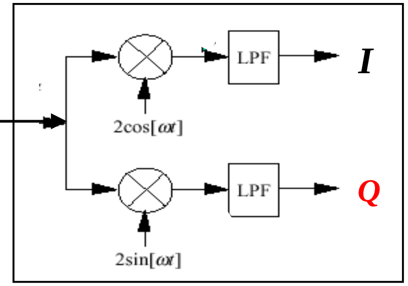
Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

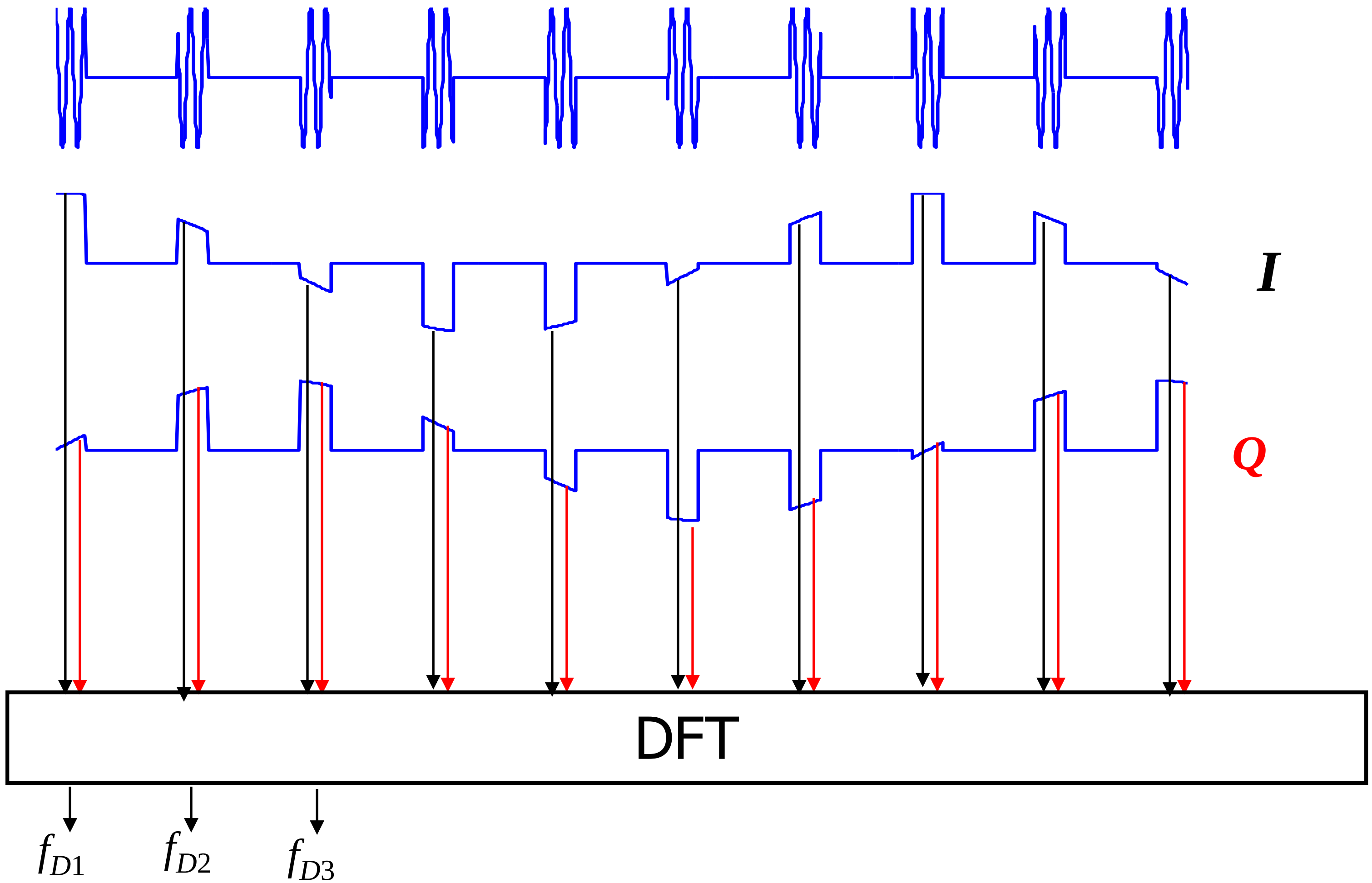
Illustration of Pulse Doppler

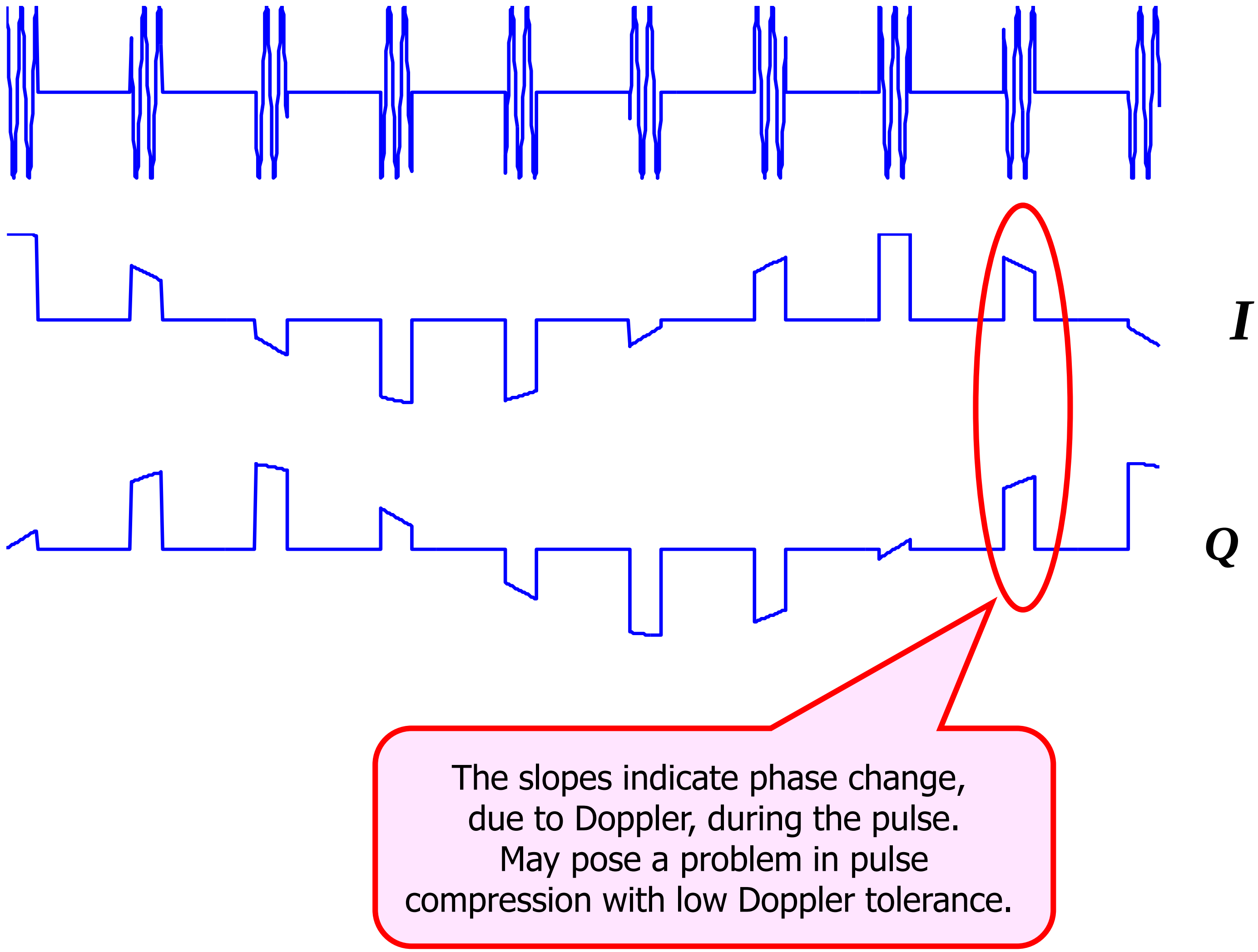


- Each bandpass filter output or DFT frequency sample is subjected to a threshold detector

$$s_r(t) = p(t)\cos\left[\left(\omega + 2\pi f_D\right)t\right]$$

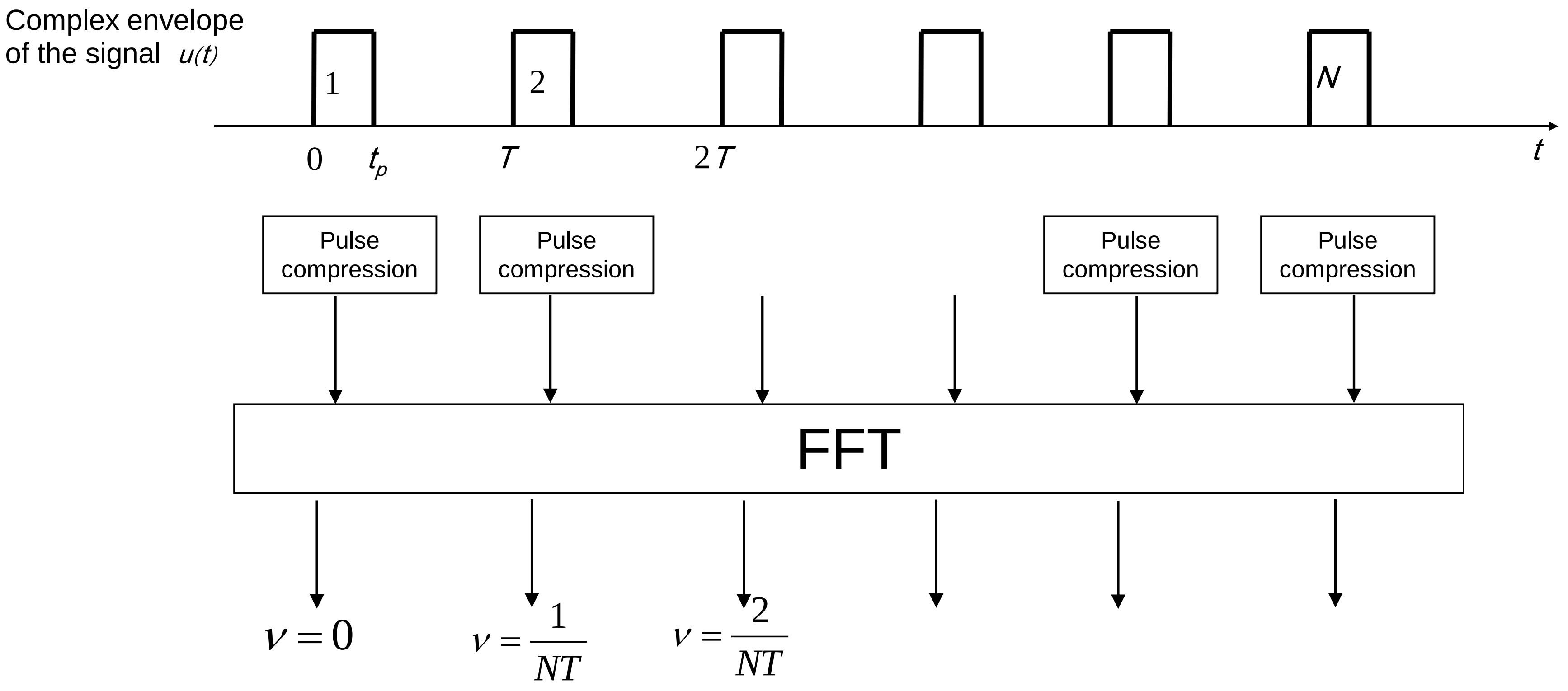


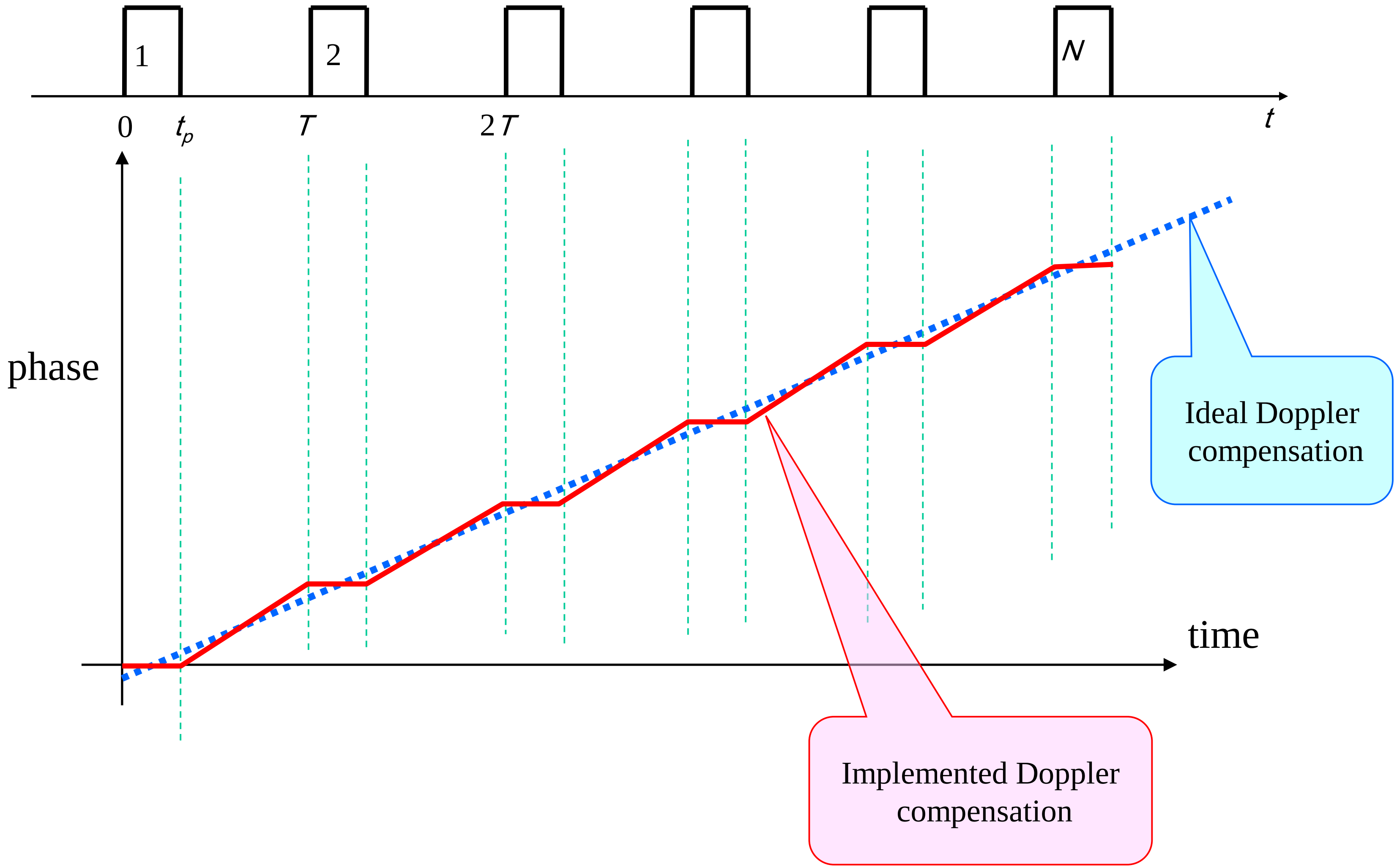



I
Q
The slopes indicate phase change, due to Doppler, during the pulse. May pose a problem in pulse compression with low Doppler tolerance.

DOPPLER TOLERANCE

An efficient implementation of filters matched to several Doppler shifts

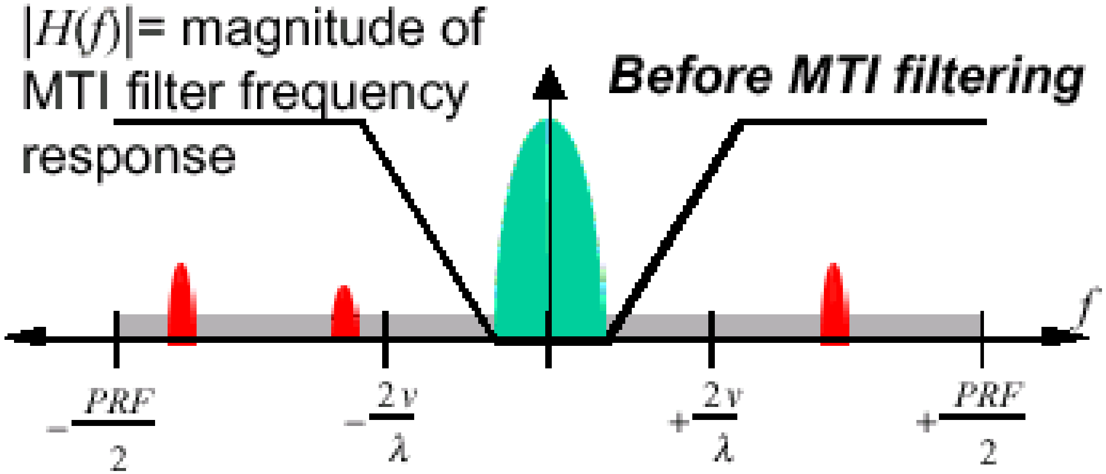


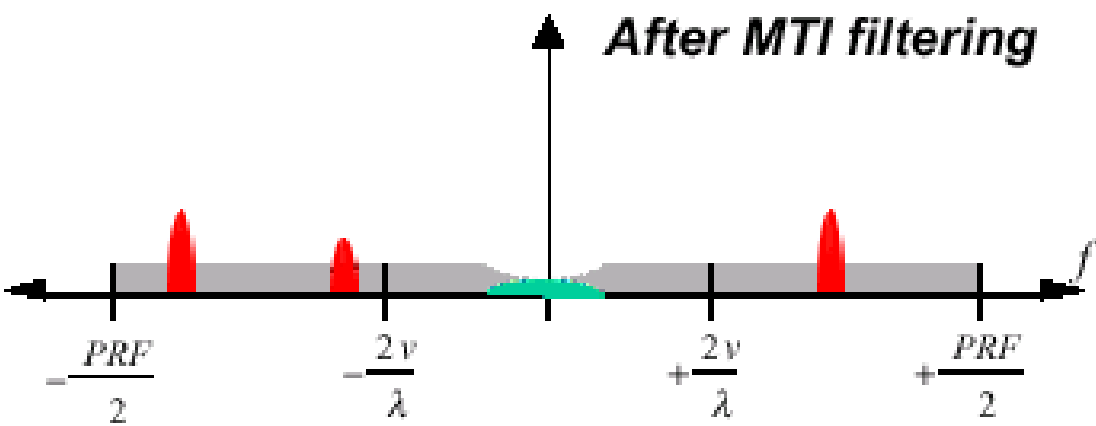


There is no Doppler compensation within each pulse.
Hence, performances degrade with Doppler

The Concept of Moving Target Indication (MTI)

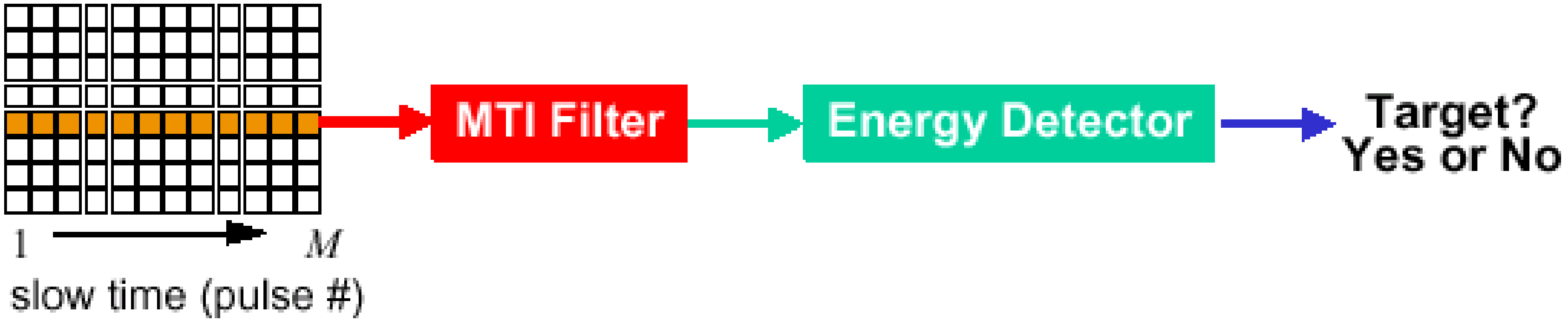
- MTI filtering applies a high-pass filter to the data in each slow-time row
- Filter output retains noise, moving target(s), but has reduced clutter
  - goes to detector next





Pros & Cons of MTI

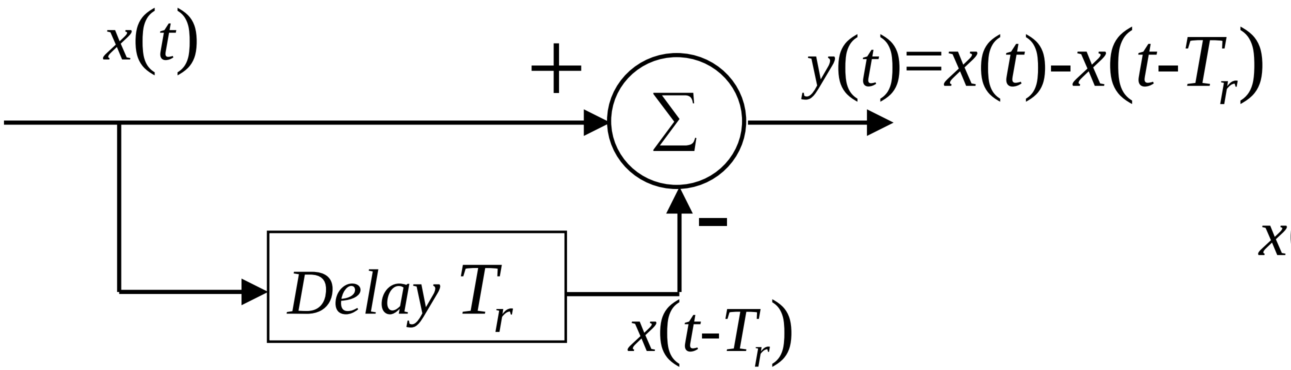
- Indicates only whether a moving target(s) is present
  - doesn't determine velocity
  - doesn't determine approaching/receding
  - doesn't indicate multiple targets
- Advantage: simplicity, light computational load

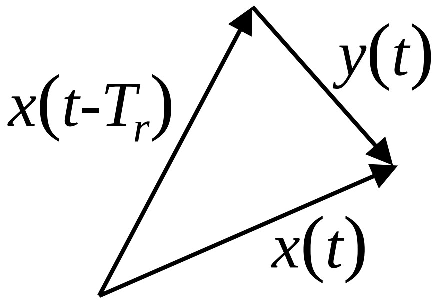


Two-Pulse Canceller Concept

- For a stationary radar and clutter (and no noise), successive pulses produce exactly the same echo
  - so subtracting them should cancel them out
- But moving targets have phase change due to range change, therefore don't cancel
- This concept leads to the two-pulse canceller MTI filter
  - example of finite impulse response (FIR) digital filter
  - periodic frequency response
  - also called "single canceller", "first-order canceller"

2-pulse canceller





$$y(t) = x(t) - x(t - T_r) \quad , \quad T_r \triangleq \text{Pulse Repetition Interval}$$

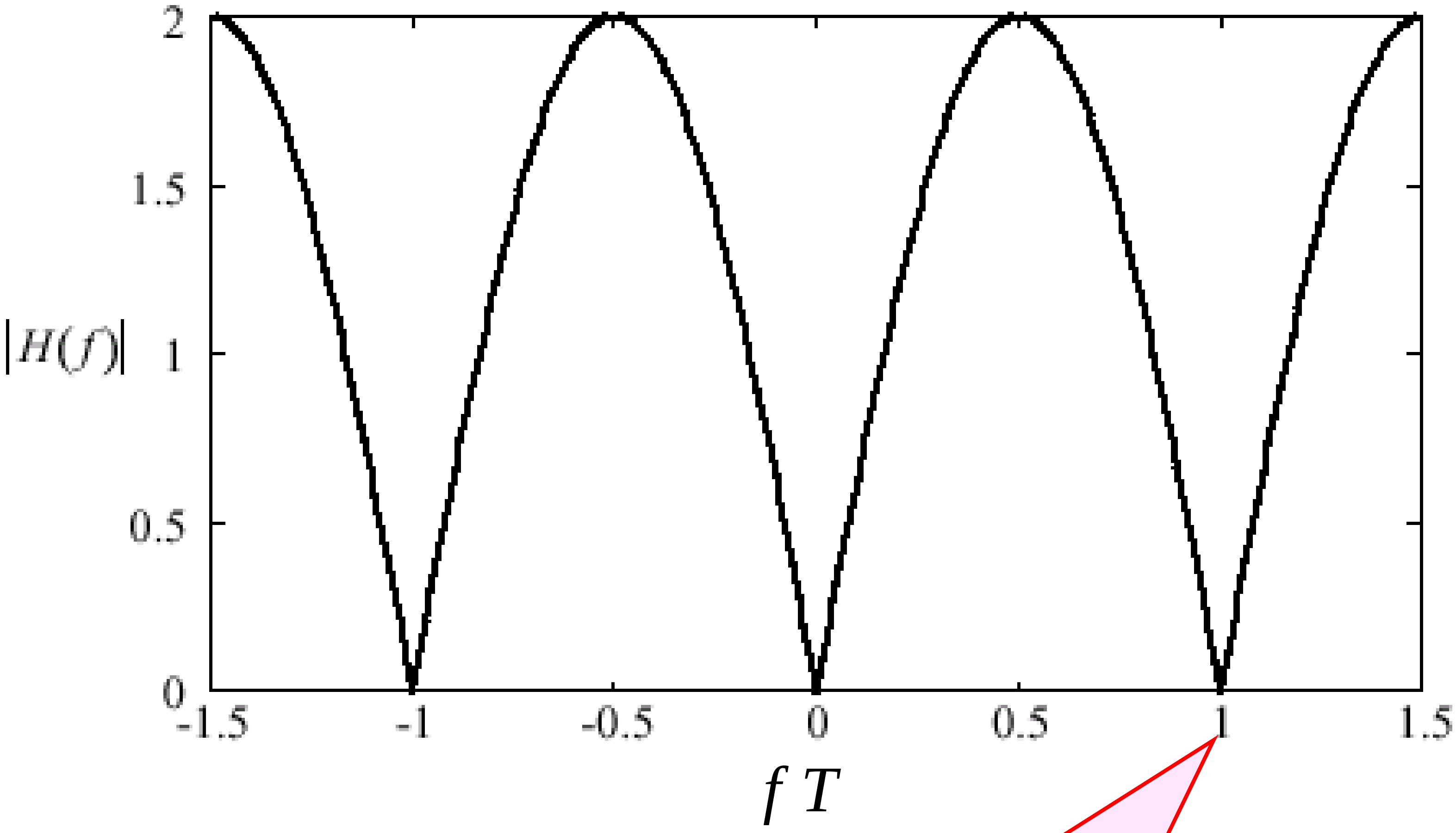
$$Y(\omega) = X(\omega)\left(1 - e^{-j\omega T_r}\right)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 - e^{-j\omega T_r} = e^{-j\omega T_r/2}\left(e^{j\omega T_r/2} - e^{-j\omega T_r/2}\right)$$

$$H(\omega) = e^{-j\omega T_r/2}\, 2j\sin\left(\omega T_r/2\right)$$

$$\left|H(\omega)\right| = 2\left|\sin\left(\omega T_r/2\right)\right|$$

$$|H(\omega)| = 2\left|\sin\left(\omega T_r / 2\right)\right|$$



1'st blind speed
$f_D = 1/T_r$

שאלה (סעיף א׳)

סעיף זה של השאלה עוסק בעיבוד דופלר ע״י FFT של החזרי רכבת פולסים הנקלטים במכ״ם קוהרנטי.
טבלה 1 מציגה את הדגימות הקומפלקסיות של ההחזרים המתקבלים מ 8 השהיות עוקבות (השורות) עבור 16 פולסים עוקבים (העמודות).
להשגת פשטות נבחר בכל ההשהיות ובכל הפולסים החזר רקע זהה (גם בעצמה וגם בפאסה) שערכו הקומפלקסי = 1.
רק בחלון השהיה מס. 12 נוסף לרקע האחיד גם החזר ממטרה נעה במהירות רדיאלית קבועה. עצמת ההחזר מהמטרה היא קבועה (= 1) ורק הפאסה שלו משתנה מפולס לפולס, כרשום בטבלה 1.

TABLE 1	Complex input data from 16 consecutive pulses															
Pulse # / Delay/dt	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	$1+e^{j0}$	$1+e^{j\pi/8}$	$1+e^{j2\pi/8}$	$1+e^{j3\pi/8}$	$1+e^{j4\pi/8}$	$1+e^{j5\pi/8}$	$1+e^{j6\pi/8}$	$1+e^{j7\pi/8}$	$1+e^{j8\pi/8}$	$1+e^{j9\pi/8}$	$1+e^{j10\pi/8}$	$1+e^{j11\pi/8}$	$1+e^{j12\pi/8}$	$1+e^{j13\pi/8}$	$1+e^{j14\pi/8}$	$1+e^{j15\pi/8}$
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	16 inputs FFT															

עבור כל השהיה (שורה בטבלה 1) מוזנים (ישירות ללא פונקציית משקול) הערכים המתאימים מ 16 הפולסים למעבד FFT בעל 16 כניסות ו 16 יציאות. מוצא ה FFT מתעדכן אחרי כל השהיה ומתקבלת טבלה 2, המציגה את הערך המוחלט של מוצאי ה FFT . התוצאה היא מפת השהייה - דופלר .

