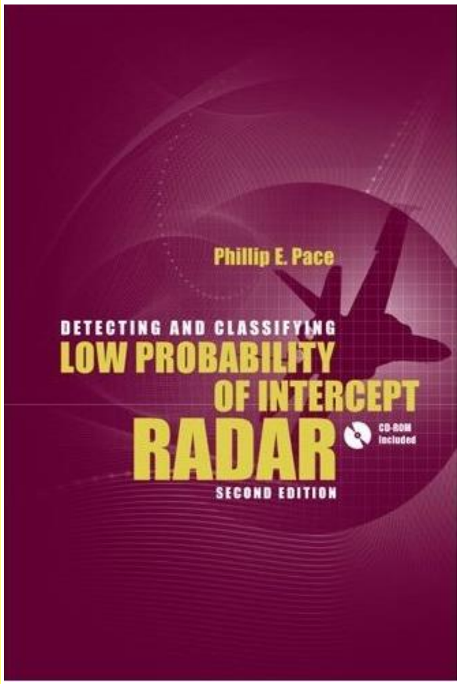
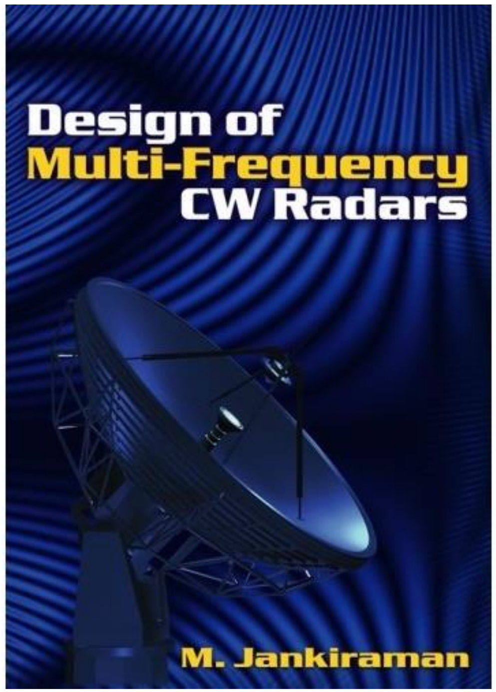
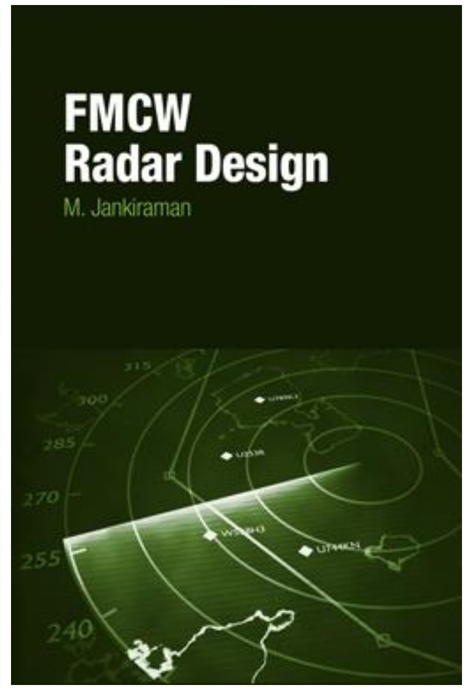
CW Radar

CW signals







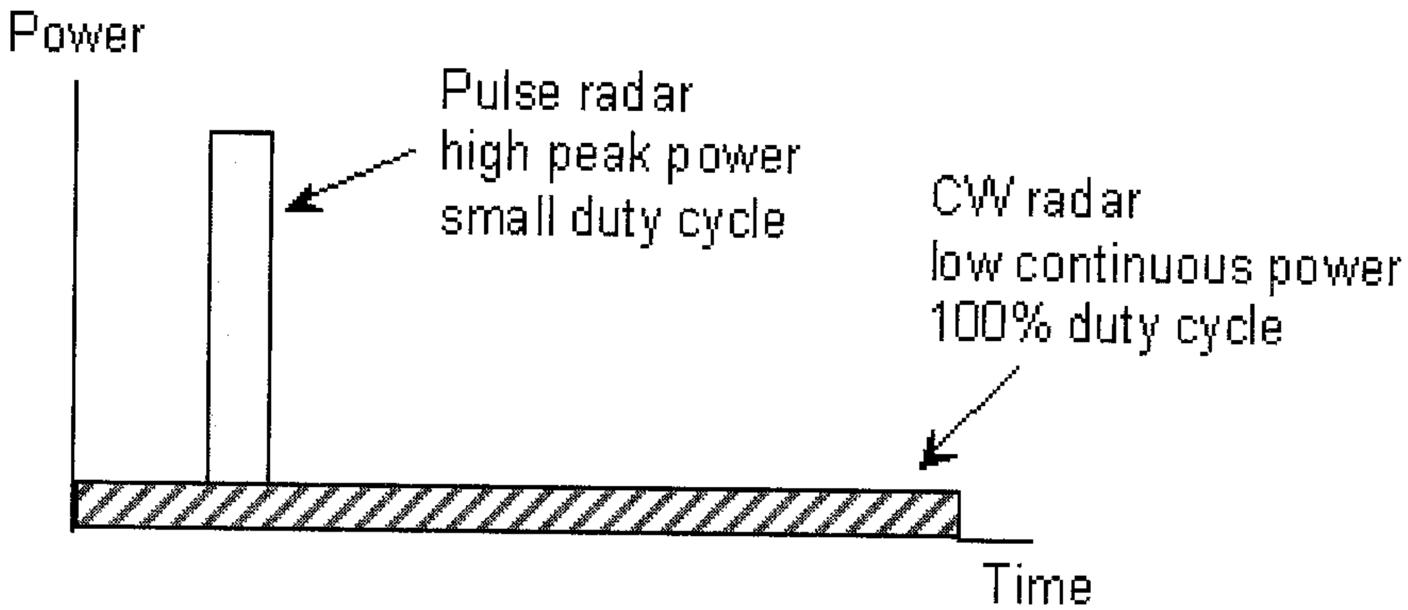
CW signals ←→ **L**ow **P**robability of **I**ntercept

Definitions (According to P. E. Pace)

LPI – A low probability of intercept radar uses a special emitted waveform intended to prevent a non-cooperative intercept receiver from intercepting and detecting its emission.

LPID – A low probability of identification radar uses a special emitted waveform intended to prevent a non-cooperative intercept receiver from intercepting and detecting its emission but if intercepted, makes identification of the emitted waveform modulation and its parameters difficult.

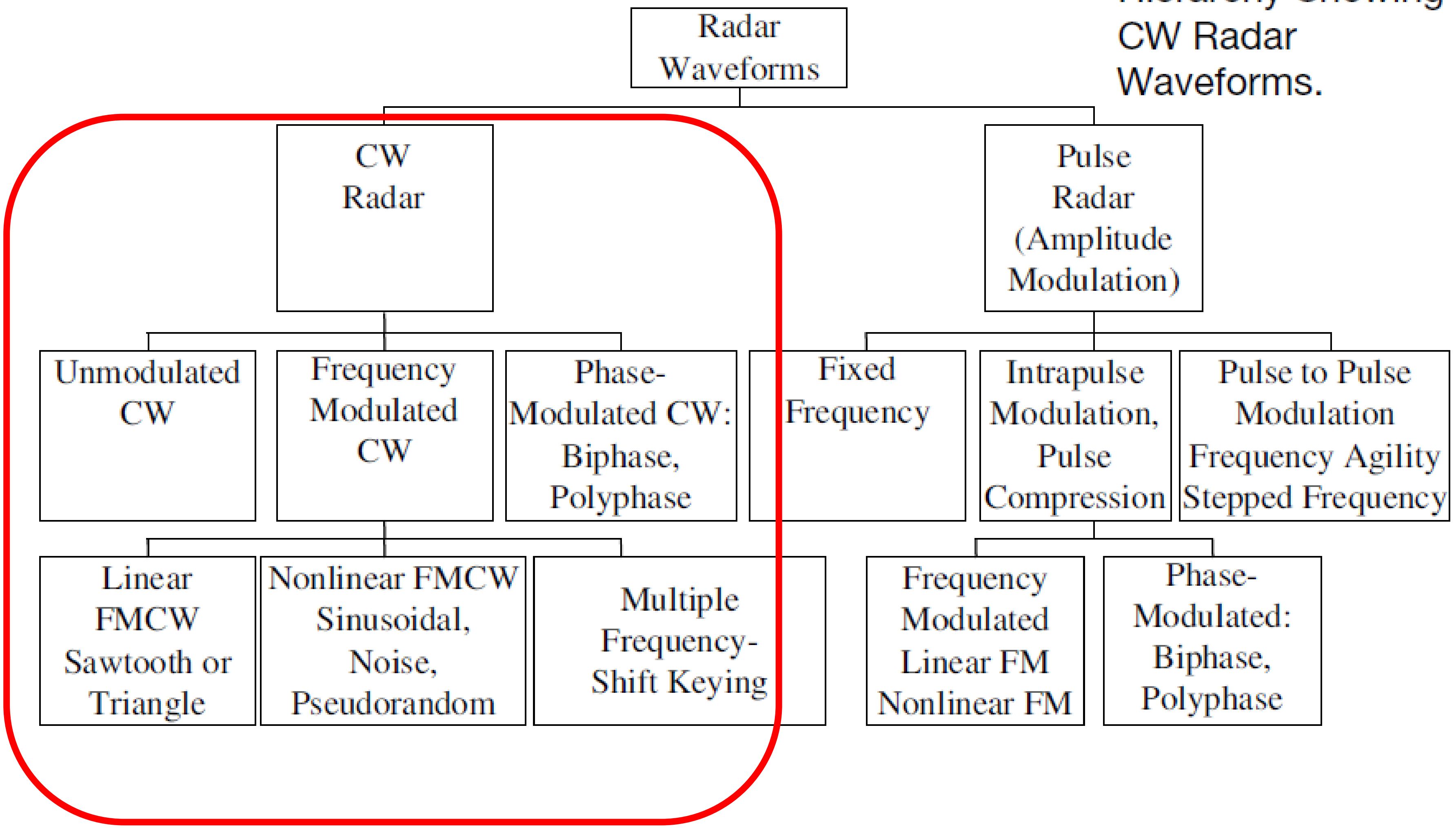
Defining a radar to be **LPI** and/or **LPID** necessarily involves the definition of the corresponding intercept receiver.



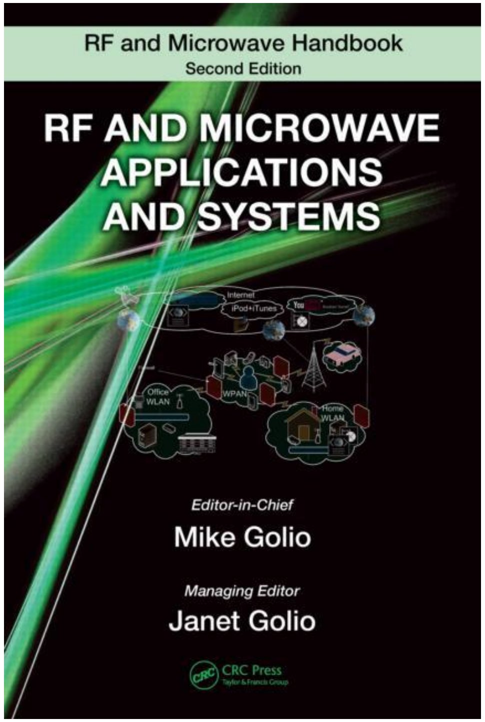
Transmitter LPI considerations:

- Low peak power ➔ CW radar
- Wide bandwidth ➔ Pulse compression (applicable also to CW)

FIGURE 2.2-2 Radar Waveform Hierarchy Showing CW Radar Waveforms.



(From: William L. Melvin; James A. Scheer, *Principles of Modern Radar: Volume 3: Radar Applications*, Scitech publishing, 2014)



14 Continuous Wave Radar

Samuel O. Piper
James C. Wiltse
Georgia Tech Research Institute

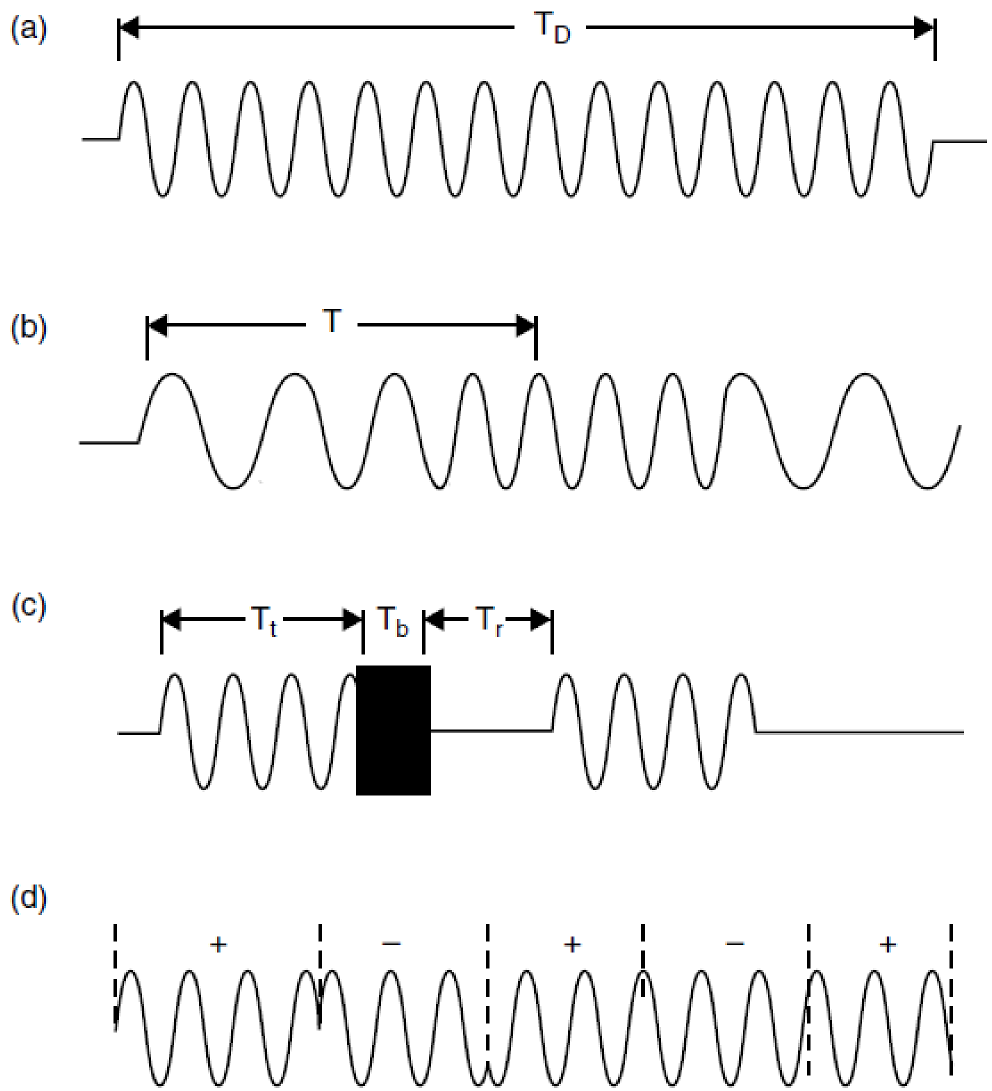


FIGURE 14.1 Waveforms for the general class of CW radar: (a) continuous sine wave CW, (b) frequency-modulated CW, (c) interrupted CW, and (d) binary phase-coded CW. (From F. E. Nathanson, *Radar Design Principles*, New York: McGraw-Hill, 1991, p. 450. With permission.)

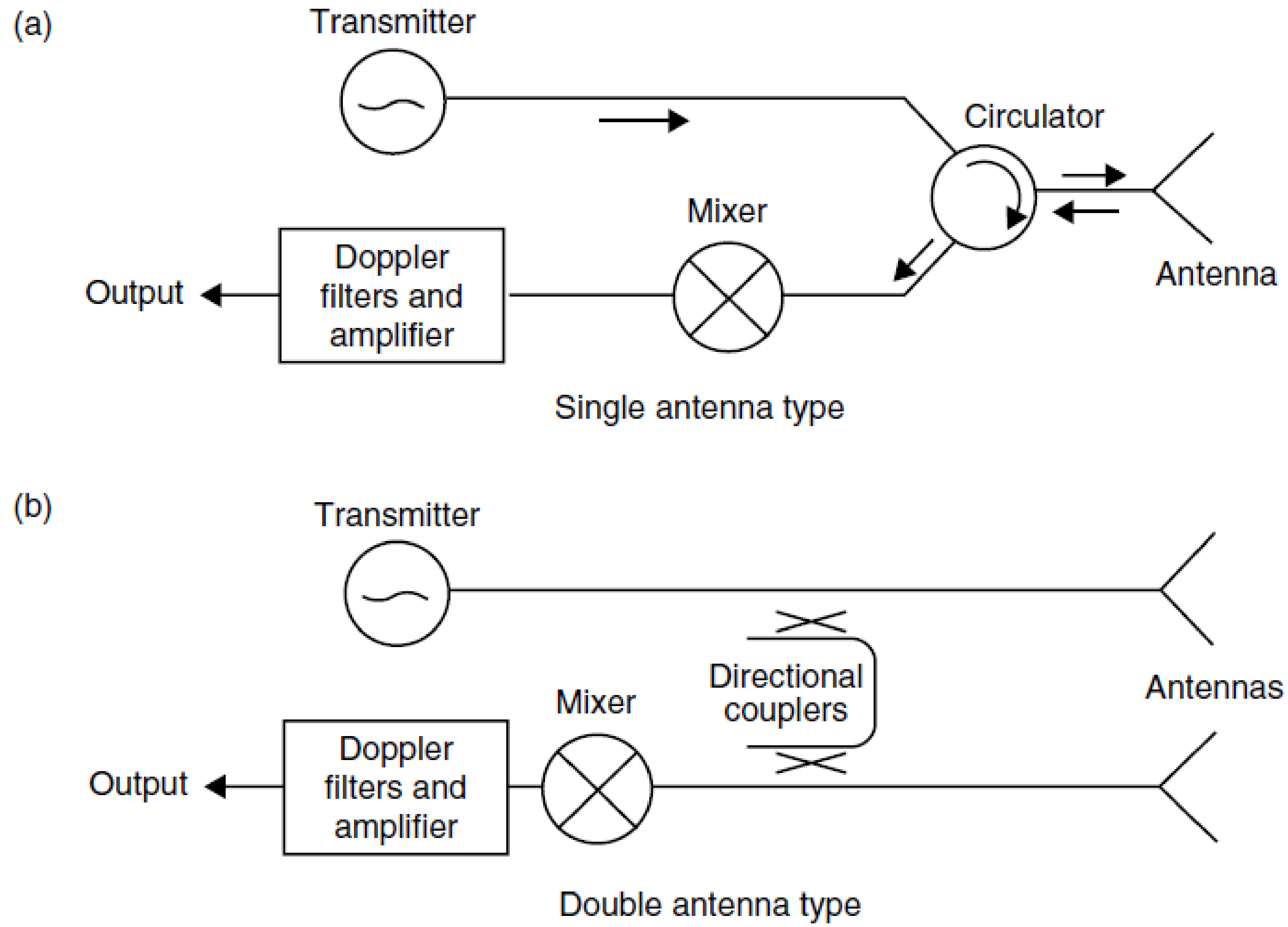


FIGURE 14.2 Block diagrams of CW-Doppler radar systems: (a) single antenna type and (b) double antenna type.

