

# DIVERSE RADAR PULSE-TRAIN WITH FAVOURABLE AUTOCORRELATION AND AMBIGUITY FUNCTIONS

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**Abstract.** A coherent train of identical Linear-FM pulses is a popular radar signal, mainly due to its good range and Doppler resolution. One of its drawbacks is the relatively high autocorrelation function (ACF) sidelobes. We show how to completely remove most of the sidelobes by diversifying the pulses through overlaying them with orthogonal coding. The technique applies to any other type of pulses – analog or digital.

## INTRODUCTION

Properties of the ambiguity function impose fundamental limitations on the ability of any radar waveform of constrained time-bandwidth product to distinguish between two or more targets closely spaced in both range and Doppler or detect targets in the presence of clutter. Designers of radar signals often use trains of compressed pulses to increase effective time bandwidth product and improve range and Doppler resolution. Linear-FM pulses is the most popular choice for the compressed pulse. Combating its drawback – the relatively high ACF sidelobes, attracts considerable attention, mostly along the line of shaping the spectrum, e.g., by introducing non-linearity.

Another approach for ACF sidelobe reduction is by adding diversity among the pulses in the train, through the use of coded waveform sets (also known as *complementary* sets [1], Welty codes [2] or  $\delta$ -codes). The important property of complementary sets is that their autocorrelation exhibits zero sidelobes for  $t_c \leq |\tau| \leq T$  and low recurrent lobes ( $T$  is the pulse duration and  $t_c$  is the coded signal chip length). Sivaswamy [3] introduced a concept of digital and analog basic *sub-complementary* sequences for pulse compression. Sub-complementary signals exhibit a zero-correlation zone for  $\tau \geq \tau_0$  and low recurrent lobes ( $\tau_0$  being the duration of the sub-complementary signal kernel).

In this paper we present a hybridised signal that, like sub-complementary signals, also results in a zero correlation zone for  $t_s \leq |\tau| \leq T$  and low recurrent lobes. The signal is constructed using a new approach based on *overlaying* an orthogonal coding over *any* analog or digital base-signal. The behaviour of the ACF sidelobes over the remaining delay  $0 \leq |\tau| \leq t_s$ , the ambiguity function, and the reduction in recurrent lobes, are determined by the specific waveform of the basic pulse and by the type of the overlaid orthogonal code.

## SIGNAL DEFINITION AND CORRELATION

We start with a train of  $P$  identical pulses with pulse length  $T$ . Assume the complex envelope of the original pulse  $s(t)$  ( $0 \leq t \leq T$ ) is divided into  $M$  slices with length  $t_s$  each ( $Mt_s = T$ ). Each slice is additionally phase-coded by the elements of an orthogonal  $P$ -by- $M$  coding matrix  $\mathbf{A} = \{\exp(j\varphi_{p,m})\}$ , where the  $P$  rows contain the coding sequences used for the  $P$  pulses in the train. The complex envelope of the new overlaid signal is given by

$$g(t) = \sum_{p=1}^P \sum_{m=1}^M \exp(j\varphi_{p,m}) s_m [t - (p-1)T_r] \quad (1)$$

Where  $\varphi_{p,m}$  is the phase coding element used for the  $m^{\text{th}}$  slice in the  $p^{\text{th}}$  pulse in the train ( $1 \leq m \leq M$ ,  $1 \leq p \leq P$ ),  $T_r$  is the pulse repetition interval and  $s_m(x)$  is the complex envelope of the  $m^{\text{th}}$  slice of the original pulse during  $(m-1)t_s \leq x < mt_s$  and zero elsewhere.

The matrix  $\mathbf{A}$  is said to be *orthogonal* when the dot product between any two columns of  $\mathbf{A}$  is zero ( $\mathbf{A}^T \mathbf{A}$  is diagonal). Note that the structure of the coding matrix is a sufficient condition for the rows to be a complementary set [4]. However, the opposite is not true – when the rows form a complementary set it does not necessarily imply that the columns are mutually orthogonal. Note also that orthogonal  $P$ -by- $M$  matrices exist only for  $M \leq P$  while complementary sets of  $P$  sequences with  $M$  bits exist also for  $M > P$ .