בטבלה 2 חסרים הערכים המתאימים לחלון ההשהיה מס. 12.

יש לחשב את כל הערכים החסרים בשורה (ולכתוב אותם כשורה במחברת הבחינה ולא בטופס שאלון הבחינה). יש להסביר את החישוב, ואת משמעות התוצאה שנתקבלה.

עזרה ותזכורת: במוצא עיבוד ע״י FFT המקבל N כניסות מN פולסים עוקבים, היציאה מספר 0 איננה מקזזת שום הסט דופלר (רמפת פאסה) אלא פשוט מסכמת את N הכניסות. יציאה מספר 1 מקזזת הסט דופלר המשלים בדיוק מחזור דופלר אחד במשךN הפולסים (בלומר רמפת פאסה המשלימה 2π במשך N הפולסים). יציאה מספר 2 מקזזת הסט דופלר המשלים בדיוק 2 מחזורי דופלר במשך N הפולסים, וכן הלאה.

TABLE 2	**Output data after 16 input FFT: Absolute intensities**															
Doppler filter # / **Delay/dt**	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
18	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	**16**	**16**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
11	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

<u>**תשובה מילולית**</u>:

ה FFT מבצע פעולה לינארית ולכן אפשר להפריד הפעולה על הרקע הקבוע בחלון השהיה 12, שהיא זהה למה שקורה בשאר חלונות ההשהיה. כלומר סיכום 16 ערכים של 1. לבין הפעולה על ההחזר מהמטרה. רמפת הפאסה עקב המטרה מתאימה <u>בדיוק</u> לקיזוז שמבצע ה FFT כשהוא יוצר את מוצא ה FFT מס. 1. אחרי הקיזוז מתבצע גם כאן סיכום 16 ערכים של 1 ולכן המוצא המתאים לדופלר 1 יהיה גם הוא שווה ל 16.

<u>**תשובה מבוססת על נוסחאת ה FFT**</u>:

ב FFT בעל N כניסות $x[n],\ n=0,1,2,\ldots,N-1$ ו N יציאות $X[k],\ k=0,1,2,\ldots,N-1$

היציאה ה k נתונה ע"י:

$$X[k]=\sum_{n=0}^{N-1}x[n]\exp\left(-j2\pi k\,n/N\right)$$

בכל ההשהיות, <u>**מלבד בהשהיה ה 12**</u>, כל 16 הכניסות ל FFT הן פאזור קבוע בעוצמה ובפאסה הנתון ע"י 1. ובהן מתקיים לכן:

$$X[0]=\sum_{n=0}^{16-1}1\bullet\exp\left(-j0\right)=\mathbf{16}\quad,\quad X[k]=\sum_{n=0}^{16-1}1\bullet\exp\left(-j2\pi k\,n/16\right)=\mathbf{0},\ k=1,2,\ldots,15$$

<u>**בהשהיה ה 12**</u> נתון כי 16 דגימות הכניסה ל FFT נתונות ע"י:

$$x[n]=1+\exp\left(j2\pi\,n/16\right),\ n=0,1,2,\ldots,15$$

הצבתן במשוואת ה FFT תתן:

$$X[k]=\sum_{n=0}^{15}\left[1+\exp\left(j2\pi\,n/16\right)\right]\exp\left(-j2\pi k\,n/16\right)=\begin{cases}\mathbf{16} & k=0,1\\ \mathbf{0} & k=2,3,\ldots,15\end{cases}$$

<u>סעיף ב׳</u>

סעיף ב׳ שונה מסעיף א׳ בכך שהעיבוד כולל תחילה מעגל MTI (מסוג 2-pulse canceller , בסיסי) שאחריו מעבד ה FFT .
טבלת ערכי הכניסה מוצגת בטבלה 3 . היא שונה מטבלה 1 רק ע״י הוספת עמודה המייצגת את פולס 0, הקודם לפולס 1. העמודה נדרשת כי מעגל ה MTI זקוק לפולס אחד קודם.

TABLE 3	Complex input data from 17 consecutive pulses (pulse 0 = pulse 16)																
Pulse #	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Delay/dt																	
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	$1+e^{j15\pi/8}$	$1+e^{j0}$	$1+e^{j\pi/8}$	$1+e^{j2\pi/8}$	$1+e^{j3\pi/8}$	$1+e^{j4\pi/8}$	$1+e^{j5\pi/8}$	$1+e^{j6\pi/8}$	$1+e^{j7\pi/8}$	$1+e^{j8\pi/8}$	$1+e^{j9\pi/8}$	$1+e^{j10\pi/8}$	$1+e^{j11\pi/8}$	$1+e^{j12\pi/8}$	$1+e^{j13\pi/8}$	$1+e^{j14\pi/8}$	$1+e^{j15\pi/8}$
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	2 – pulse canceller																
		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
		16 inputs FFT															

בעיבוד זה מוזנים ל FFT 16 הערכים הקומפלקסים המתקבלים ממוצא מעגל ה MTI . (שוב ישירות ללא פונקציית משקול.) מוצא ה FFT מתעדכן אחרי כל השהיה ומתקבלת טבלה 4, המציגה את הערך המוחלט של מוצאי ה FFT . התוצאה היא מפת השהייה - דופלר חדשה השונה מהמפה שהתקבלה בסעיף א׳.

TABLE 4	Output data after a 2-pulse canceller followed by a 16 inputs FFT: Absolute intensities															
Doppler filter # / Delay/dt	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
18	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	**0**	**6.243**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
11	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

בטבלה 4 חסרים הערכים בעמודת דופלר 0 ובשורה המתאימה לחלון השהייה מס. 12 (בו נמצאת המטרה הנעה).

יש לחשב את כל הערכים החסרים בשורה ובעמודה. יש להסביר את החישוב, ואת כל משמעויות התוצאה שנתקבלה (גם חסרונות וגם יתרונות של העיבוד בסעיף ב׳ לעומת סעיף א׳).

<u>תשובה מילולית</u> :

ה MTI מחסיר את הרקע הזהה מכל שני פולסים עוקבים ולכן המוצא שלו = 0 וזה מה שמוזן ל FFT.

לכן גם כל מוצאי ה FFT בכל ההשהיות הם 0.

החריג הוא מוצא ה 2-pulse canceller לגבי ההחזר מהמטרה הנעה בהשהיה 12. כאן מוזנים ל FFT 16 פאזורים זהים בעצמתם (שאינה שווה ל 1) והמסתובבים גם הם באותו קצב כמו הפאזורים בכניסה ל MTI.

את הערך המוחלט של הפאזורים הללו אפשר לחשב בשתי דרכים:

א. מציאת הערך המוחלט של ההפרש בין שני פאזורים שכנים, למשל:

$$\left|e^{j\pi/8}-e^{j0}\right|=0.3902 \quad \Rightarrow \quad 16*0.3902=6.243$$

ב. ניצול נוסחאת תגובת התדר של 2-pulse canceller :

$$\left|H(\omega)\right|=2\left|\sin\left(\omega T_r/2\right)\right|$$

$$\omega T_r=2\pi/16 \;\Rightarrow\; \left|H(\omega)\right|=2\left|\sin\left(\pi/16\right)\right|=0.3902 \Rightarrow \quad 16*0.3902=6.243$$

$$X\left[k\right]=\sum_{n=0}^{N-1}x\left[n\right]\exp\left(-j2\pi k\,n/N\right)$$

<u>תשובה מבוססת על נוסחאת ה FFT</u> :

לאחר מעגל ה MTI בכל שורה, מכל דגימה בשורה מוחסרת (קומפלקסית) הדגימה שלשמאלה שהיא הדגימה מהפולס הקודם. לפיכך כל החזרי הרקע הנתונים ע״י הפאזור הקבוע (=1) מתאפסים. כאשר כל הכניסות ל FFTשוות לאפס גם כל היציאות שוות לאפס.
רק בשורה 12 הכניסות לא מתאפסות הודות למטרה הנעה והדגימות נתונות ע״י הביטויים :

$$x\left[n\right]=\exp\left(j2\pi\,n/16\right)-\exp\left(j2\pi\left(n-1\right)/16\right),\ n=1,\ 2,\ \ldots,16$$

הצבה במשוואת ה FFT תתן

$$X\left[k\right]=\sum_{n=0}^{15}\left[\exp\left(j2\pi\,n/16\right)-\exp\left(j2\pi\left(n-1\right)/16\right)\right]\exp\left(-j2\pi k\,n/16\right)=\begin{cases}\boldsymbol{y} & k=1\\ \mathbf{0} & k=0,2,3,\ldots,15\end{cases}$$

חישוב הערך המוחלט של y המתקבל כאשר $k=1$:

$$y=X\left[1\right]=\sum_{n=0}^{15}\left[\exp\left(j2\pi\,n/16\right)-\exp\left(j2\pi\left(n-1\right)/16\right)\right]\exp\left(-j2\pi\,n/16\right)$$

$$=\sum_{n=0}^{15}\left[1-\exp\left(-j\,2\pi/16\right)\right]=16\left[1-\exp\left(-j\,2\pi/16\right)\right]$$

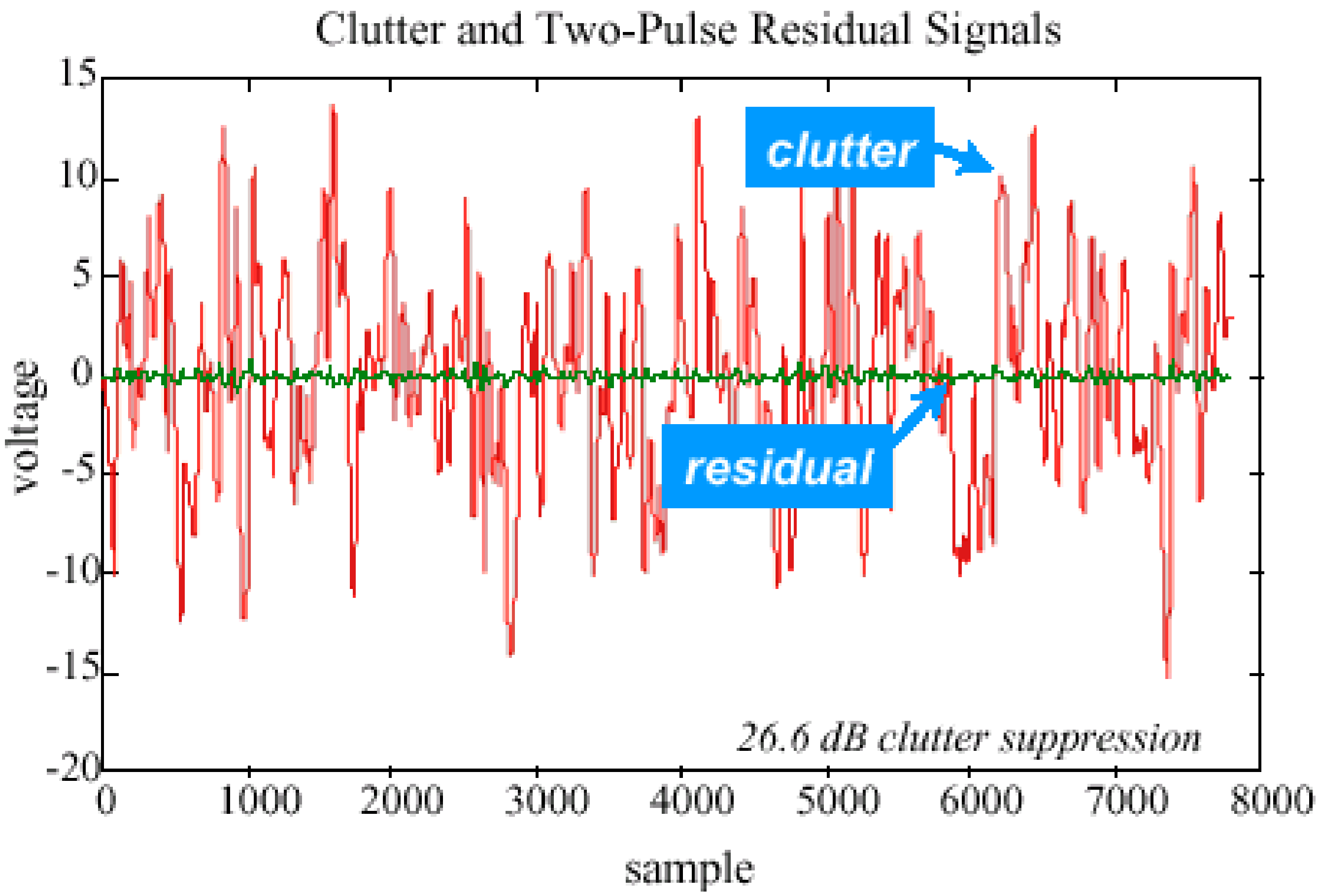
$$=16\exp\left(-j\,\pi/16\right)\left[\exp\left(j\,\pi/16\right)-\exp\left(-j\,\pi/16\right)\right]=16\exp\left(-j\,\pi/16\right)2\,j\sin\left(\pi/16\right)$$

$$\left|\boldsymbol{y}\right|=\left|X\left[1\right]\right|=32\sin\left(\pi/16\right)=\mathbf{6.2429}$$

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Example: Two-Pulse Canceller Clutter Attenuation

- Clutter represented by correlated noise
- 26.6 dB clutter energy reduction



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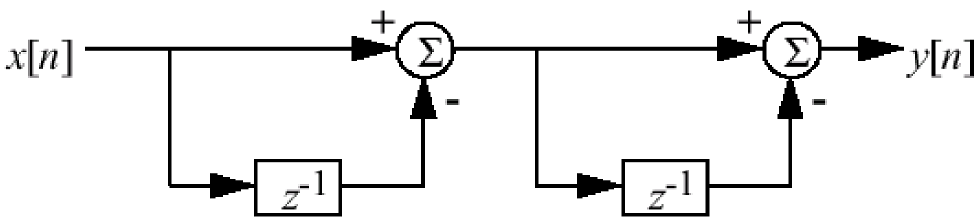
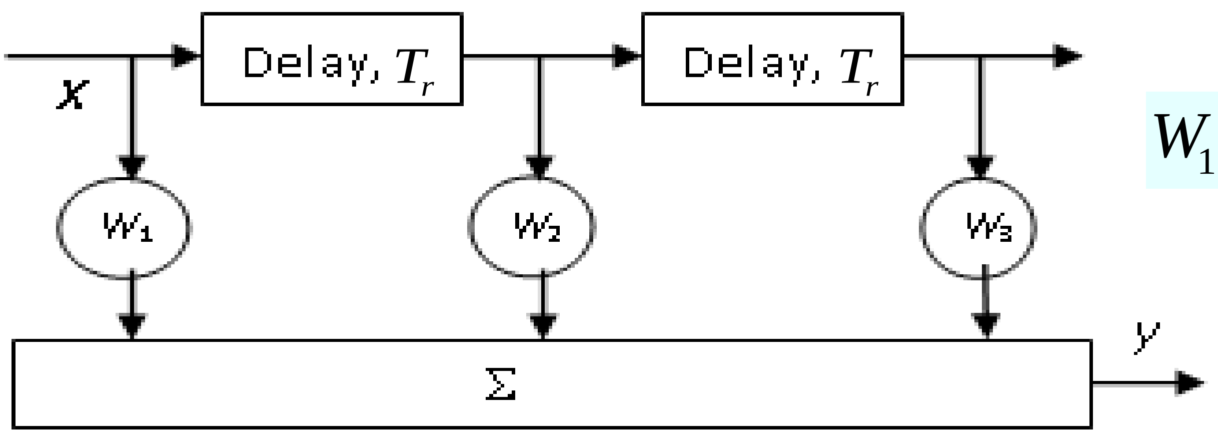
Three-Pulse Canceller

$$\exp(-j\omega T_r) = z^{-1}$$

- A three-pulse (double, second-order) canceller can be formed by cascading two first-order sections
- Transfer function $H(z) = 1\text{-}2z^{-1}+z^{-2}$

$$\left(1-z^{-1}\right)\left(1-z^{-1}\right) = 1-2z^{-1}+z^{-2}$$





$$W_1 = 1,\ W_2 = -2,\ W_3 = 1$$

$$T_r = \begin{cases} \text{Pulse Repetition Interval (PRI) ,} & \text{Pulse Radar} \\ \text{Waveform period ,} & \text{Continuous Wave (CW) radar} \end{cases}$$

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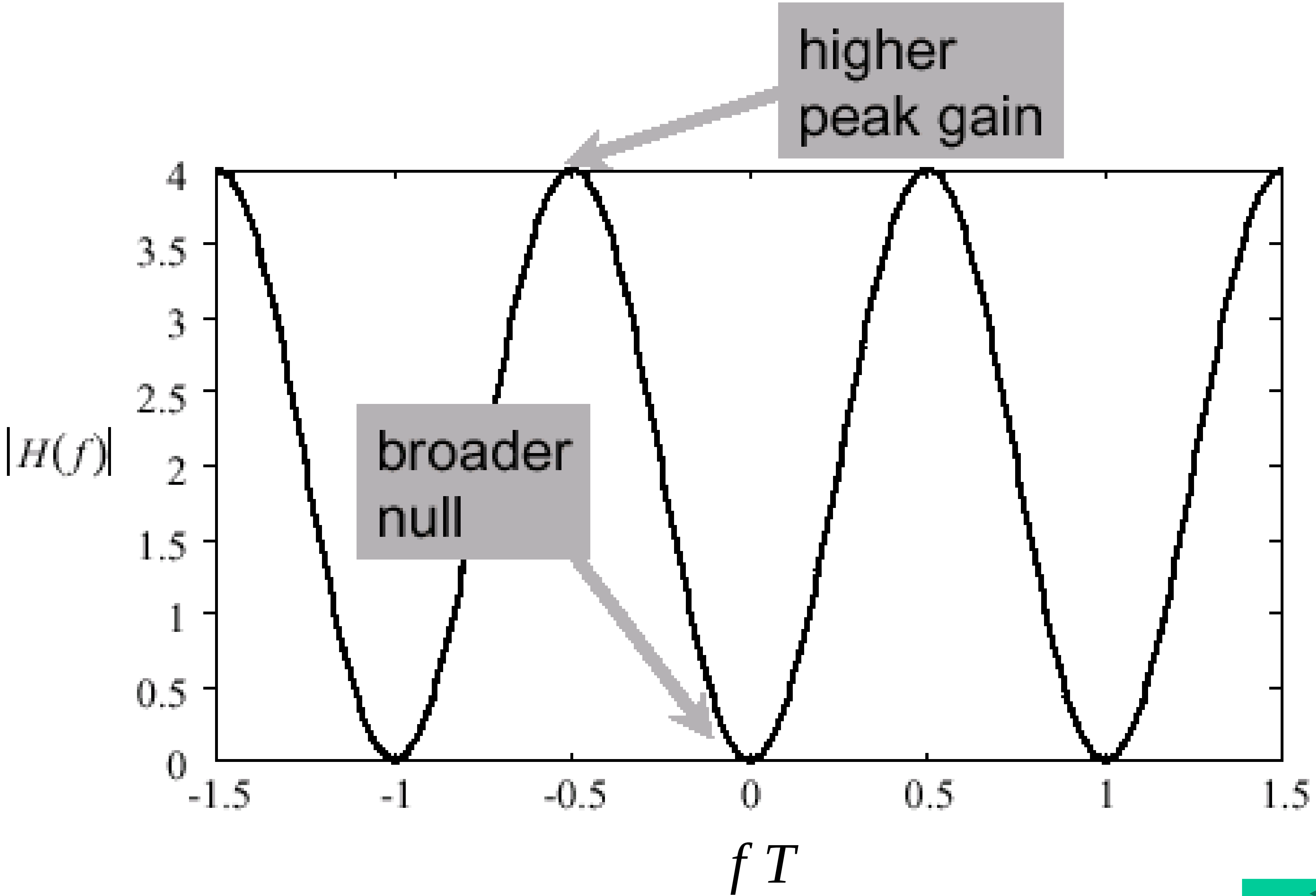
Three-Pulse Canceller

$$\left|H(\omega)\right| = 4\sin^2\left(\omega T_r / 2\right)$$



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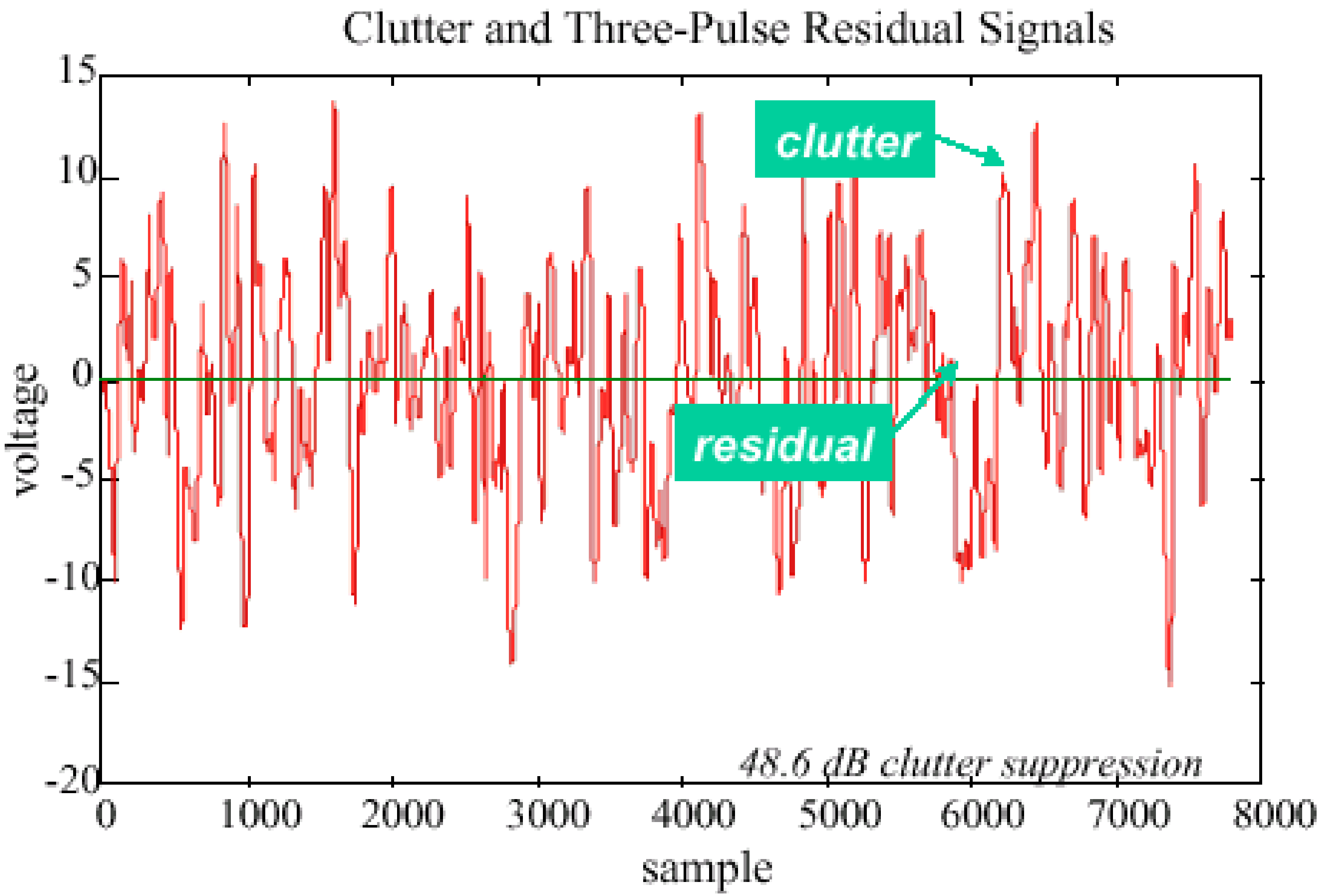
Example: Three-Pulse Canceller

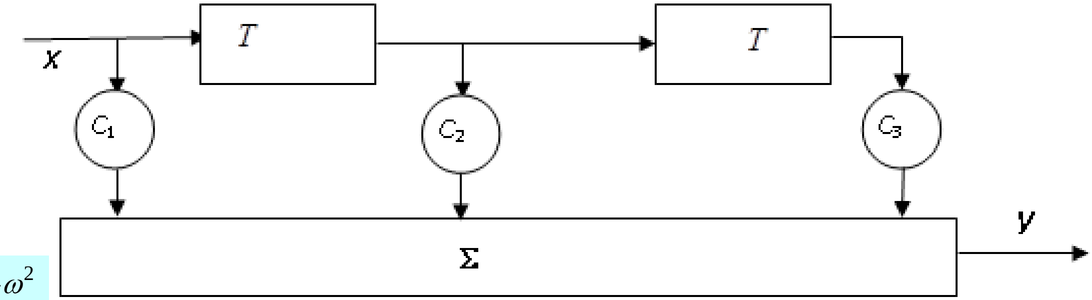
- 48.6 dB clutter attenuation



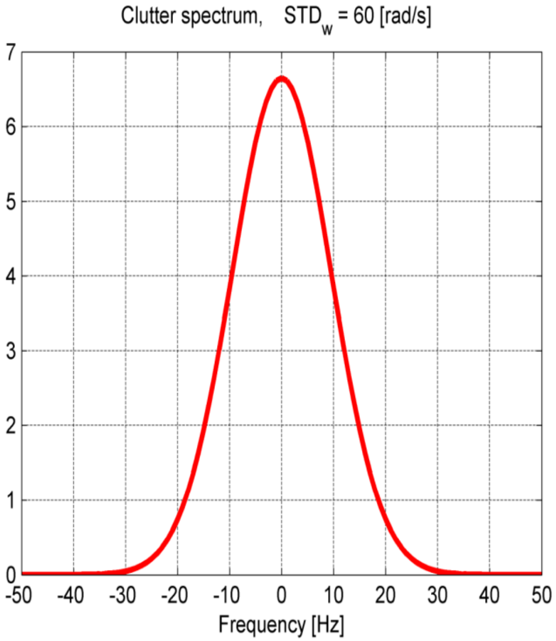
Modified 3-pulse canceller

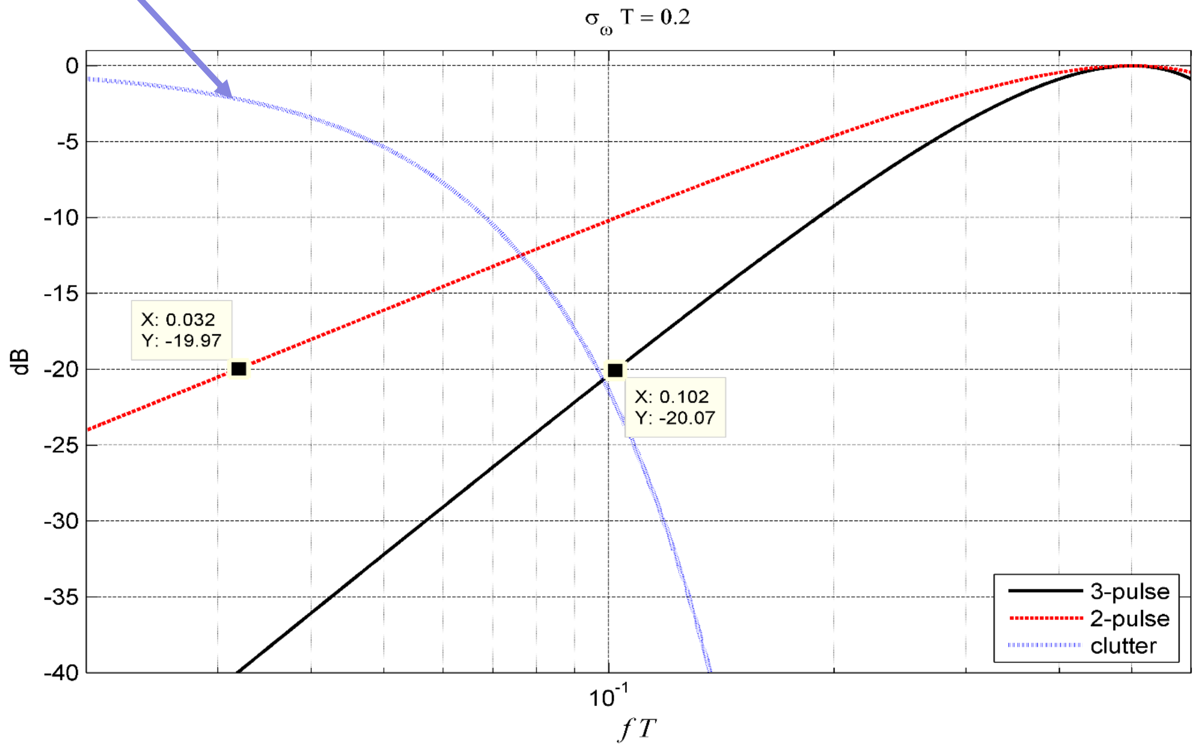
$$c_1 = 1, \quad c_2 = -(1+a), \; c_3 = a$$