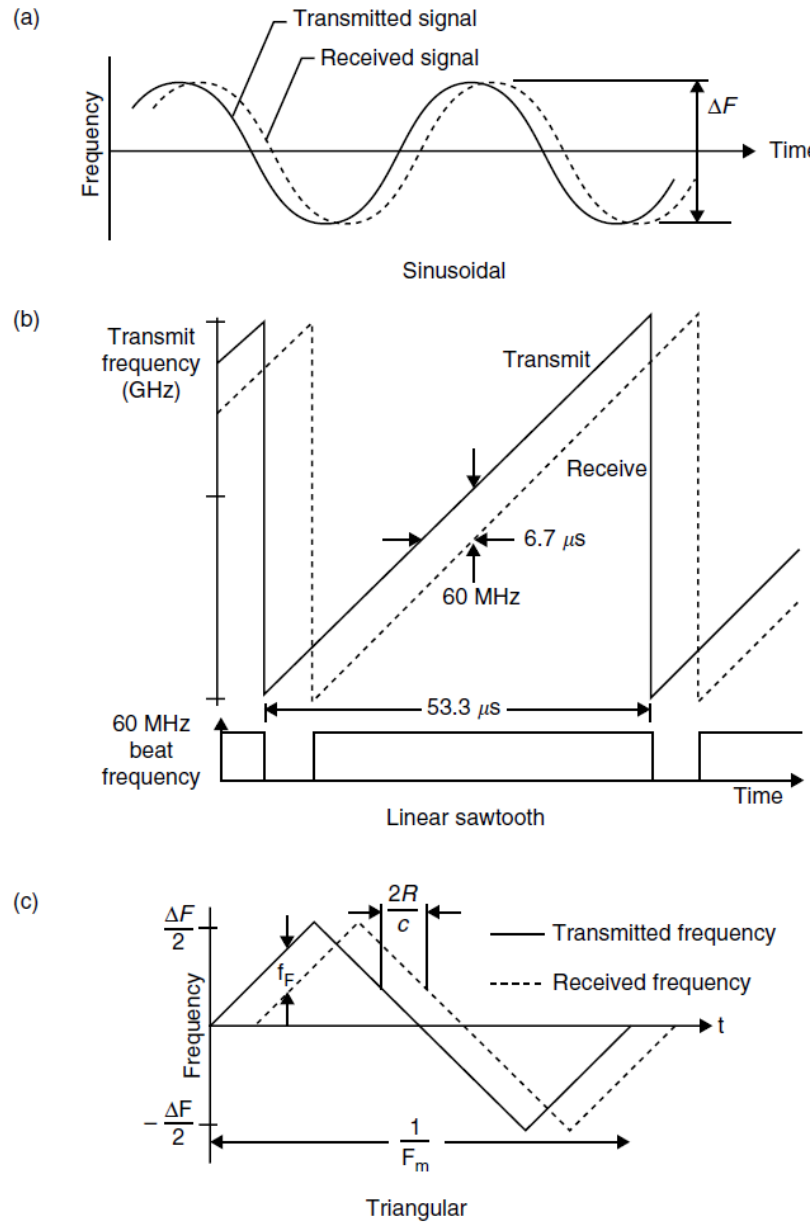
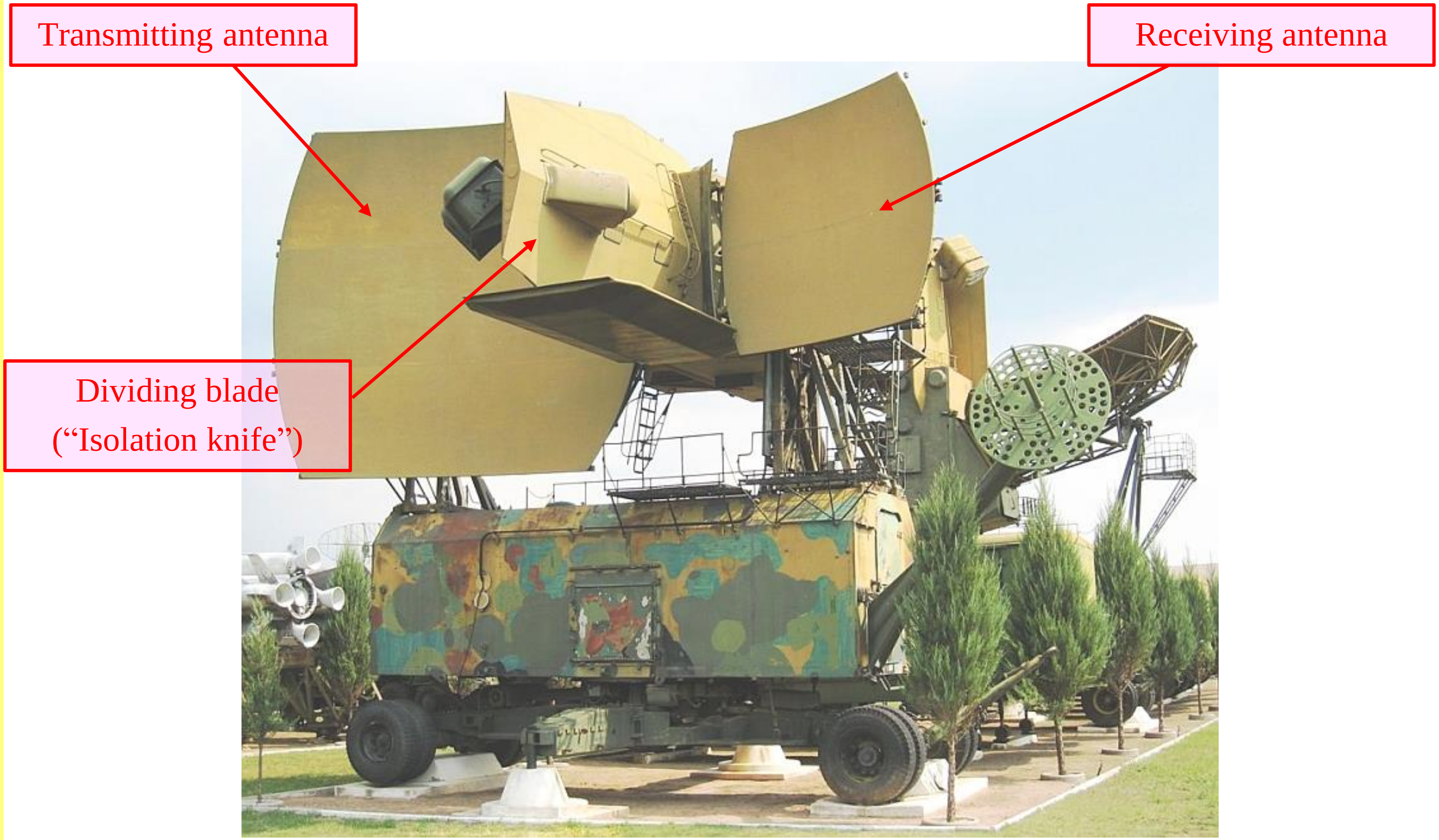


FIGURE 14.3 Frequency versus time waveforms for FMCW radar: (a) sinusoidal, (b) linear sawtooth, and (c) triangular modulations.



5N62 "Square Pair" Guidance and Illumination FMCW Radar

Transmit power: 100 kW CW (!)

Frequency: 6-8GHz (C-band)

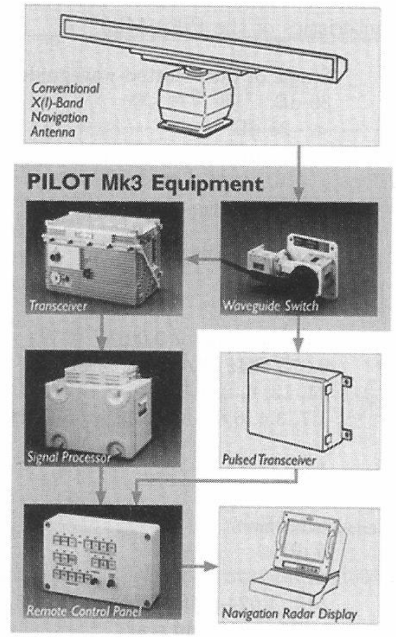
Modulation: phase coded CW

Reported detection range: 240 km (fighters) – 290 km (large aircrafts)

To See and Not Be Seen 3

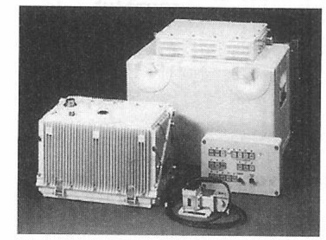
Table 1.3: Technical Characteristics of the Pilot Mk3

Antenna	Type:	Single or dual slotted-waveguide
	Gain:	30 dB
	Side lobes:	< −25 dB
		< −30 dB
	Beamwidth (3 dB)	
	horizontal:	1.2 deg
	vertical:	20 deg
	Rotational speed:	24/48 RPM
	Polarization:	horizontal
Transmitter	Output power:	1.0, 0.1, 0.01, or 0.001W (CW)
	Frequency:	9.375 GHz (X-band)
	Range selection:	24, 12, 6, 3, 1.5, 0.75 nmi
	Frequency sweep:	1.7, 3.4, 6.8, 13.75, 27.5, 55 MHz
	Sweep repetition frequency:	1 kHz
Receiver	IF bandwidth:	512 kHz
	Noise figure:	5 dB
Processor Unit	No. of range cells:	512 (1,024-point FFT)
	Range resolution:	< 75m at 6 nmi scale
	Range accuracy:	< ±25m at 6 nmi scale
	Azimuth accuracy:	±2 degrees
	Azimuth resolution:	1.4 degrees
Display System	Type:	Color
	Minimum effective PPI diameter:	250 mm
	Resolution:	768× 1,024V
	Tracking capacity:	40
	Range ring accuracy:	1.5% of selected scale or 50m, whichever is greater



PILOT

FMCW tactical navigation radar



Doppler (velocity) resolution is not specified

Pengelley, R. "Philips' Pilot, covert naval radar," *International Defense Review*, pp. 1177-1178, Sept. 1988.

THALES

MARITIME LPI SURVEILLANCE RADAR

- Automatic detection and tracking
- Helicopter detection and guidance during approach and landing
- True LPI: See, but not be seen
- Low life cycle costs, high availability

SCOUT MK3

Unrivalled small target detection

- SCOUT Waveform and Receiver Parameter Summary

Range setting	2.4 (4.4)	6 (11)	24 (44)	nmi (km)
FMCW waveform	Sawtooth	Sawtooth	Sawtooth	
Frequency deviation, peak to peak	70	28	7	MHz
Range resolution (at 6nmi)	2.4	6.0	24.0	m
Modulation frequency	1	1	1	kHz
FFT length	4,096	4,096	4,096	Points

FIGURE 14.5 Honeywell 35-GHz biphase modulated CW obstacle avoidance radar [http://ccf.arc.nasa.gov/dx/basket/pix/RASCAL.jpg] (Photo—Dominic Hart).

TABLE 14.3 Honeywell Phase-Modulated CW Radar Parameters

Parameter	Value
RF center frequency	35 GHz
Transmit power	35 mW
Receiver noise figure	6 dB
Antenna gain	34 dBi
Beamwidth	3°
Antenna sidelobes	25 dB
Azimuth field of view	±45°
Elevation field of view	±10°
Bi-Phase code chip	32 ns or 4.8 m (16 ft)
Codes transmitted	1, 5, 7, 11, and 13 bit
Receiver bandwidth	25 kHz

TABLE 14.4 Honeywell Phase-Modulated CW Radar Waveform

Biphase Modulation Parameter	Performance	Requirements
32 ns Chip length (1,120 35 GHz cycles per chip)	4.8 m (16 ft) range resolution	31.3 MHz bandwidth phase modulator 62.5 MHz ADC sample rate: 8- to 10-bit (48–60 dB) dynamic range (ADC technology limitation) 62.5 MHz memory
190 Code length (190 code length $\times$32 ns chip length ~6 µs code repetition interval)	0.9 km (3 kft) Unambiguous range	190 Element correlator 190 Multiplies every 16 ns 11.9×10^9 multiplies per second

As described in Table 14.4, the 32 ns biphase code chip corresponds to 4.8 m (16 ft) range resolution. This will require a 31.3 MHz phase modulator and an analog-to-digital converter (ADC) sample rate of at least 62.5 MHz along with 62.5 MHz memory. A code sequence of at least 190 chips is required for 0.9 km maximum range. For a maximal length code of 190, the range sidelobe levels will be approximately 23 dB. The radar receiver must perform a 190-element correlation. For a brute force correlator, this will require 190 multiplies every 16 ns or 11.9×10^9 multiplies per second.

10*log10(1/190)= -22.79 🡐 error

Should be: 20*log10(1/190)= -45.58

Honeywell

HG9550 Radar Altimeter System

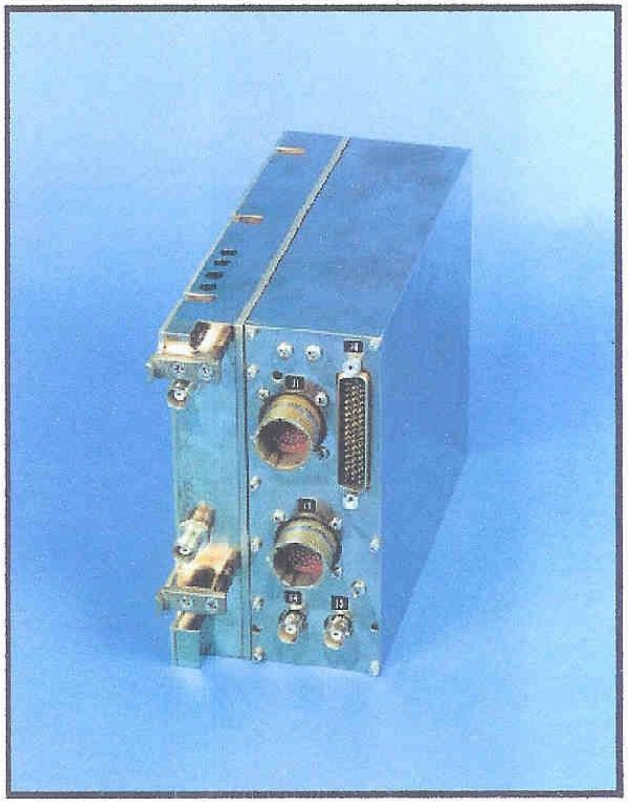
An advanced low probability of intercept altimeter system for high-performance aircraft and missile applications

The Honeywell HG9550 represents a quantum leap in altimeter capabilities—in reliability, covertness, size, weight, and cost.

The HG9550 was developed by Honeywell and jointly qualified by Honeywell, Lockheed Martin, and the U.S. Air Force. The HG9550 provides form, fit, function (F^3) replaceability of 1553 versions of the U.S. Air Force CARA altimeter while also providing significant performance and reliability improvements at a lower cost.

A Honeywell-patented design combines the high accuracy and programmable features of pulsed altimeter designs with the sensitivity and low probability of intercept (LPI) advantages of coherent systems. With less than one watt peak transmit power the HG9550 is virtually undetectable.

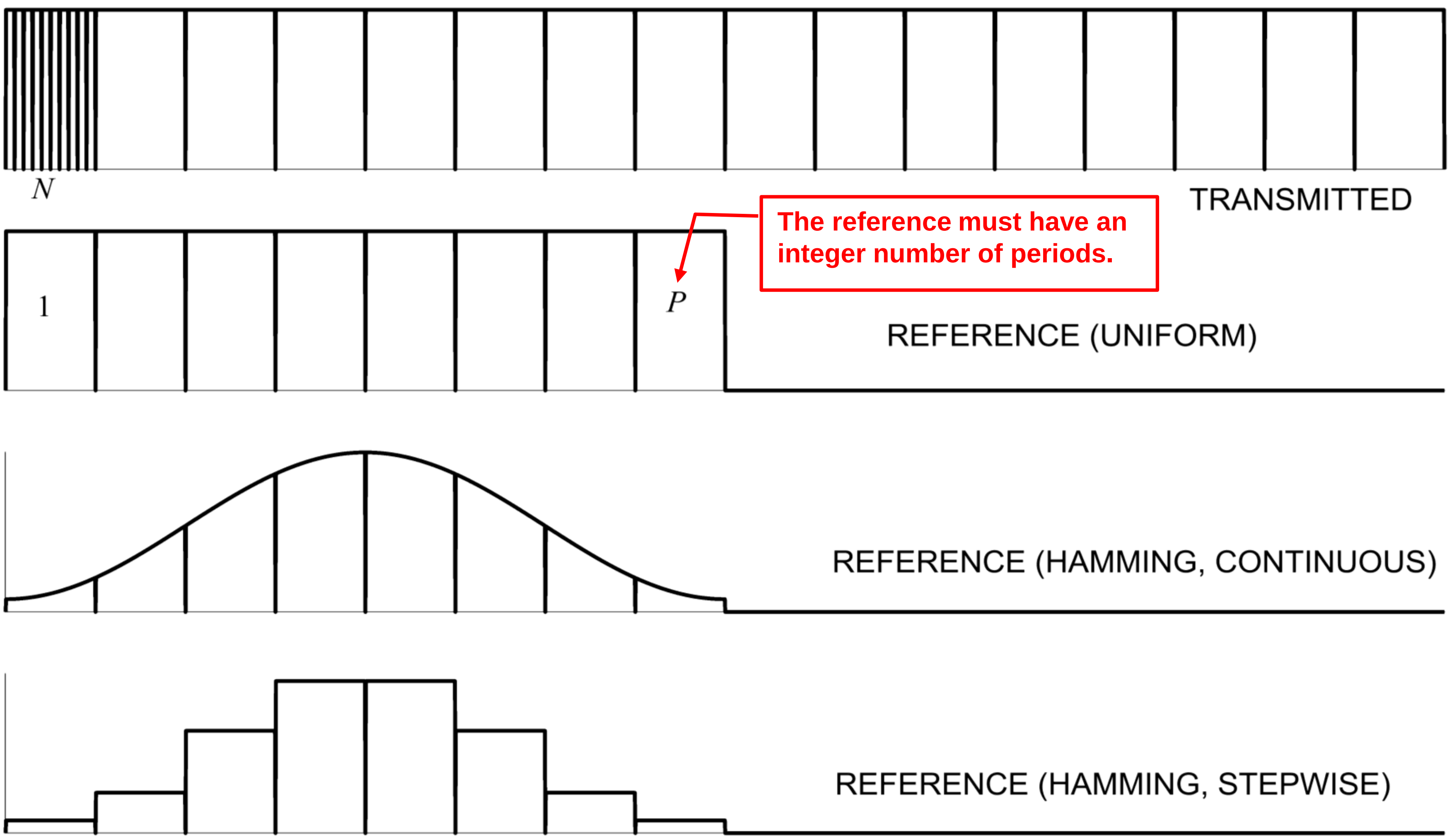
A micro processor-based design permits system characteristics such as track rate and ECCM response to be varied as a function of real-time inputs, or to be pre-programmed according to specific mission requirements.