Two types of orthogonal codes are demonstrated in (2). The first is a binary orthogonal phase code constructed using nested operations on a binary 2-element complementary pair. Note that due to the construction method the code exhibits a random like nature. The second type of orthogonal code is based on the complementary set formed from all 8 cyclic shifts of a P4 sequence [4][5]. Note that the second code is a quaternary code and that it is of regular nature since the P4 itself is of regular nature and the use of ordered cyclic shifts adds additional order to the code. The two explicit examples in (2) will be used throughout the paper demonstrating some properties of the new design. For simplicity of writing we will denote the first code as the *binary code* and the second code as the *P4-based code*.

$$\varphi_{\text{binary}} = \pi \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$\varphi_{\text{P4}} = \frac{\pi}{8} \begin{bmatrix} -1 & -4 & 7 & 0 & 7 & -4 & -1 & 0 \\ -4 & 7 & 0 & 7 & -4 & -1 & 0 & -1 \\ 7 & 0 & 7 & -4 & -1 & 0 & -1 & -4 \\ 0 & 7 & -4 & -1 & 0 & -1 & -4 & 7 \\ 7 & -4 & -1 & 0 & -1 & -4 & 7 & 0 \\ -4 & -1 & 0 & -1 & -4 & 7 & 0 & 7 \\ -1 & 0 & -1 & -4 & 7 & 0 & 7 & -4 \\ 0 & -1 & -4 & 7 & 0 & 7 & -4 & -1 \end{bmatrix}$$

It can be shown that the partial ACF of the overlaid signal in the area of the main lobe (close to zero delay) is given by

$$R(\tau) = \begin{cases} P \sum_{m=1}^M R_m(\tau) & 0 \leq |\tau| < t_s \\ 0 & t_s \leq |\tau| < T_r - T \end{cases} \quad (3)$$

where  $R_m(\tau)$  is the autocorrelation of  $s_m(t)$ . Note that the autocorrelation function in the area of the mainlobe is not a function of the order of the slices in the original signal or the orthogonal phase coding matrix used.

### ORTHOGONAL-CODED LFM PULSE-TRAIN

An example of a stepwise linear frequency modulated (LFM) pulse with 40 steps (bits) and a bandwidth set such that the first null in the autocorrelation plot is at  $T/40 = t_s/5 = t_b$  is given. Fig. 1 shows the frequency (top) of the basic pulse, and phase history of all pulses overlaid with the binary code (solid) and P4-based code (dotted). The ACF, ACF mainlobe zoom, and signal spectrum are shown in Fig. 2, using log scale. The repetition interval ( $T_r$ ) used for the plots was  $3T$  (33% Duty cycle).

Though not shown, the ACF and spectrum do not change much when continuous LFM is used instead of 5 steps/slice or when one frequency value is used for each slice (frequency hopping from slice to slice). Notice that the first ACF sidelobe level is -15.9 dB while for a train of identical LFM pulses the sidelobe level is -13.6 dB.

The spectrum shape of an LFM pulse-train (not shown) has the same general shape as the spectrum of the orthogonal phase-coded train, shown in Fig. 2, but is more efficient. It exhibits a -20 dB point at  $\nu T = 29$  (39 and 45 for the two types of codes used here) and lower 60% power point at  $\nu T = 16$  (instead of 26 for the binary code and 24 for the P4-based code).

Note also that the autocorrelation recurrent lobe level shown in Fig. 2 is misleading since for non-zero Doppler there could be a much higher peak (close to 0 dB) due to the cyclic shift structure of the overlaid code and the original pulse frequency history. The sidelobes at higher Doppler can be reduced by permutations of the

slices and changes to the slice slope direction of the basic LFM pulse without changing the autocorrelation function in the area of the mainlobe but with an effect on the partial ambiguity function in that area. Note that reducing these sidelobes is not always necessary since their Doppler value can be made much higher than the expected Doppler returns.