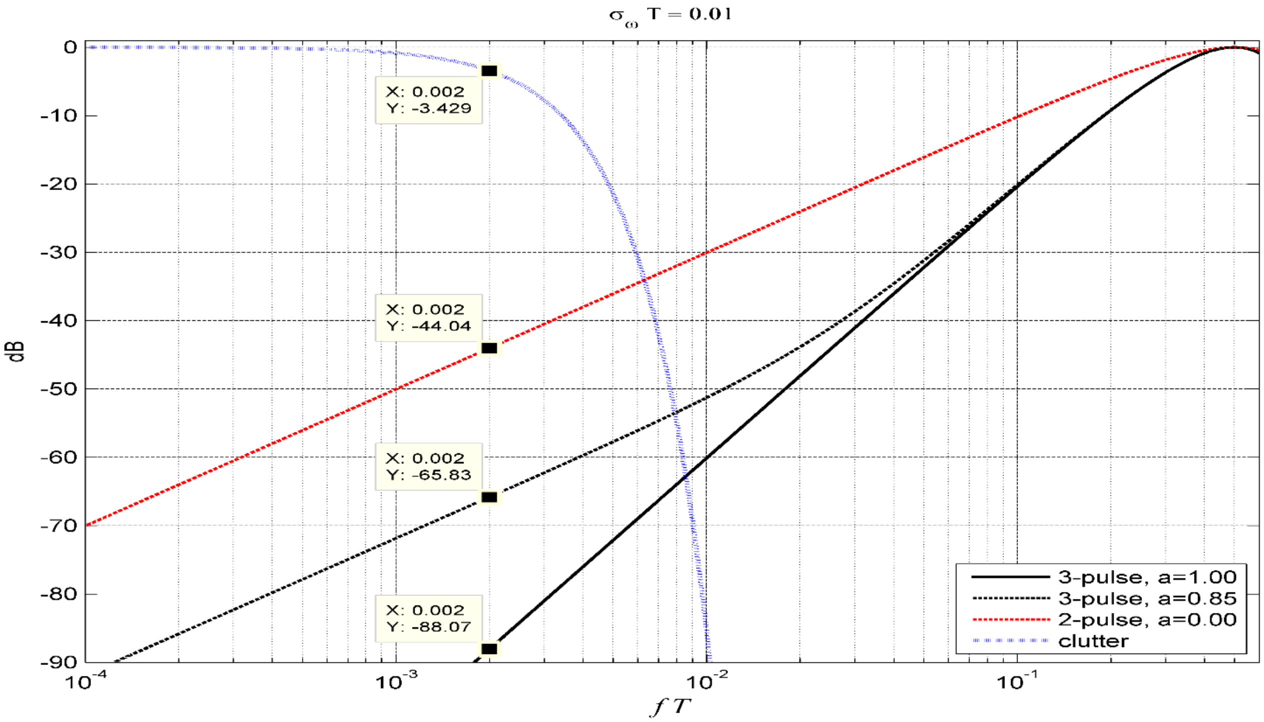
$$a = 1$$

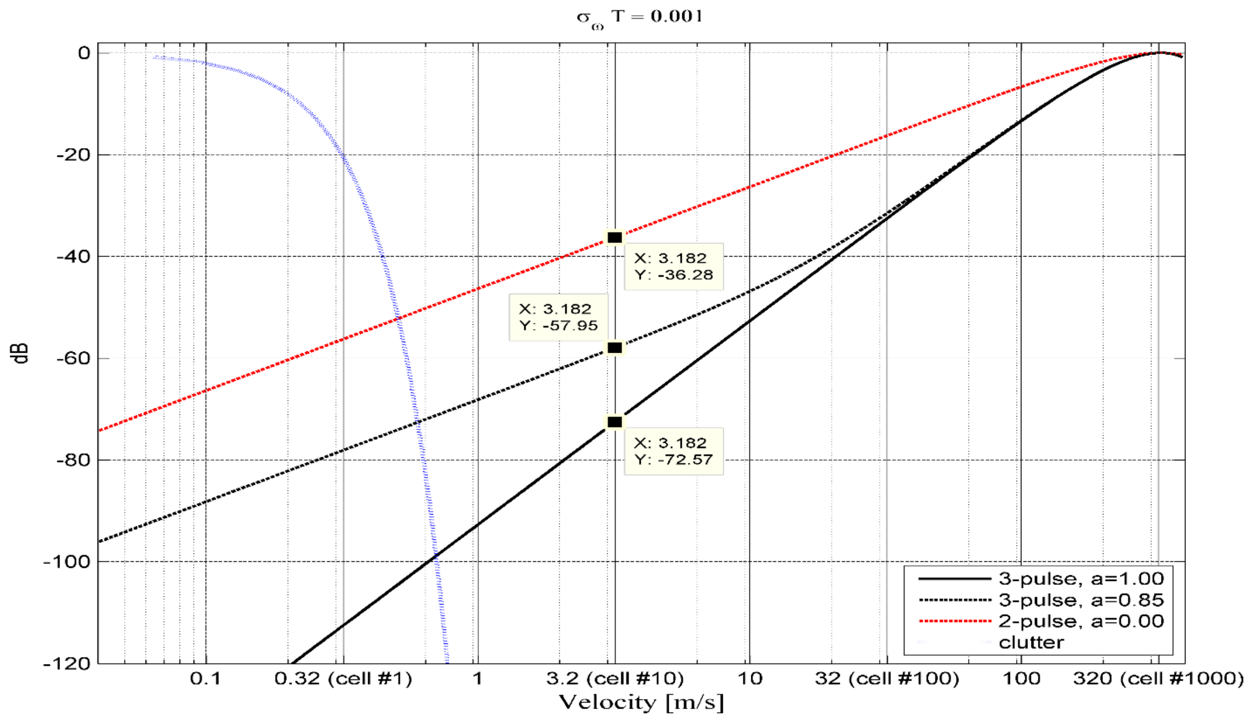


$$S_c(\omega) = \frac{P_c}{\sigma_\omega \sqrt{2\pi}} e^{\frac{-\omega^2}{2\sigma_\omega^2}}$$





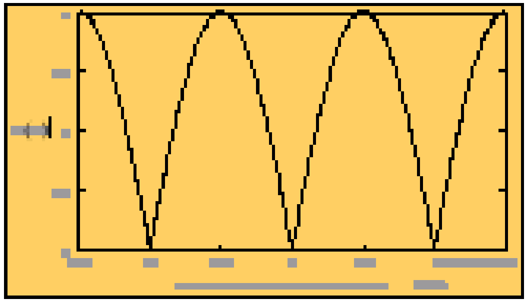

σ_ω T = 0.01
dB
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
10^{-4}
10^{-3}
10^{-2}
10^{-1}
fT
X: 0.002
Y: -3.429
X: 0.002
Y: -44.04
X: 0.002
Y: -65.83
X: 0.002
Y: -88.07
3-pulse, a=1.00
3-pulse, a=0.85
2-pulse, a=0.00
clutter


σ_ω T = 0.001
dB
0
-20
-40
-60
-80
-100
-120
0.1
0.32 (cell #1)
1
3.2 (cell #10)
10
32 (cell #100)
100
320 (cell #1000)
Velocity [m/s]
X: 3.182
Y: -36.28
X: 3.182
Y: -57.95
X: 3.182
Y: -72.57
3-pulse, a=1.00
3-pulse, a=0.85
2-pulse, a=0.00
clutter

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

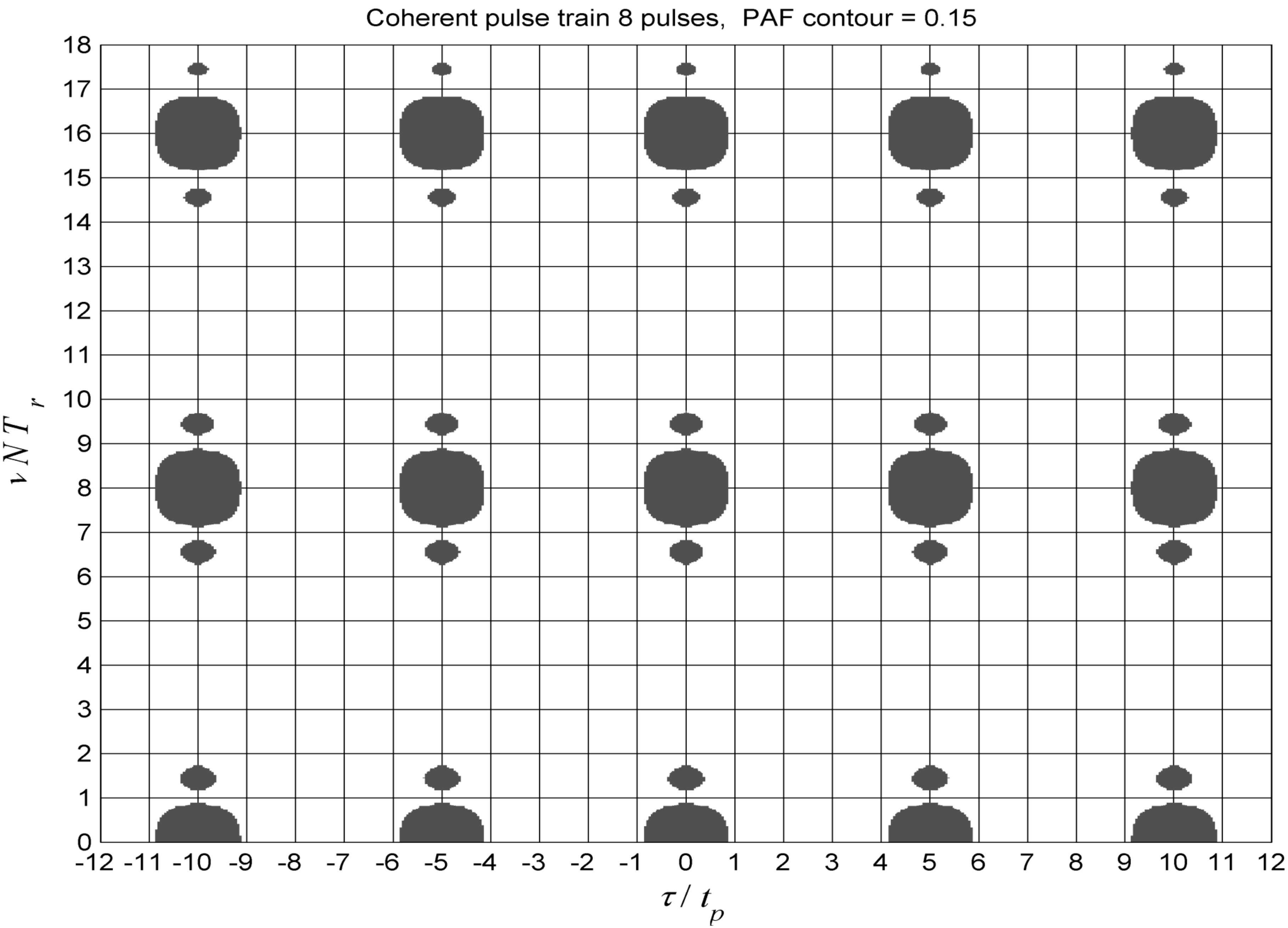
Blind Speeds

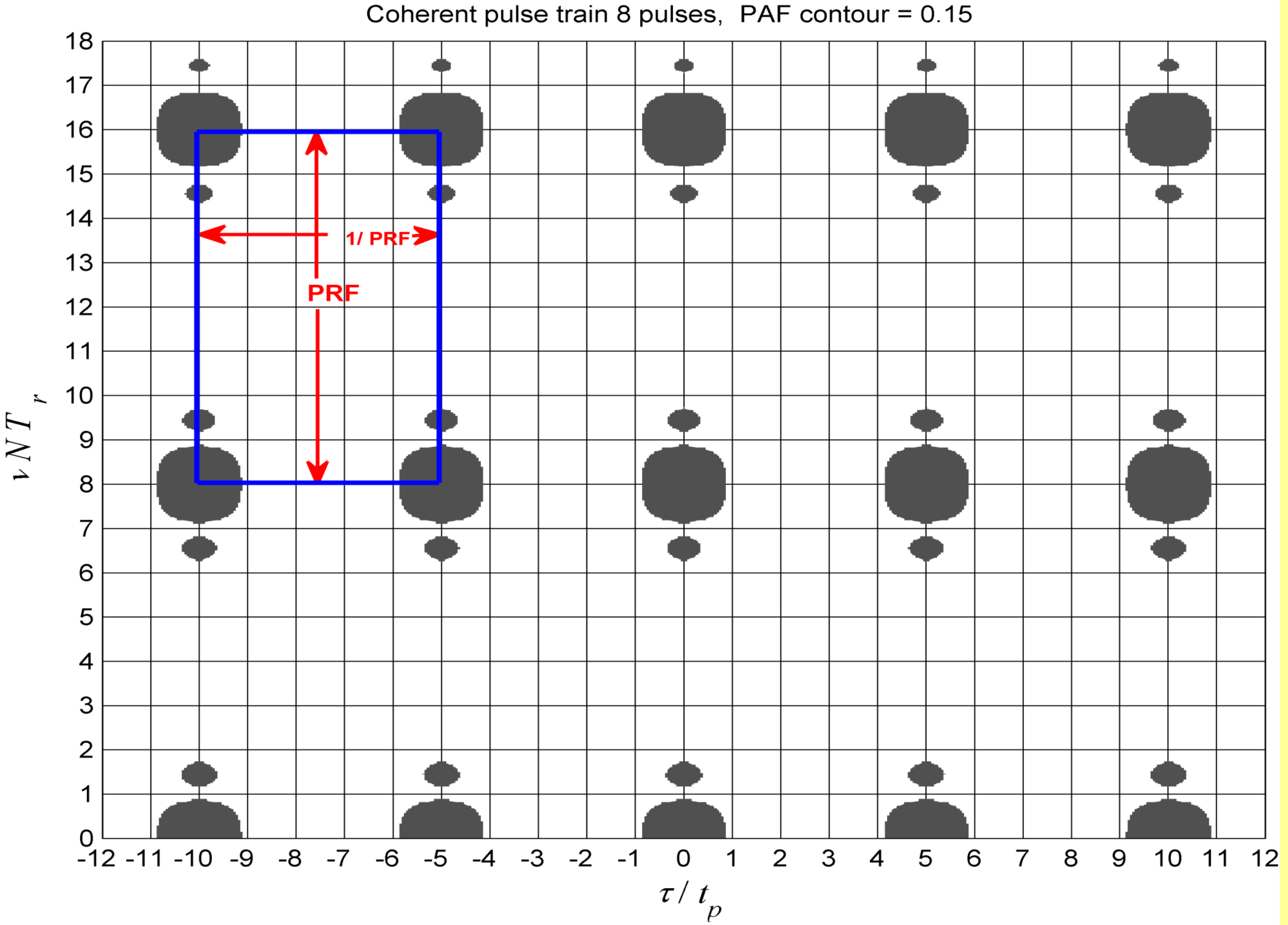
- MTI filters are digital filters, so frequency response is periodic
  - Nulls at multiples of PRF Hz
  - "Blind" to targets at corresponding radial velocity:
  - Could fix by raising PRF
- Unambiguous range is inversely proportional to PRF:
- Tradeoff in PRF choice required



$$v_{blind} = \frac{\lambda\, PRF}{2} = \frac{c\, PRF}{2 f_0}$$

$$R_{ua} = \frac{c}{2 PRF}$$


Coherent pulse train 8 pulses, PAF contour = 0.15
$\nu N T_r$
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
τ / t_p


Coherent pulse train 8 pulses, PAF contour = 0.15
1/ PRF
PRF
$\nu N T_r$
τ / t_p
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12

$$R_{UA} = \frac{C}{2} PRI = \frac{C}{2\,PRF}$$

$$PRF = \frac{C}{2R_{UA}}$$

UA = **un-ambiguous**

$$PRF = \frac{2V_{BS}}{\lambda} = \frac{2V_{BS} f_c}{C}$$

***BS*= blind speed**

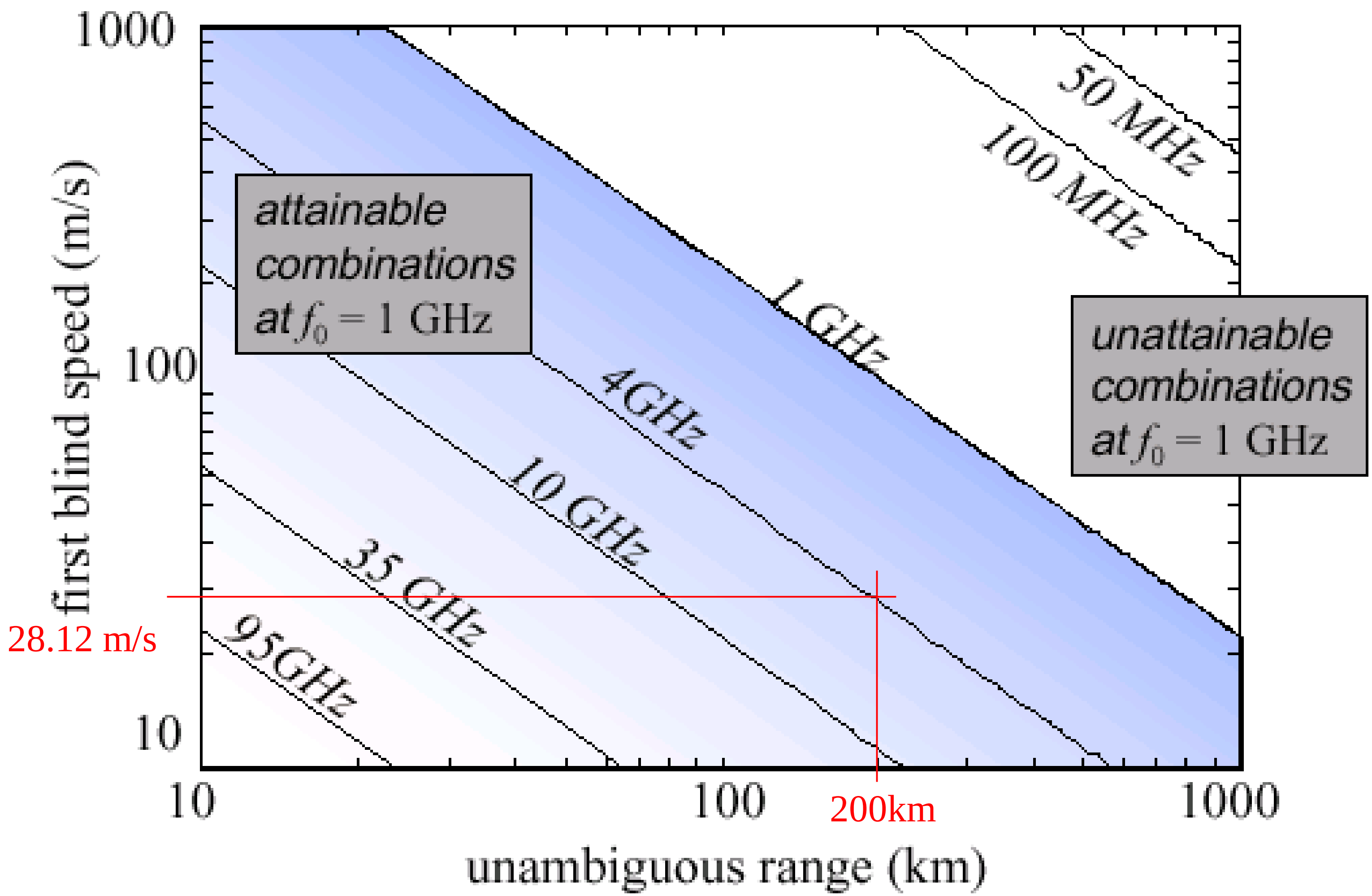
$$\frac{C}{2R_{UA}} = \frac{2V_{BS} f_c}{C}$$

$$V_{BS} R_{UA} = \frac{C^2}{4 f_c}$$

$$V_{\mathrm{BS}}\,[\mathrm{m/s}]\,R_{\mathrm{UA}}\,[\mathrm{km}] = \frac{22500}{f_c\,[\mathrm{GHz}]}$$

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Blind Speeds *vs.* RF



$$V_{\mathrm{BS}}\,[\mathrm{m/s}]R_{\mathrm{UA}}\,[\mathrm{km}]=\frac{22500}{f_c\,[\mathrm{GHz}]}$$

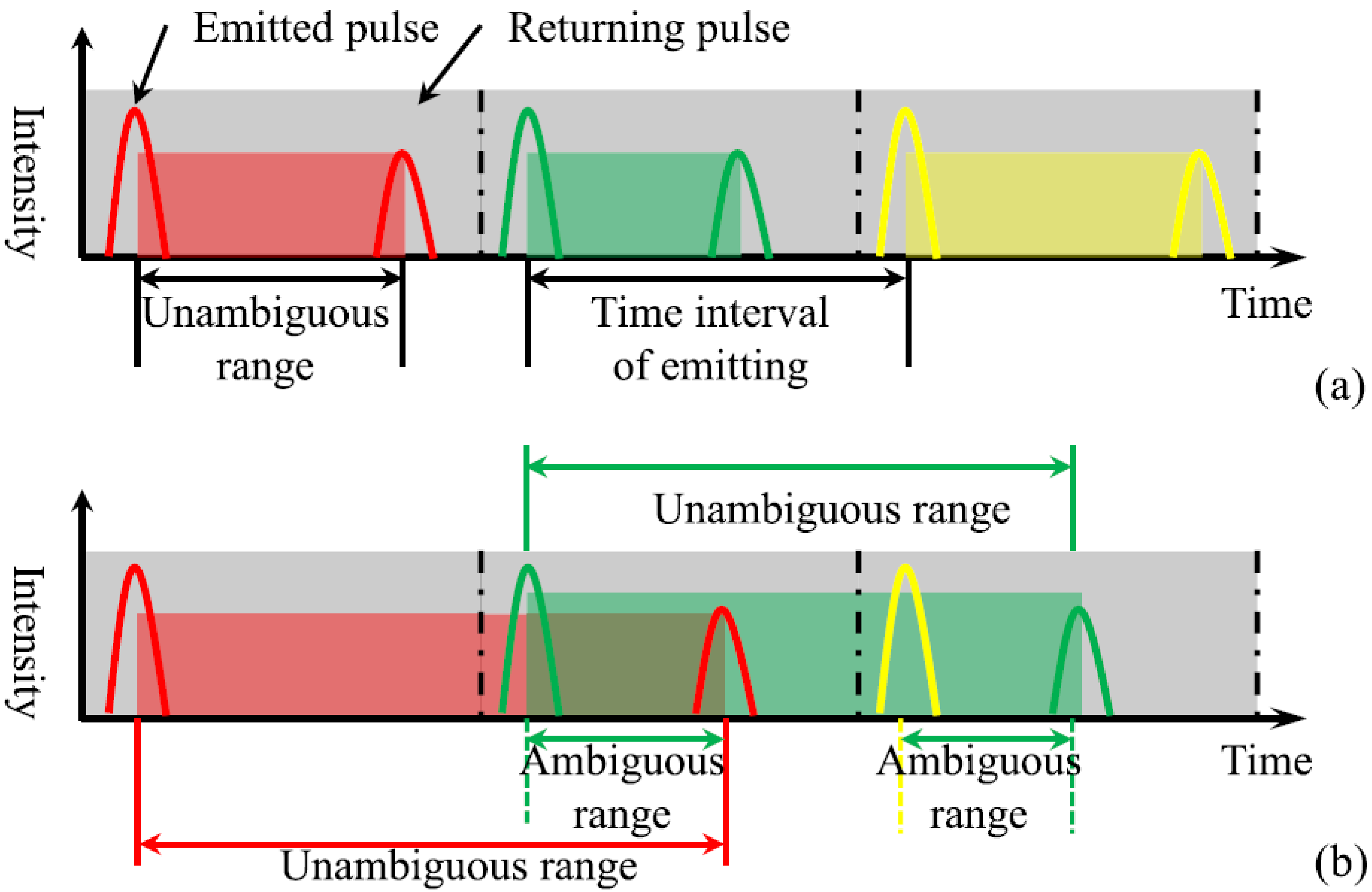


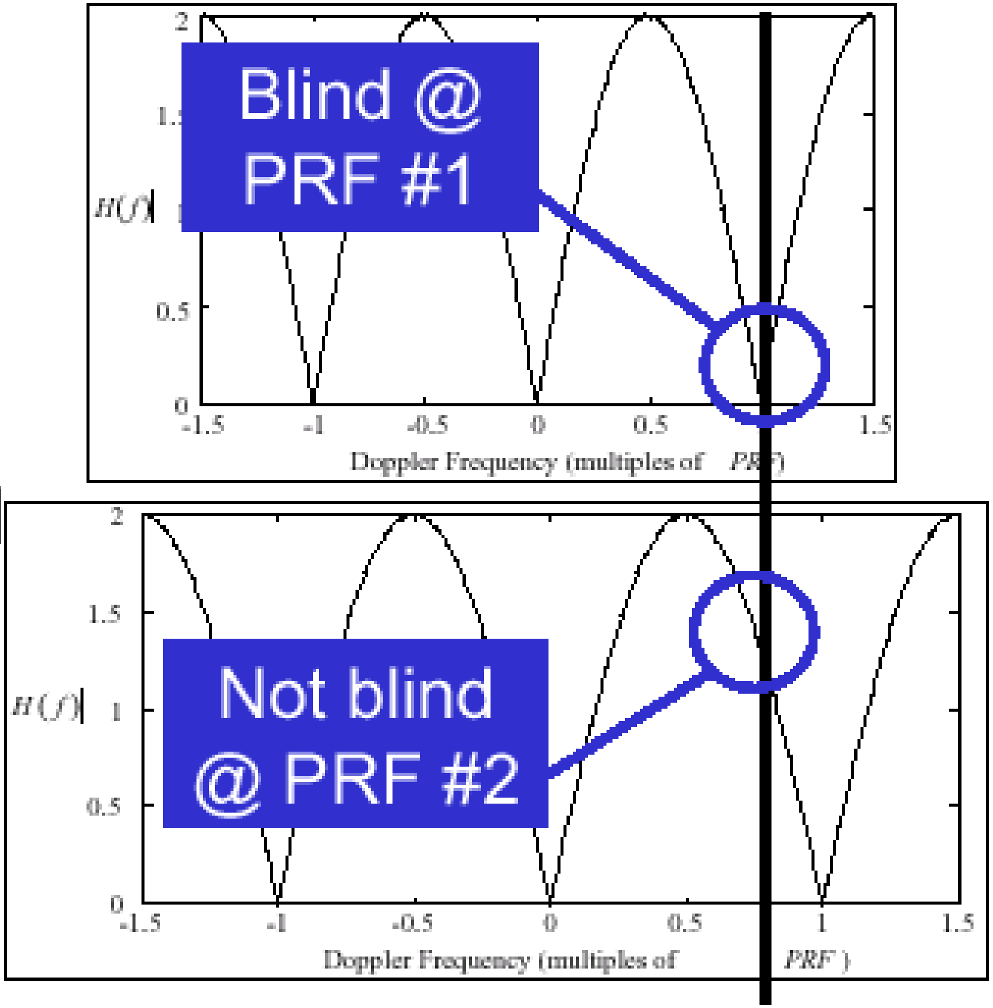
Fig. 1. (a) Range determination without ambiguity. (b) Range determination with ambiguity.