The HG9550 is an off-the-shelf, fully qualified solution that is currently in production for the U.S. Air Force C-130J, United Kingdom C-130J, Argentine A-4 upgrade, Korean T-50, and the F-16 Block 60.

Nadav Levanon, Tel-Aviv University

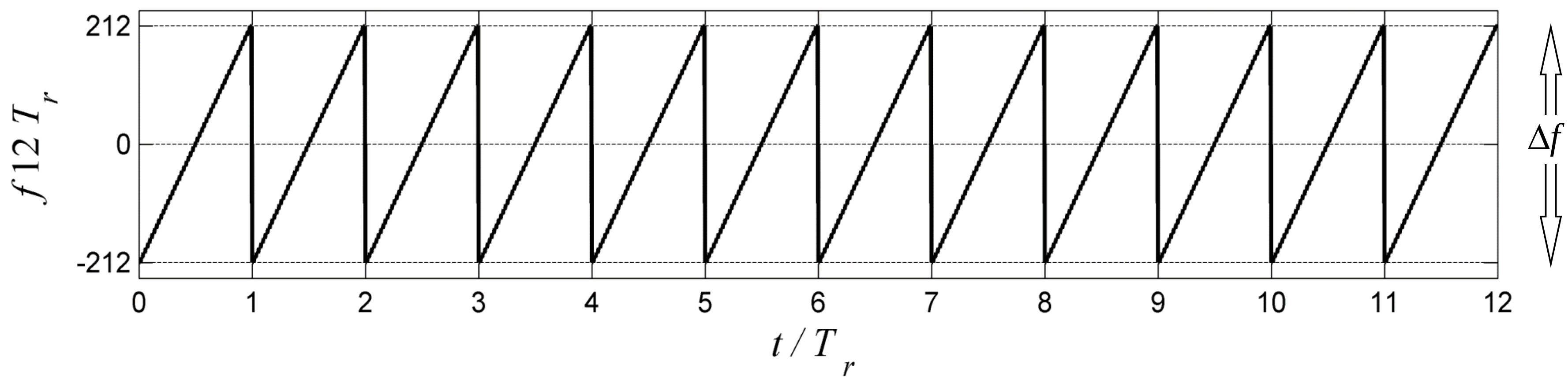
CW signals

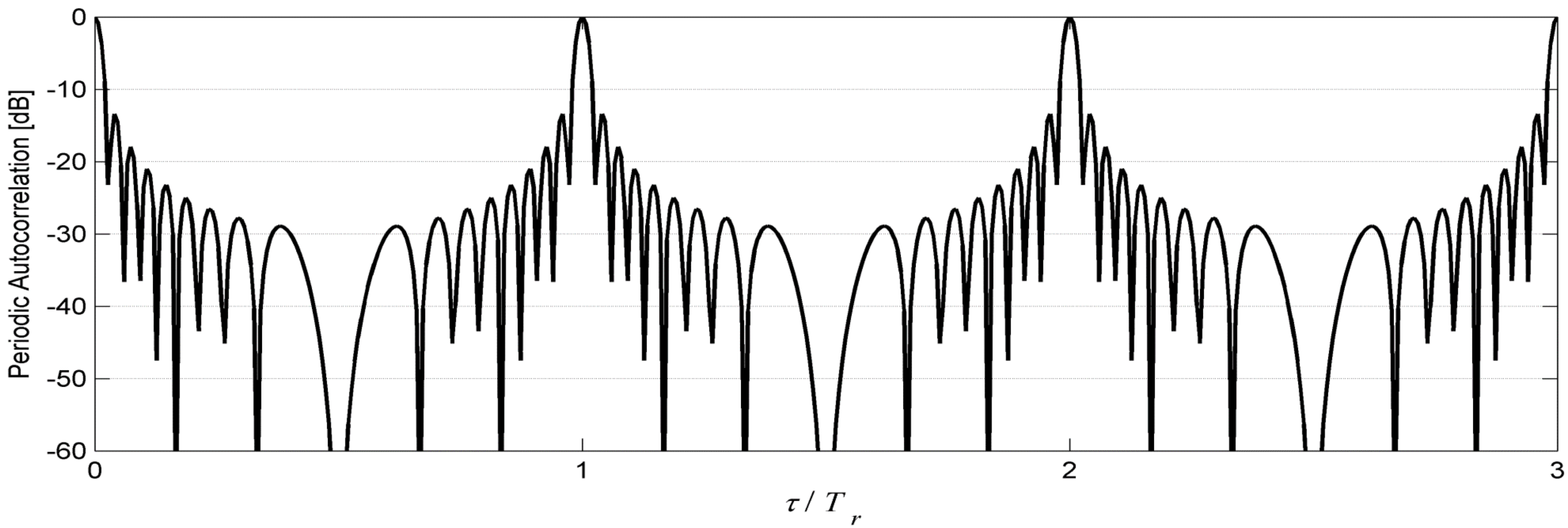


LFM CW RADAR

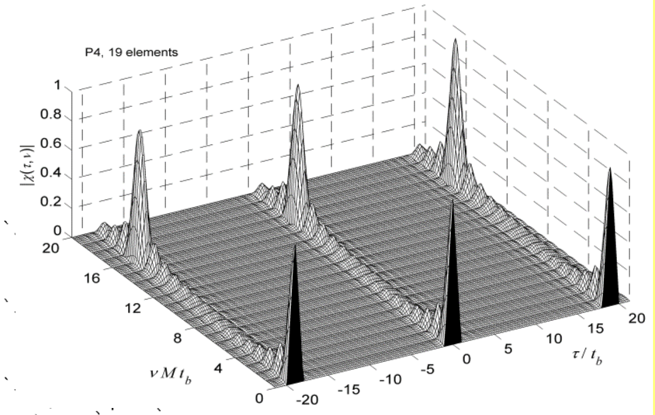
Transmit: Saw-toothed infinite periodic CW
Receiver ref: 12 periods (no amplitude weighting)

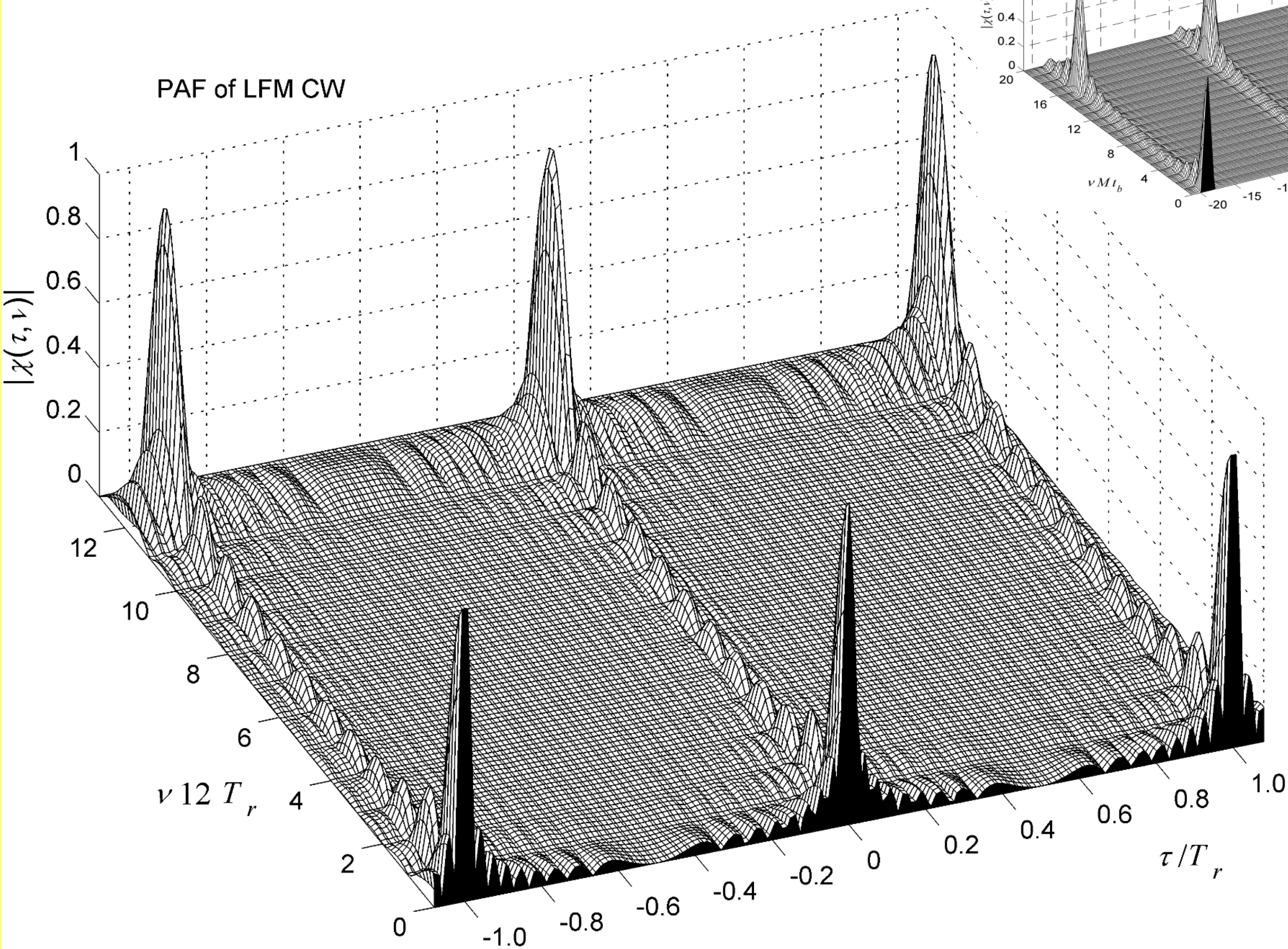
$\Delta f \; T_r = 17.7$





In LFM-CW the delay response is not ideal
(as in some phase-coded periodic CW waveforms)





LFM CW receiver processing methods

- Optimal receiver
  - Matched filter + Doppler processor
- Stretch processing
  (also termed "de-chirping" or "unramping" method)
  Sub-optimal receiver with some limitations
  - Stretch processing with non coherent inter-period integration
  - Coherent stretch processing receiver

Stretch: A Time-Transformation Technique

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Institute of Science and Technology
University of Michigan
Ann Arbor, Mich. 48107

Abstract

Stretch is a passive, linear, time-variant technique for performing temporal operations on many classes of signals. The technique employs three dispersive networks and a mixer. Signal slowdown, speedup, or time reversal can be attained by choice of network slopes. These temporal operations are performed within a signal "window," and the duration of the window is determined by the network time-bandwidth products. Both heuristic argumentation and rigorous analysis are presented, as are the results of a simple laboratory experiment.

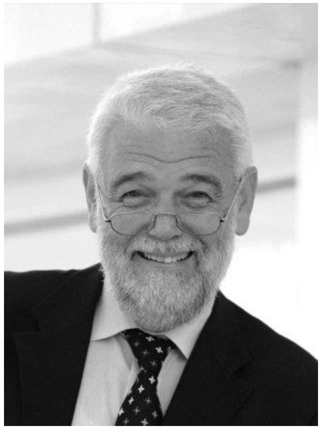
IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS VOL. AES-7, NO. 2 MARCH 1971

Old approach to LFM-CW

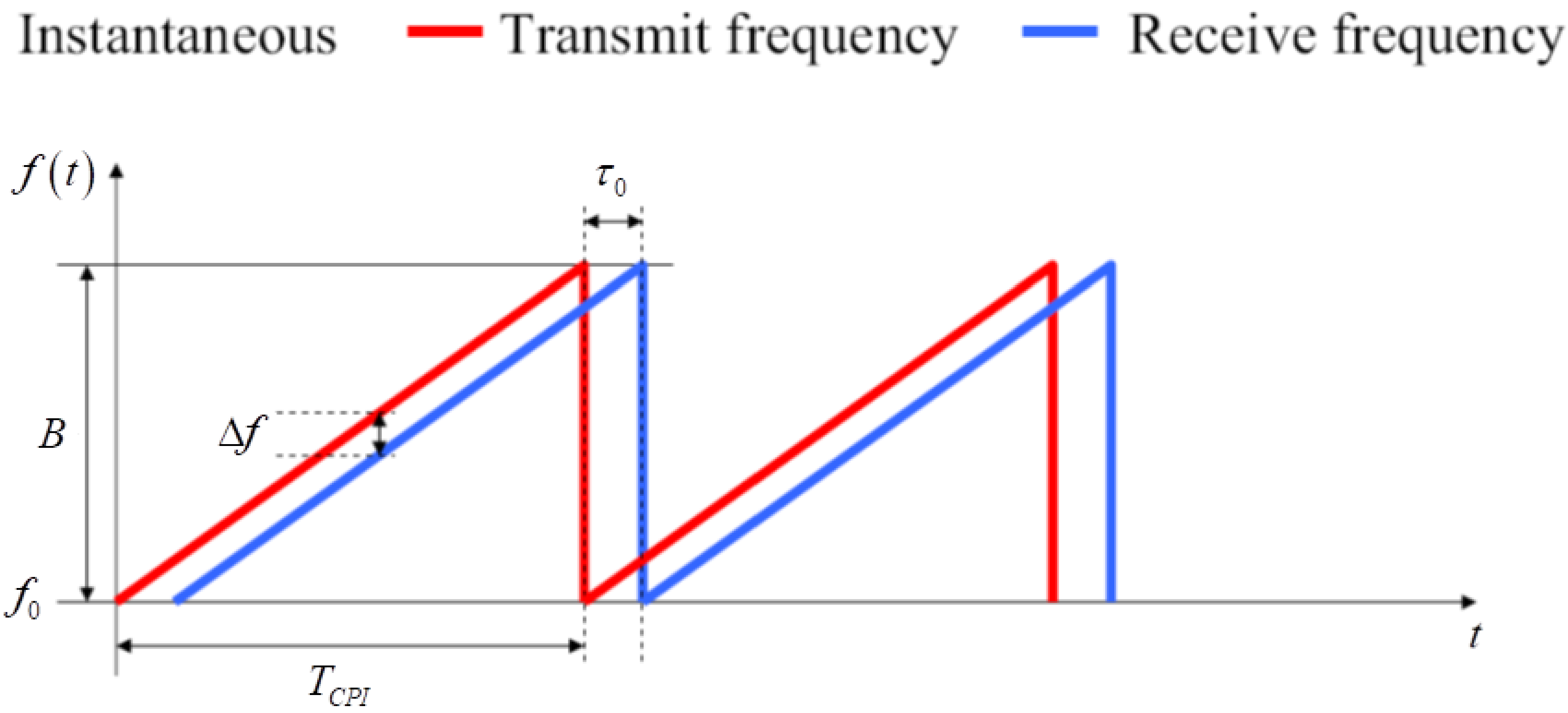
From a tutorial at RADAR 2008, Rome, by

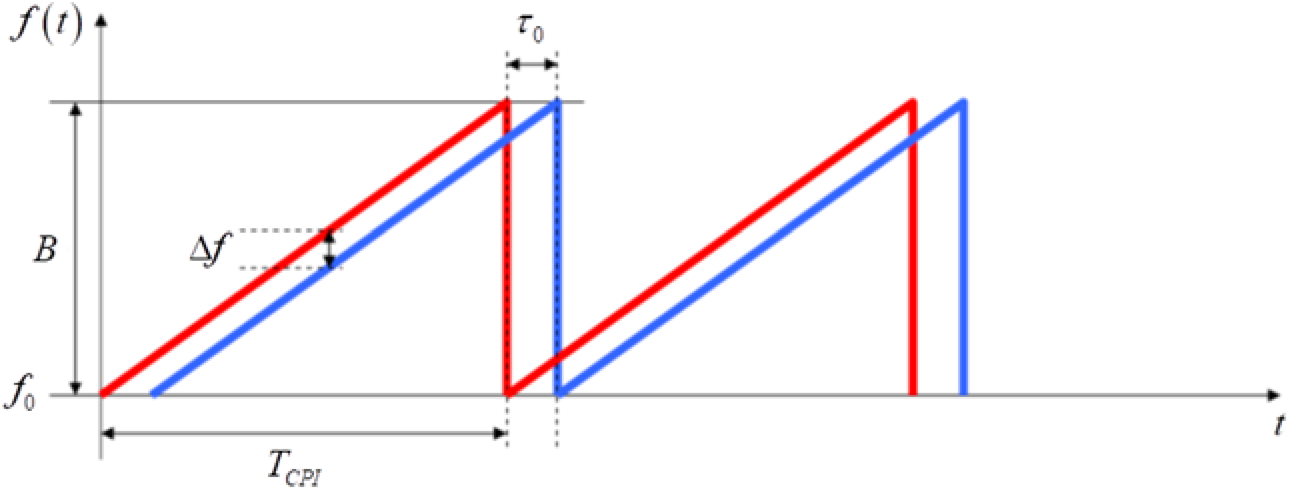
Prof. Dr. Hermann Rohling

TUHH Technische Universität Hamburg-Harburg



Measurement of range for fixed targets:





Range

$$R=\frac{c}{2}\tau_0=\frac{c}{2}\frac{T_{CPI}}{B}\Delta f$$

Unambiguous range

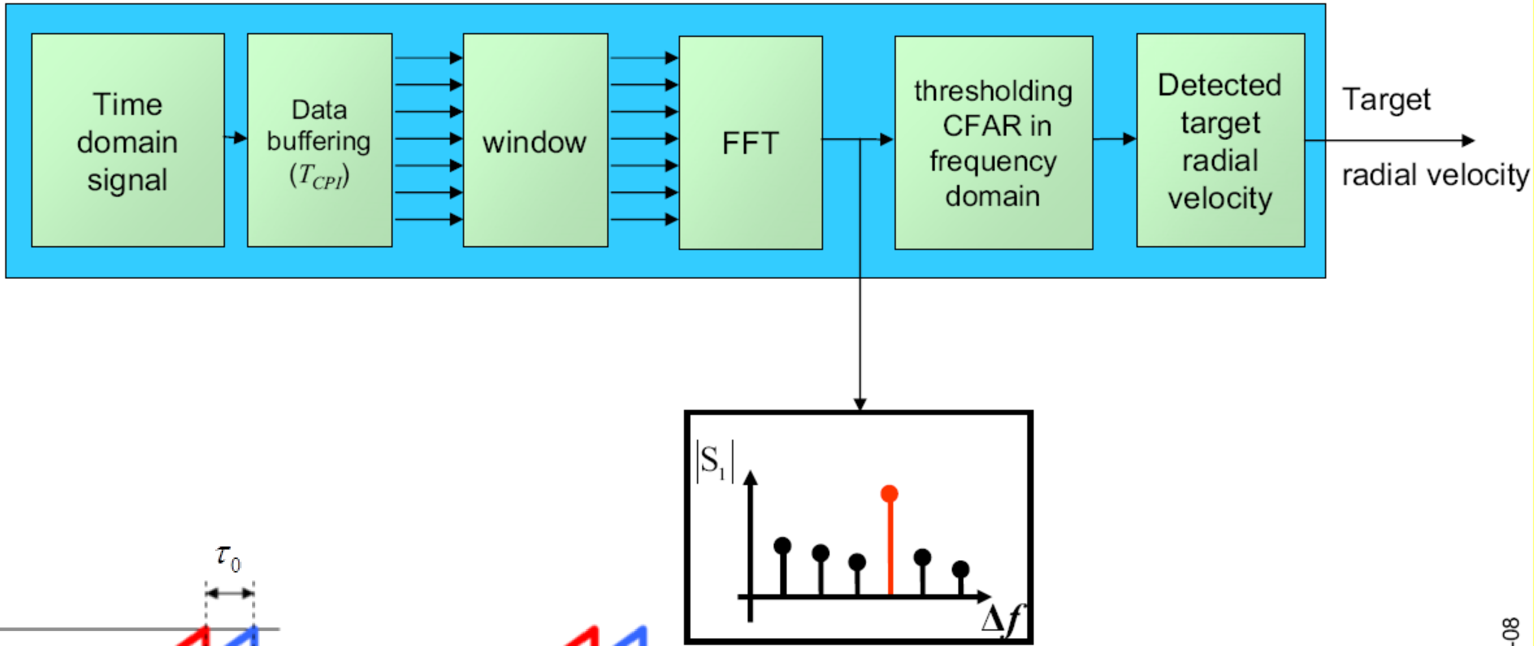
$$R_{UA}=\frac{c}{2}T_{CPI}$$

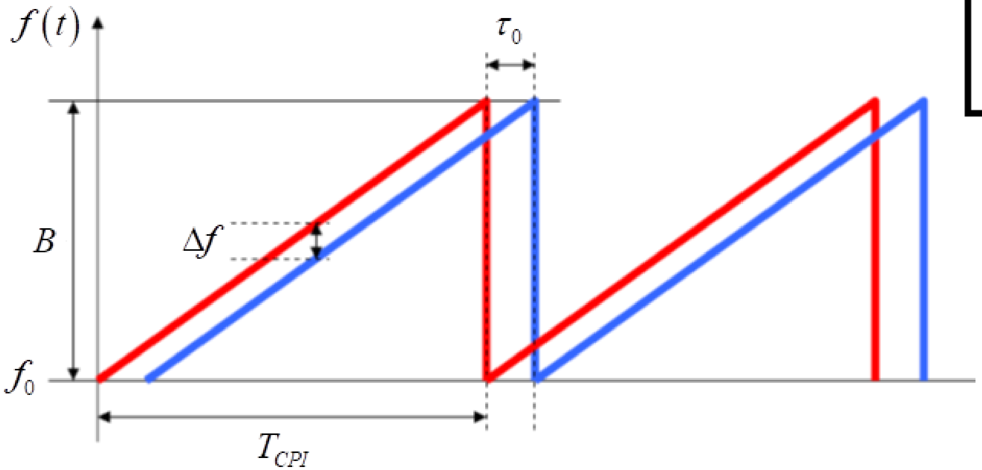
Range resolution

$$\Delta R=\frac{c}{2B}$$

Range estimation accuracy

$$\tilde{R}\sim\frac{c}{2B}$$

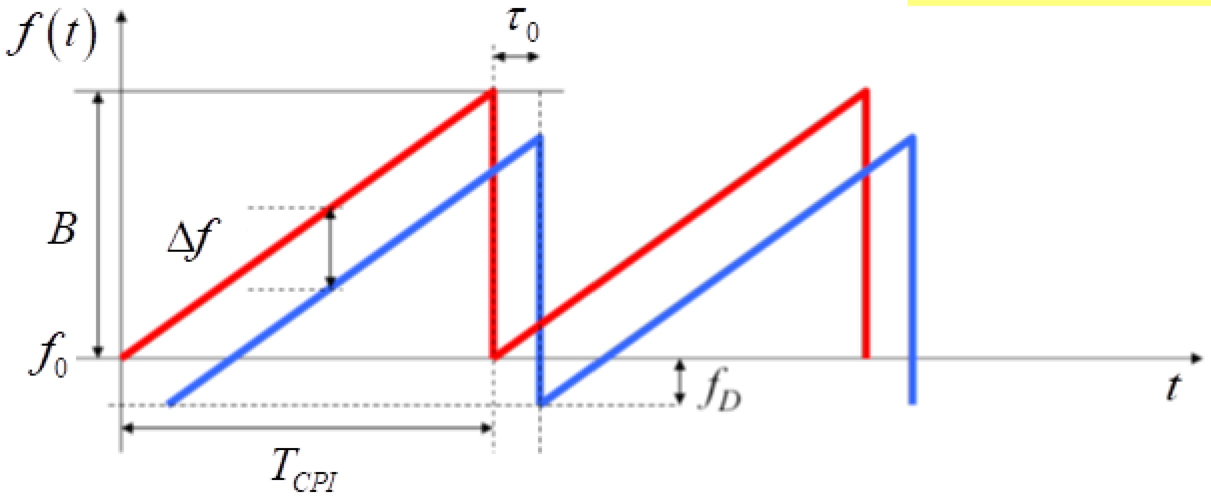




z-08

In this simplified approach Δf is measured using FFT. The coherent processing interval (CPI) is shorter than one period. This will perform far less than optimal processing (matched filter), in the presence of noise and other targets.

Furthermore ….

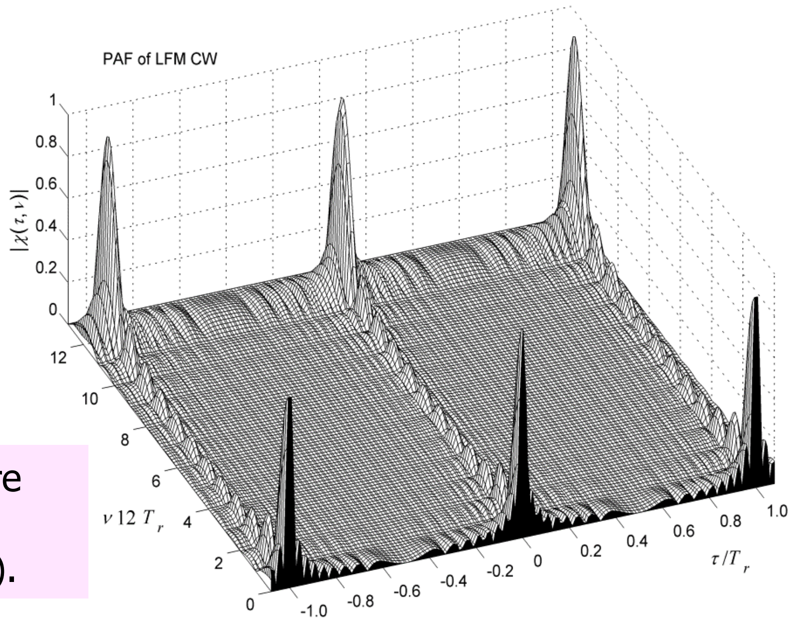


Moving single target:

$$\Delta f = f_D - f_{\tau_0}$$

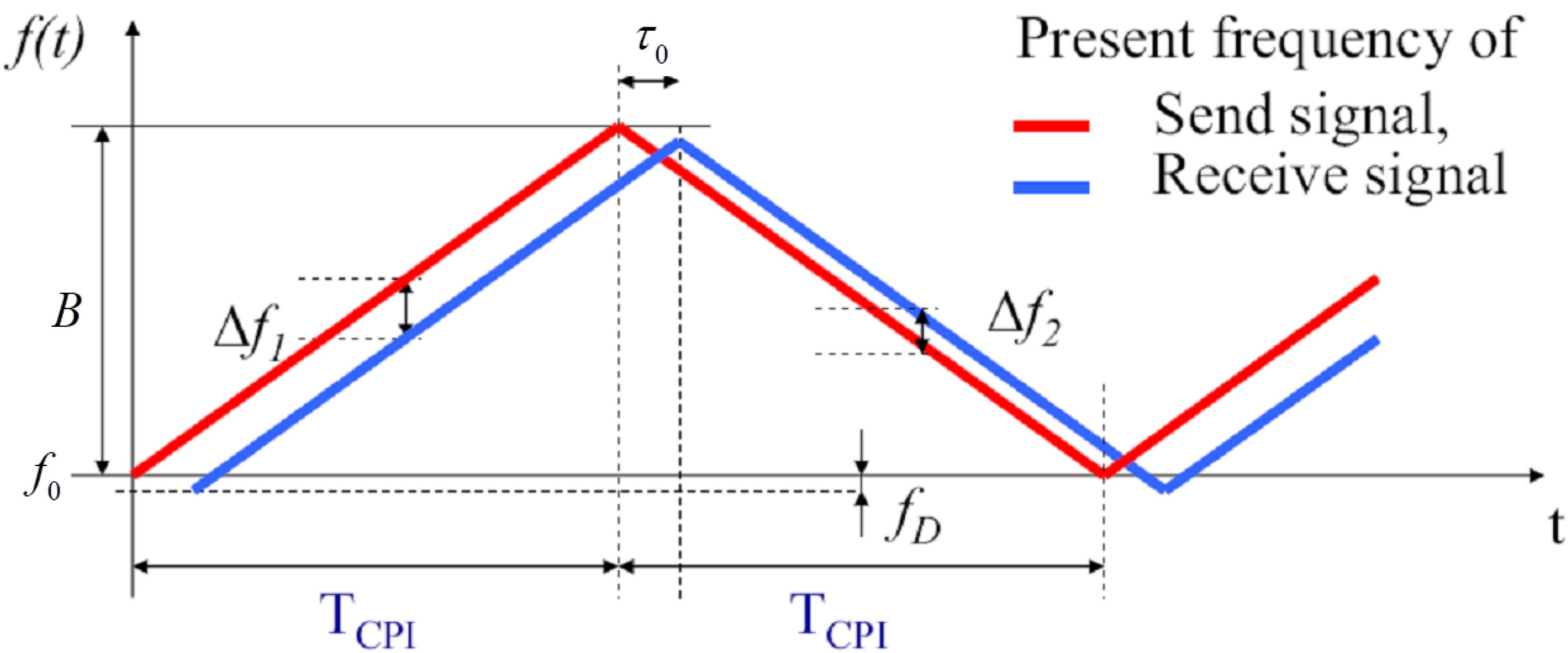
$$f_{\tau_0} = 2\frac{B}{c \cdot T_{CPI}} R \qquad f_D = -\frac{2}{\lambda} v_r$$

⇨ Range and radial velocity <u>can not be resolved</u>



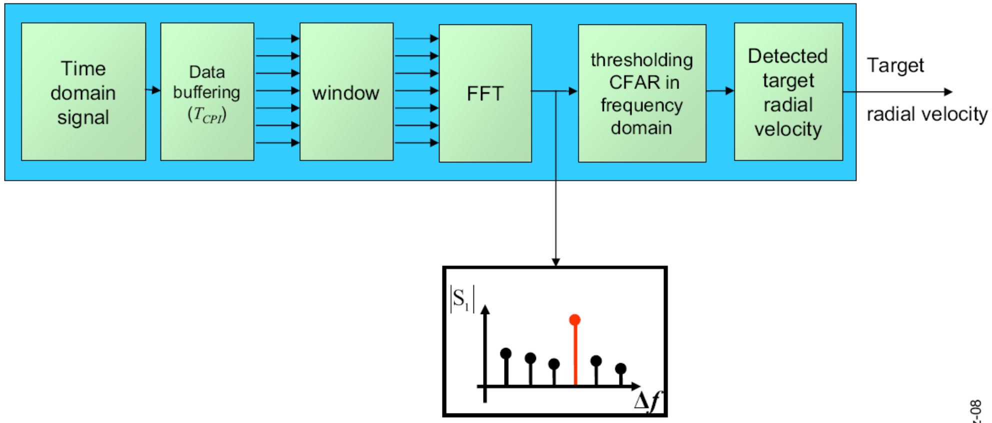
Delay and Doppler <u>can be resolved</u>, if enough periods (M) are processed coherently by a matched filter for M periods, as demonstrated by the response of such a matched filter (PAF).

FM CW radar up- and downchirp

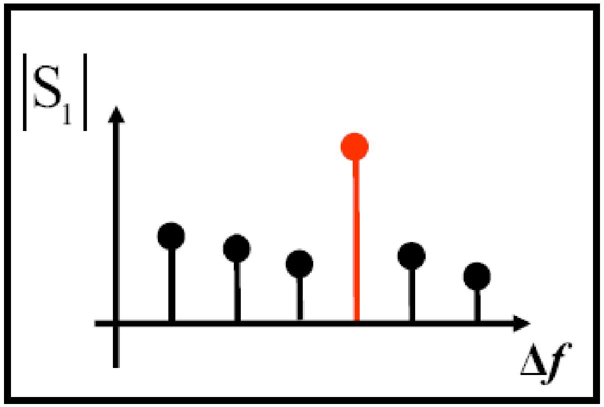


- Simultaneous measurement of target range and radial velocity with one up- and down-chirp in a single target situation:

$\Delta f_1 = f_D - f_\tau$ $\Delta f_2 = f_D + f_\tau$

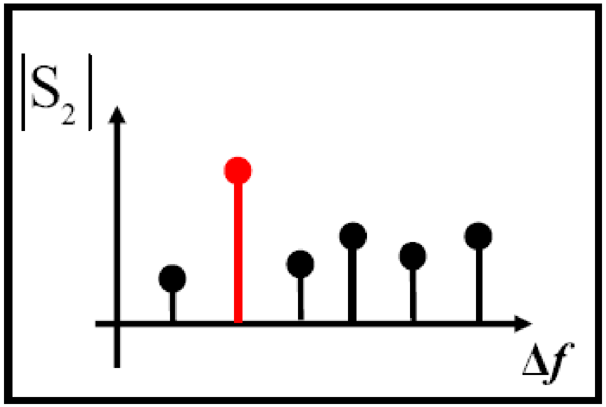


upchirp

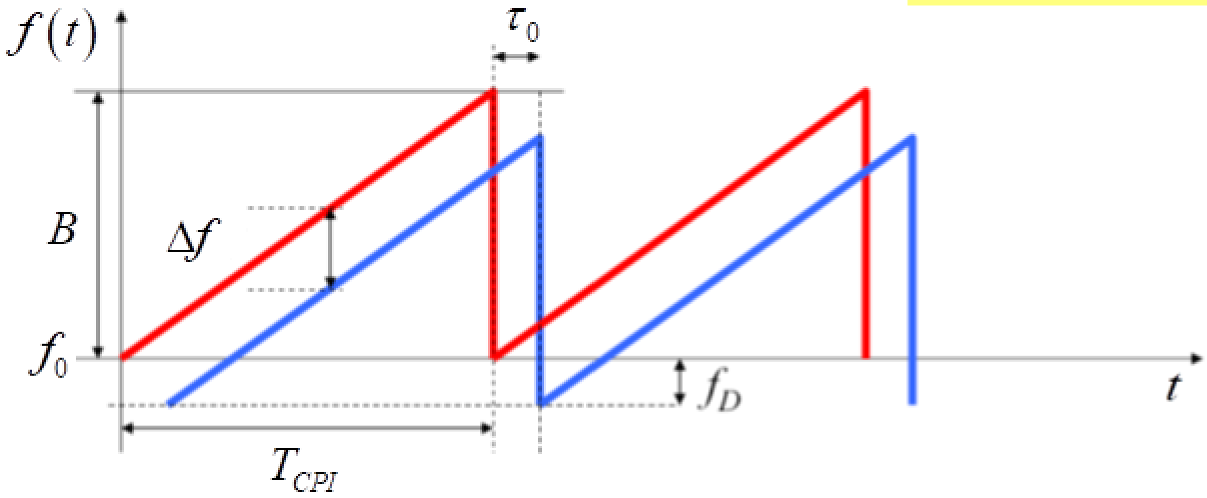


$$\Delta f_1 = f_D - f_\tau$$

downchirp



$$\Delta f_2 = f_D + f_\tau$$



Moving single target:

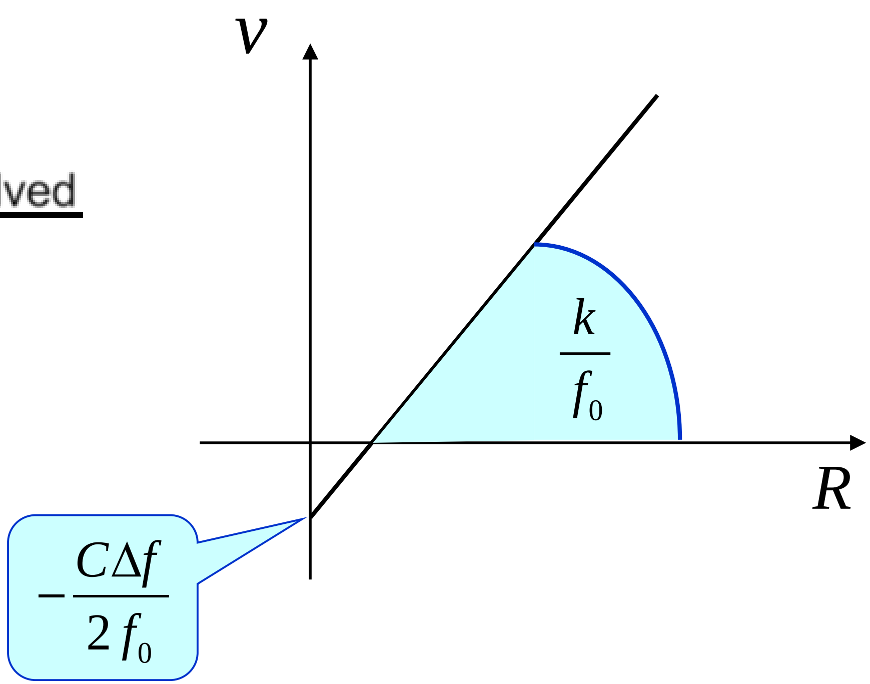
$$\Delta f = f_D - f_{\tau_0}$$

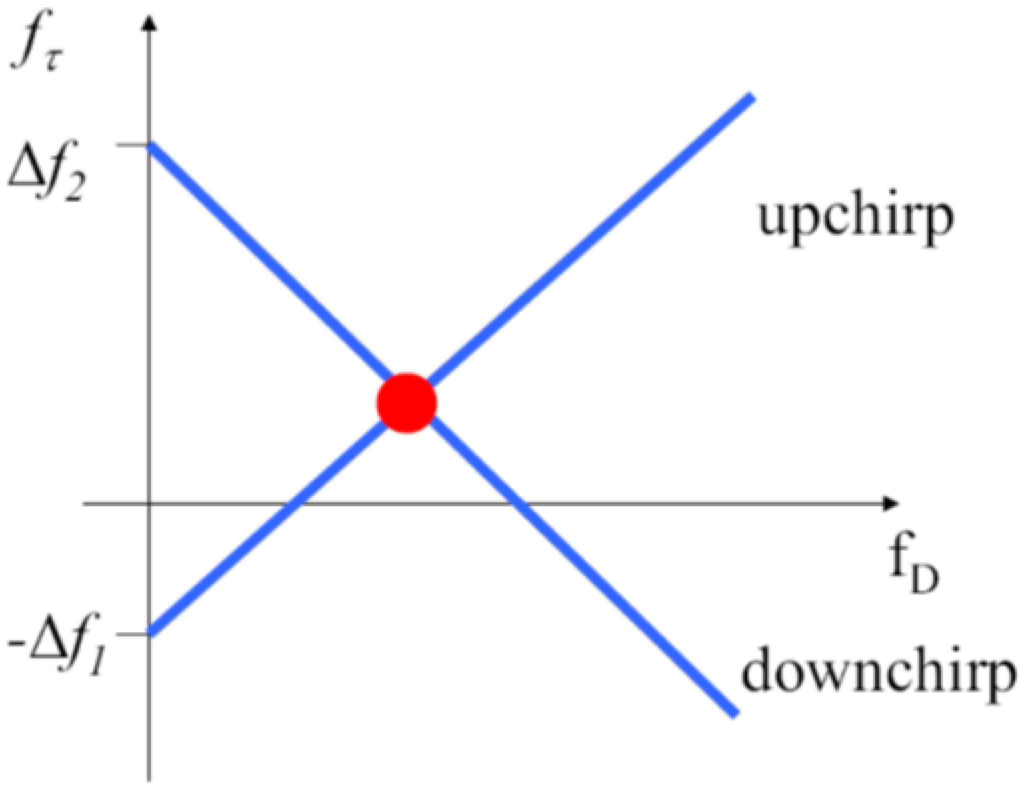
$$f_{\tau_0} = 2\frac{B}{c \cdot T_{CPI}} R \qquad f_D = -\frac{2}{\lambda} v_r$$

⇨ Range and radial velocity <u>can not be resolved</u>

$$\Delta f = -\frac{2v}{\lambda} + 2\frac{k}{C}R, \quad k = \frac{B}{T_{CPI}}, \quad \lambda = \frac{C}{f_0}$$

$$v = \frac{k}{f_0}R - \frac{C\Delta f}{2f_0}$$





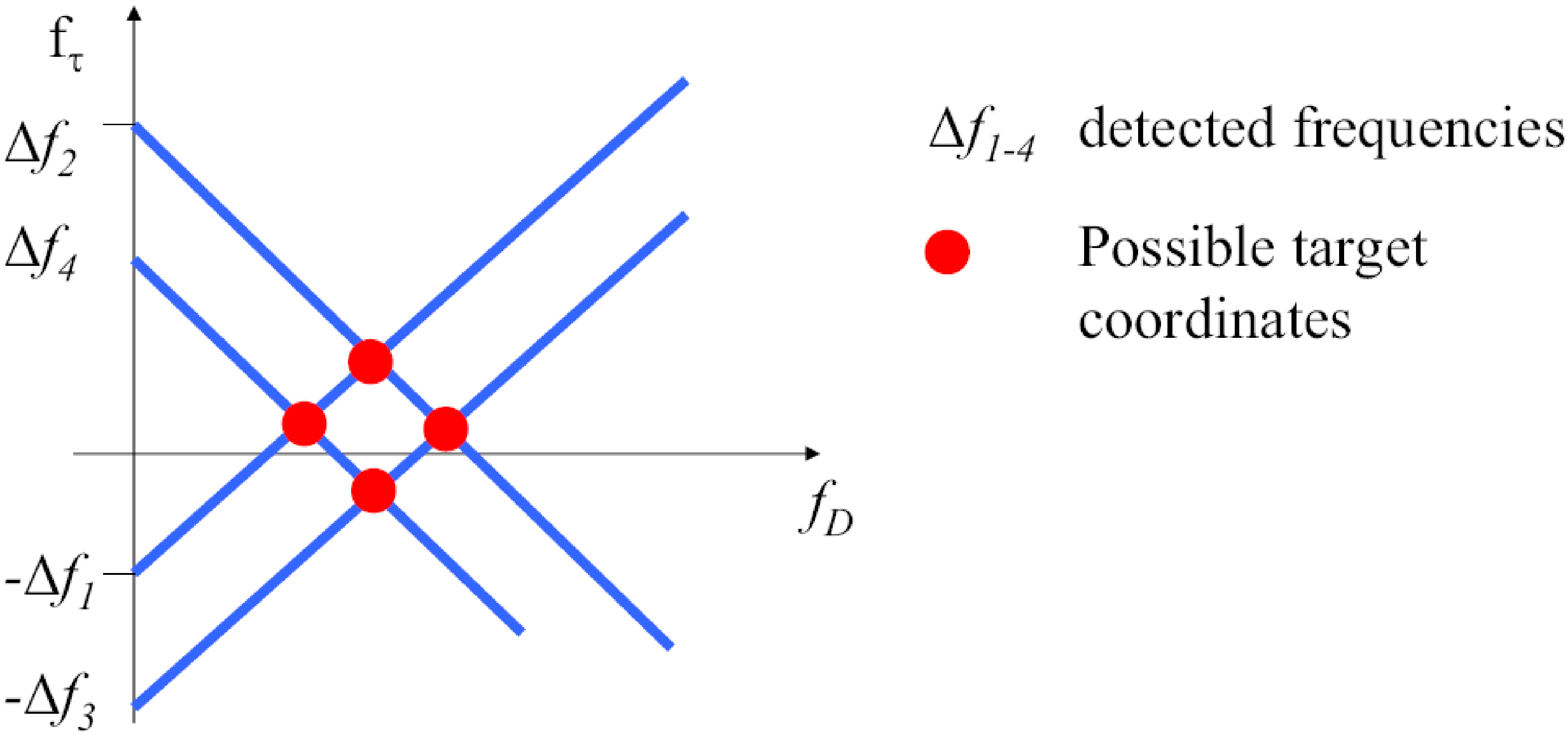
$$f_{\tau_0} = \frac{\Delta f_2 - \Delta f_1}{2}$$

$$f_D = \frac{\Delta f_2 + \Delta f_1}{2}$$

$$f_{\tau_0} = \frac{2B}{cT_{CPI}} R$$

$$f_D = -\frac{2}{\lambda} v_r$$

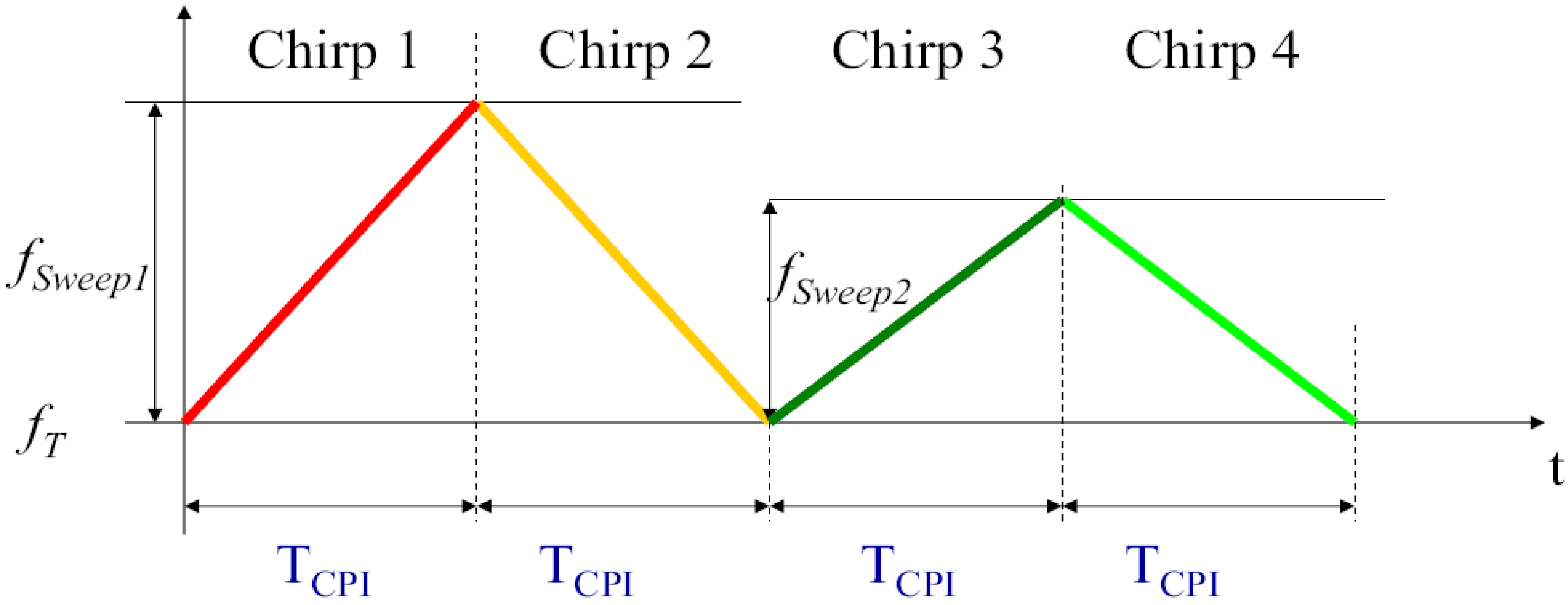
FM CW radar Two target situation



- Range and radial velocity of two targets can not be resolved simultaneously

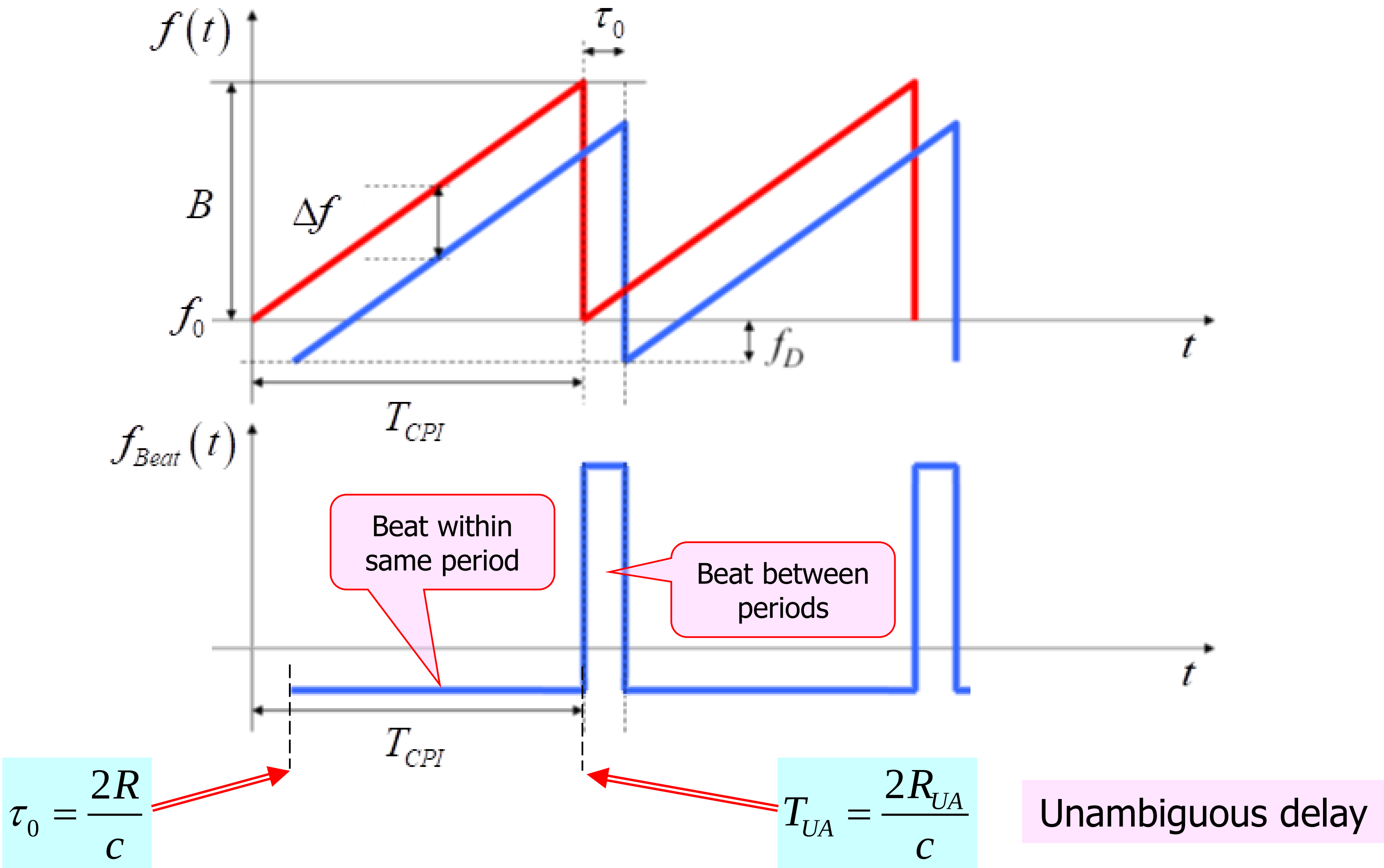
Nadav Levanon, Tel-Aviv University

FM CW radar multitarget situation



$\Rightarrow$ Usage of 2 up- and down-chirps in order to resolve 2 simultaneous detected targets

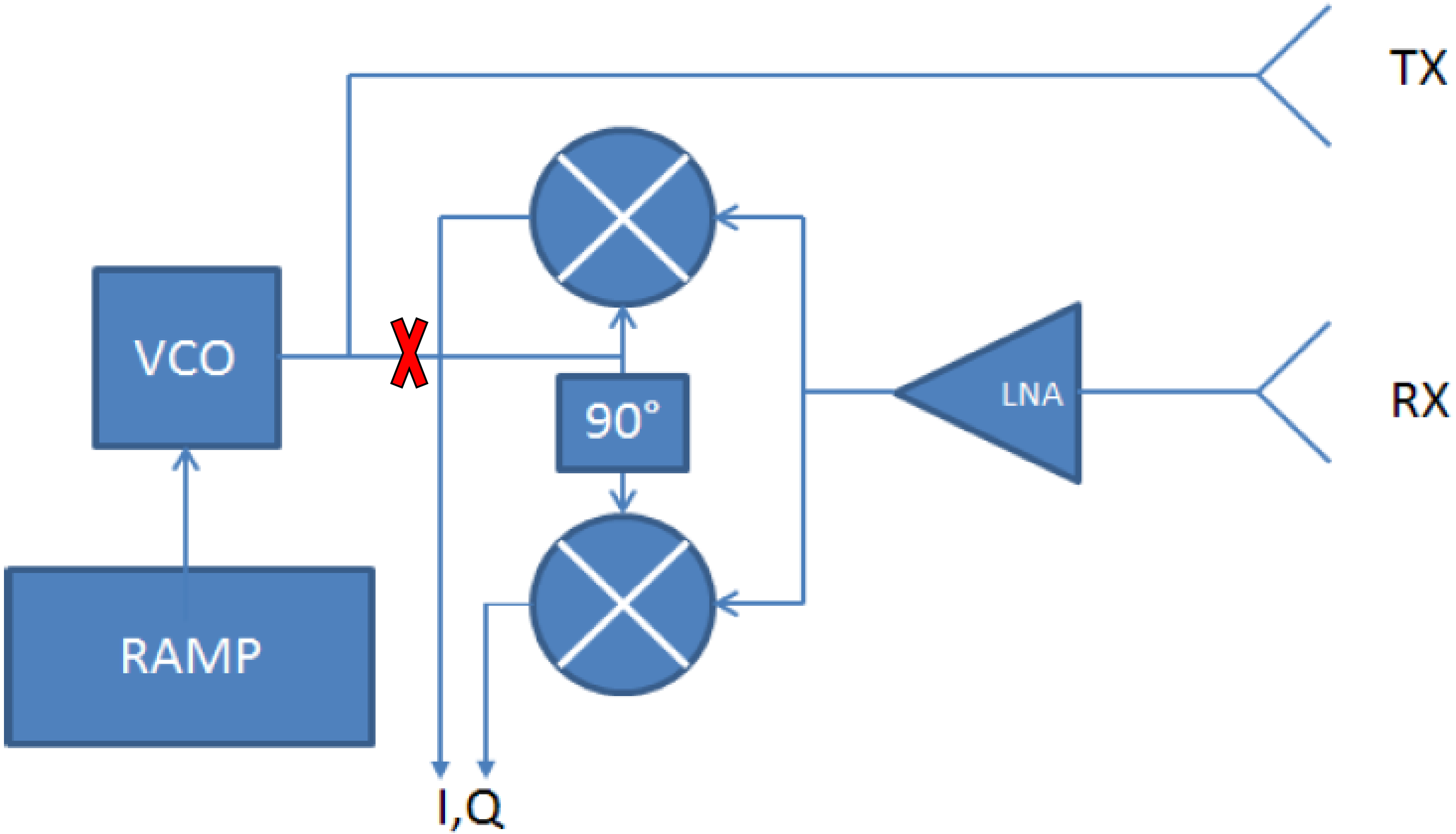
The prevailing multi-target scene terminated the use of this approach !



- Only the overlapping part from same period is of interest
- The overlap is decreasing with target range

➔ Effective measurement length is decreasing with range

➔ Measurement accuracy of beat frequency is range dependent



In practice, the reference chirp is the transmitted chirp.

In bi-static radar scene CW-LFM looses it's main advantage - Simple stretch processing.