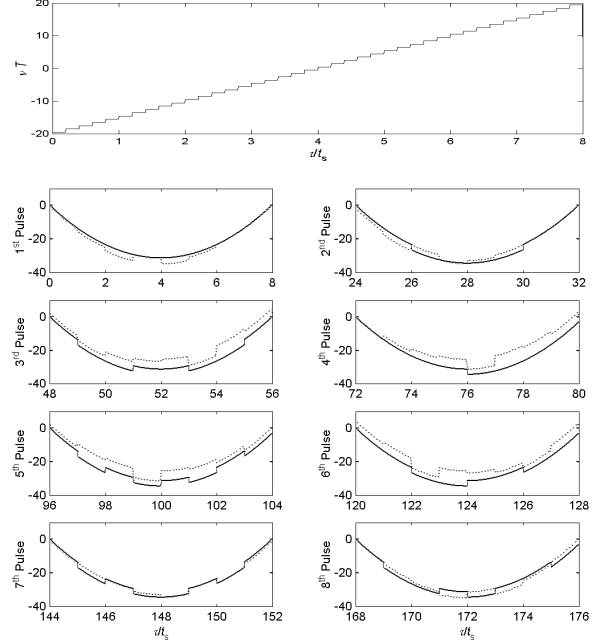


Fig. 1: Original pulse frequency history (top) and all pulse phase history for a stepwise LFM pulse-train overlaid with the 8x8 binary (solid) or P4-based (dotted) orthogonal phase code.

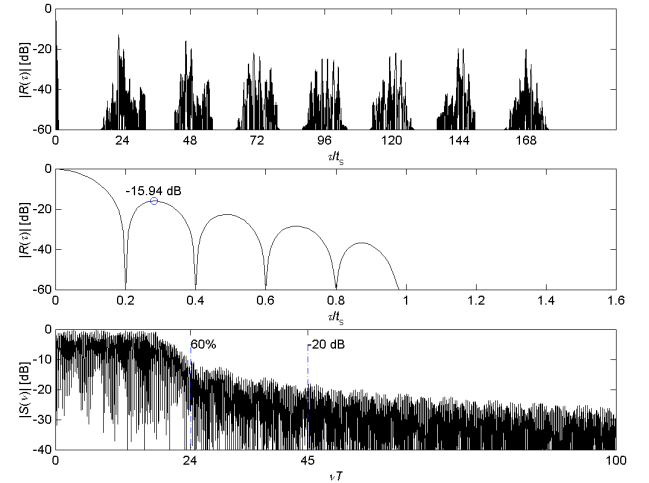


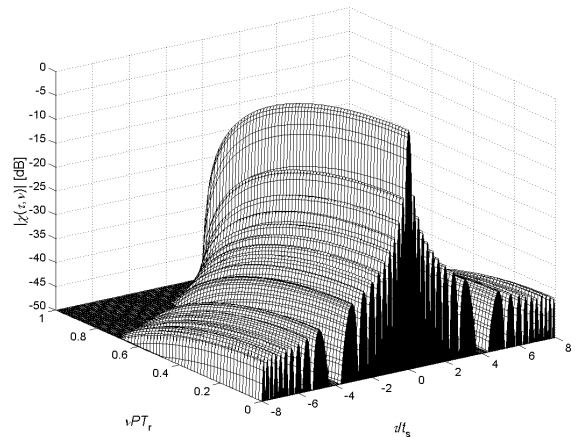
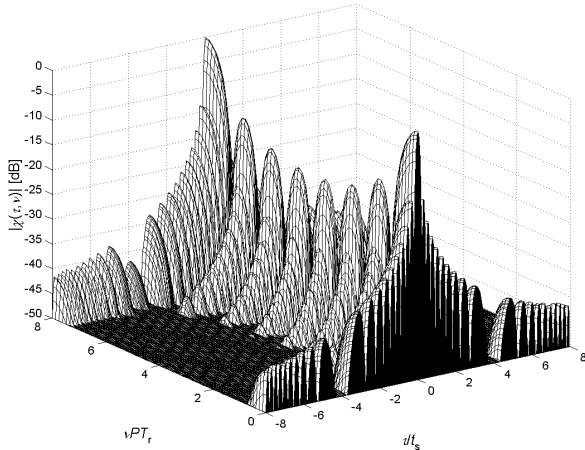
Fig. 2: Autocorrelation, autocorrelation mainlobe zoom and spectrum of linear stepwise frequency pulse-train overlaid with an 8x8 P4-based orthogonal phase code

The partial ambiguity plot of a LFM pulse-train with and without orthogonal coding is shown in Fig. 3. The figure shows the partial ambiguity plot of the signal with the two types of overlaid orthogonal coding (center and bottom part) and without coding (top) for  $0 \leq \nu \leq 1/T_r$  (left) and  $0 \leq \nu \leq 1/PT_r$  (right).