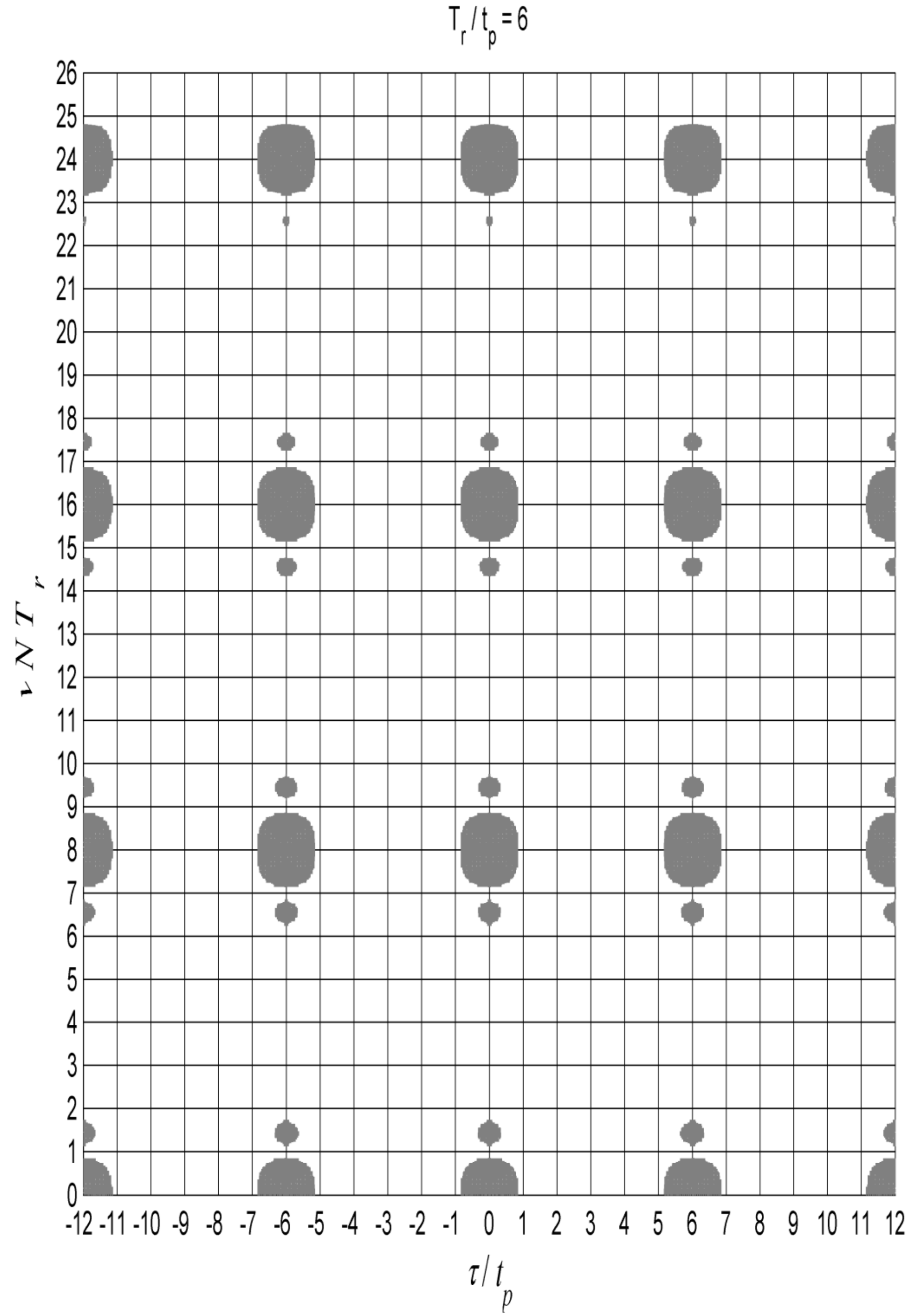
Staggered PRFs

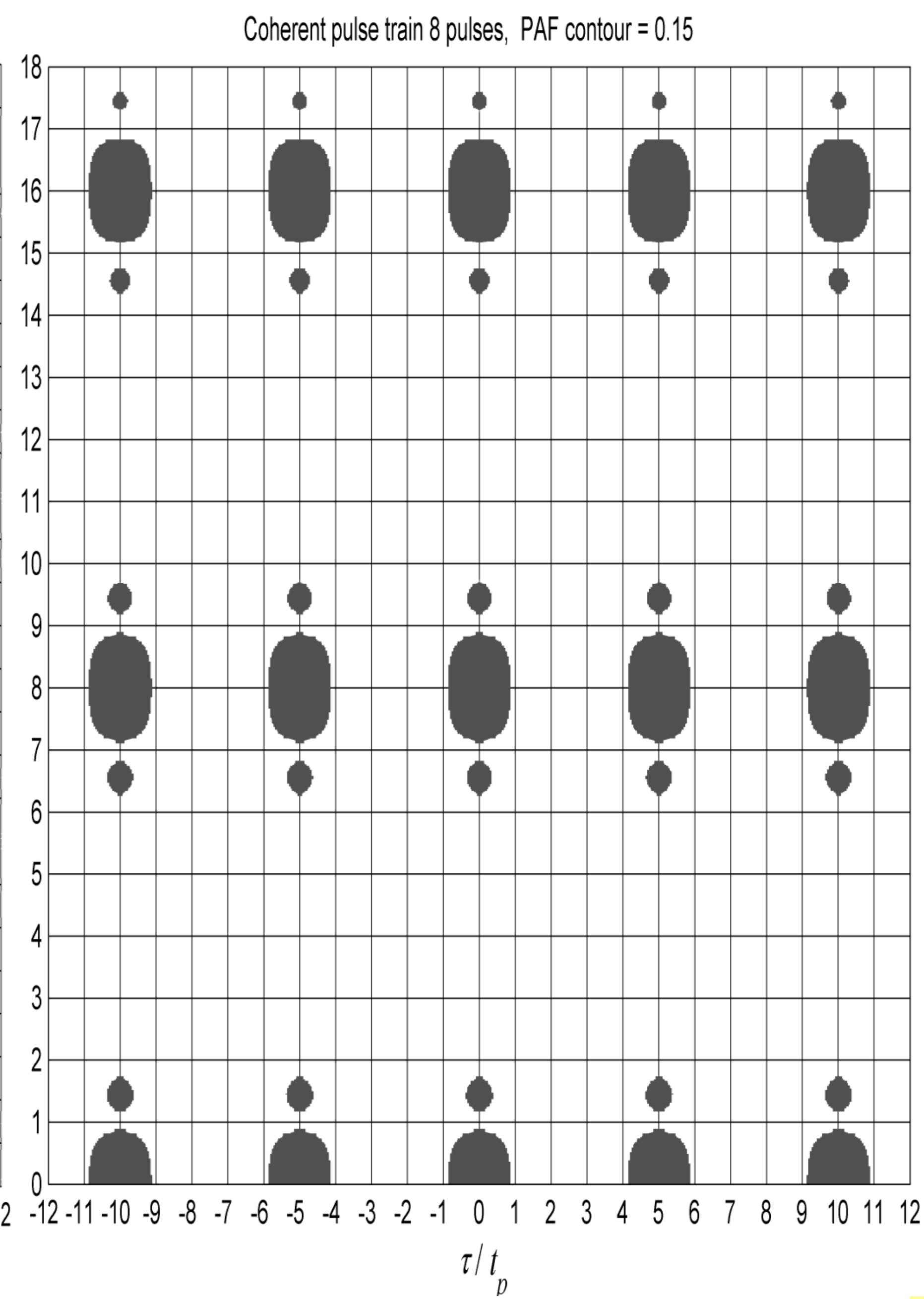
- A way to raise blind speed without significant effect on unambiguous range
- Concept: combine data from multiple PRFs
  - target is not simultaneously blind on all of them
- Two basic varieties:
  - pulse-to-pulse
  - block-to-block

Block-to-Block Stagger

- Two or more dwells (coherent processing intervals (CPIs)) transmitted and processed
- "M of N" logic applied to detections
- Velocity ambiguities complicates velocity estimate in pulse Doppler version




$T_r / t_p = 6$
$\nu N T_r$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
τ / t_p


Coherent pulse train 8 pulses, PAF contour = 0.15
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
τ / t_p

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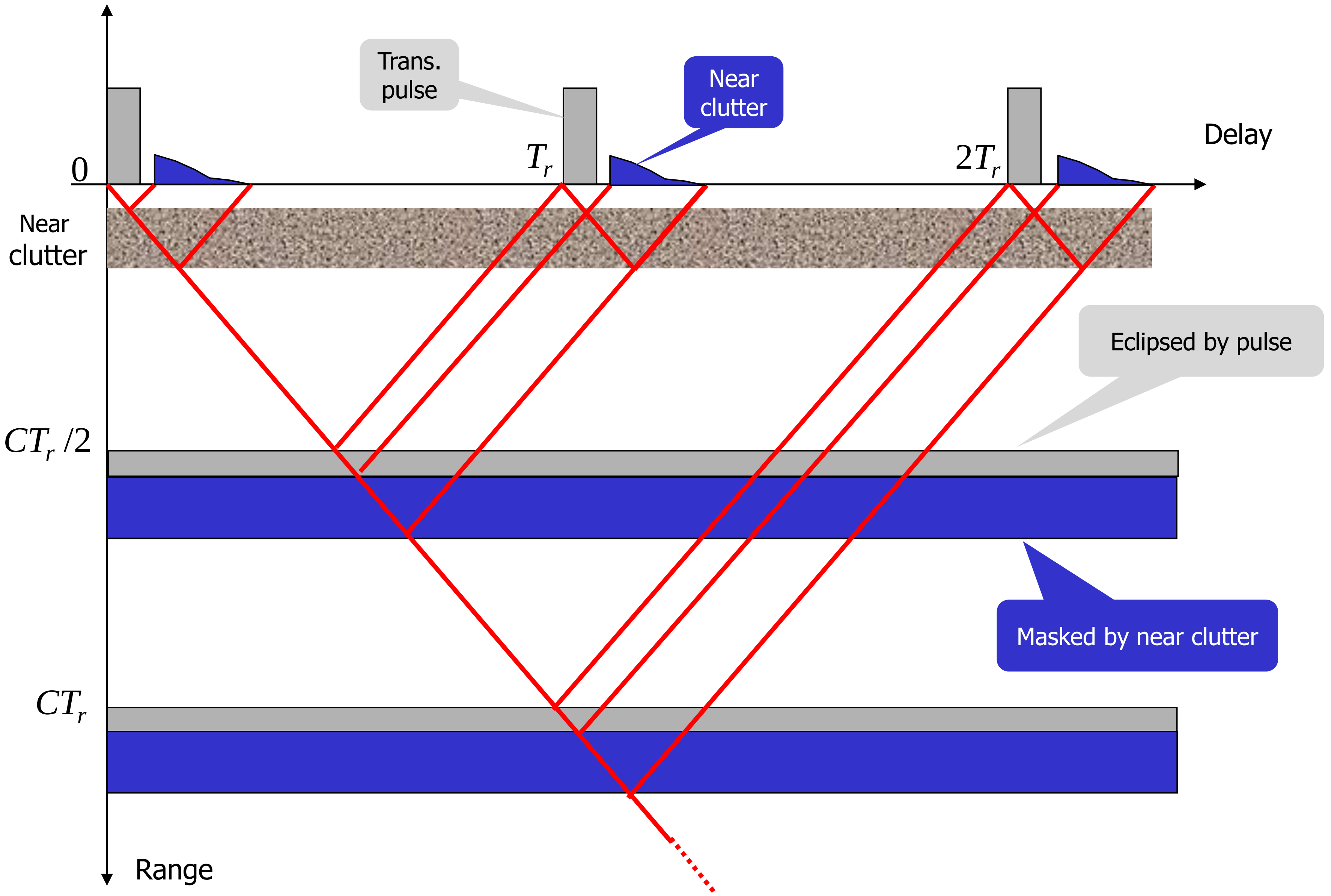
Eclipsing joins blind speeds



Davis, P.G. and Hughes, E.J. "Medium PRF set selection using evolutionary algorithms", *IEEE Trans. on AES*, Vol. 38, No. 3, (July 2002), pp. 933-939.

$f_c = 10$ GHz, $\lambda = 3$ cm

10.4 KHz $\le$ PRF ≤ 20 KHz

$\mathrm{PRI}/t_p = \{50, 51, 52, ..., 95, 96\}$

$t_p = 1\mu\mathrm{s} \Rightarrow$ range cell $= 150$ m

Number of range cells $= 1000$, max range $= 150$ km

Doppler resolution $= 100$ Hz $\Rightarrow$ velocity cell $= 1.5$ m/s

Number of velocity cells $= 200$, max velocity $= 300$ m/s

Clutter notch $= \pm 17$ velocity cells

Pulse eclipsing $= 1$ range cell

Near clutter masking $= 10$ range cells

i.e., loss of 5/8'th of the energy is tolerated. Used often, but very inefficient !

8 different PRIs are used. A cell in the blind-zone map is considered clear if it is not blinded for 3 or more PRIs. (M out of N detection.)

Which set of 8 PRIs yields the clearest blind-zone map?

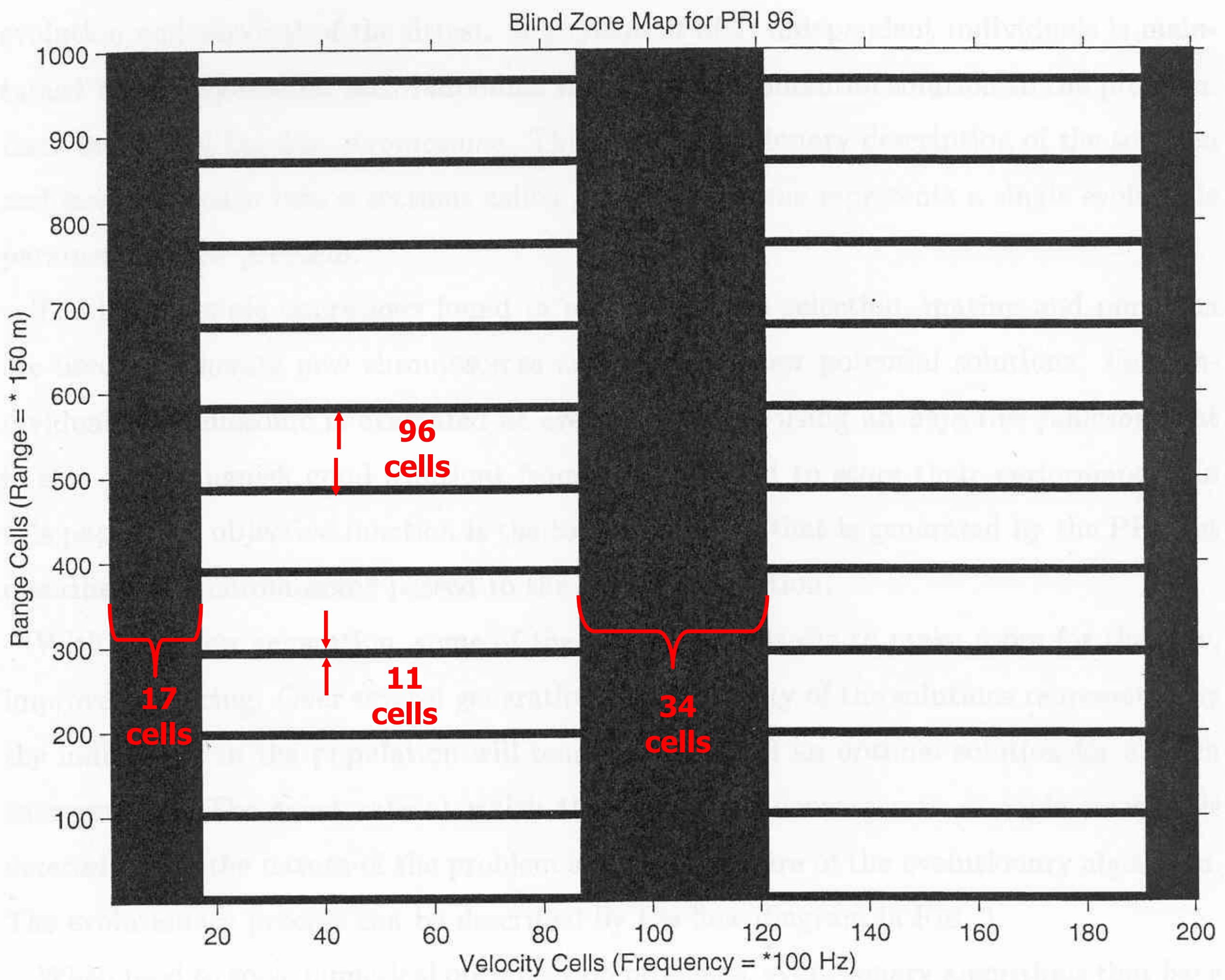


Fig. 2. Blind Map for PRI 96μS

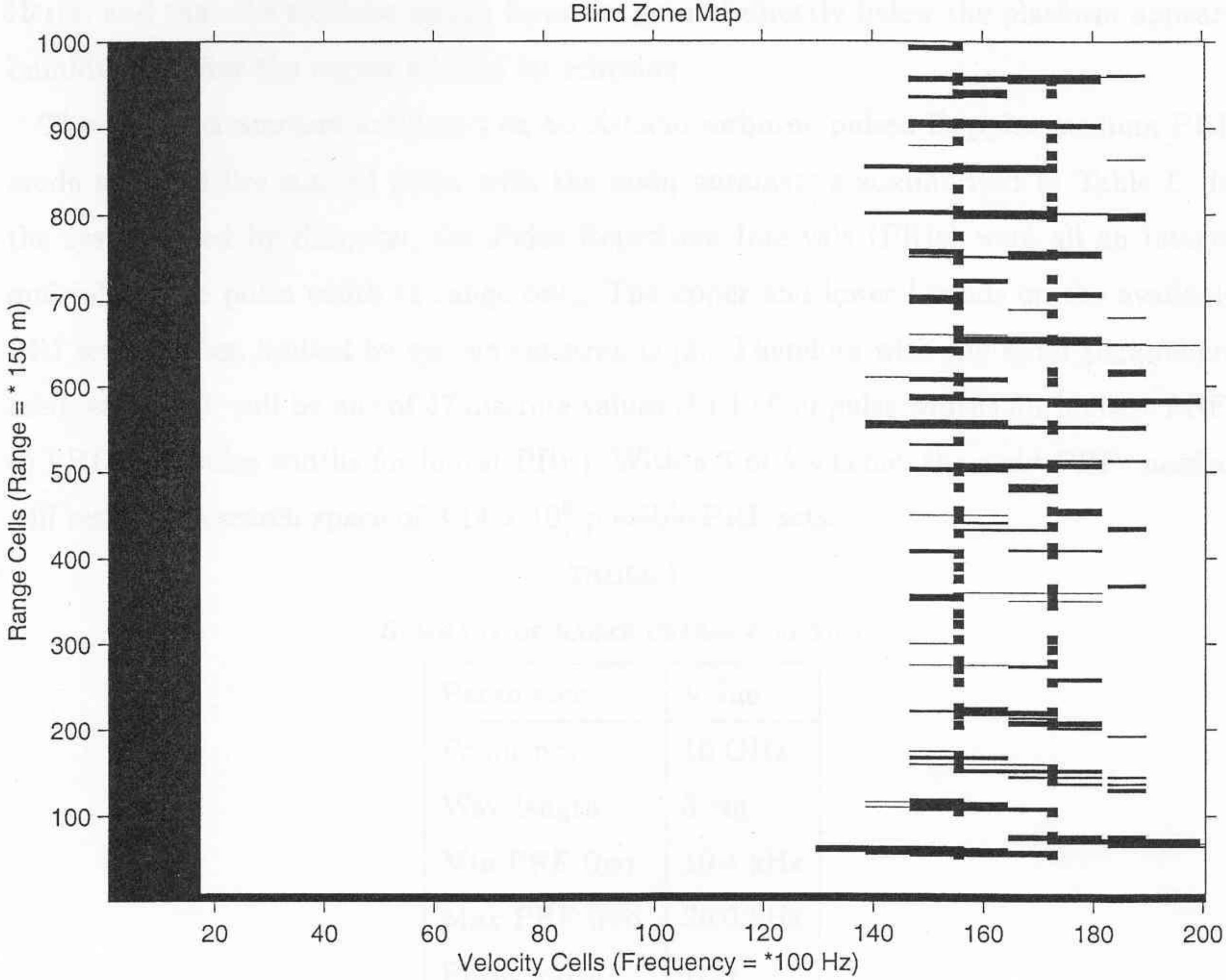


Fig. 1. Typical 3 of 8 PRF blind-zone map

PRI = { 50 53 55 58 61 64 68 72 }

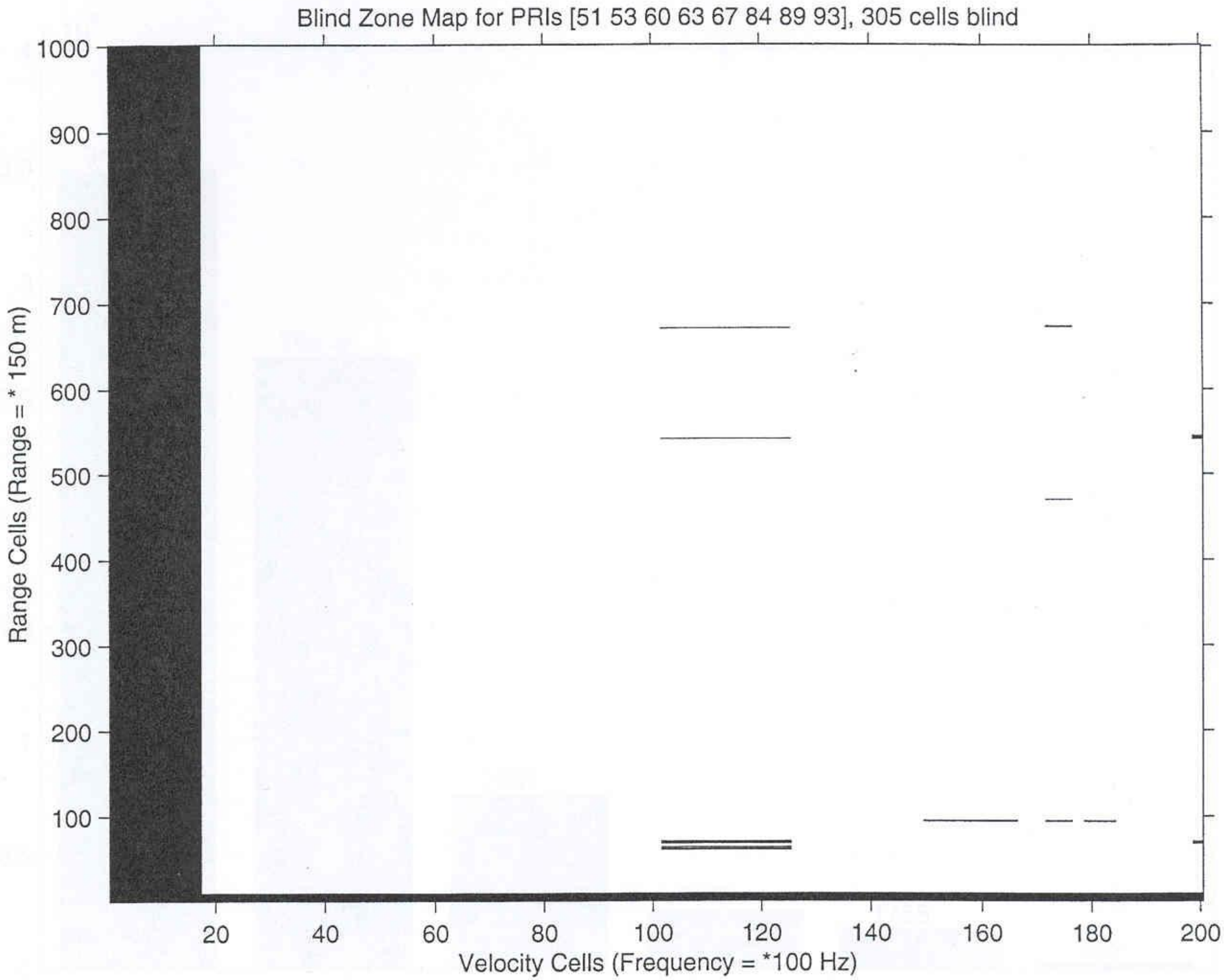


Fig. 4. Blind-zone map for 8 PRFs found by evolution

PRI = { 51 53 60 63 67 84 89 93 }

Combined MTI and Pulse Doppler

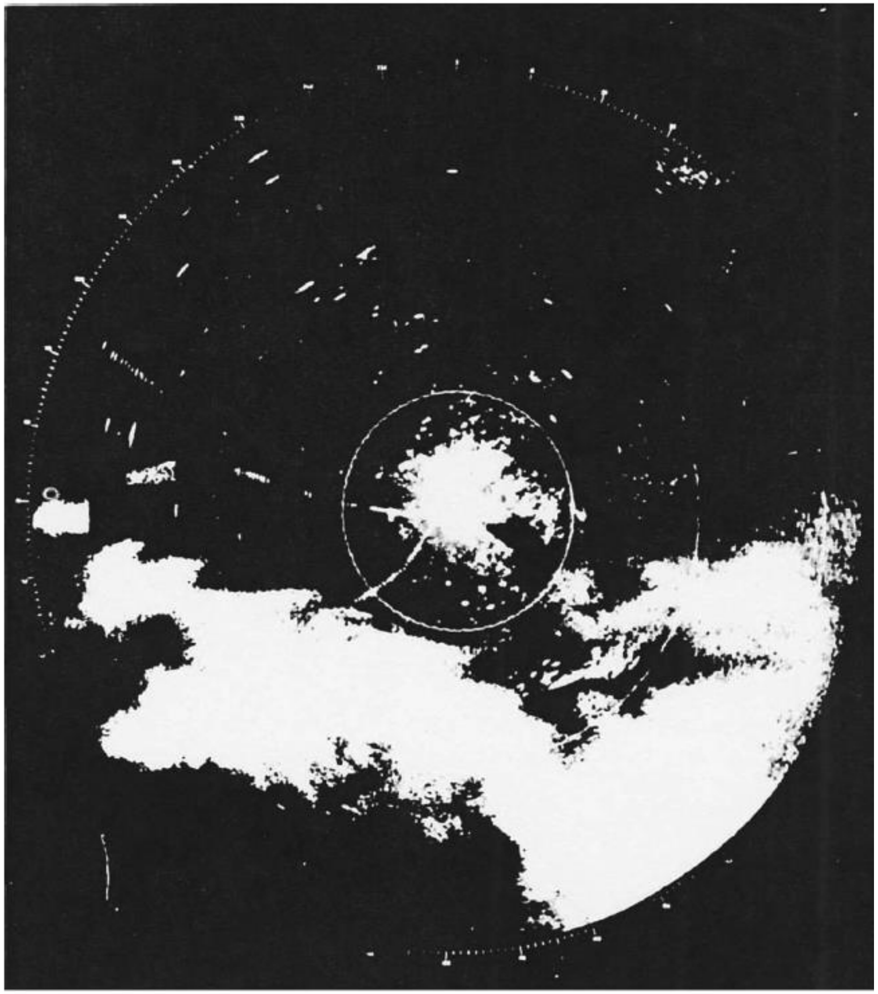
- 2- or 3-pulse MTI canceller and pulse Doppler processing often used together
  - MTI for gross clutter suppression
  - pulse Doppler for finer-grained spectral analysis
- Order matters, even though both are linear filters
  - strong clutter sidelobes can swamp nearby targets
  - if processor dynamic range is limited, strong clutter may saturate it, driving small targets into the quantization noise
    - primarily an issue in fixed-point processors
- Therefore MTI usually precedes pulse Doppler

The Pioneer Award Committee of the IEEE Aerospace and Electronic Systems Society
has named

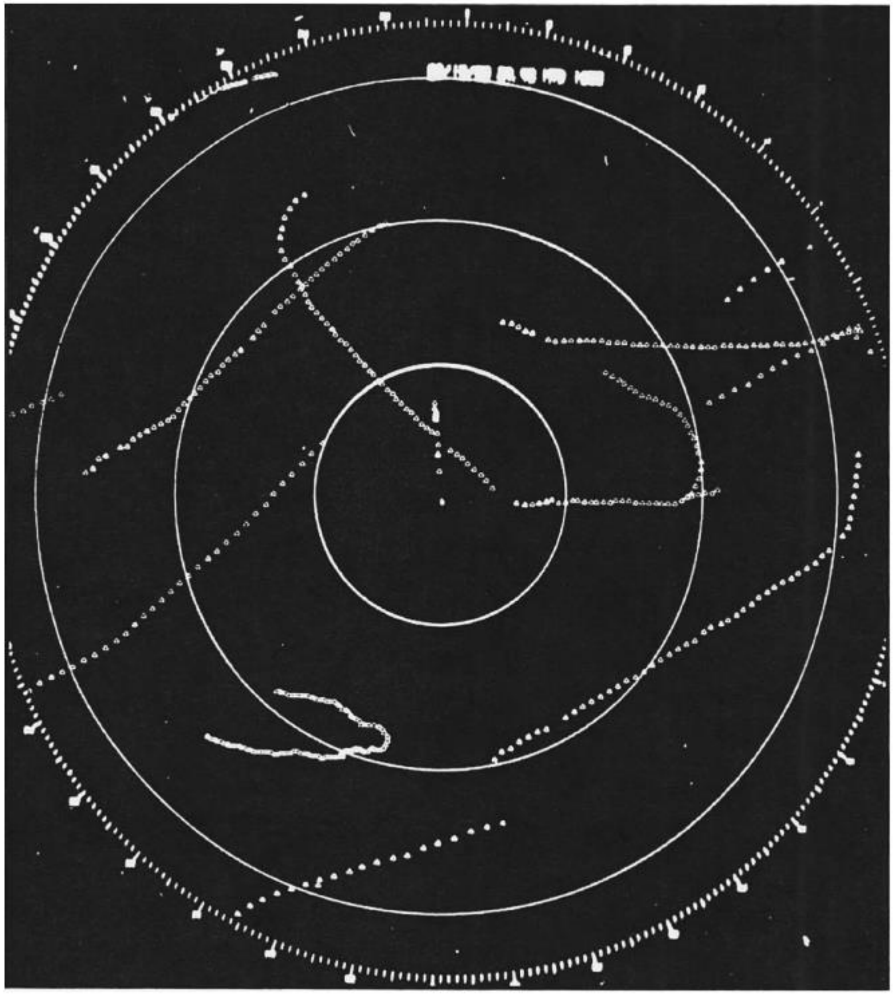
CHARLES E. MUEHE

as the recipient of the 2005 Pioneer Award. The citation reads:

For the invention of the Moving Target Detector (MTD) digital signal processor for aircraft surveillance radar.



(a)



(b)

Fig. 2. Detection of aircraft flying in rain. (a) Output of a conventional MTI radar taken with a 5-min exposure. The aircraft flying within the rain echoes is obscured. (b) Output of moving target detector (MTD). The screen is free of rain clutter and the overall track of the aircraft is clearly delineated. Because only one display was available, the photographs were made sequentially.