Nadav Levanon, Tel-Aviv University

Initial freq. of ramp

Freq. slope

BW=Freq. sweep

Signal duration

$$s_1(t) = \cos(2\pi f_0 t + \pi\mu t^2), \quad 0 \le t \le t_p \ , \quad \mu = B/t_p$$

Transmitted signal

Delay $\tau_0 = 2R/C$

Static target (zero Doppler)

Attenuation

$$s_r(t) = a\, s_1(t - \tau_0) = a\cos\left[2\pi f_0 (t - \tau_0) + \pi\mu (t - \tau_0)^2\right]$$

Received signal

Initial freq. of ref. ramp

Reference sig. duration

$$s_{ref}(t) = \cos\left(2\pi f_r t + \pi\mu t^2\right), \quad T_{\min} \le t \le T_{UA}$$

Reference signal

$T_{rec} = T_{UA} - T_{\min}$ Duration of Reference signal

$f_r = f_0$

$$s_0(t) = a'\cos\left[2\pi f_0 \tau_0 + 2\pi\mu\tau_0 t - \pi\mu\tau_0^2\right]$$

Post mixer and LPF signal

("beat signal")

$$\mu = B/t_p$$

$$s_0(t) = a'\cos\left[2\pi f_0\tau_0 + 2\pi\mu\tau_0 t - \pi\mu\tau_0^2\right]$$

Post mixer and LPF signal

$$\tau_0 = 2R/C$$

$$s_0(t) = a'\cos\left[\frac{4\pi RB}{Ct_p}t + \frac{4\pi RB}{C}\left(\frac{f_0}{B} - \frac{\tau_0}{2t_p}\right)\right]$$

Carrier

Delay

$$\frac{f_0}{B} >> 1, \quad \frac{\tau_0}{t_p} << 1$$

Bandwidth

Signal duration

$$\therefore \frac{f_0}{B} >> \frac{\tau_0}{2t_p}$$

Instantaneous frequency

$$\therefore s_0(t) \approx a'\cos\left[2\pi\left(\frac{2RB}{Ct_p}\right)t + \frac{4\pi Rf_0}{C}\right]$$

The frequency of the signal following the mixer and LPF is proportional to the range of the target (or to the delay)

$$f_{inst} = \frac{2B}{Ct_p}R$$

$$f_{inst} = \frac{B}{t_p}\tau_0$$

Nadav Levanon, Tel-Aviv University

Moving target – delayed and Doppler shifted

Attenuation

Doppler

$f_D = -\frac{2}{\lambda} v_r$

Delay $\tau_0 = 2R/C$

$$s_r(t) = a\, s_1(t-\tau_0) = a \cos\left[2\pi\, f_D t + 2\pi\, f_0 (t-\tau_0) + \pi\mu (t-\tau_0)^2\right]$$

Received signal

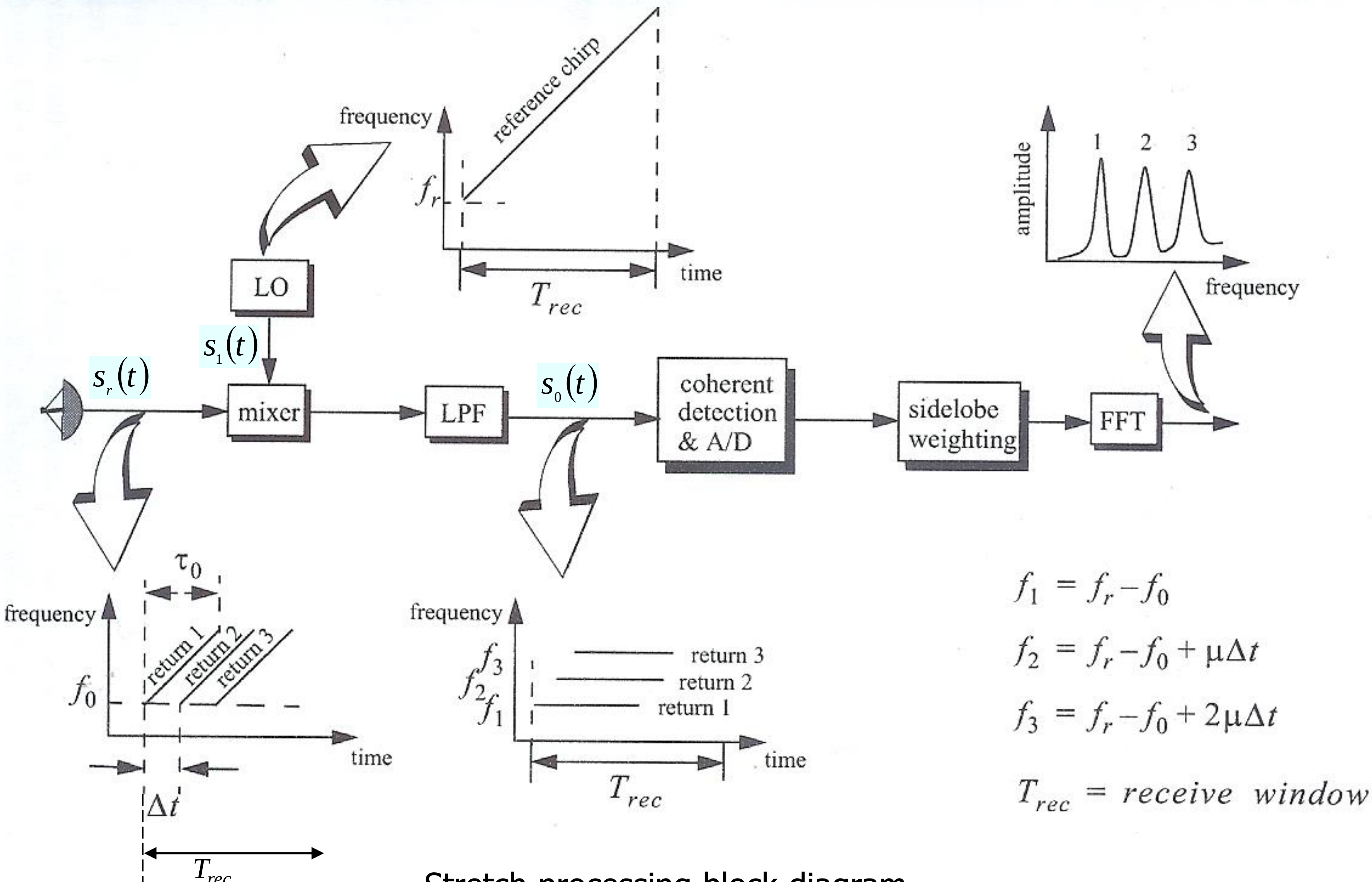
$$s_0(t) = a' \cos\left[2\pi\mu\tau_0 t - 2\pi f_D t + 2\pi\, f_0 \tau_0 - \pi\mu\tau_0^2\right]$$

$$s_0(t) = a' \cos\left[2\pi\left(\frac{2B}{ct_p} R - \frac{2}{\lambda} v_r\right) t + 2\pi\, f_0 \tau_0 - \pi\mu\tau_0^2\right]$$

Post mixer and LPF signal

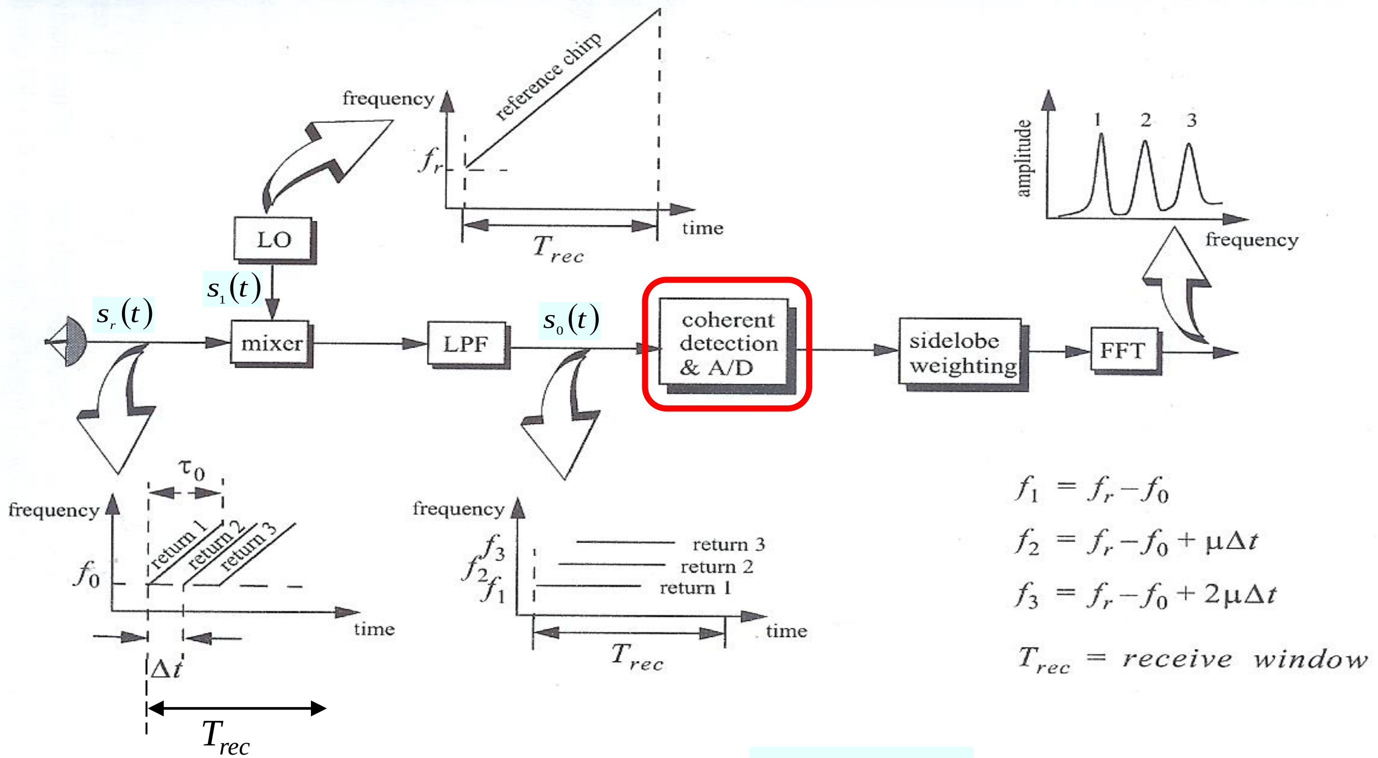
("beat signal")

Instantaneous frequency



Stretch processing block diagram

From: Mahafza, B. R., *Radar Signal Analysis and Processing Using MATLAB*, CRC Press, 2009, Sec. 8.5



Assuming the maximal expected range $R_{\max} \ll R_{UA}$, the maximal expected frequency of the beat signal $s_0(t)$ is relatively small:

$$f_{beat} = \frac{2B}{ct_p} R - \frac{2}{\lambda} v_r \ll B$$

Therefore, the analog to digital converter can use a sample rate much lower than the signal's BW (Shannon-Nyquist sampling theorem).

$$2\left(\frac{2B}{ct_p} R_{\max} + \left|\frac{2}{\lambda} v_{r\max}\right|\right) \le f_s \ll B$$

Main advantage of stretch processing

Considerations in designing the FFT

$$f_{inst} = \frac{2B}{Ct_p} R$$

Range resolution : $\Delta R = \frac{C}{2B}$

Given parameters

Range resolution : ΔR

Range span : $R_{max} - R_{min}$

$\frac{\text{Pulse energy}}{\text{Pulse power}} = \text{Pulse width} : t_p$

Calculated parameters

B - Bandwidth

N - FFT size

T_s - Sampling interval

Frequency resolution : $\Delta f = \frac{2B}{Ct_p} \Delta R = \frac{2B}{Ct_p} \frac{C}{2B} = \frac{1}{t_p}$

FFT frequency span $= \pm \frac{N\Delta f}{2}$

$$\frac{N\Delta f}{2} > \frac{2B}{Ct_p}\left(R_{max} - R_{min}\right) = \frac{2B}{Ct_p} \frac{CT_{rec}}{2} = \frac{BT_{rec}}{t_p}$$

$$\frac{N\Delta f}{2} = \frac{N}{2t_p} > \frac{BT_{rec}}{t_p}$$

$$N > 2BT_{rec}$$

$N_{FFT} = 2^m \geq 2BT_{rec}$

m is an integer

$$\Delta f = \frac{1}{T_s N_{FFT}} \Rightarrow$$

$$T_s = \frac{1}{N_{FFT} \Delta f}$$

Required Energy $\Rightarrow t_p \quad (= 1/\Delta f)$

$\Delta R \Rightarrow B$

$R_{max} - R_{min} \quad \& \quad \Delta R \Rightarrow N_{FFT}$

$N_{FFT} \quad \& \quad t_P \Rightarrow T_s$

Example:

THALES

- SCOUT Waveform and Receiver Parameter Summary

Range setting	2.4 (4.4)	6 (11)	24 (44)	nmi (km)
FMCW waveform	Sawtooth	Sawtooth	Sawtooth	
Frequency deviation, peak to peak	70	28	7	MHz
Range resolution (at 6nmi)	2.4	6.0	24.0	m
Modulation frequency	1	1	1	kHz
FFT length	4,096	4,096	4,096	Points

$t_p = 1ms$ Modulation period

$\lambda \approx 3.3cm$ Wavelength

$R_{\max} = 11km$

$B = 28MHz$

Maximum delay

$$\tau_{0,\max} = \frac{2R_{\max}}{c} \simeq 73.3\mu s$$

Maximum beat frequency

$$f_{beat,\max} = \frac{2B \cdot R_{\max}}{c \cdot t_p} \simeq 2.05MHz$$

Minimal sampling rate

$$f_s \geq 2 f_{beat,\max} = 4.1MHz$$

Ideal range resolution

$$\Delta R = \frac{c}{2B} \simeq 5.35m$$

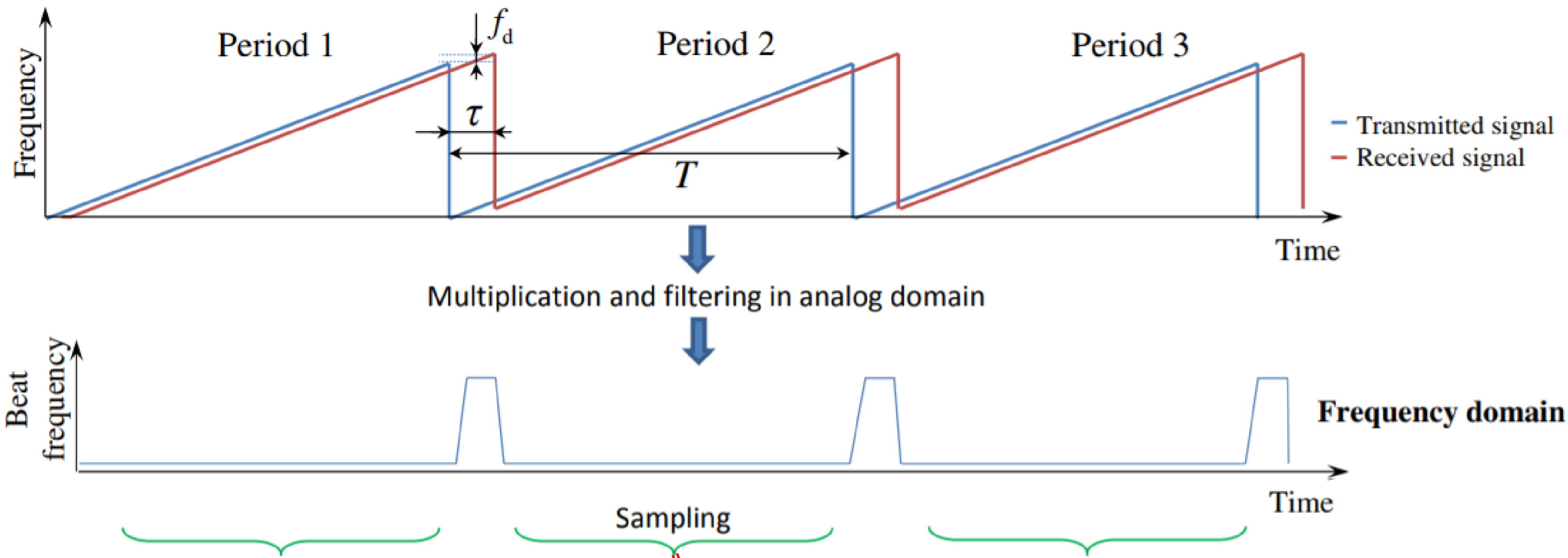
Range resolution at R_{max}

$$\Delta R = \frac{c}{2B} \frac{t_p}{t_p - t_{0,\max}} \simeq 5.78m$$

Claimed range resolution is worse than calculated due to:

- Actual processing length $T_{rec} < t_p$
- Pre-FFT weighting window (Hamming, Hann, Blackman…)

Coherent processing of LFM-CW



$$s_0(t) = a'\cos\left[2\pi\left(f_D - \mu\tau_0\right)t + \varphi(\tau_0)\right]$$

Beat signal of 1st frame

$$s_0(t) = a'\cos\left[2\pi\left(f_D - \mu\tau_0\right)\left(t - t_p\right) + \varphi(\tau_0) + 2\pi f_D t_p\right]$$

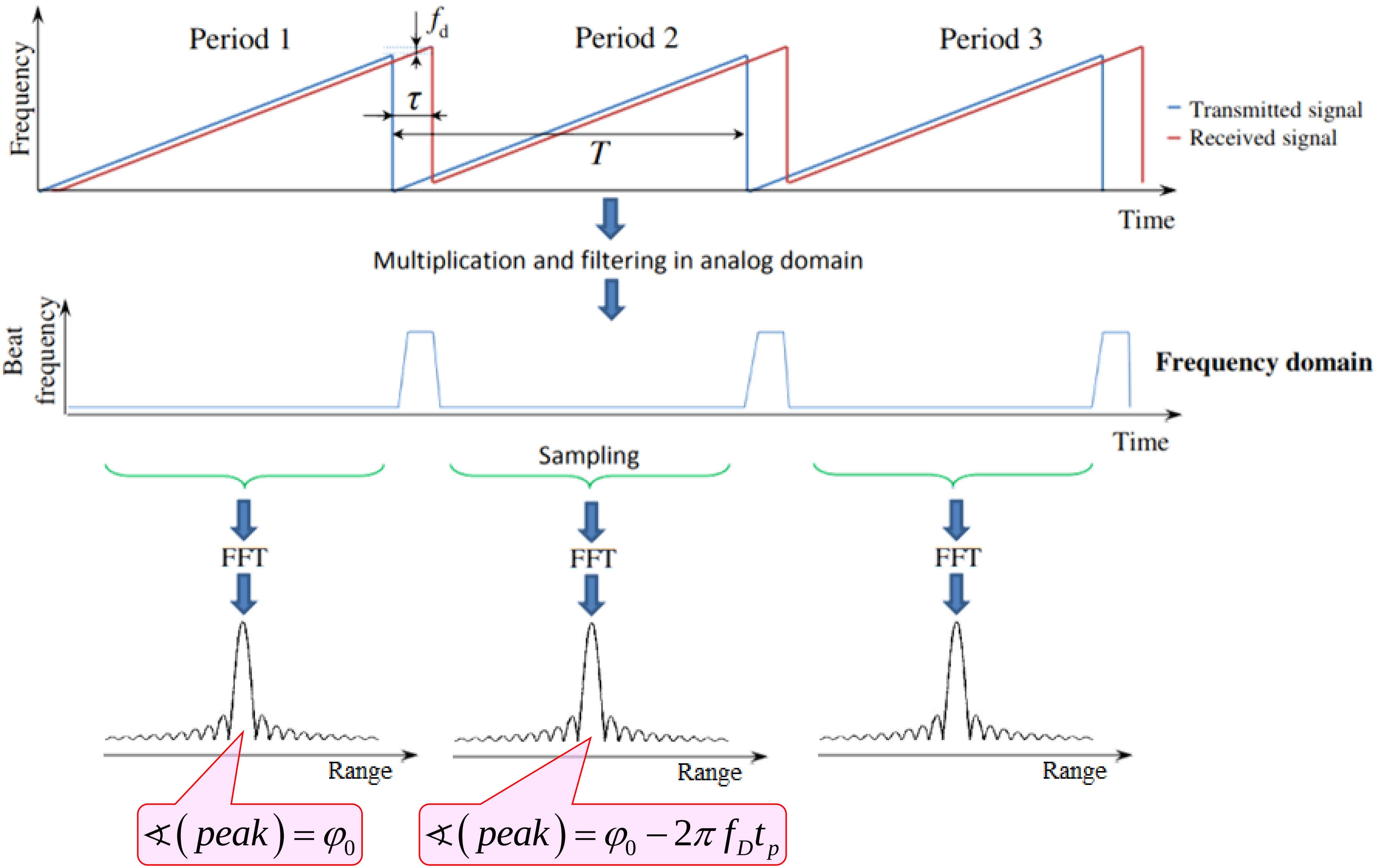
Beat signal of 2nd frame

$$s_0(t) = a'\cos\left[2\pi\left(f_D - \mu\tau_0\right)\left(t - (m-1)t_p\right) + \varphi(\tau_0) + 2\pi(m-1) f_D t_p\right]$$