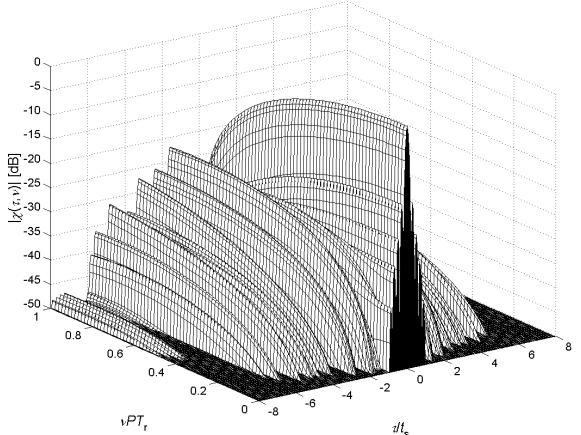
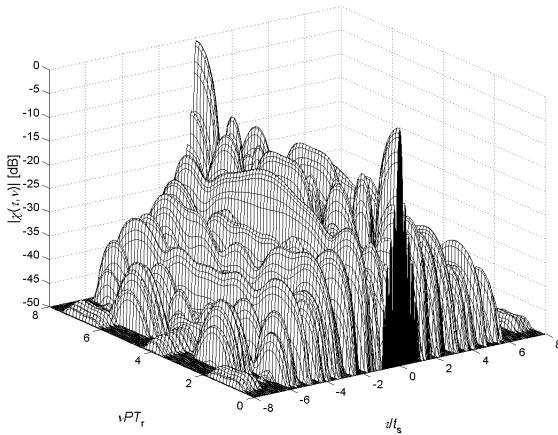
Note in Fig. 3 how orthogonal phase-coding moves the ambiguity volume from the zero-Doppler axis to higher Doppler. With binary coding the volume is distributed almost evenly over all non-zero Doppler values (Fig. 3, center) while with ordered P4-based coding the volume is moved to a diagonal ridge (Fig. 3, bottom). Note also the secondary peaks close to zero delay in the lower left partial ambiguity plot. These peaks are a by-product of the P4 code and the diagonal ridge that results from it. While not shown, note also that cyclically permuting the

pulses in the pulse-train or changing the direction of the P4 code (flipping the original P4 code left-right) affects portions of the partial ambiguity function, the signal spectrum and the autocorrelation recurrent lobes but does not change their general behaviours. On the other hand some simple permutations can result in significant changes to the partial ambiguity function of a P4-coded LFM pulse-train. Using opposite direction cyclic shifts or the conjugate code results in rotating the diagonal ridge in  $90^\circ$  (extending from the central peak to the first

Identical LFM pulse-train (no coding)