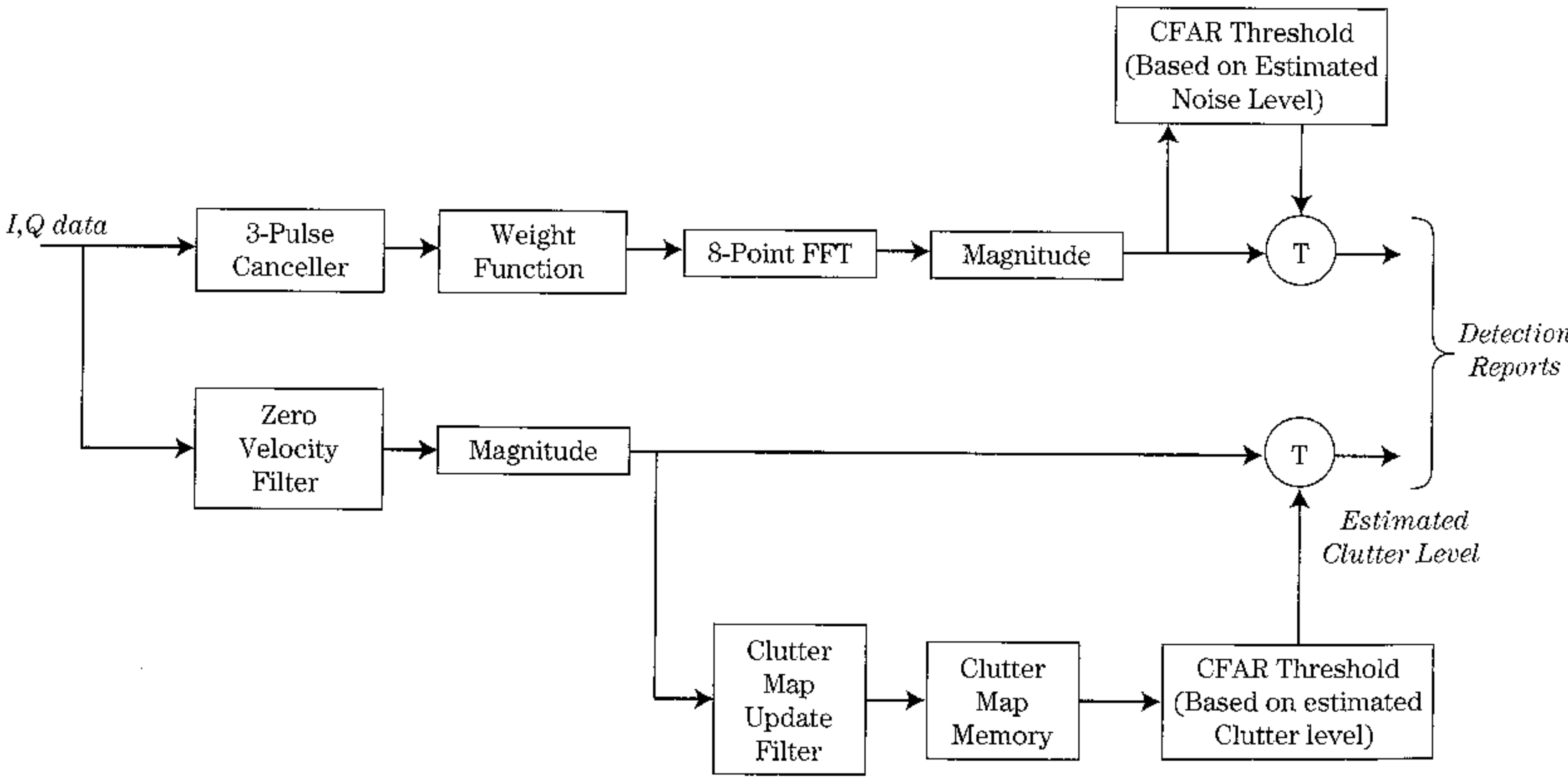


FIGURE 17-34 Block diagram of a complete "moving target detector" system combining MTI, pulse-Doppler, and clutter mapping.

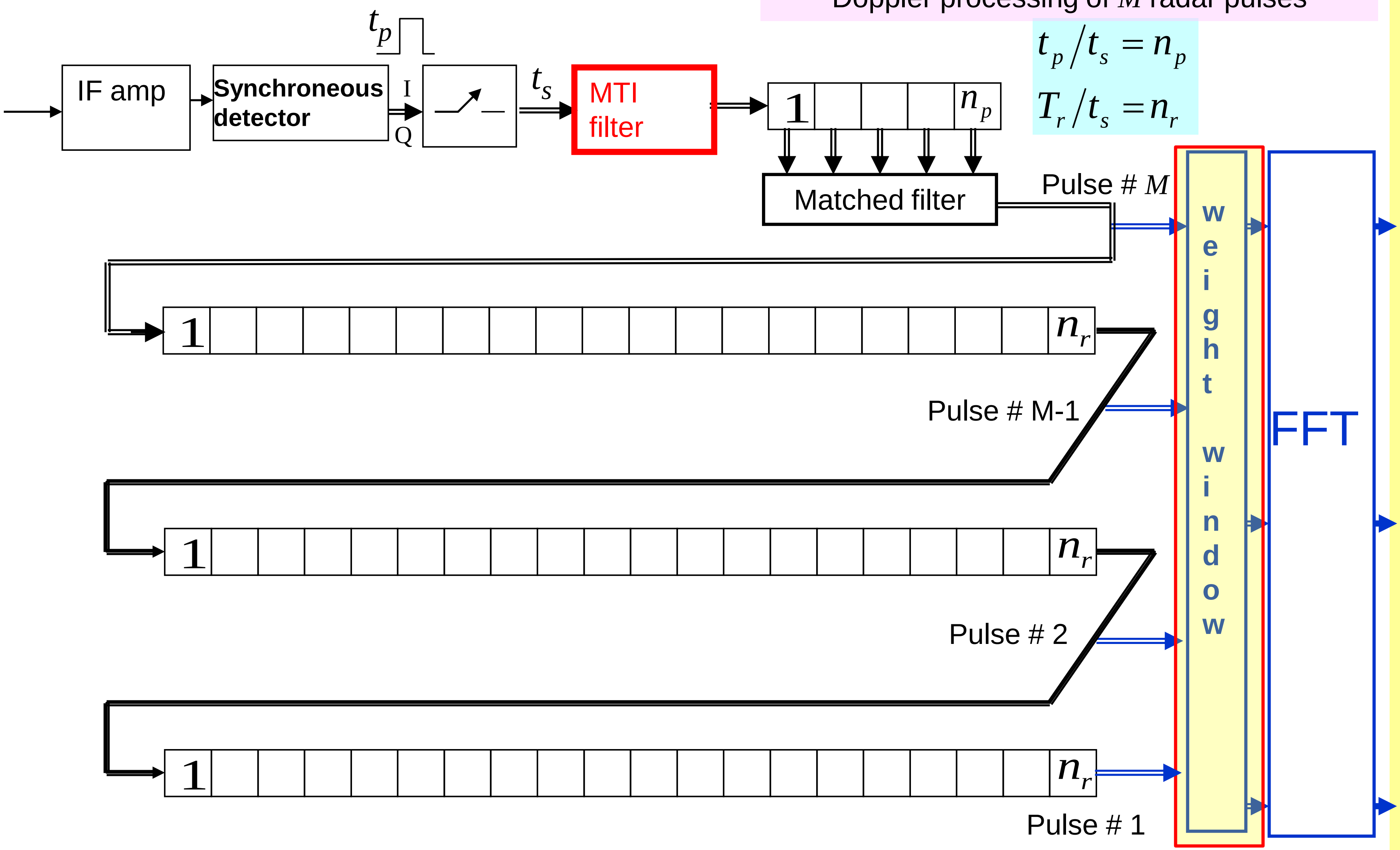
Copied from *Principles of Modern Radar – Basic Principles*.
Originally from F. E. Nathanson *Radar Design Principles* 2nd ed. Mc-Graw Hill, New York, 1991

Nadav Levanon, Tel-Aviv University

Doppler processing of M radar pulses

$t_p/t_s = n_p$

$T_r/t_s = n_r$



Integration of M pulses

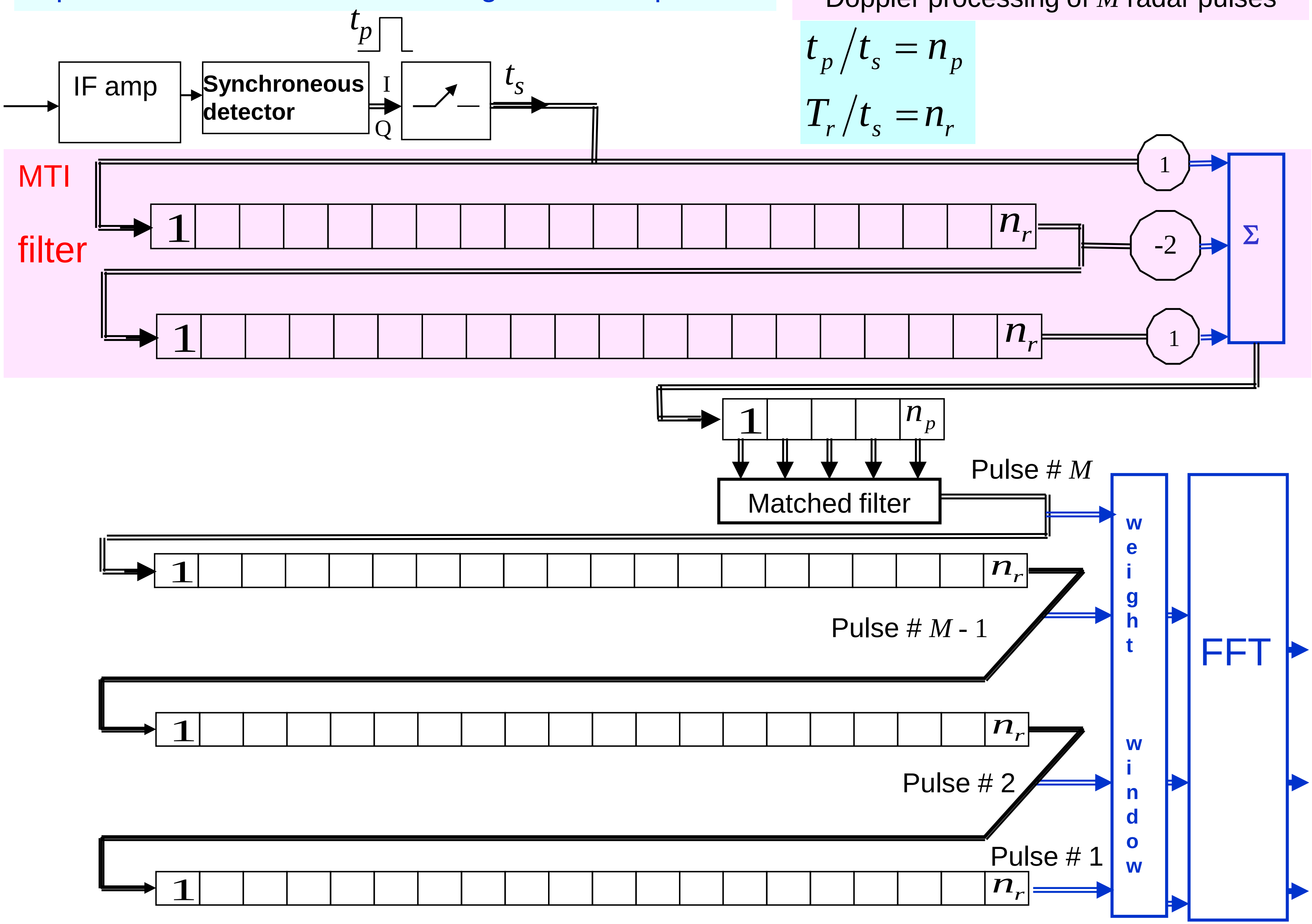
Nadav Levanon, Tel-Aviv University

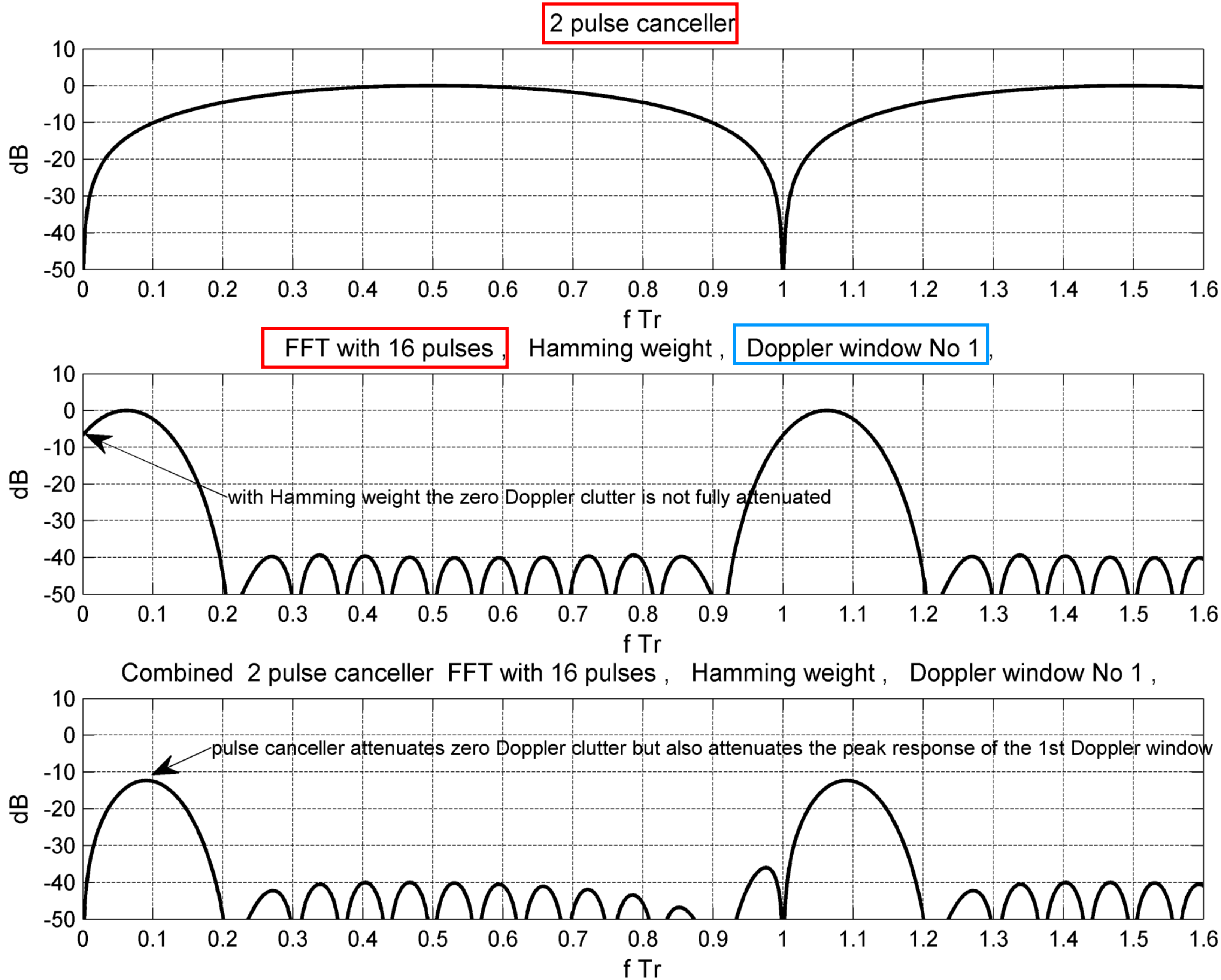
3-pulse MTI canceller & FFT Integration of M pulses

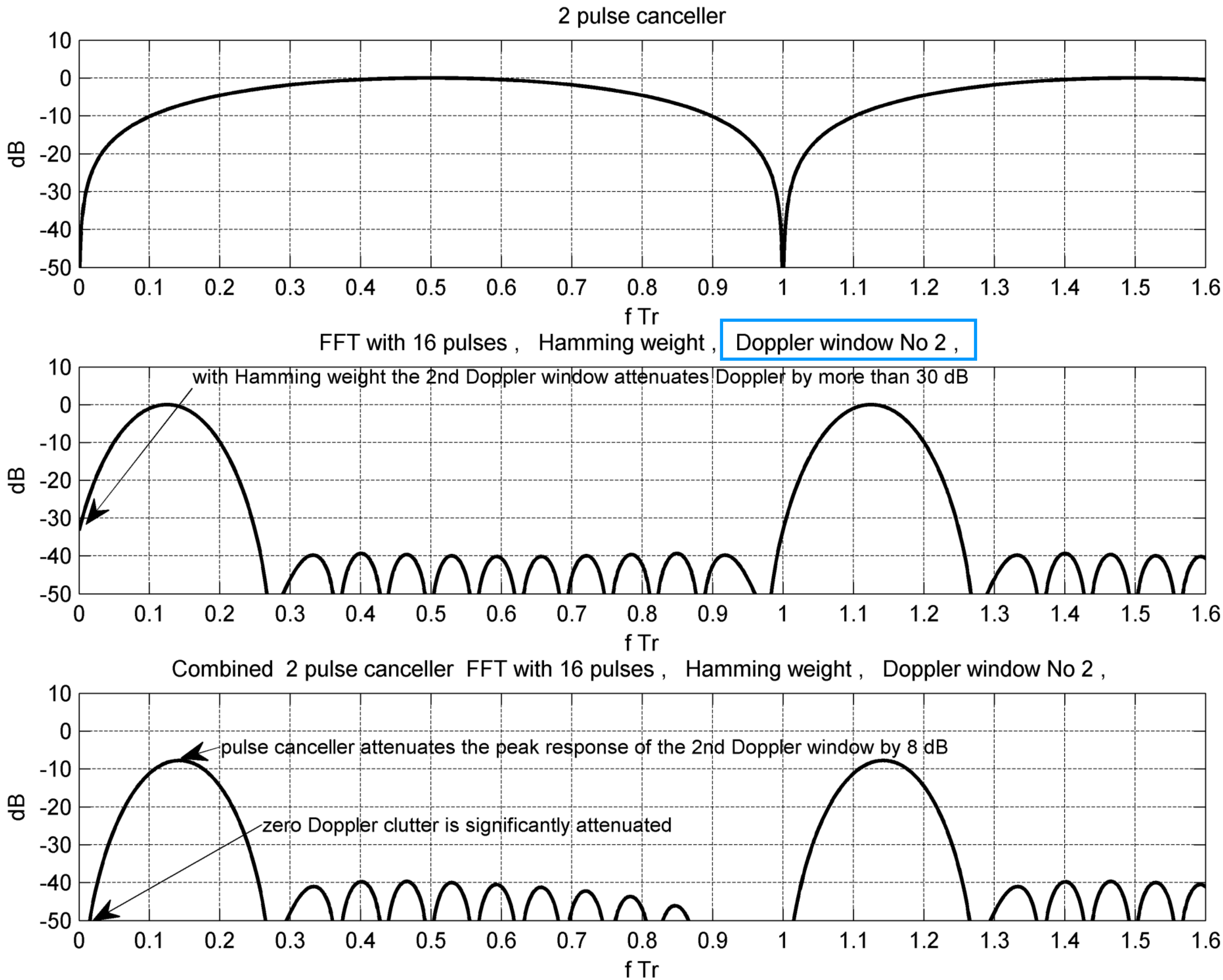
Doppler processing of M radar pulses

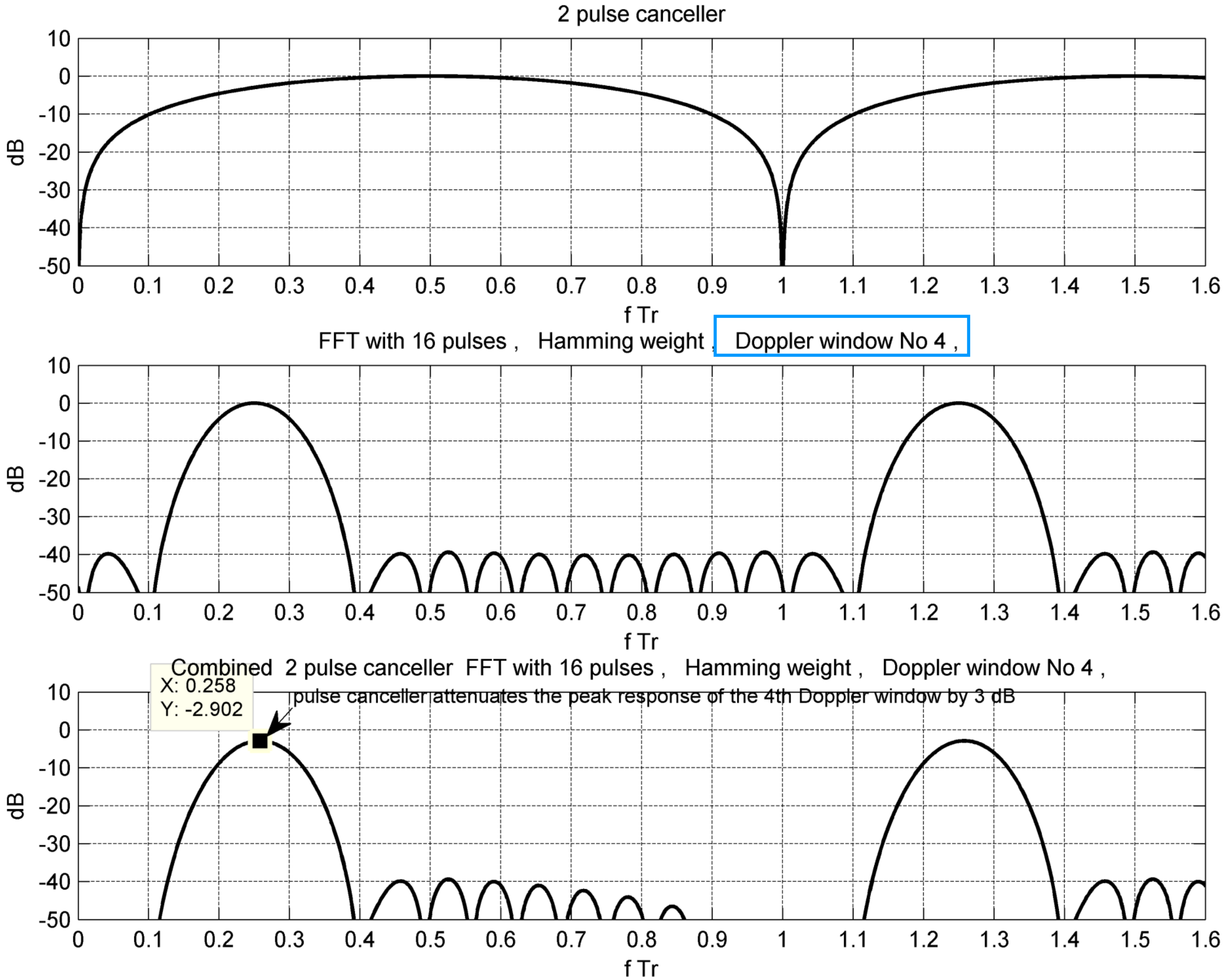
$$t_p / t_s = n_p$$

$$T_r / t_s = n_r$$

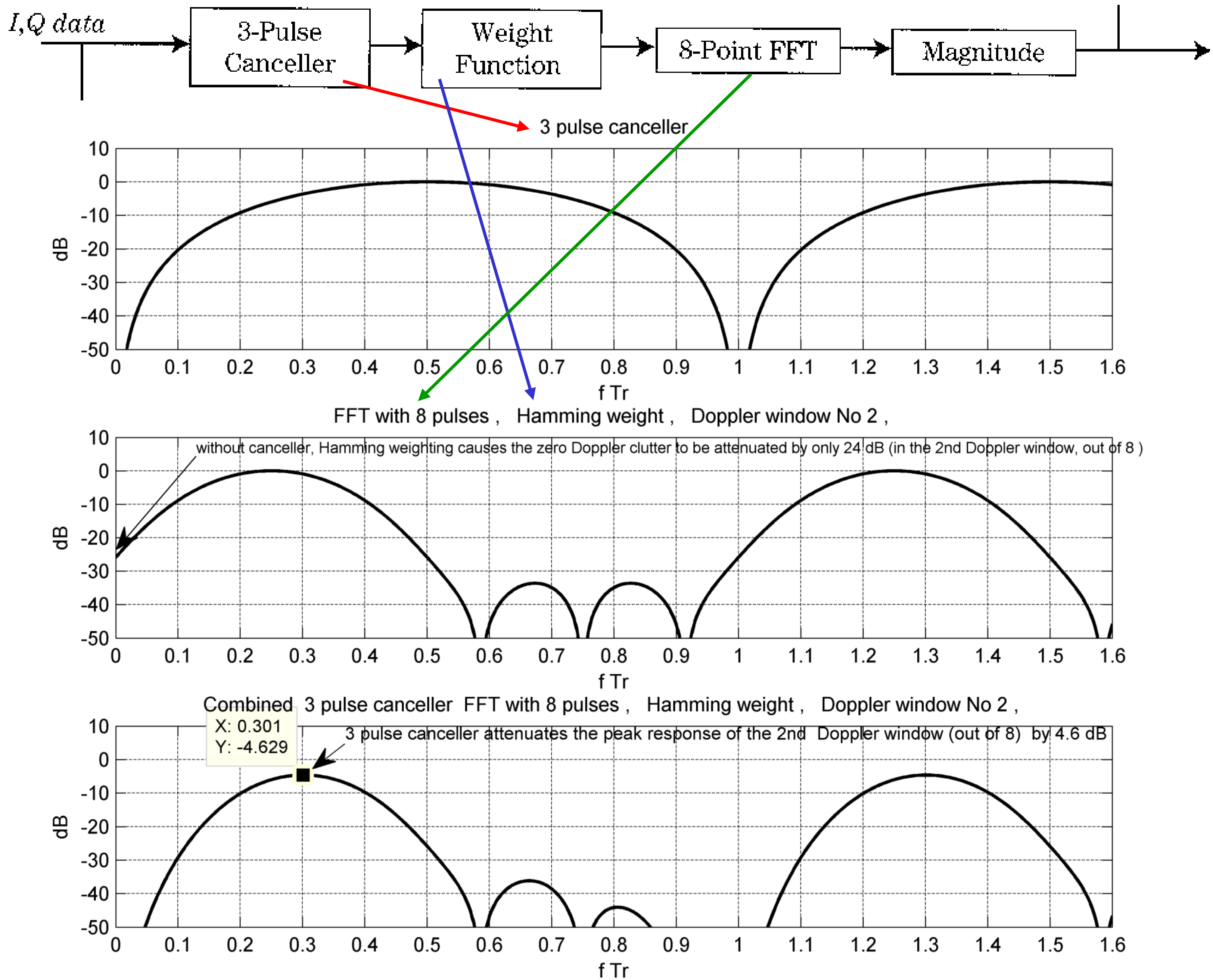


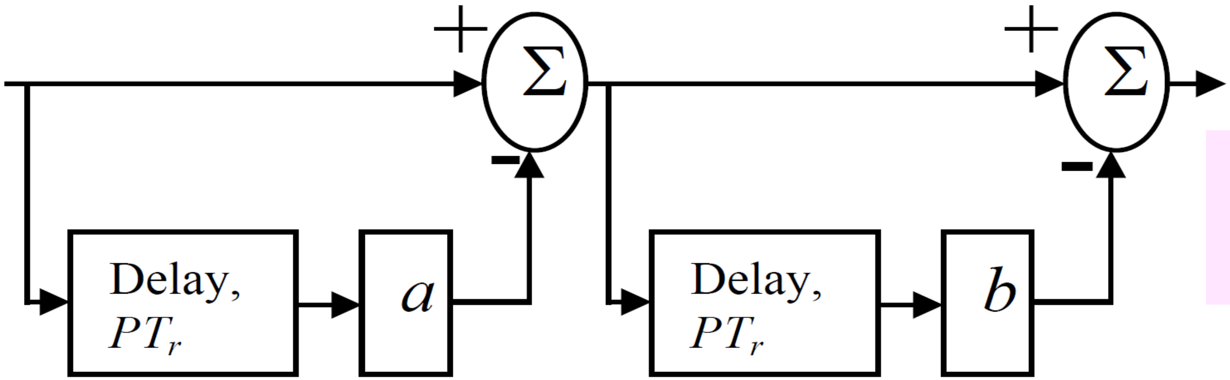

2 pulse canceller
10
0
-10
-20
-30
-40
-50
dB
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6
f Tr
FFT with 16 pulses , Hamming weight , Doppler window No 1 ,
with Hamming weight the zero Doppler clutter is not fully attenuated
Combined 2 pulse canceller FFT with 16 pulses , Hamming weight , Doppler window No 1 ,
pulse canceller attenuates zero Doppler clutter but also attenuates the peak response of the 1st Doppler window


2 pulse canceller
dB
f Tr
FFT with 16 pulses , Hamming weight , Doppler window No 2 ,
with Hamming weight the 2nd Doppler window attenuates Doppler by more than 30 dB
dB
f Tr
Combined 2 pulse canceller FFT with 16 pulses , Hamming weight , Doppler window No 2 ,
pulse canceller attenuates the peak response of the 2nd Doppler window by 8 dB
zero Doppler clutter is significantly attenuated
dB
f Tr