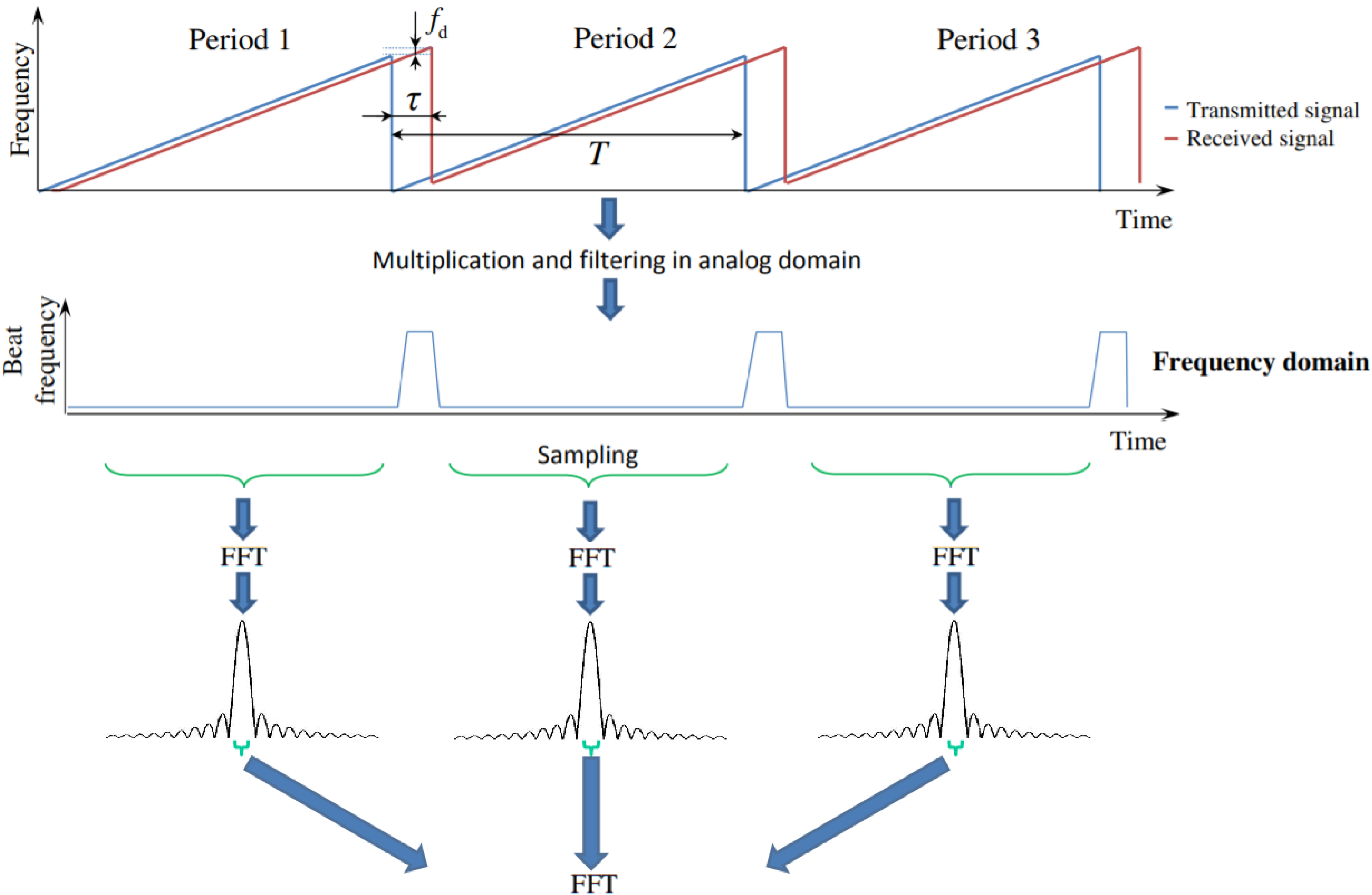
Beat signal of mth frame

The phase of the beat signal is increasing between periods

Coherent processing of LFM-CW



Coherent processing of LFM-CW



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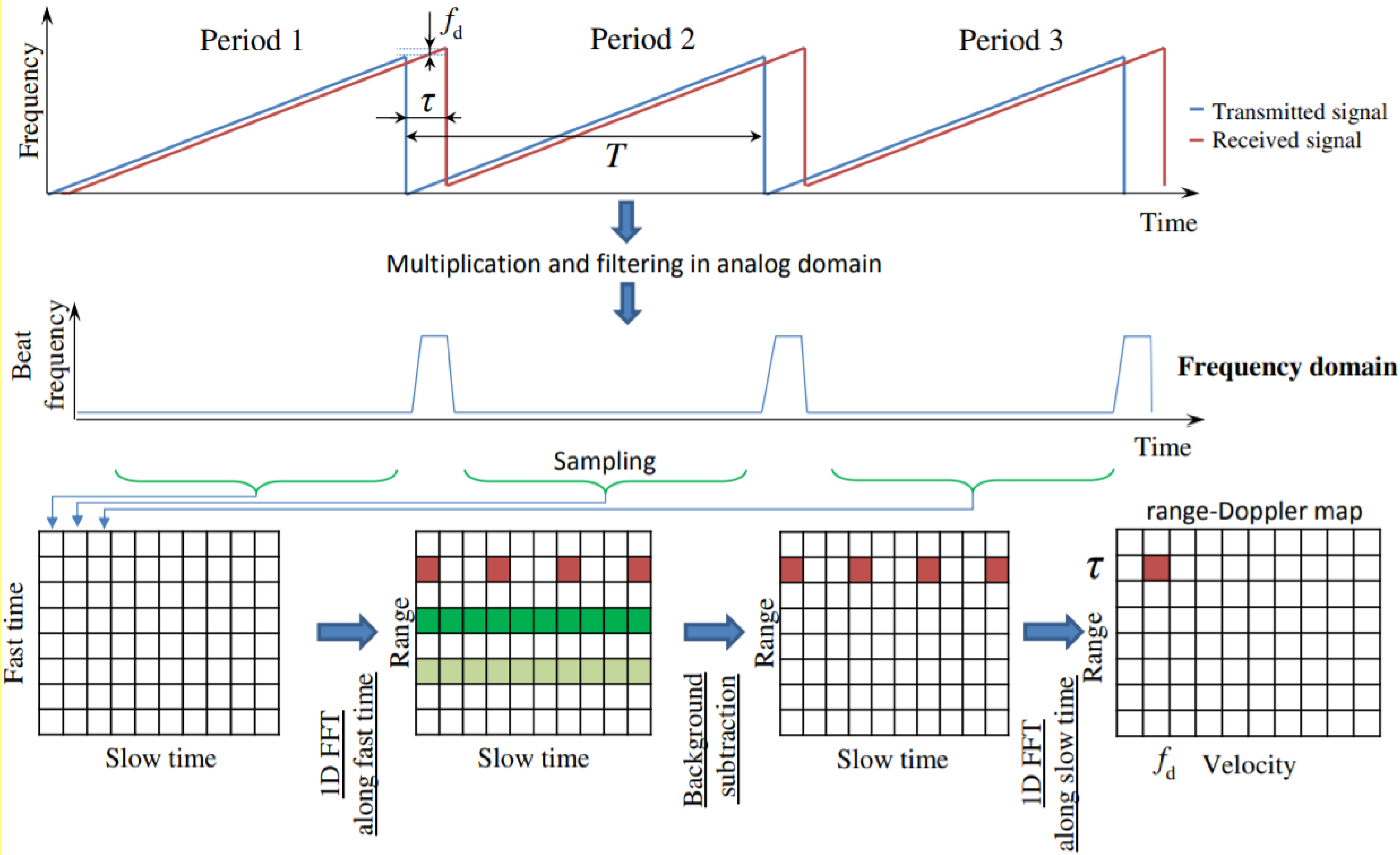
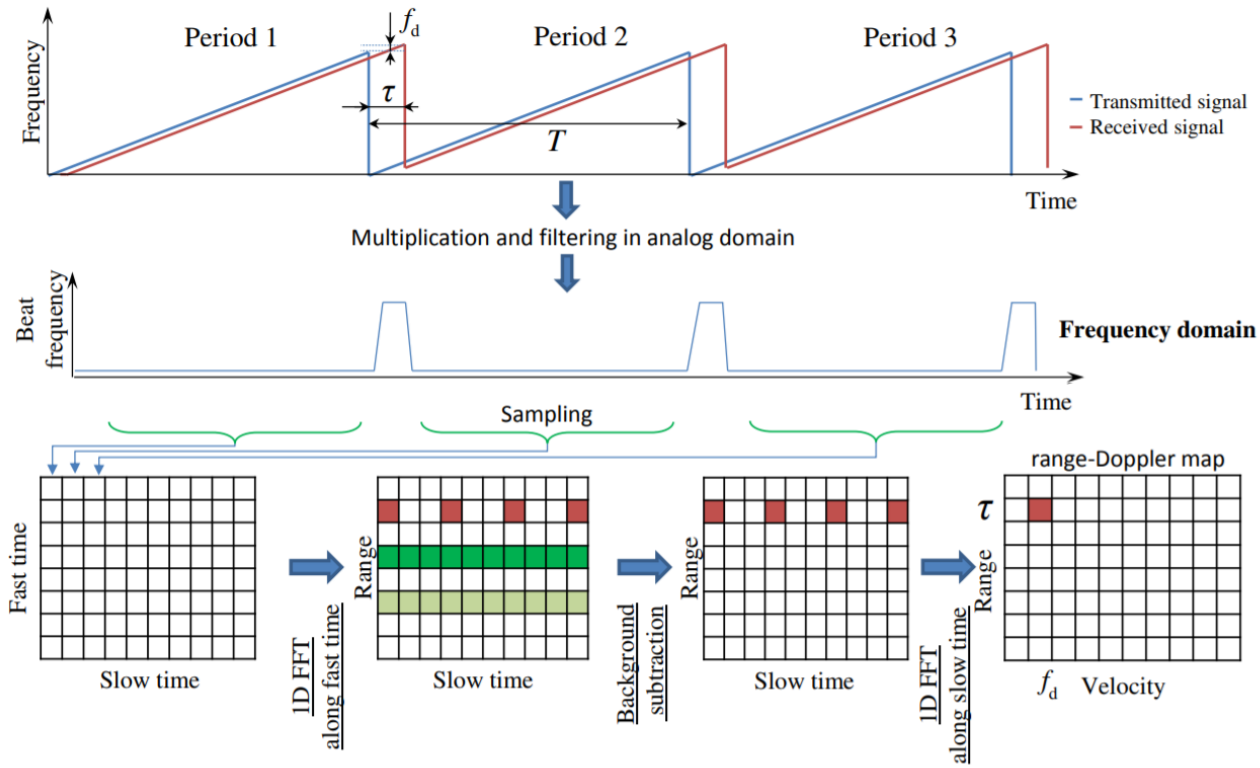
Coherent processing of LFM-CW



Coherent processing of LFM-CW



Similarly to pulse Doppler processing:

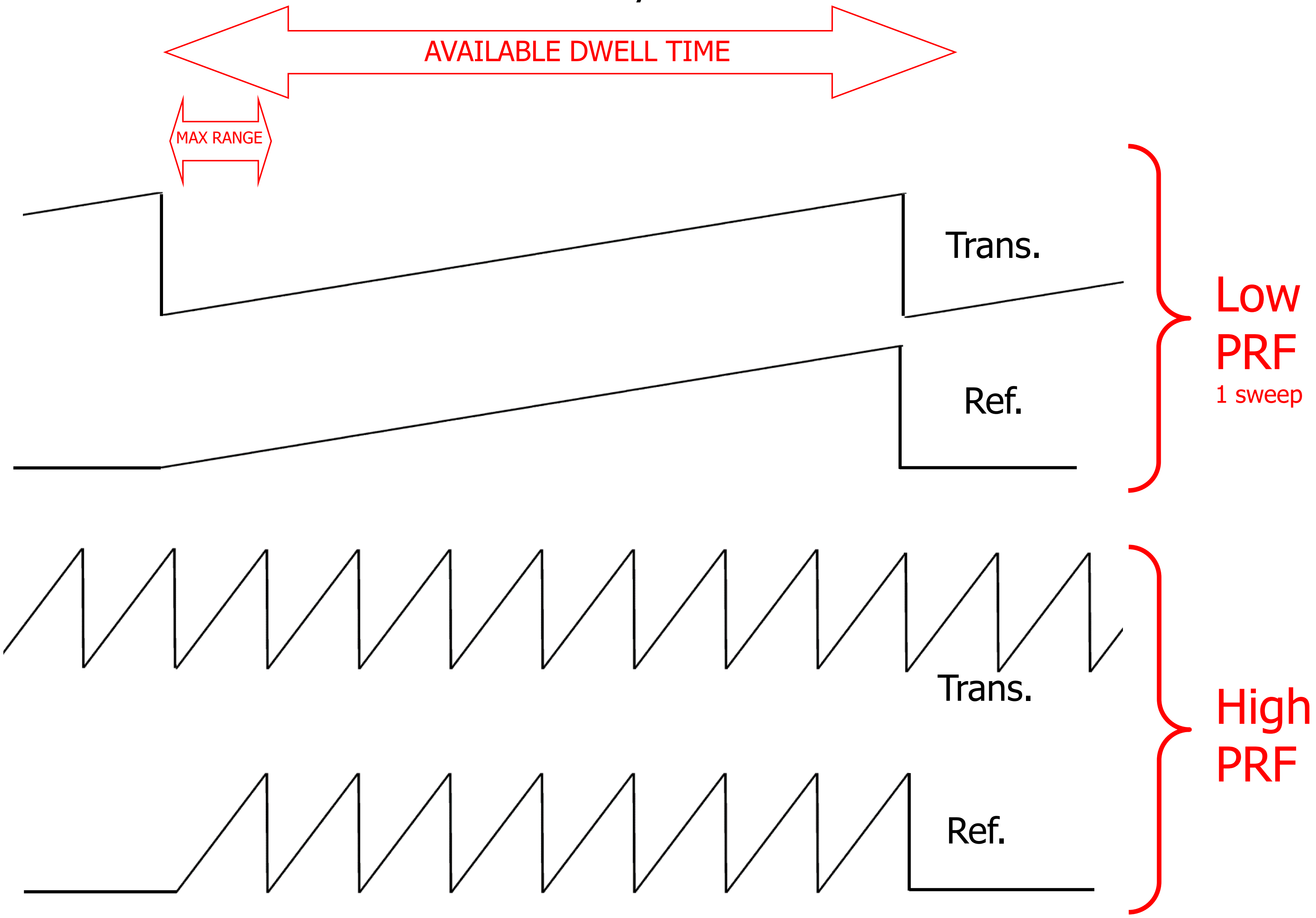
Unambiguous Doppler: $f_{UA} = \pm \frac{1}{2t_p}$

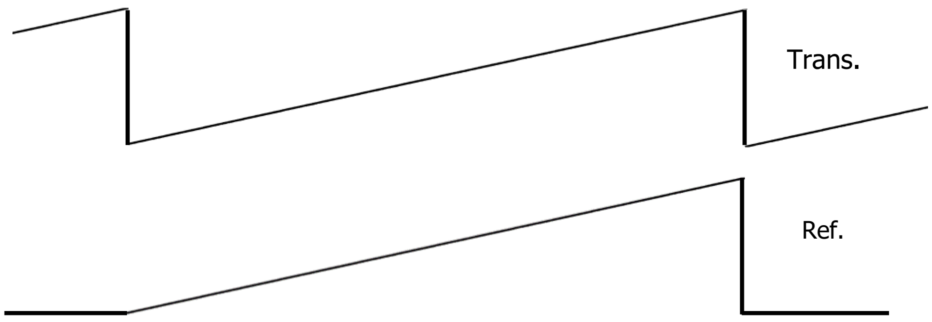
Doppler resolution (Doppler span/number of periods): $\Delta f_D = \frac{1}{N \cdot t_p}$

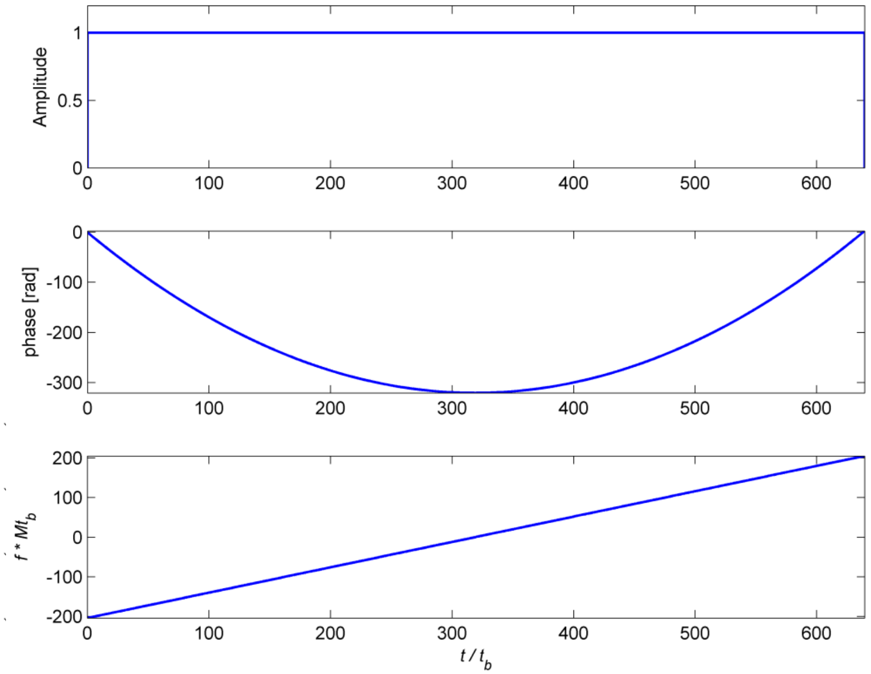
Unambiguous velocity: $v_{UA} = \pm \frac{\lambda}{4t_p}$