8x8 binary coding of identical LFM pulse-train



8x8 P4-based coding of identical LFM pulse-train

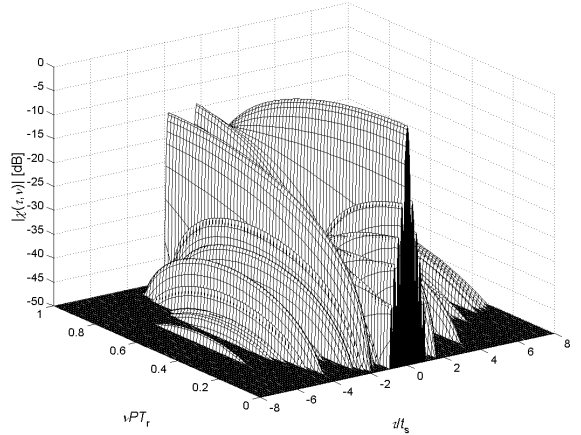
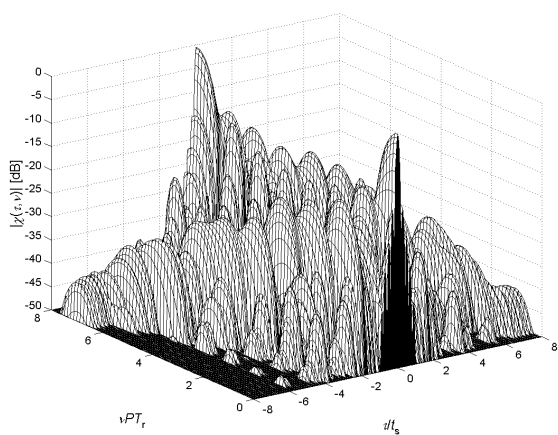


Fig. 3: Partial ambiguity plot of LFM pulse-train without orthogonal coding (top) and for the two types of orthogonal coding (binary – center, P4-based – bottom) for  $0 \leq \nu \leq 1/T_r$  (left) and  $0 \leq \nu \leq 1/PT_r$  (right)

and third quadrants instead of the second and fourth quadrant). Using generalized P4 [4] code, where the relatively prime integer to  $M$  is not  $\pm 1 \pmod{M}$ , results in splitting the ridge to several smaller parallel ridges such that the ambiguity plot has a shape closer to that of the binary-coded LFM pulse-train.

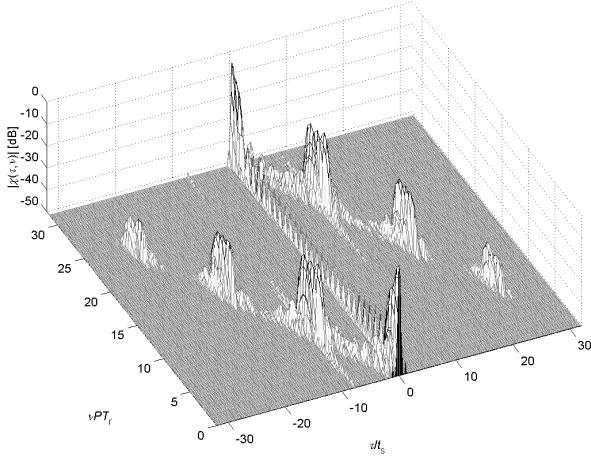


Fig. 4: The partial ambiguity function of a 32x32 LFM P4-based orthogonal-coded pulse-train with intrapulse and interpulse weighting

The effect of increasing the number of slices and pulses in the train as well as using intrapulse and interpulse weighting is demonstrated in Fig. 4. Note that increasing  $M$  and  $P$  results in sharper ridges and lower sidelobes between the ridges. The weighting used in Fig. 4 were Hamming windows on receive. Intrapulse weighting reduces the sidelobe level along the diagonal ridge (between the peaks) and some of the low sidelobes at high delay and low Doppler. Interpulse weighting reduces the low sidelobes forming ridges parallel to the Doppler axis (e.g.  $\tau_s=4$  or  $\tau_s=8$ ) but damages the orthogonality of the slices.

Note also that the ridge shape of the partial ambiguity function for a P4-based code is a function of using both P4 and *consecutive* cyclic shifts. When the pulses are permuted (all cyclic shifts are used but not in a consecutive order), the volume is spread over all non-zero Doppler values. Relatively uniform volume spread is also observed when all consecutive cyclic shifts of other random-like ideal sequence are used, e.g., a Golomb sequence [6] with  $M=P=7$ .

### LFM-NLFM AND LFM-LFM

We can further decrease the peak sidelobe level by allowing the FM slope and central carrier to change from slice to slice (we name the new signal LFM-NLFM). Fig. 5 is an example of an orthogonal phase-coded LFM-NLFM signal with minimum peak-sidelobe for  $\tau_s \geq t_s/5$ , where the pulse time-bandwidth product is limited to 75. (The global minimum was found using numerical methods.) The spectrum shown in

Fig. 5 was calculated for the signal coded with the binary orthogonal code given in (2).