2 pulse canceller
dB
f Tr
FFT with 16 pulses , Hamming weight , Doppler window No 4 ,
dB
f Tr
Combined 2 pulse canceller FFT with 16 pulses , Hamming weight , Doppler window No 4 ,
X: 0.258
Y: -2.902
pulse canceller attenuates the peak response of the 4th Doppler window by 3 dB
dB
f Tr

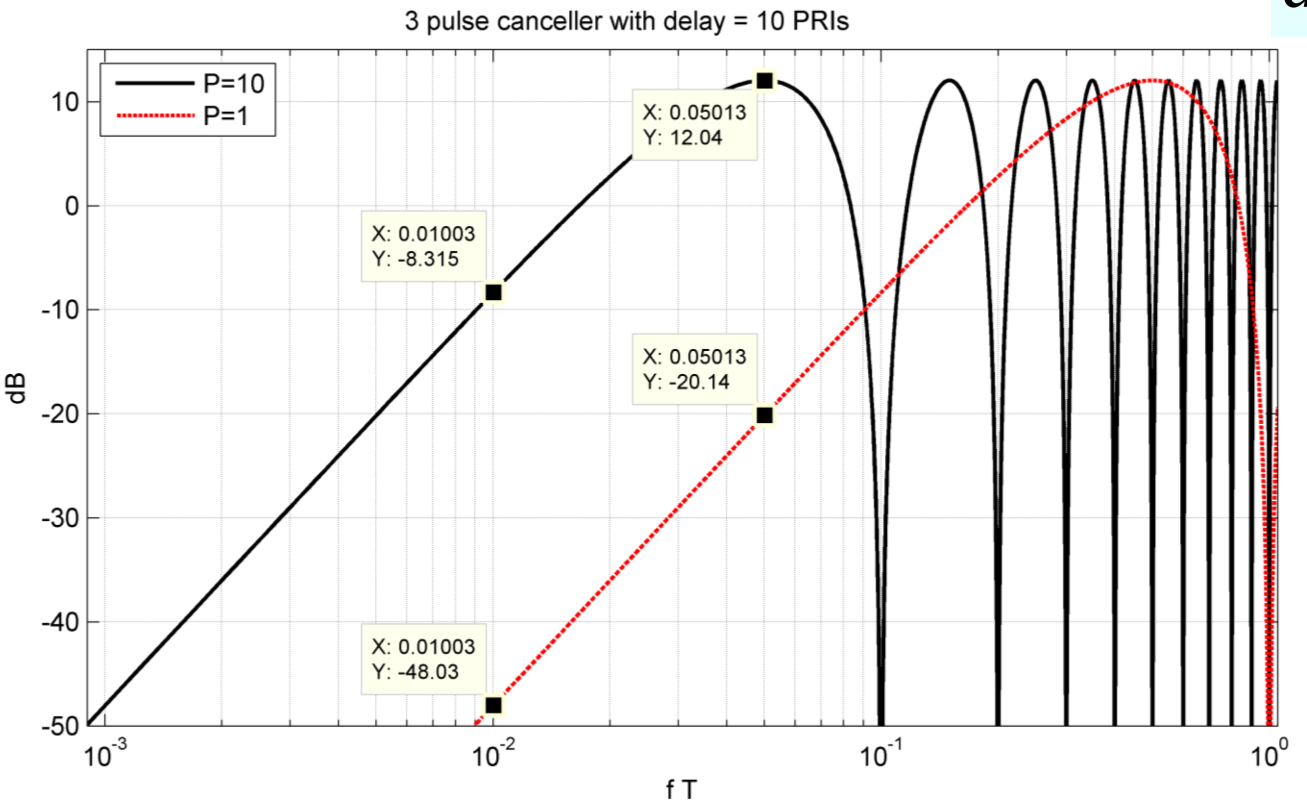
Nadav Levanon, Tel-Aviv University



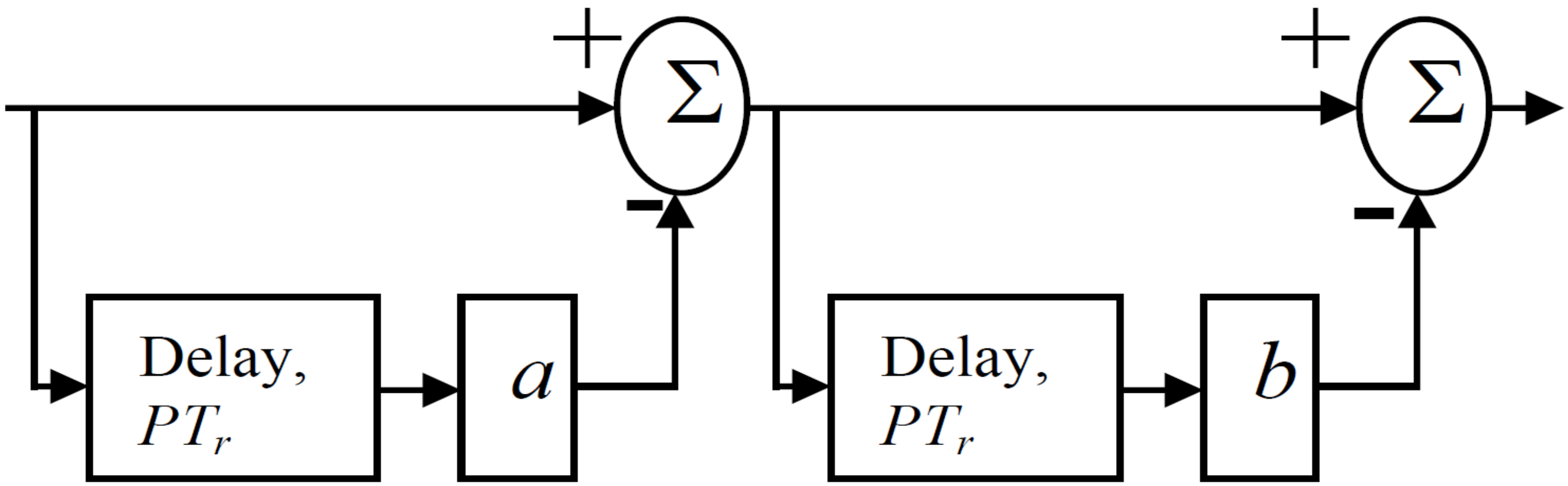


Generalization of 3-pulse canceller

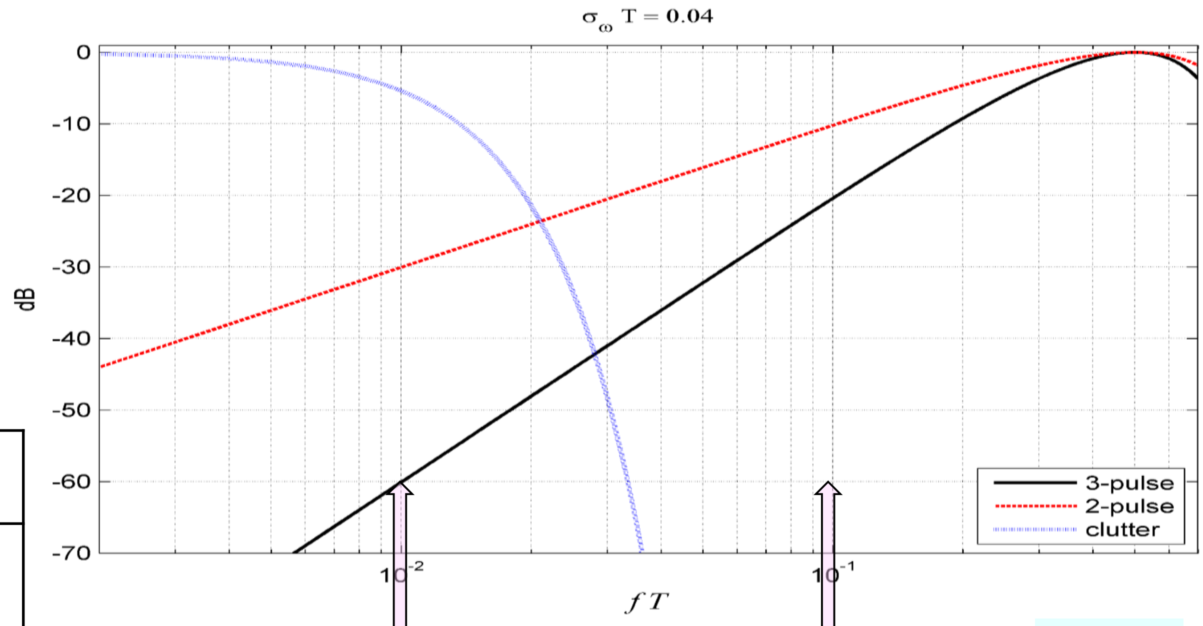
$$a=1, b=1, P=10$$



Generalization of 3-pulse canceller

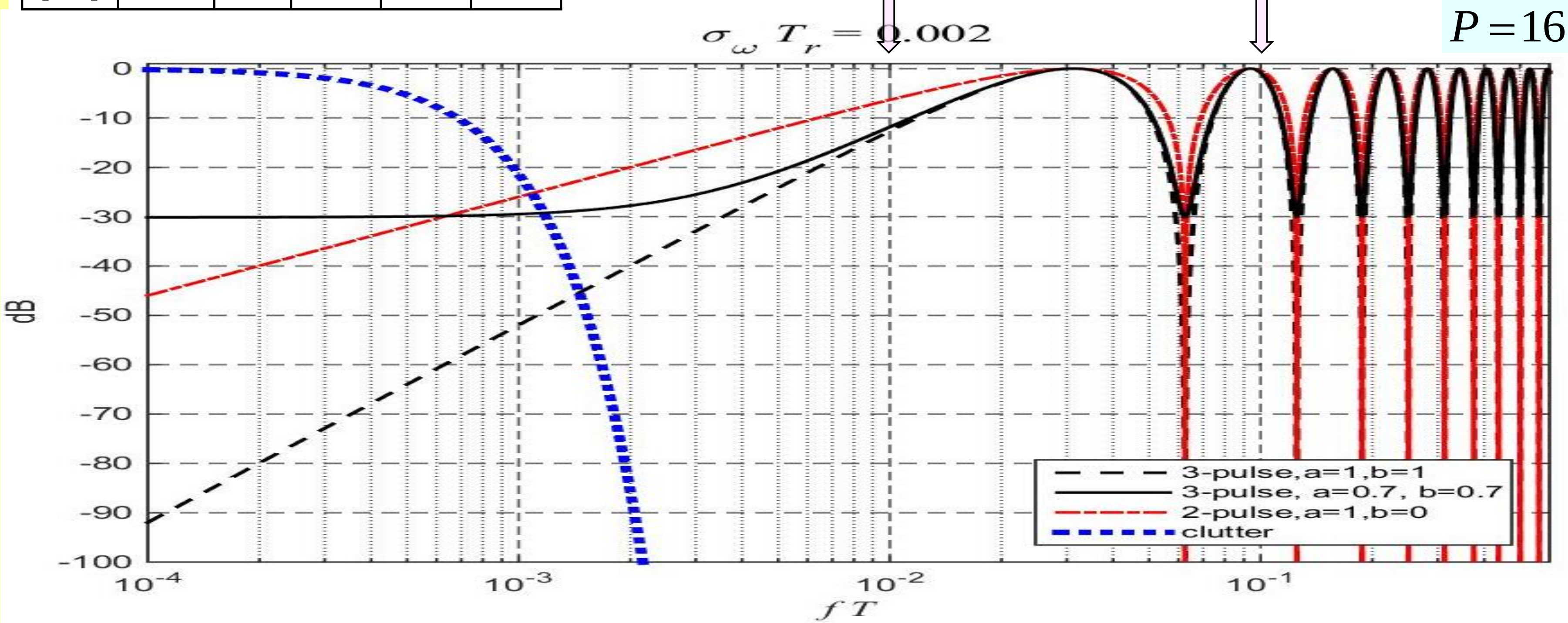


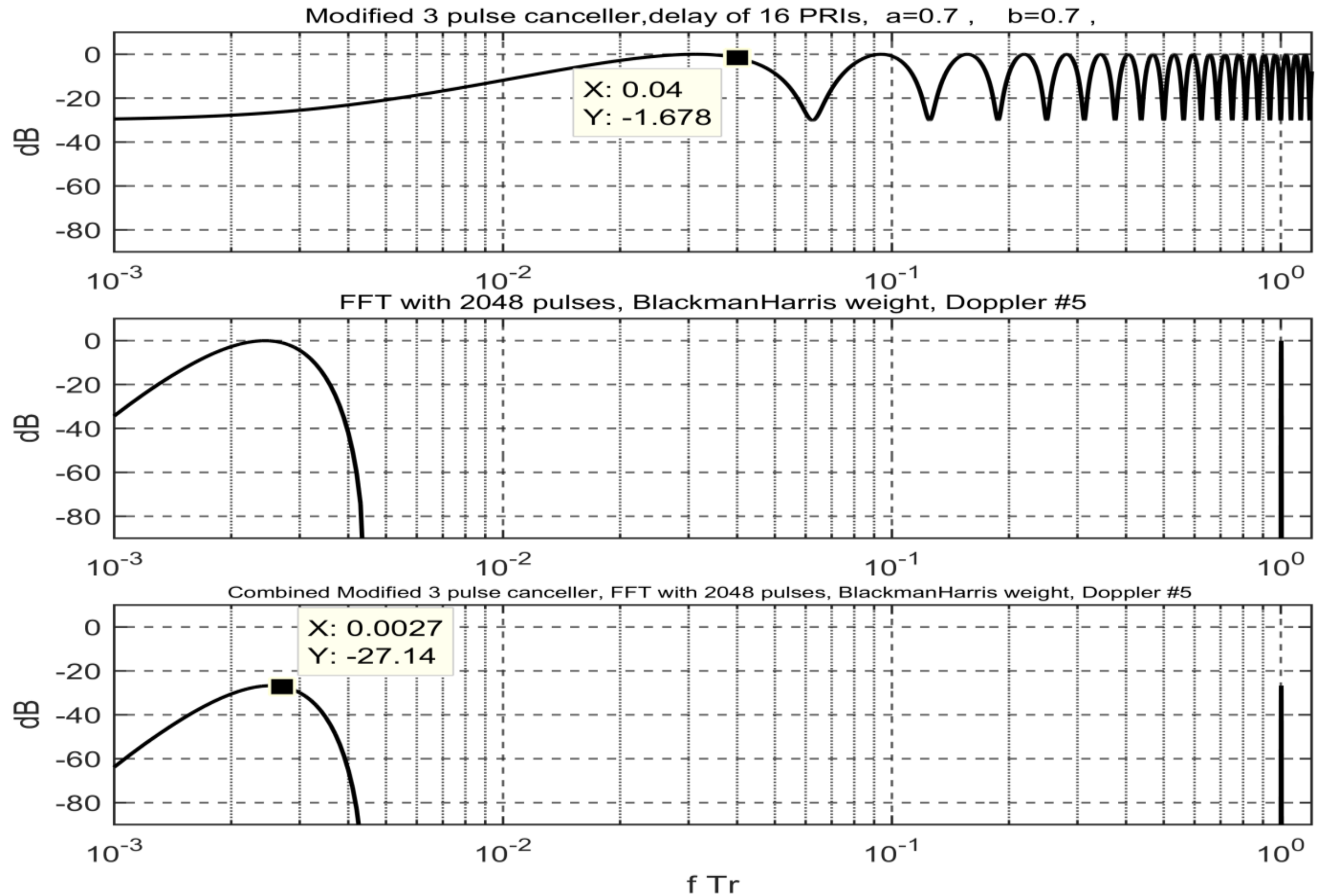
Type	***P***	***a***	***b***
2-pulse canceller	1	1	0
3-pulse canceller	1	1	1
Modified 3-pulse canceller for slow targets	16	0.7	0.7



Conversion from $f\,T_r$ to v[m/s], (f_c=9GHz, T_r=25.6μs).

$f\,T_r$	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
V [m/s]	0.065	0.65	6.5	65	650





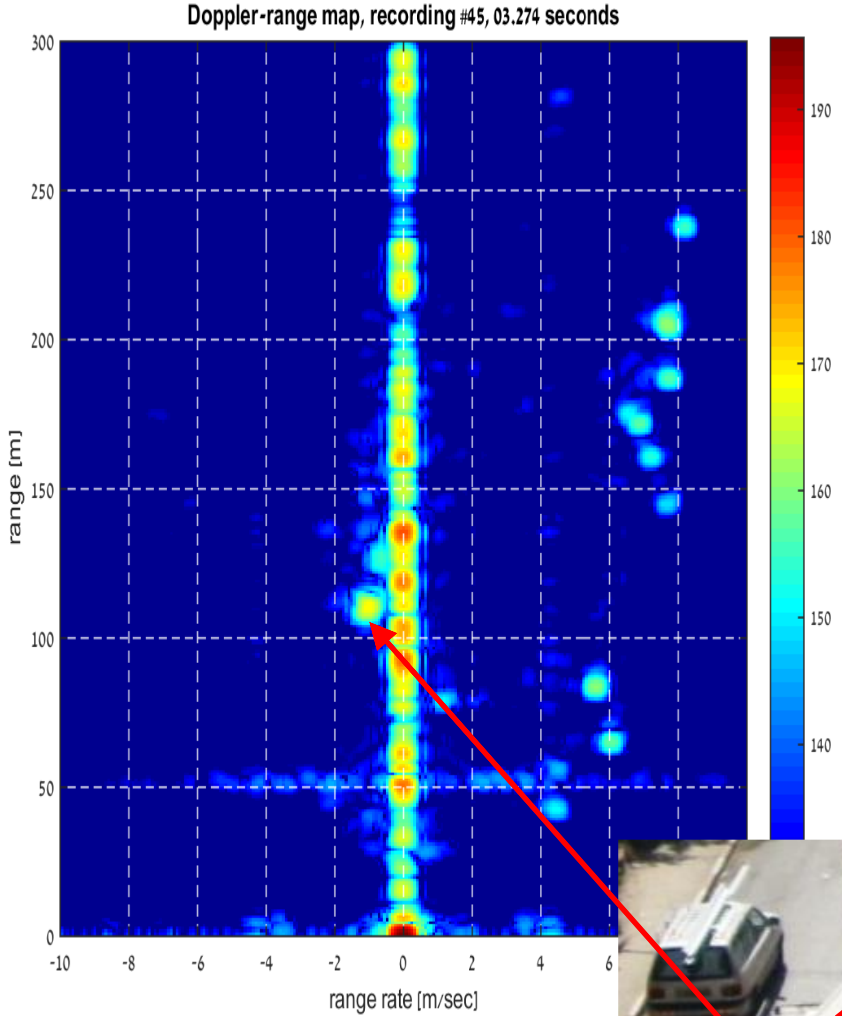
Frequency response of a <u>modified</u> 3-pulse canceller (a=0.7, b=0.7, P=16), followed by a BlackmanHarris weighted 2048-pulse FFT (Doppler window #5)

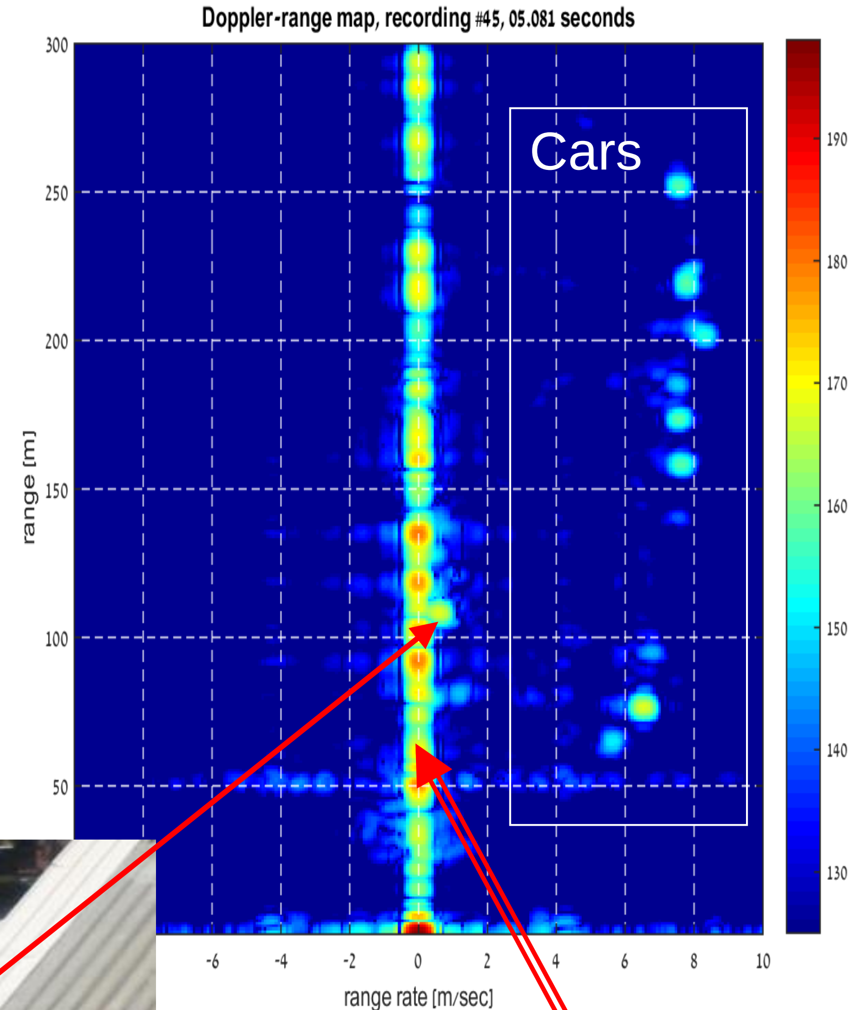
$f\ T_r$	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
v [m/s]	0.065	0.65	6.5	65	650

Moving target

Slow moving targets

Clutter

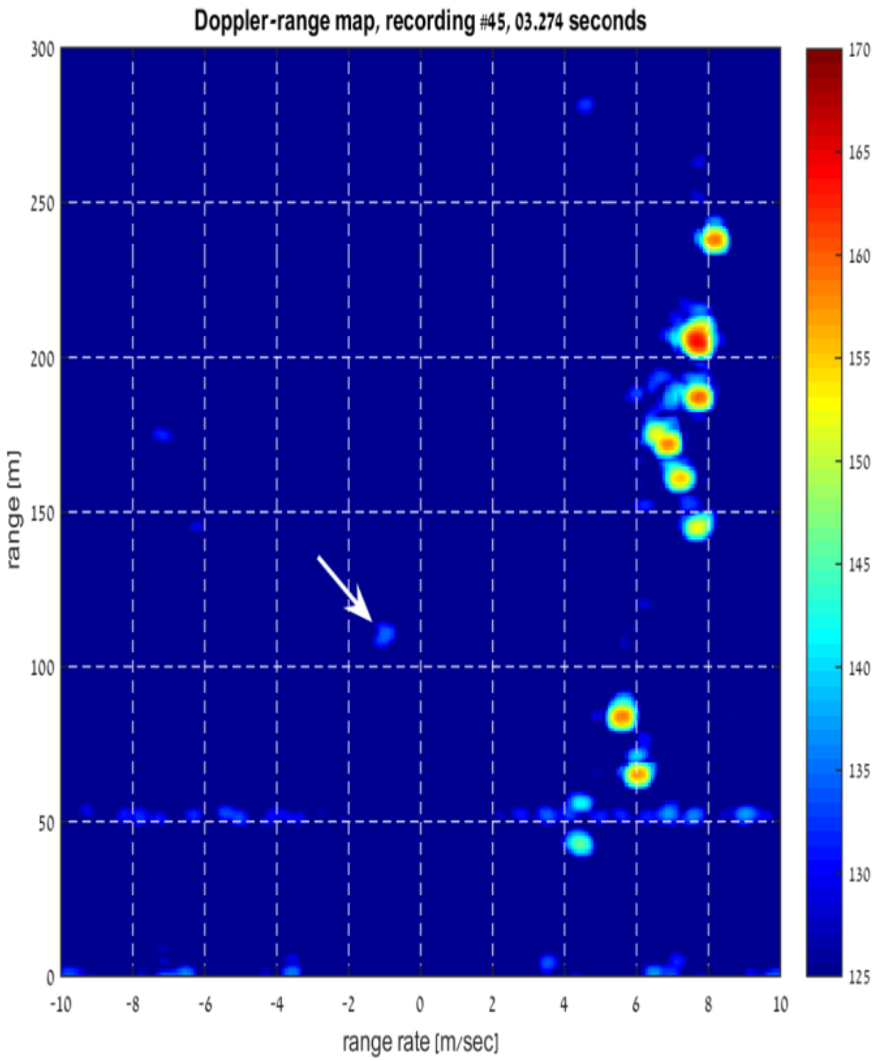


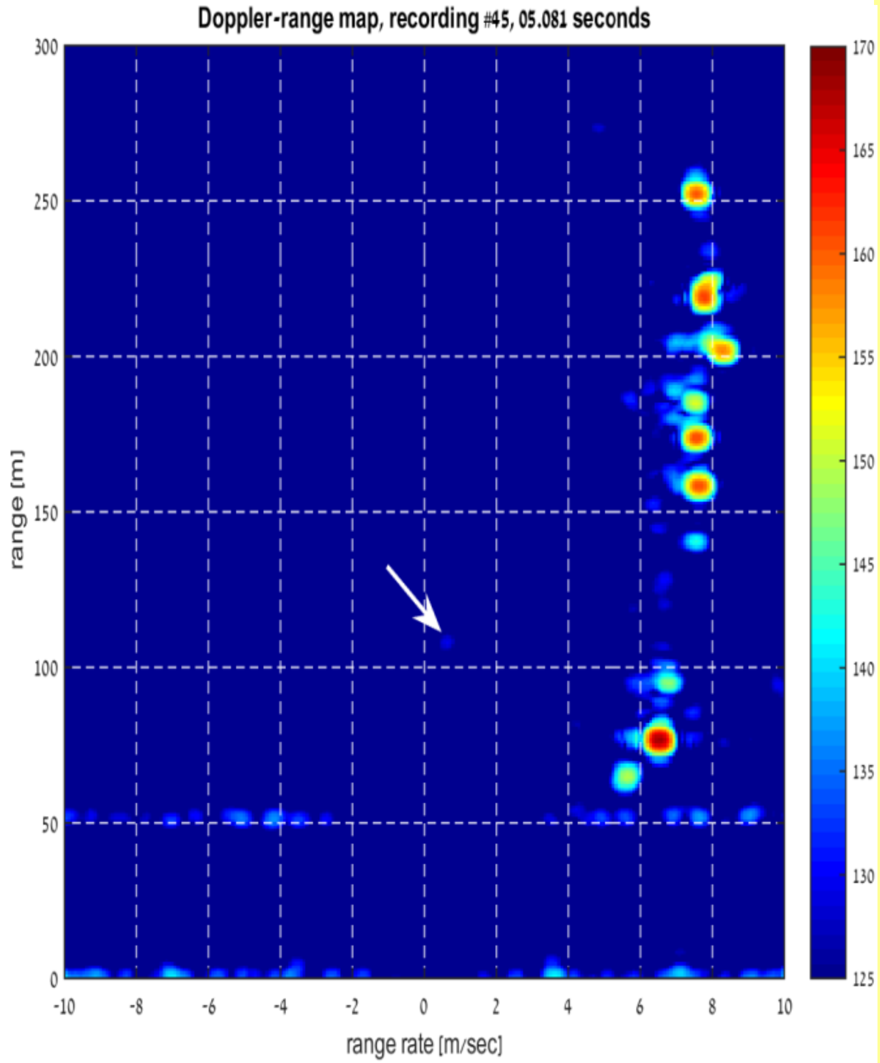


Doppler processing
Without MTI

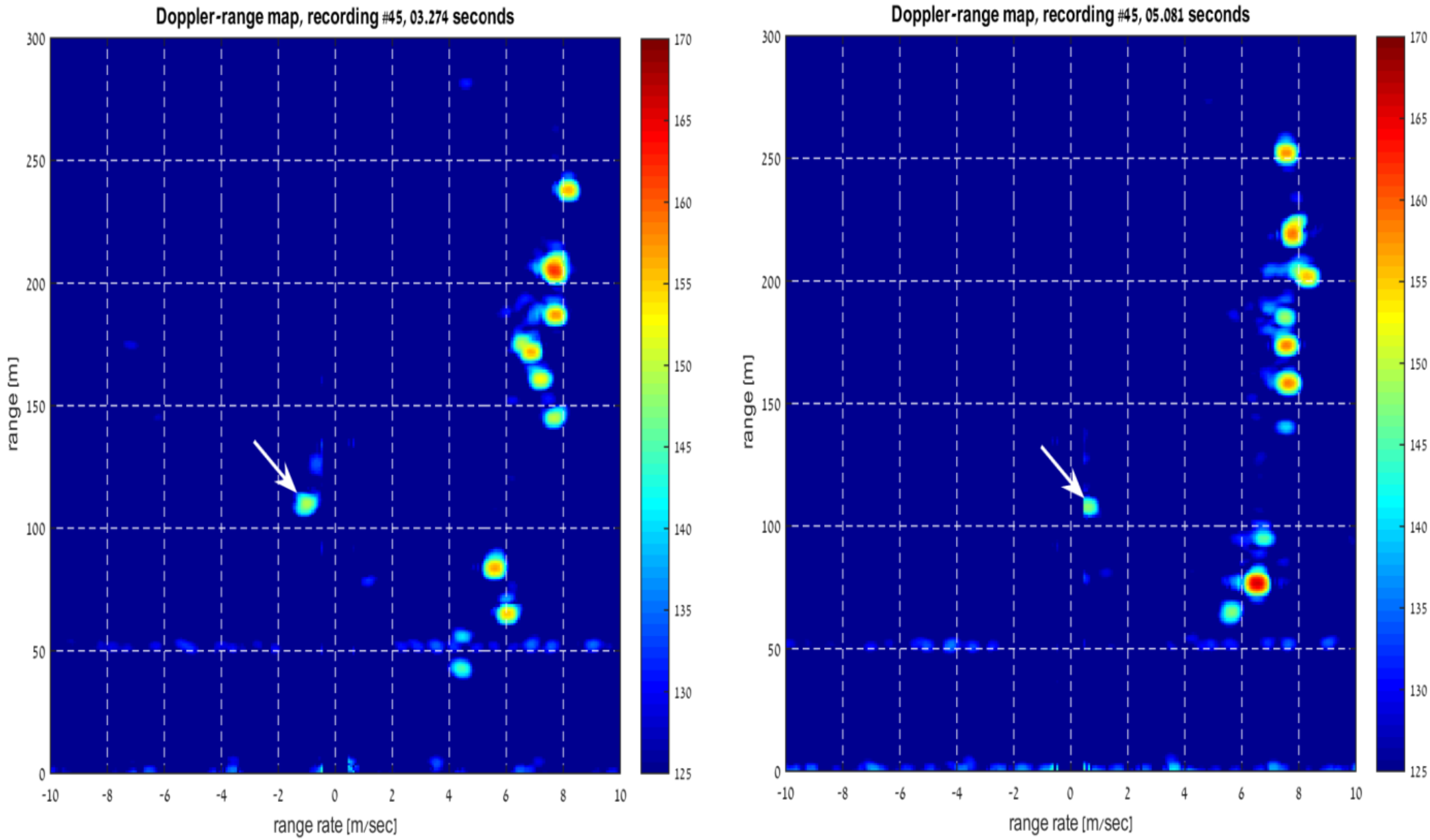
?