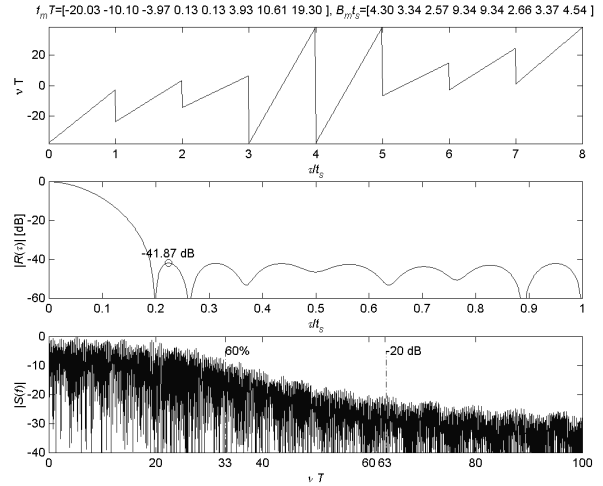


Fig. 5: LFM-NLFM orthogonal phase-coded pulse frequency history (top), autocorrelation mainlobe (middle) and spectrum (bottom)

Allowing the signal frequency deviation to take higher values further decreases the obtainable minimum peak sidelobe level with only little change in the spectrum width. For maximal time-frequency span product of 80 and 90 we found minimum peak sidelobes of  $-43.6$  dB and  $-47.3$  dB respectively.

A degenerated version where all slices have the same bandwidth and are evenly shifted in frequency (named LFM-LFM) can be easier to implement. The LFM-LFM signal gives peak sidelobe level of  $-37.4$  dB for  $M=P=8$  and  $-41.7$  dB for  $P=M=16$  with a similar bandwidth.

### DOPPLER TOLERANCE

In many practical situations (e.g., in surveillance radar) target Doppler is not known and it is required to use several filters (at least  $P$ ) at the receiver (each matched to a different target Doppler) in order to detect the target return. Generally implementing  $P$  filters requires replicating the matched filters construction,  $P$  times. For Doppler-tolerant signals such as a wide bandwidth LFM pulse, an FFT-based filtering method can be used to simplify receiver complexity with almost no loss in the central peak and only a small decrease in the peak-to-sidelobe level ratio. FFT-based Doppler filtering compensates for the Doppler-induced phase-shift between pulses, but not within each pulse.

We present simulation results showing the FFT based receiver central peak loss for LFM pulse-train (no coding), orthogonal 8x8 P4-based coded LFM pulse-train, orthogonal-coded LFM-LFM pulse-train and a train of complementary pulses with the same mainlobe width. All signals have 8 pulses and were designed to give a first null at  $T/40$ . Using binary coding instead of a P4-based code gives similar results. Fig. 6 gives the

receiver central peak loss caused by the waveform. The peak-sidelobe level (normalized to the receiver central peak at zero Doppler) in the area of the central peak (the same Doppler filter) is shown in Fig. 7.

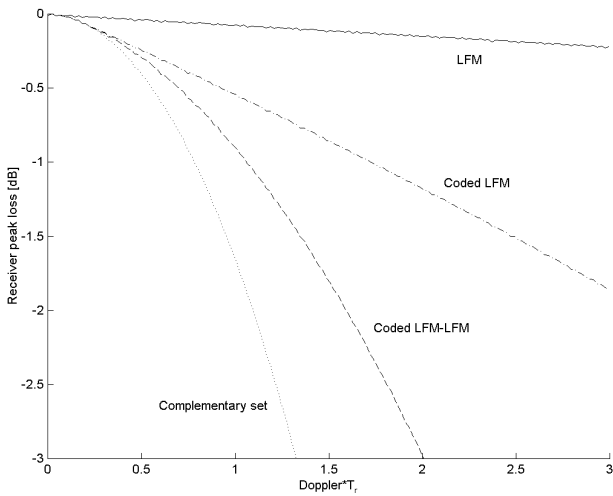


Fig. 6: Central peak FFT based receiver loss for LFM pulse-train, orthogonal-coded LFM pulse-train, orthogonal-coded LFM-LFM pulse-train and a train of complementary coded pulses

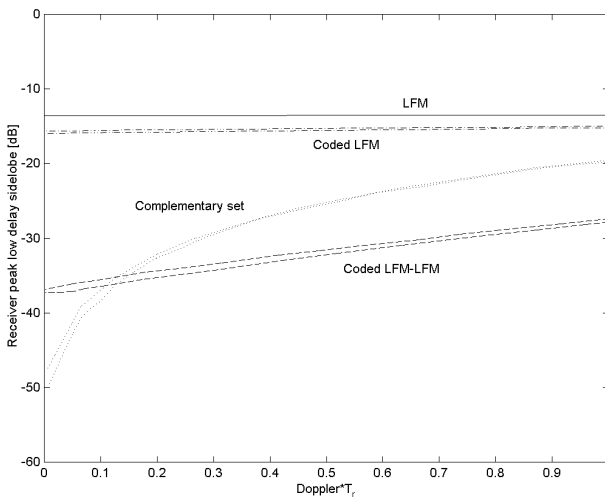


Fig. 7: Near-sidelobes peak-level as a function of target Doppler for 8 pulse 8 slice orthogonal-coded LFM, orthogonal-coded LFM-LFM, complementary set and a train of identical LFM pulses

Fig. 7 gives the near-delay peak-sidelobe level (delayed  $T/40$  to  $T/8$  from the central peak) for Doppler values up to  $1/T_r$ . Both P4-based coding and binary-based coding peak-sidelobe levels are shown. The P4-based coding is the one with the slightly higher peak sidelobes. Note that all signals exhibit high peak-sidelobes for Doppler values higher than  $1/T$ . Note also that the peak sidelobes for delays from  $T/8$  to  $T$  are higher than  $-25$  dB for the LFM pulse-train, but are only around  $-45$  dB for the coded LFM, coded LFM-LFM and complementary pulse-trains.

## CONCLUSIONS

A method of diversifying any train of  $P$  identical pulses was applied to a train of frequency modulated pulses. The original pulse was divided into  $M$  slices with width  $t_s$ , and diversity was obtained by overlaying orthogonal coding on the  $M$  slices in the  $P$  pulses. Orthogonal overlay removes the autocorrelation sidelobes over  $(M-1)/M$  of the pulse duration. Two types of overlay coding were used – a binary noise-like coding and an ordered coding, based on all cyclic shifts of P4. The use of a code based on cyclic shifts of P4 resulted in lower ambiguity function sidelobes in the area of the central peak and higher recurrent lobe peaks off zero-Doppler.

A significant practical advantage of the new FM-based waveforms over complementary sets or other phase-coded signals with thumbtack ambiguity function is in the improved Doppler tolerance, which enables the use of FFT to simplify receiver complexity.

Finally, the different distribution of ambiguity function volume, between a train of identical LFM pulses and a train of orthogonal-coded ones, points to a possible advantage in using the two signals side by side.

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## REFERENCES

1. Tseng, C. C. and Liu, C. L, 1972, "Complementary sets of sequences", *IEEE Trans. Information Theory*, 18, 644-652
2. Luke H. D, 1985, "Sets of one and higher dimensional Welti codes and complementary codes", *IEEE Trans. Aerosp. Electron. Syst.*, 21, 170-179
3. Sivaswamy R, 1978, "Digital and analog sub-complementary sequences for pulse compression", *IEEE Trans. Aerosp. Electron. Syst.*, 14, 343-350
4. Kretschmer, F. F. Jr, Gerlach, K, 1991, "Low sidelobe radar waveforms derived from orthogonal matrices", *IEEE Trans. Aerosp. Electron. Syst.*, 27, 92-102
5. Popovic B. M, 1990, "Complementary sets based on sequences with ideal periodic autocorrelation", *Electronic Letters*, 26, 1428-1430
6. Golomb S. W, 1992, "Two-valued sequences with perfect periodic autocorrelation", *IEEE Trans. Aerosp. Electron. Syst.*, 28, 383-386