Stationary clutter at zero Doppler and all ranges





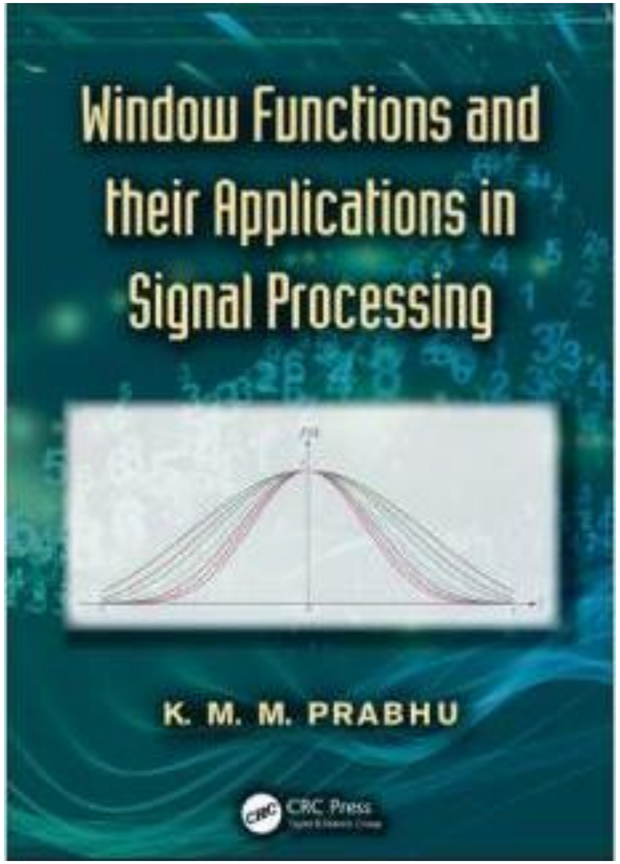
Delay-Doppler display at 3.274 seconds (left); and at 5.081 seconds (right). (***a*=1**, ***b*=1**, *P*=16), Colorbar in dB.

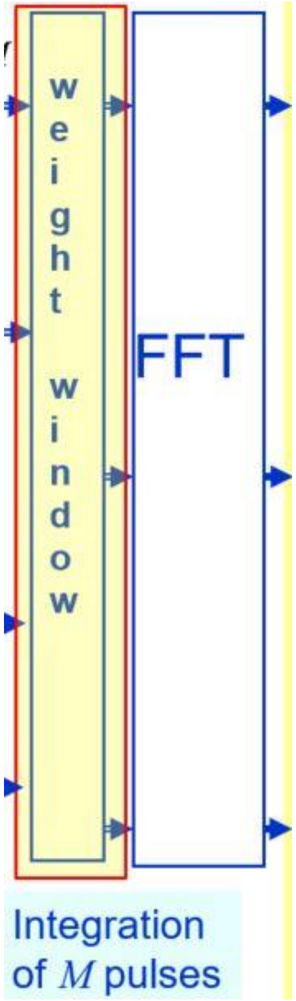


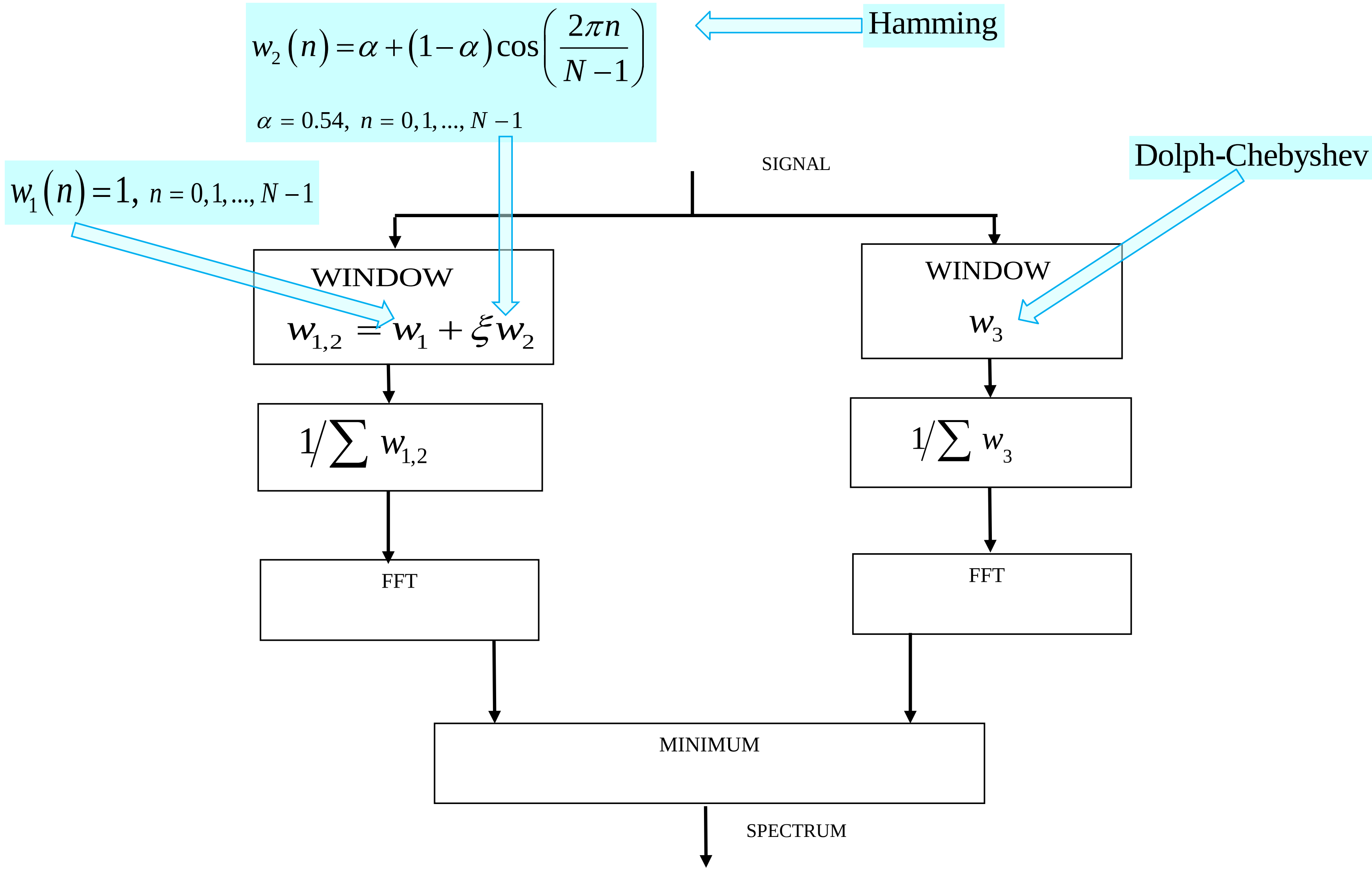
Delay-Doppler display at 3.274 seconds (left); and at 5.081 seconds (right). (***a*=0.7**, ***b*=0.7**, *P*=16), Colorbar in dB.

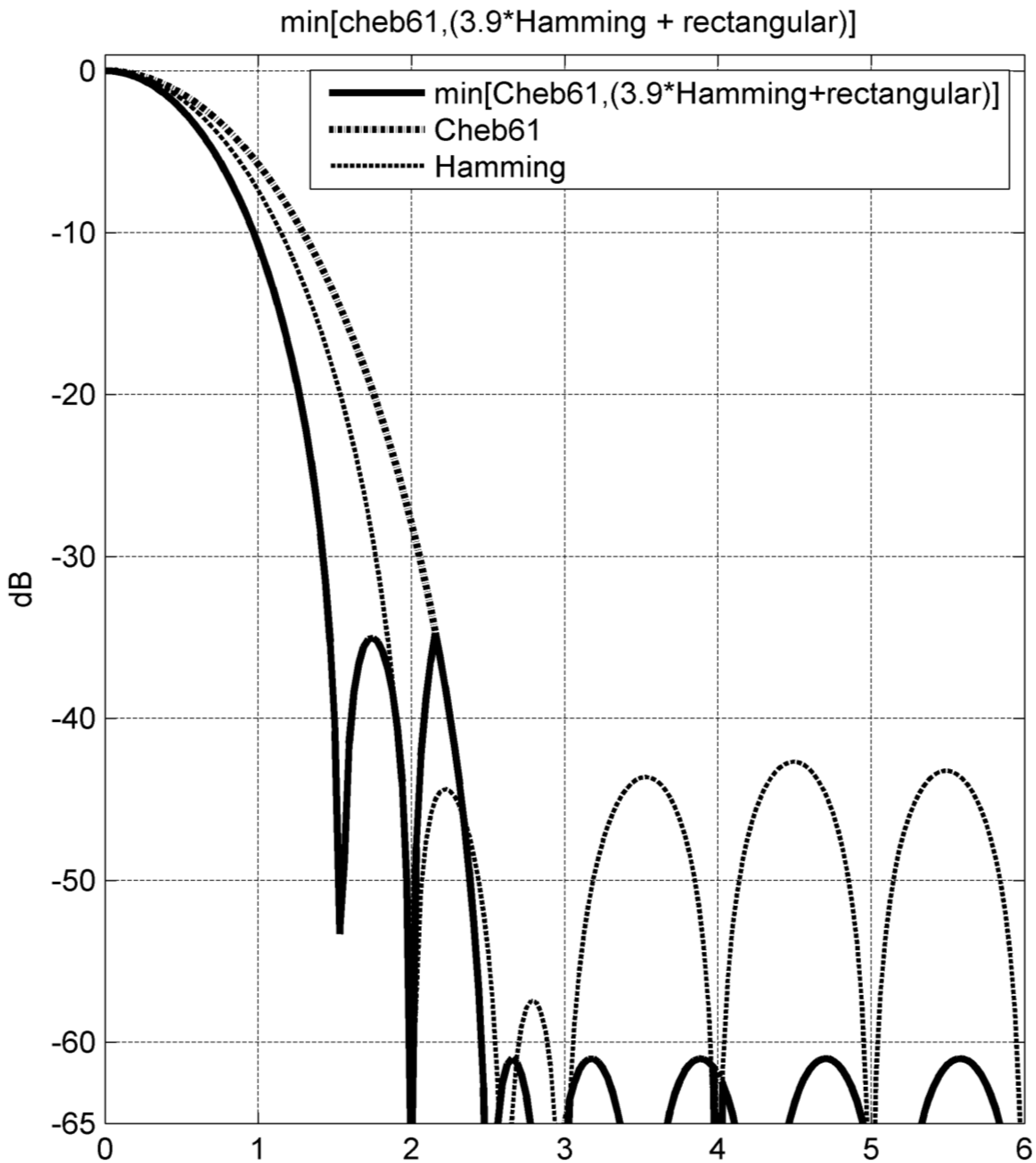
F. J. Harris, "On the use of Windows for harmonic analysis with the discrete Fourier transform," IEEE Proc., vol. 68, no. 1, pp. 51-83, Jan. 1978.

H.C. Stankwitz, R. J. Dallaire and J. R. Fienup, "Nonlinear apodization for sidelobe control in SAR imagery," IEEE Trans. Aerospace and Electronic Systems, vol. 31, no. 1, pp. 267-279, Jan. 1995.






$w_2(n) = \alpha + (1-\alpha)\cos\left(\frac{2\pi n}{N-1}\right)$
$\alpha = 0.54,\ n = 0, 1, \ldots, N-1$
Hamming
$w_1(n) = 1,\ n = 0, 1, \ldots, N-1$
SIGNAL
Dolph-Chebyshev
WINDOW
$w_{1,2} = w_1 + \xi w_2$
WINDOW
w_3
$1/\sum w_{1,2}$
$1/\sum w_3$
FFT
FFT
MINIMUM
SPECTRUM

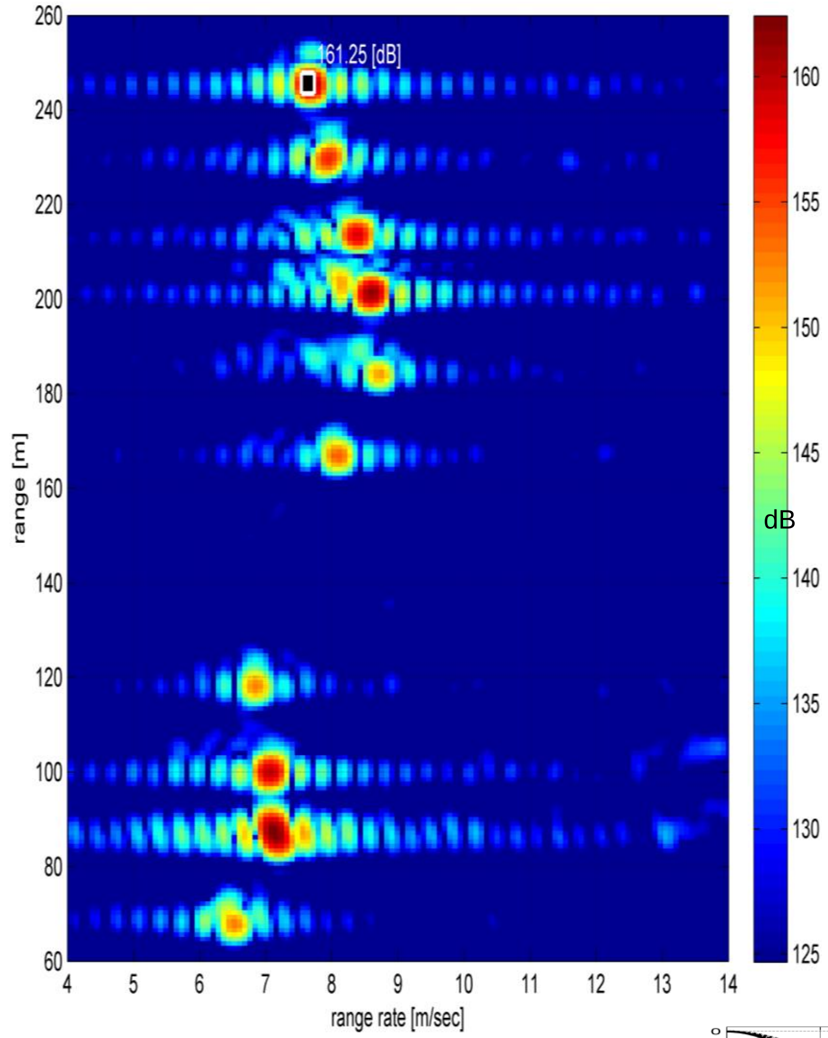


Responses of:

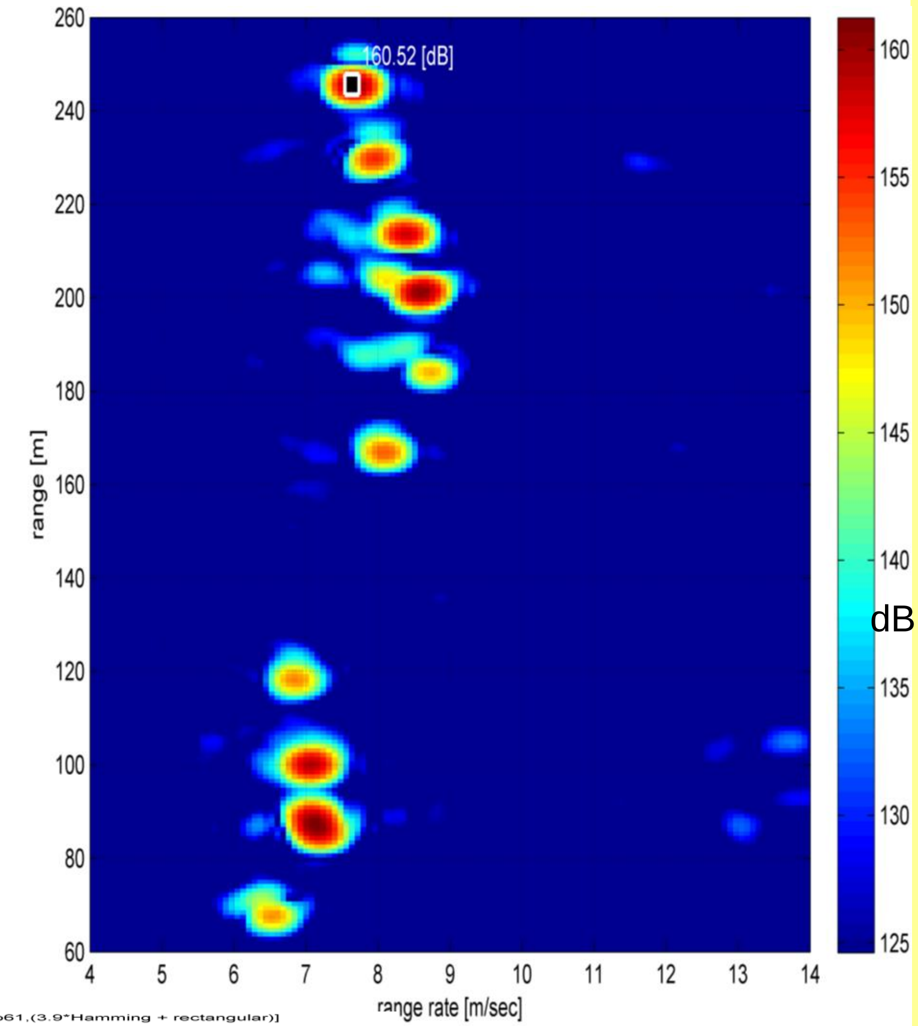
- The new window (solid),
- Dolph-Chebyshev, 61dB (dash-dot)
- Hamming (dots).

N=512.

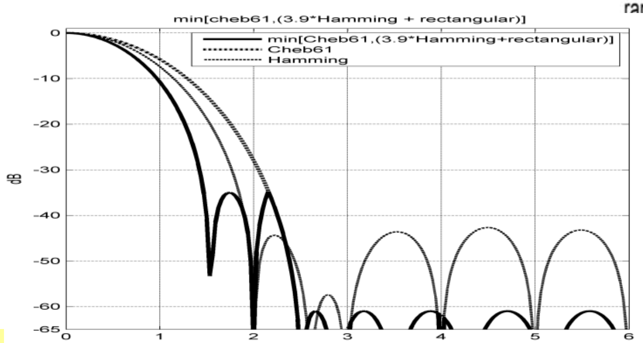
$$N = 512$$



Rectangular window



The new window



Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

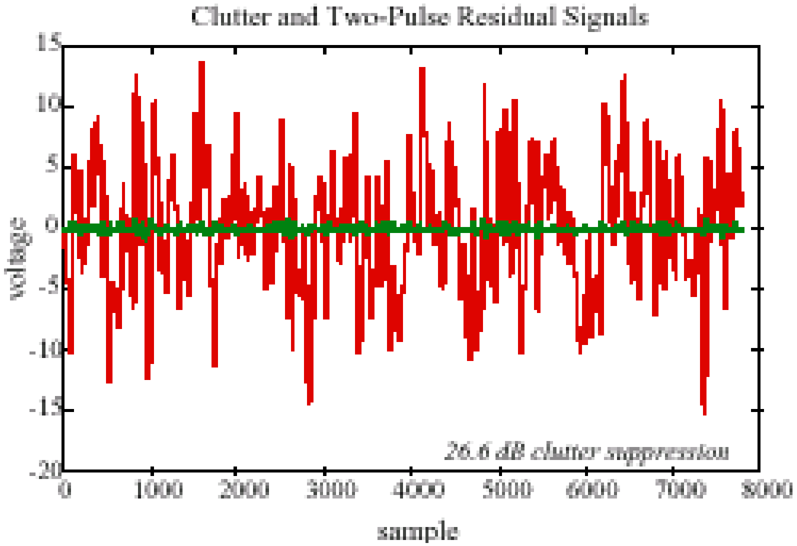
MTI Figures of Merit

- There are three reasonably well-defined figures of merit for quantifying clutter filtering performance
  - Clutter attenuation, CA
  - Improvement Factor, I
  - Subclutter Visibility, SCV

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Clutter Attenuation

- Reduction in clutter power due from the input to the output of the clutter filter
  - no target assumptions necessary
  - measures filter performance only, not including detector & integrator
  - depends not only on filter design, but also on clutter spectrum shape
  - can be several tens of dB



Improvement Factor

- Improvement Factor is the increase in S/C ratio at the output of the clutter filter over that at the input, averaged over all target radial velocities of interest:

$$I = \mathbf{E}\left\{ \frac{(S/C)_{out}}{(S/C)_{in}} \right\}_{v_r}$$

  - usually assume all radial velocities equally likely
  - assumes non-fluctuating target
  - measures filter performance only, not including detector & integrator
  - depends not only on filter design, but also on clutter spectrum shape
  - can be several tens of dB

Subclutter Visibility

- Subclutter Visibility is the maximum value of input clutter to signal (*sic*) ratio, averaged over all target radial velocities, for which specified detection and false alarm probabilities can be obtained
  - $SCV < I$
  - difference is the clutter visibility factor, V_c
  - depends on same quantities as I, plus
    - detection and false alarm specifications
    - detector and integrator characteristics
    - clutter to noise ratios
  - can also be several tens of dB

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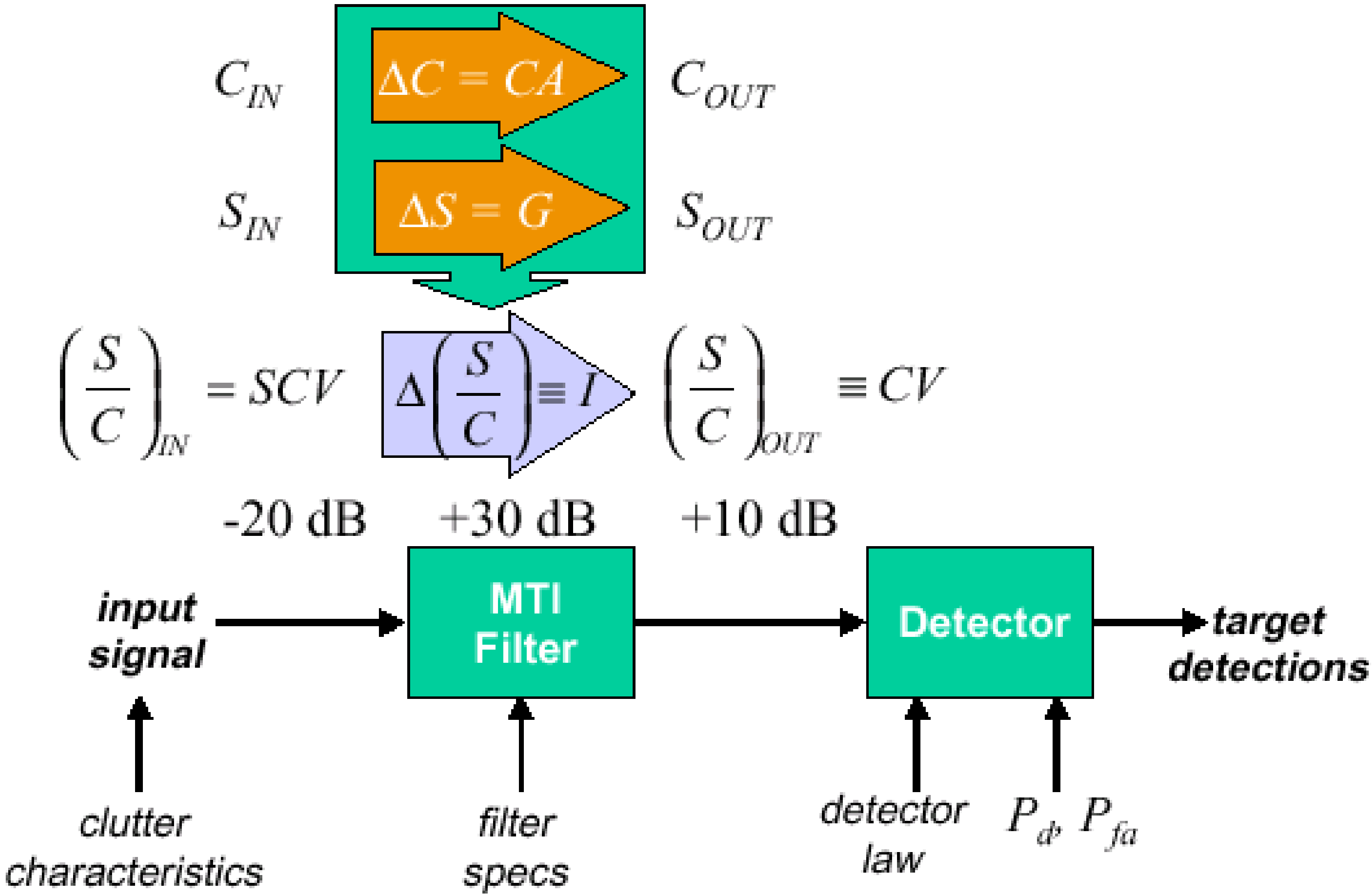
From M.A. Richards, Georgia Tech

Relation Between Figures of Merit



Factoring Improvement Factor

- Improvement factor is the product of the filter's effect on the clutter and on the target:

$$I = \frac{(S/C)_{out}}{(S/C)_{in}} = \left(\frac{S_{out}}{S_{in}}\right)\left(\frac{C_{in}}{C_{out}}\right) = G \bullet CA$$

- Can compute gain and clutter attenuation in time or frequency domain

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech

Computing Gain - 1

- Gain is the increase in target power due to the clutter filter

$G =$

$$E\left\{\frac{S_{OUT}}{S_{IN}}\right\} = E\left\{\frac{S_{IN}\left|H\left(f_{IN}\right)\right|^2}{S_{IN}}\right\} = E\left\{\left|H\left(f_{IN}\right)\right|^2\right\}$$

  – usually averaged over target velocities by considering f_{IN} to be a uniform random variable over $(-PRF/2,+PRF/2)$

- Gain is usually averaged over target velocities by considering f_{IN} to be a uniform random variable over $(-PRF/2,+PRF/2)$:

$$G = \int_{-PRF/2}^{PRF/2} \frac{1}{PRF}\left|H\left(f\right)\right|^2 df$$

- More restrictive assumptions used, with better results, when possible

Two-Pulse Canceller Gain

$$G = \int_{-\mathrm{PRF}/2}^{\mathrm{PRF}/2} \frac{1}{\mathrm{PRF}} \left|H(f)\right|^2 df$$

$$H(f) = 2\left|\sin\left(\pi f / \mathrm{PRF}\right)\right|$$

$$\omega = 2\pi f T = 2\pi f / \mathrm{PRF}$$

- Equivalently, in normalized radian frequency,

$$G = \int_{-\pi}^{\pi} \frac{1}{2\pi} \left|H(\omega)\right|^2 d\omega$$

$$\left|H(\omega)\right|^2 = 4\sin^2(\omega/2)$$

$$G = \int_{-\pi}^{\pi} \frac{1}{2\pi} 4\sin^2(\omega/2) d\omega$$

$$= \frac{2}{\pi}\int_{0}^{\pi} \sin^2(\omega/2) d\omega + \frac{2}{\pi}\int_{-\pi}^{0} \sin^2(\omega/2) d\omega$$

$$= \frac{4}{\pi}\int_{0}^{\pi} \sin^2(\omega/2) d\omega$$

$$= \frac{4}{\pi}\left(\frac{\pi}{2}\right) = 2$$

$$G = 2$$

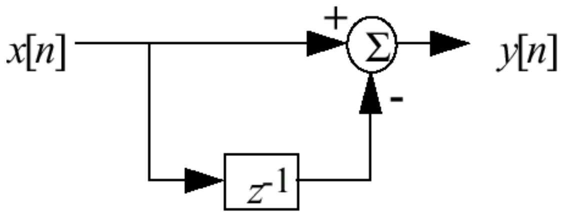
Improvement factor of two-pulse canceller

$$I = \frac{(S/C)_{out}}{(S/C)_{in}} = \left(\frac{S_{out}}{S_{in}}\right)\left(\frac{C_{in}}{C_{out}}\right) = G \bullet CA$$

$$G = 2$$

$$CA = \frac{clutter_{in}}{clutter_{out}} = ?$$

Improvement factor of two-pulse canceller



$$CA = \frac{Pclutter_{in}}{Pclutter_{out}} = ?$$

$$\mathrm{E}\left[y^2(t)\right] = \mathrm{E}\left\{\left[x(t) - x(t-T)\right]^2\right\}$$
$$= \mathrm{E}\left[x^2(t)\right] - 2\mathrm{E}\left[x(t)x(t-T)\right] + \mathrm{E}\left[x^2(t-T)\right]$$
$$= 2R(0) - 2R(T)$$

$$CA = \frac{\mathrm{E}\left[x^2(t)\right]}{\mathrm{E}\left[y^2(t)\right]} = \frac{R(0)}{2R(0) - 2R(T)}$$
$$= \frac{1}{2}\,\frac{1}{1-\rho(T)}$$

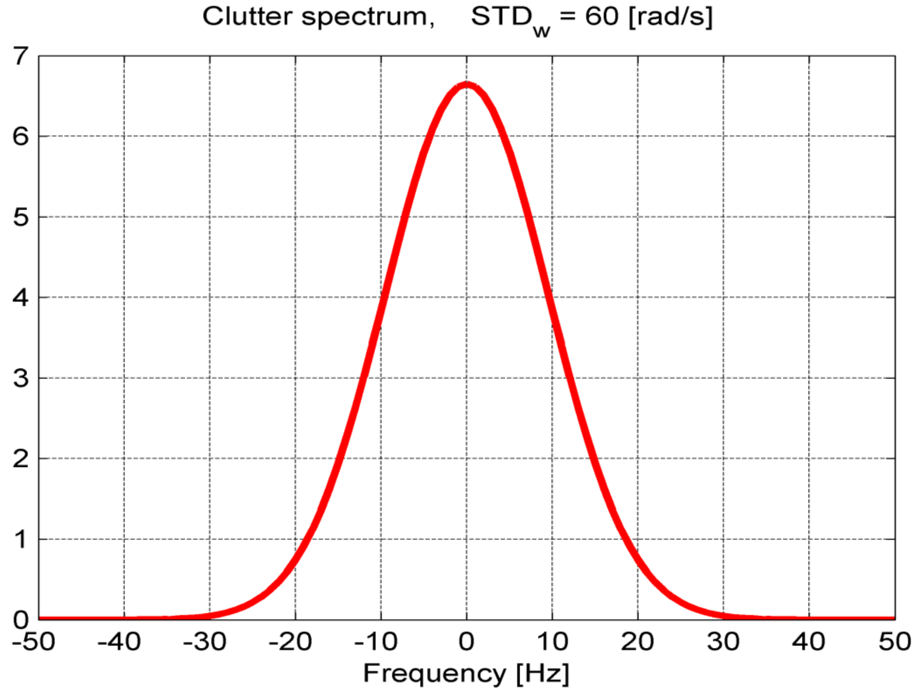
$$\frac{R(T)}{R(0)} = \rho(T)$$

$$G = 2$$

$$I = G \cdot CA = \frac{1}{1-\rho(T)}$$

Assume clutter spectrum is Gaussian

$$S_c(\omega) = \frac{P_c}{\sigma_\omega \sqrt{2\pi}} e^{\frac{-\omega^2}{2\sigma_\omega^2}}$$



$$R(\tau) = \int_{-\infty}^{\infty} S_c(\omega) e^{-j\omega\tau} d\omega = P_c e^{\frac{-\sigma_\omega^2 \tau^2}{2}}$$

$$R(0) = P_c$$

$$\rho(\tau) = \frac{R(\tau)}{R(0)} = e^{\frac{-\sigma_\omega^2 \tau^2}{2}}$$

$$I = \frac{1}{1 - e^{\frac{-(\sigma_\omega T)^2}{2}}}$$

$$I = G \cdot CA = \frac{1}{1 - \rho(T)}$$

Two-pulse canceller

Three-pulse canceller

$$\mathrm{E}\left[y^2(t)\right] = \mathrm{E}\left\{\left[x(t) - 2x(t-T) + x(t-2T)\right]^2\right\}$$
$$= 6R(0) - 8R(T) + 2R(2T)$$

$$G = 6$$

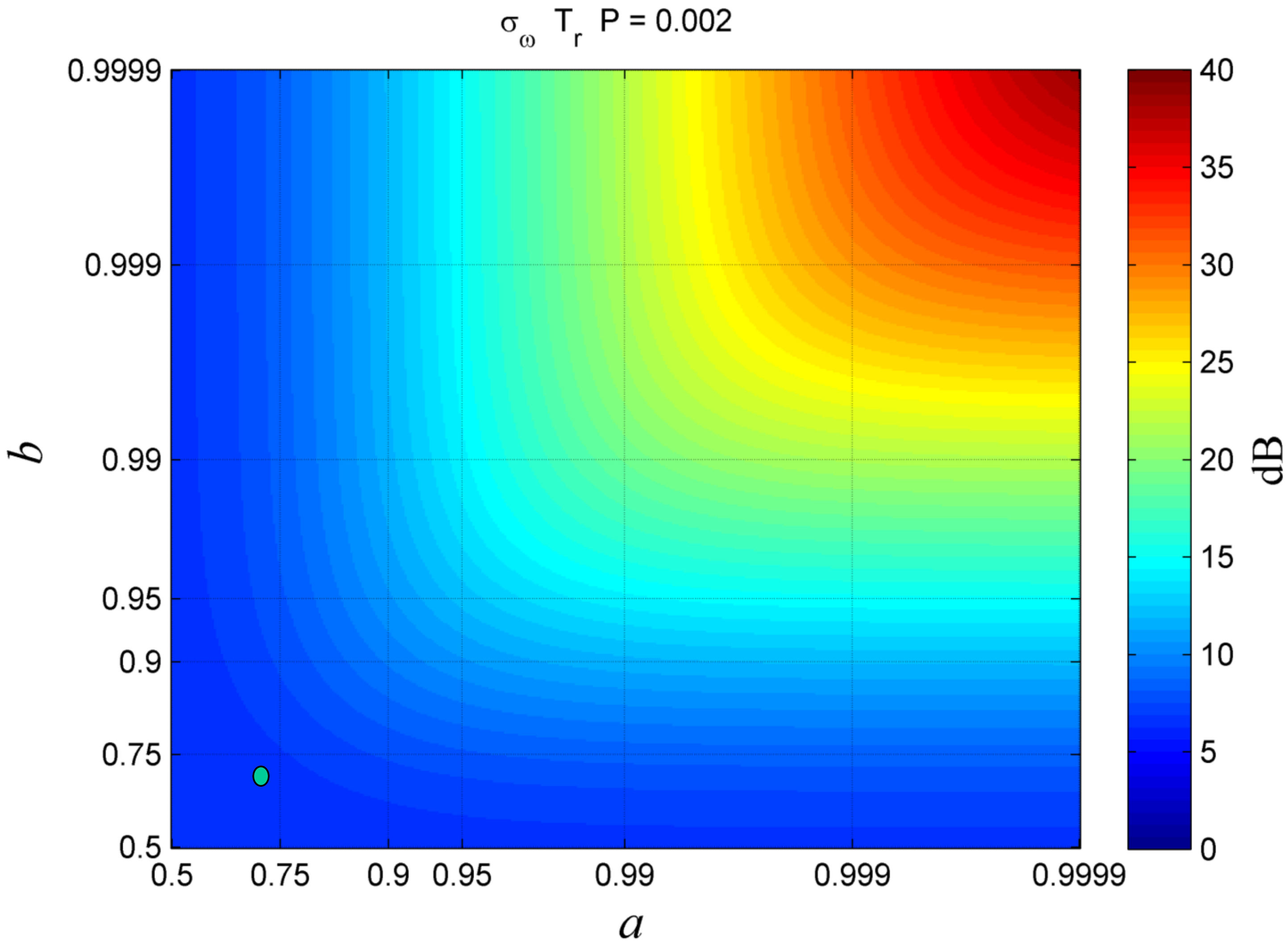
$$I = \frac{1}{1 - \frac{4}{3}e^{\frac{-(\sigma_\omega T)^2}{2}} + \frac{1}{3}e^{\frac{-(2\sigma_\omega T)^2}{2}}}$$

Improvement factor – conventional 3-pulse canceller

$$I = \mathrm{E}\left[\frac{(S/C)_{\text{out}}}{(S/C)_{\text{in}}}\right] = \left[1 - \frac{4}{3}e^{-\frac{1}{2}(\sigma_\omega T_r)^2} + \frac{1}{3}e^{-\frac{1}{2}(2\sigma_\omega T_r)^2}\right]^{-1}$$

Improvement factor –Modified 3-pulse canceller (a,b,P)

$$I = \left[\begin{array}{l} 1 - \dfrac{2(a+b)\left(1+a^2b^2\right)}{1+(a+b)^2+a^2b^2}e^{-\frac{1}{2}(\sigma_\omega PT_r)^2} \\ + \dfrac{2ab}{1+(a+b)^2+a^2b^2}e^{-\frac{1}{2}(2\sigma_\omega PT_r)^2} \end{array}\right]^{-1}$$



Improvement factor as function of *a* and *b*. Gaussian clutter spectrum with $\sigma_\omega P T_r = 0.002$

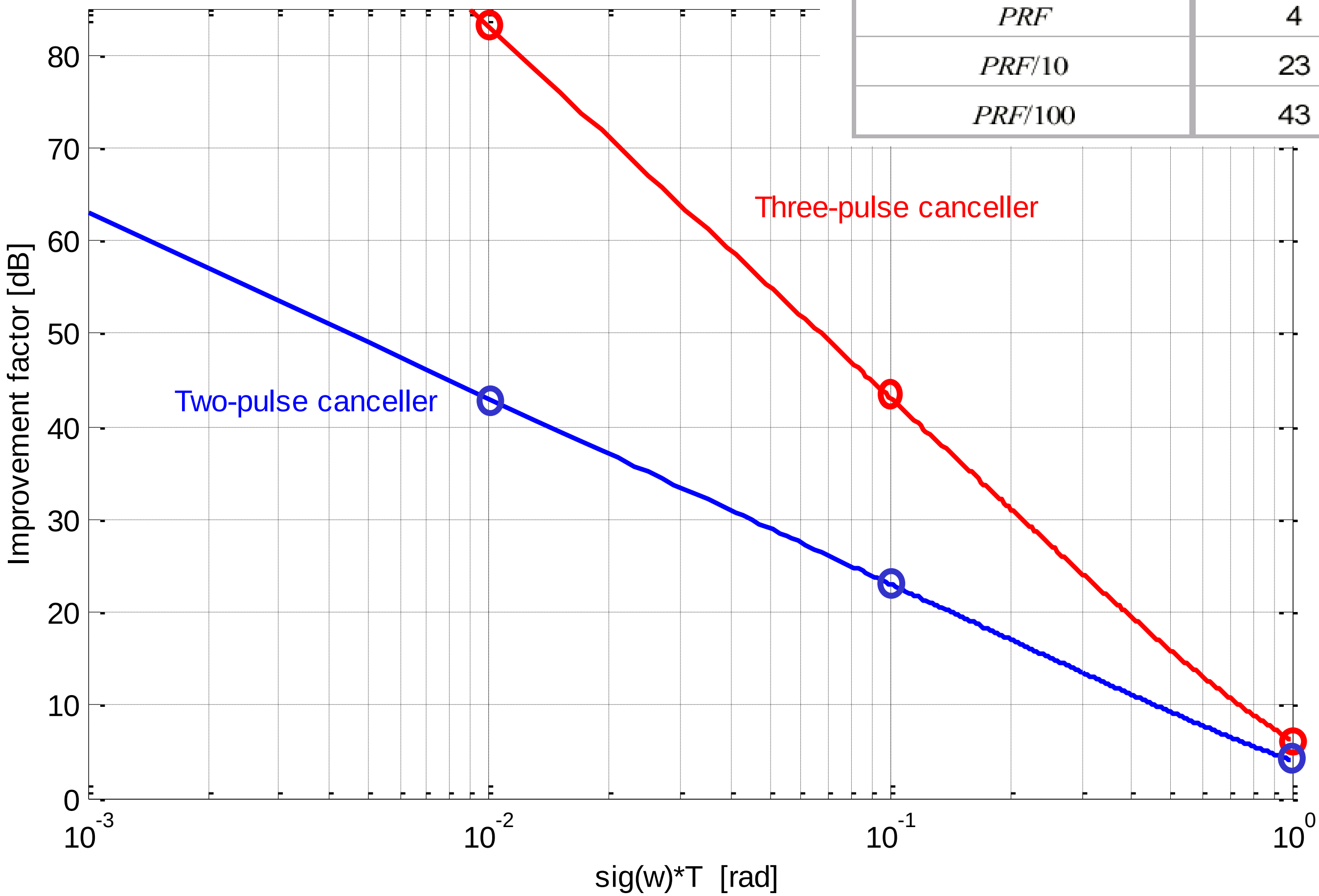
Choosing *a* = *b* =0.7, gives up much of the improvement factor, but allows observing very slow targets.

From M.A. Richards, Georgia Tech

Improvement Factor for Gaussian Clutter Power Spectrum

- **Effectiveness of MTI filtering improves for narrower clutter spectra**

Standard Deviation of Clutter Power Spectrum	Improvement Factor (dB)	
	Two-Pulse Canceller	Three-Pulse Canceller
PRF	4	6.3
PRF/10	23	43
PRF/100	43	83

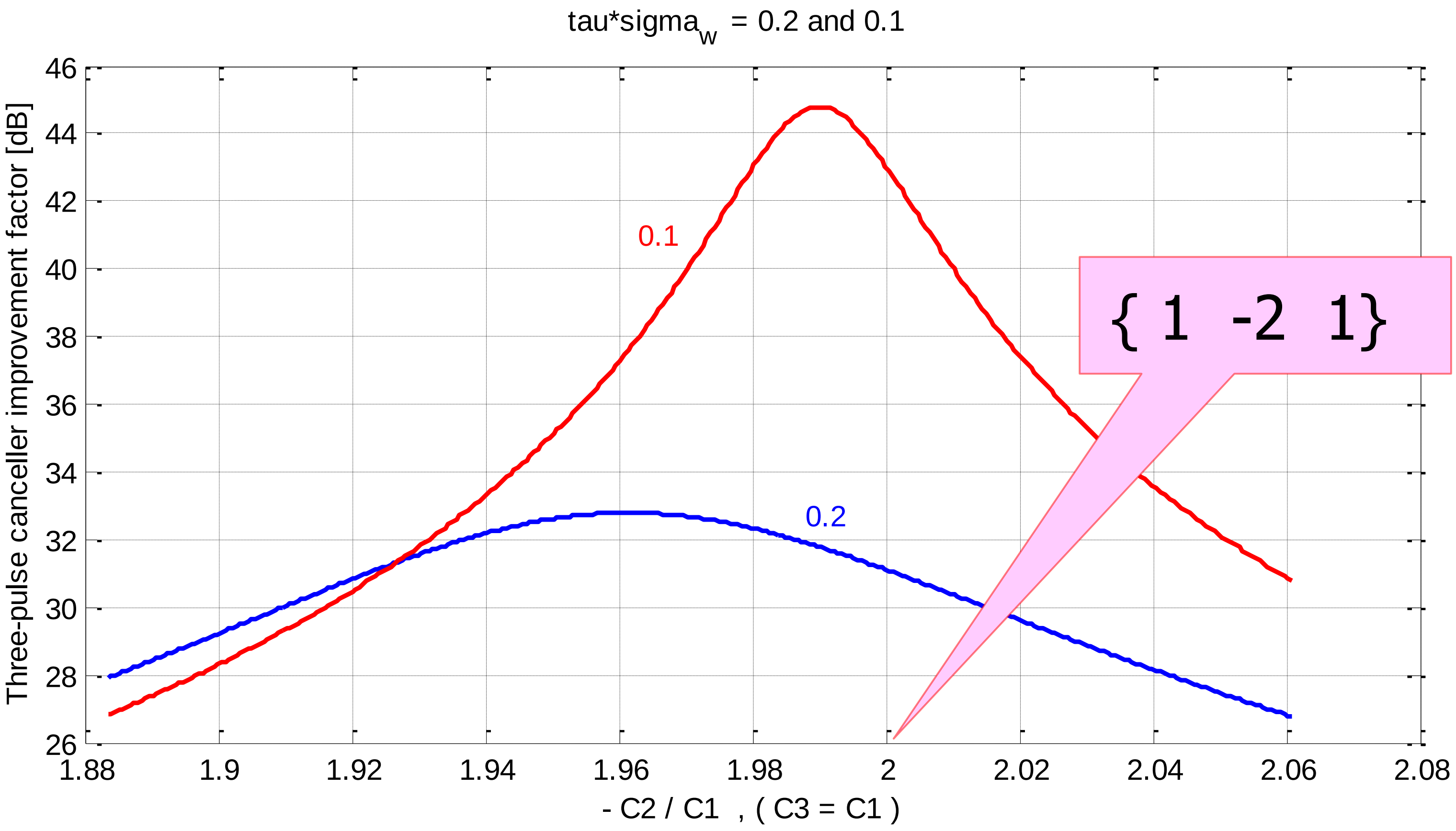


Nadav Levanon, Tel-Aviv University

Are binomial coefficients {1 -2 1}, { 1 -3 3 -1} optimal?

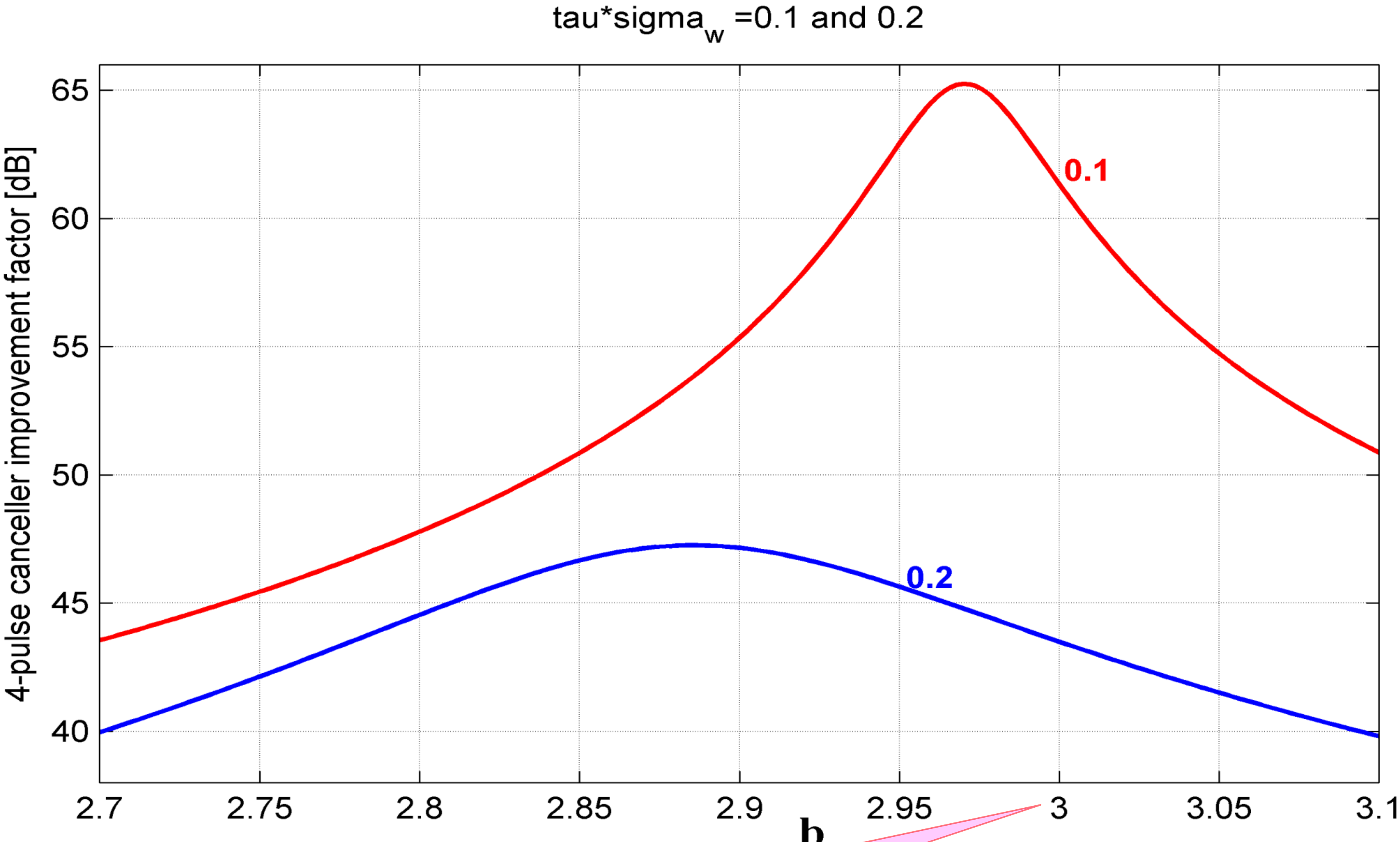
$$y(t) = x(t) - C_2 x(t-T) + x(t-2T)$$

$$I = \left\{ 1 - \frac{4C_2}{2+C_2^2} \exp\left[-\frac{(T\sigma_\omega)^2}{2} \right] + \frac{2}{2+C_2^2} \exp\left[-\frac{(2T\sigma_\omega)^2}{2} \right] \right\}^{-1}$$



$$y(t)=x(t)-\mathrm{b}x(t-T)+\mathrm{b}x(t-2T)-x(t-3T)$$

4-pulse canceller



{ 1 -3 3 -1}

Capon, J. "Optimum weighting functions for the detection of sampled signals in noise", *IEEE Trans. Information Theory*, IT-10 (2), April 1962, pp. 152-159.

Hsiao, J.K. "On the optimization of MTI clutter rejection", *IEEE Trans. Aerospace and Electronic Systems*, AES-10 (5), Sep. 1974, pp. 622-629.

As pointed out in insert 11A, the normalized correlation function of clutter with zero-centered Gaussian distribution is given by

$$\rho(\tau) = \exp\left[-\frac{(\tau\sigma_\omega)^2}{2}\right] \quad ; \quad \rho(0) = 1 = \text{average input power} \qquad (11B.3)$$

For equally spaced pulses, T seconds apart, the clutter samples covariance matrix is therefore

$$\mathbf{A} = \begin{bmatrix} 1 & \rho & \rho^4 & \rho^9 \; .... \\ \rho & 1 & \rho & \rho^4 \; ..... \\ \rho^4 & \rho & 1 & \rho \; ..... \\ \rho^9 & \rho^4 & \rho & 1 \; ..... \end{bmatrix} \qquad (11B.4)$$

where $\rho = \rho(T)$. The elements a_{ij} of **A**, are real and positive.

Let the filter coefficients be described by the vector

$$\mathbf{c} = [C_1 \; C_2 \; ..... \; C_N]^{\mathrm{T}} \qquad (11B.5)$$

The clutter average power at the output of the filter is proportional to

$$\frac{P_{c_{out}}}{P_{c_{in}}} = \sum_i \sum_j C_i C_j^* a_{ij} \qquad (11B.6)$$

Since a_{ij} are real and, as will be shown, the filter coefficients are also real, the average clutter output power can be represented also by the inner product

$$\frac{P_{c_{out}}}{P_{c_{in}}} = (\mathbf{c}, \mathbf{A}\mathbf{c}) \qquad (11B.7)$$

If the vector of filter coefficients **c** is an eigenvector of **A,** corresponding to an eigenvalue λ (a scalar, not to be confused with the wavelength) then by definition

$$\mathbf{A}\mathbf{c} = \lambda\,\mathbf{c} \qquad (11B.8)$$

Using (11B.8) in (11B.7) yields

$$\frac{P_{c_{out}}}{P_{c_{in}}} = (\mathbf{c}, \lambda\mathbf{c}) = \lambda(\mathbf{c}, \mathbf{c}) = \lambda \sum_{n=1}^{N} C_n^2 = \lambda \overline{G} \qquad (11B.9)$$

Assuming that the Doppler signal is uniformly distributed at all frequencies, the signal power gain is $\overline{G}$, hence for the given coefficient vector, the improvement factor is given by

$$I = \frac{\overline{G}}{\frac{P_{c_{out}}}{P_{c_{in}}}} = \frac{1}{\lambda} \qquad (11B.10)$$

The improvement factor will be maximum when the eigenvalue λ is the smallest of the eigenvalues of **A**. Hence the optimal filter coefficients are given by the eigenvector of **A** which corresponds to the minimum eigenvalue of **A**.

EXAMPLE 1

What should be the optimal double canceller coefficients when $T\sigma_\omega = 0.1$, and what is the corresponding improvement factor?

The covariance matrix is given by

$$\mathbf{A} = \begin{bmatrix} 1.0000 & 0.9950 & 0.9802 \\ 0.9950 & 1.0000 & 0.9950 \\ 0.9802 & 0.9950 & 1.0000 \end{bmatrix}$$

The three eigenvalues of **A** are:

3.3222e-005 , 0.0198 , 2.9802

of which the first one is clearly the smallest. The eigenvector corresponding to the minimum eigenvalue is

$$\mathbf{c} = [\ 0.4096 \quad -0.8151 \quad 0.4096\ ]^T$$

which, when normalized with respect to the first coefficient, becomes

$$\mathbf{c}/C_1 = [\ 1.0000 \quad -1.9901 \quad 1.0000\ ]^T$$

Finally, the improvement factor in dB is obtained from the inverse of the minimum eigenvalue

$$I = -10 \log_{10}(3.3222\text{e-}5) = 44.79 \text{ dB}$$

EXAMPLE 2

What should be the optimal coefficients of a canceller of length 5 when $T\sigma_\omega = 0.1$, and what is the corresponding improvement factor?

$\mathbf{c}/C_1 = [\ 1.0000 \quad -3.9409 \quad 5.8821 \quad -3.9409 \quad 1.0000\ ]^T$

$I = 84.6742$ dB

What is the improvement factor of a corresponding filter with binomial {1 -4 6 -4 1} coefficients?

$I_B = 78.37$ dB

which is about 6.3 dB less than the improvement factor of an optimal 5 element FIR filter.

Fig. 11.17 displays the magnitude of the frequency response of a 5 element FIR MTI filter with the above optimal coefficients (solid line), compared to 5 element filter with binomial coefficients $\{1 \ \ -4 \ \ 6 \ \ -4 \ \ 1\}/\sqrt{70}$ (dash line). The dotted line presents clutter spectrum with a frequency spread ($T\sigma_\omega = 0.1$) to which the filter coefficients were optimized. The higher improvement factor is probably due to the extended band-stop section in the frequency response of the optimized filter.

```
% mti_coef.m
% written by Nadav Levanon on 1 Dec 2008
tw=input(' Spectrum S.D.*PRI (typ. = 0.1) = ?   ');
nn=input(' No of pulses = ?   ');
rho=exp(-0.5*tw^2);
q=(0:nn-1).^2;
vv=rho.^q;
aa=toeplitz(vv);
[v,d]=eig(aa);
coef=v(:,1).'./v(1,1)
improvement_dB=-10*log10(d(1))
```

```
tw = 0.1000
rho = 0.9950
vv = 1.0000   0.9950   0.9802   0.9560   0.9231
aa =
    1.0000    0.9950    0.9802    0.9560    0.9231
    0.9950    1.0000    0.9950    0.9802    0.9560
    0.9802    0.9950    1.0000    0.9950    0.9802
    0.9560    0.9802    0.9950    1.0000    0.9950
    0.9231    0.9560    0.9802    0.9950    1.0000
v =
   -0.1216    0.3200    0.5377   -0.6306    0.4428
    0.4791   -0.6306   -0.2618   -0.3200    0.4494
   -0.7151    0.0000   -0.5336   -0.0000    0.4516
    0.4791    0.6306   -0.2618    0.3200    0.4494
   -0.1216   -0.3200    0.5377    0.6306    0.4428
d =
    0.0000         0         0         0         0
         0    0.0000         0         0         0
         0         0    0.0007         0         0
         0         0         0    0.0967         0
         0         0         0         0    4.9026
10*log10(diag(d))` =
  -84.6742  -56.2680  -31.6551  -10.1465    6.9043

v(:,1).'./v(1,1) =
    1.0000   -3.9409    5.8821   -3.9409    1.0000
```

```
>> mti_coef
 Spectrum S.D. x PRI (typ. = 0.1) = ?   .1
 No of pulses = ?   5
 coef =
   1.0000   -3.9409    5.8821   -3.9409    1.0000
 improvement_dB =
  84.6742
```

5-pulse canceller

1.0000
-4.0000
6.0000
-4.0000
1.0000
improvement_factor_dB = 78.37

1.0000
-3.9409
5.8821
-3.9409
1.0000
improvement_factor_dB = 84.6742