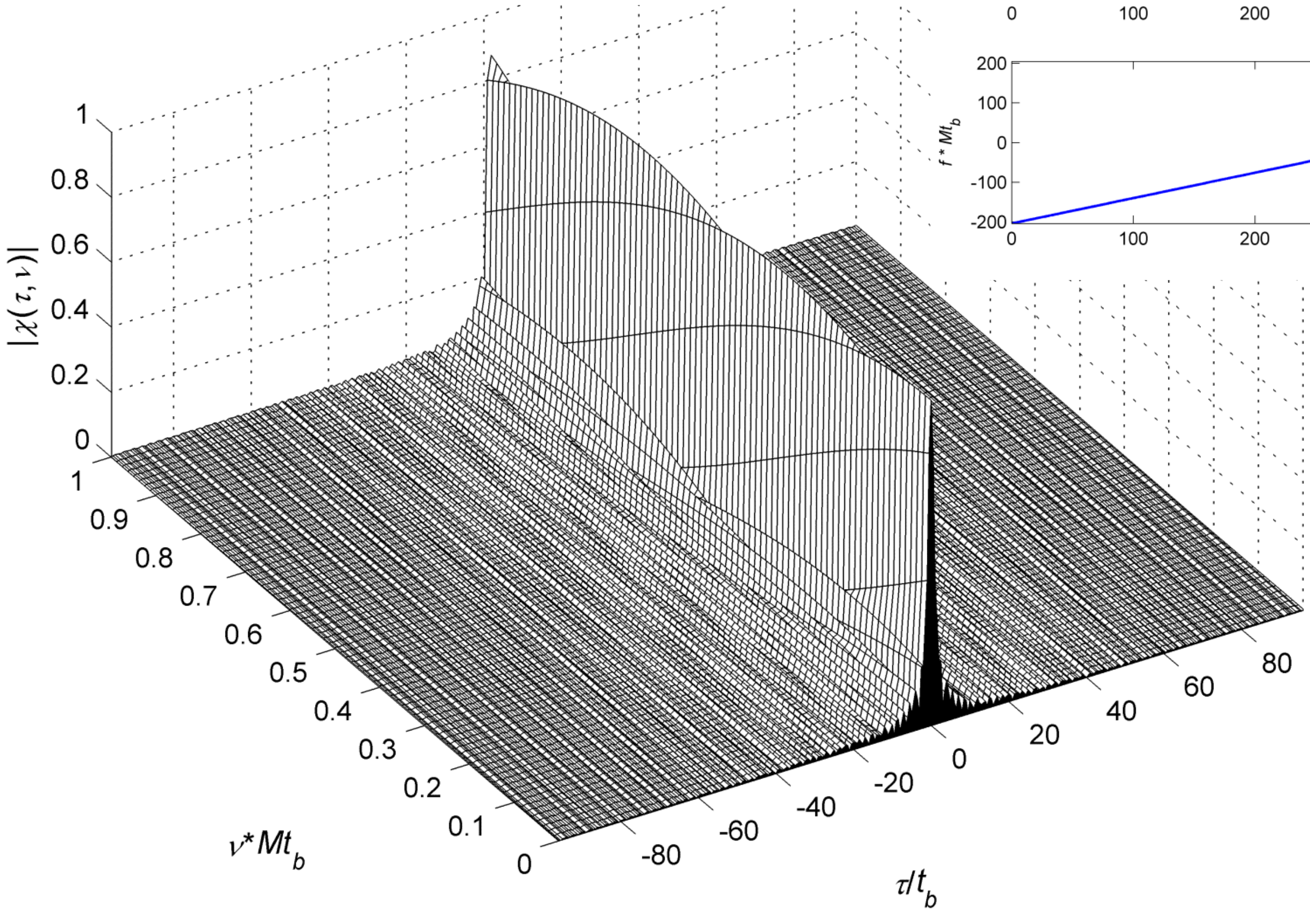
Velocity resolution: $\Delta v = \frac{\lambda}{2Nt_p}$

Summary of LFM-CW









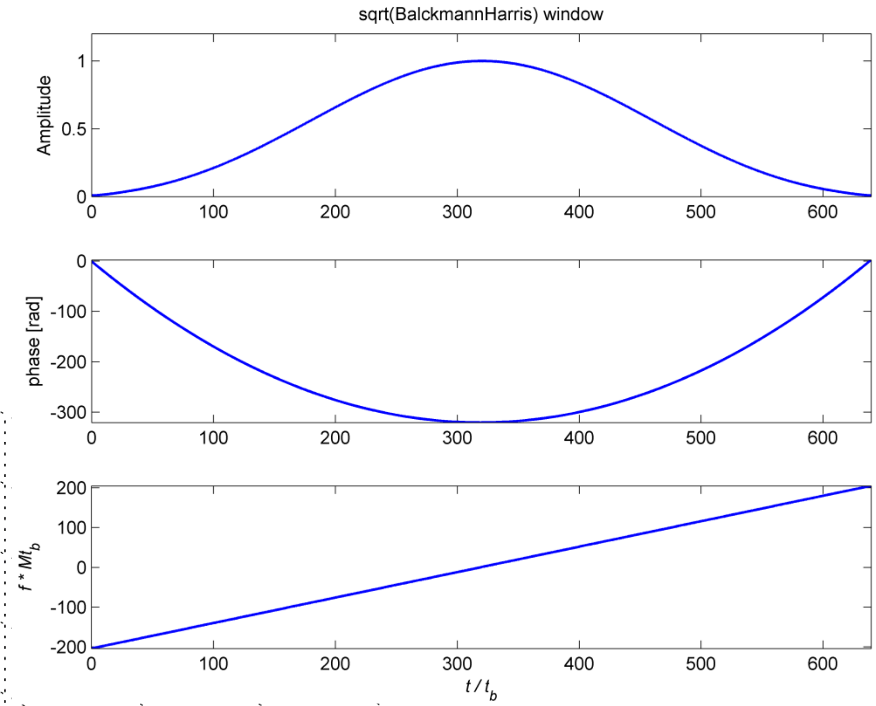
Low PRF

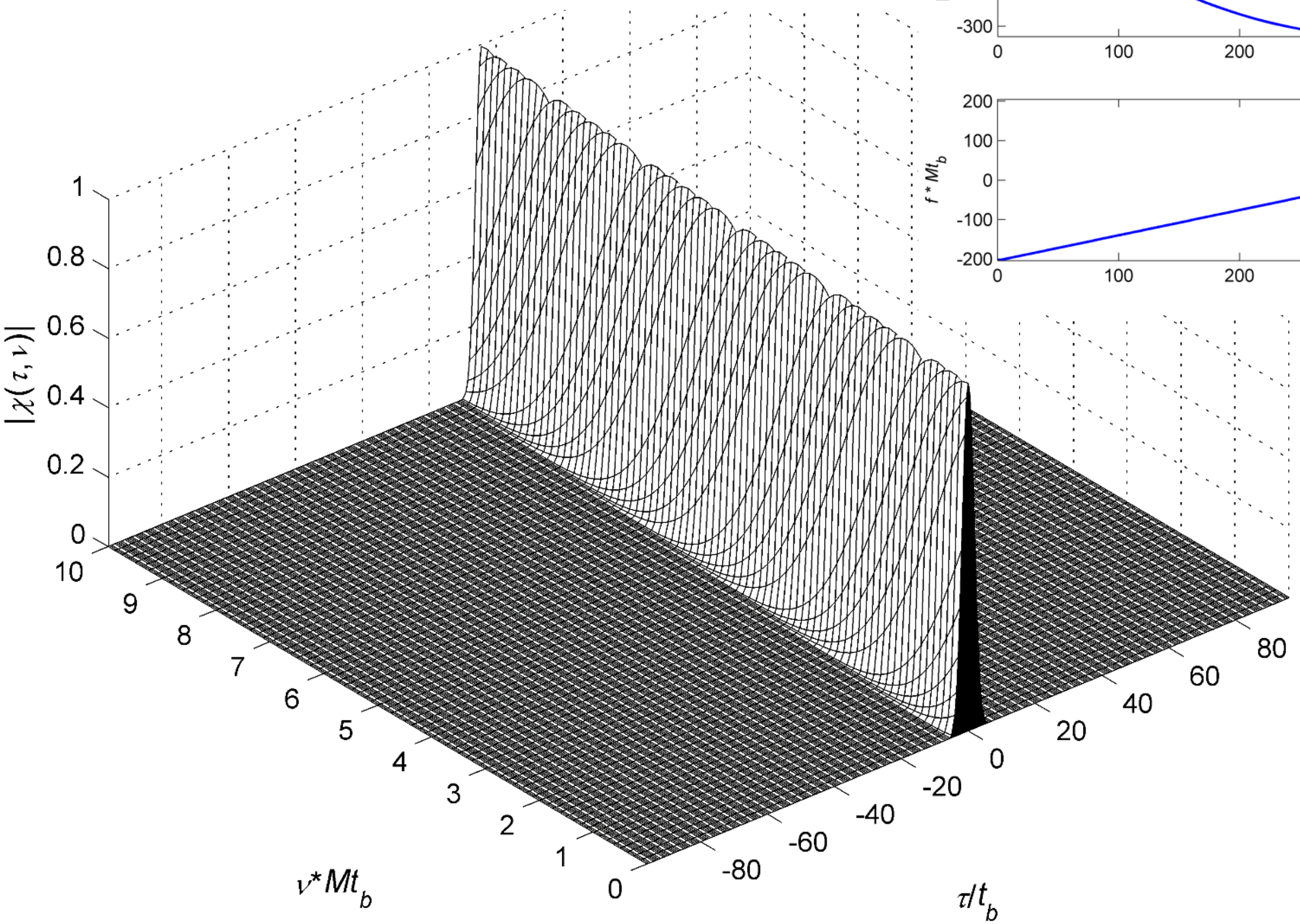
No weighting

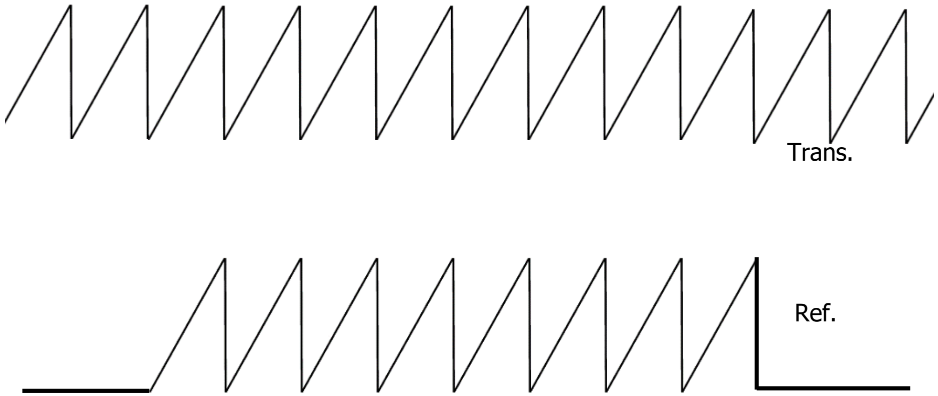
Low PRF

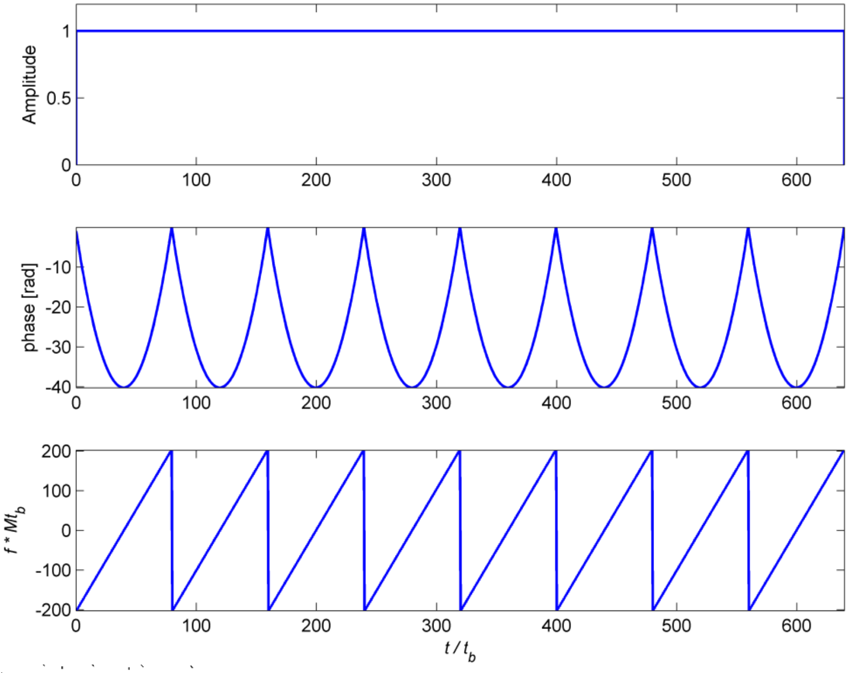
BlackmanHarris weighting

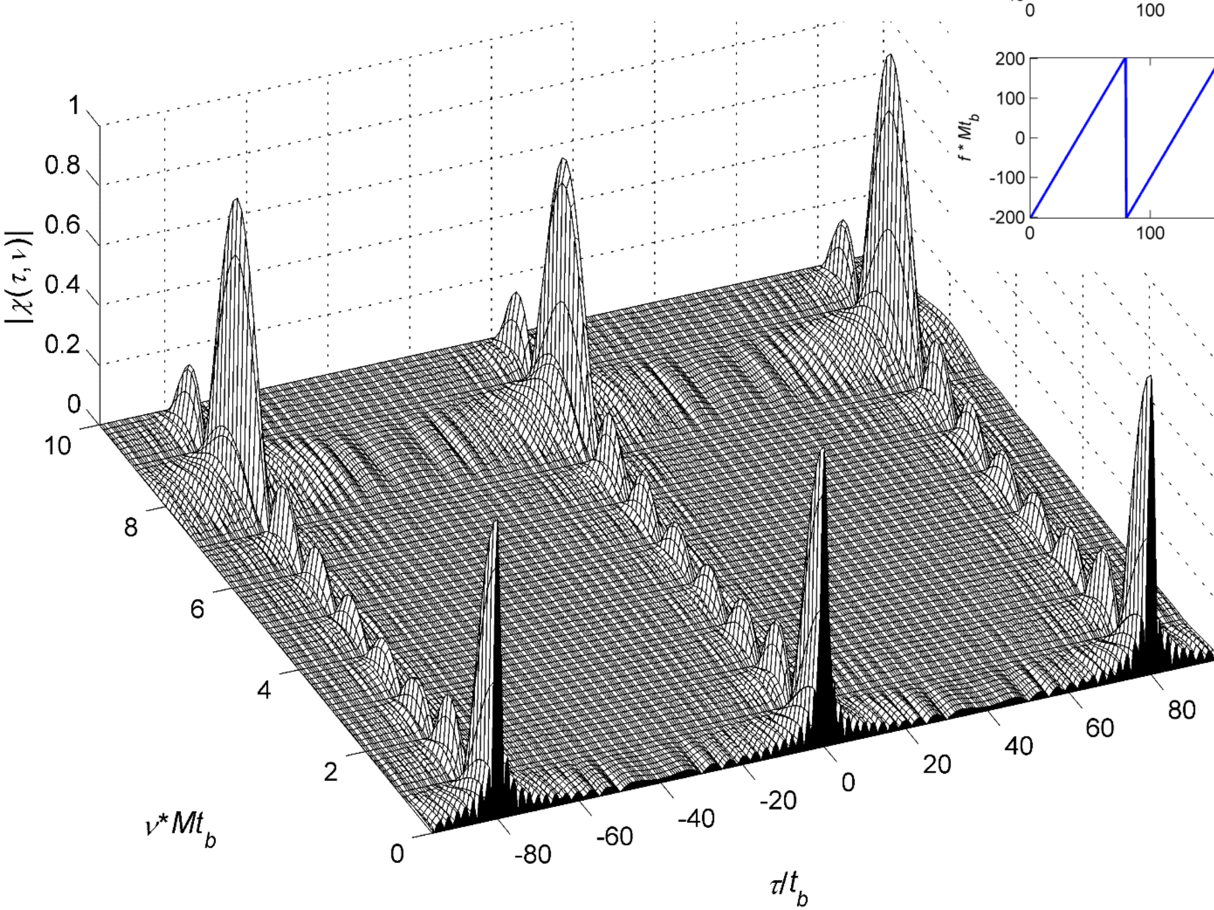
Coherent processing of a single period is simple but yields no Doppler resolution.









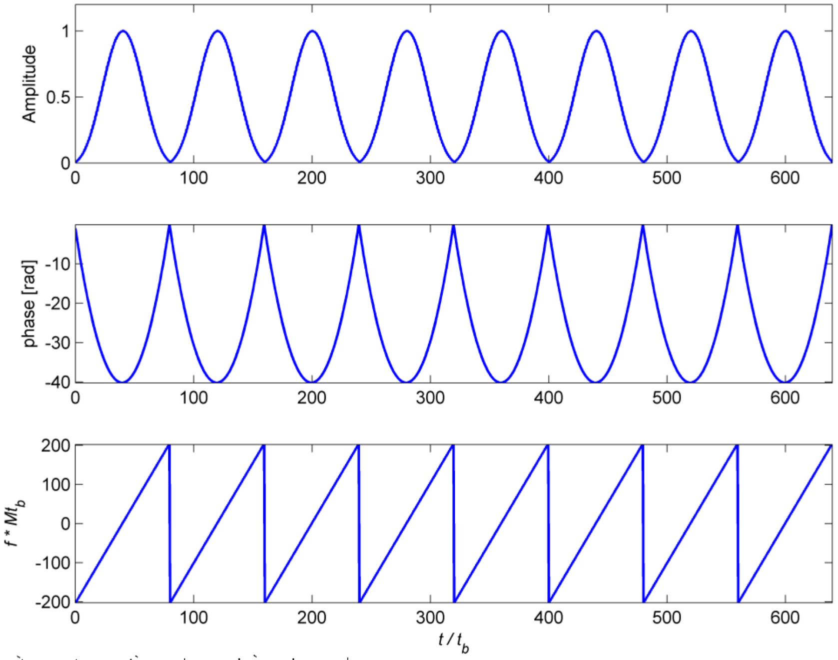


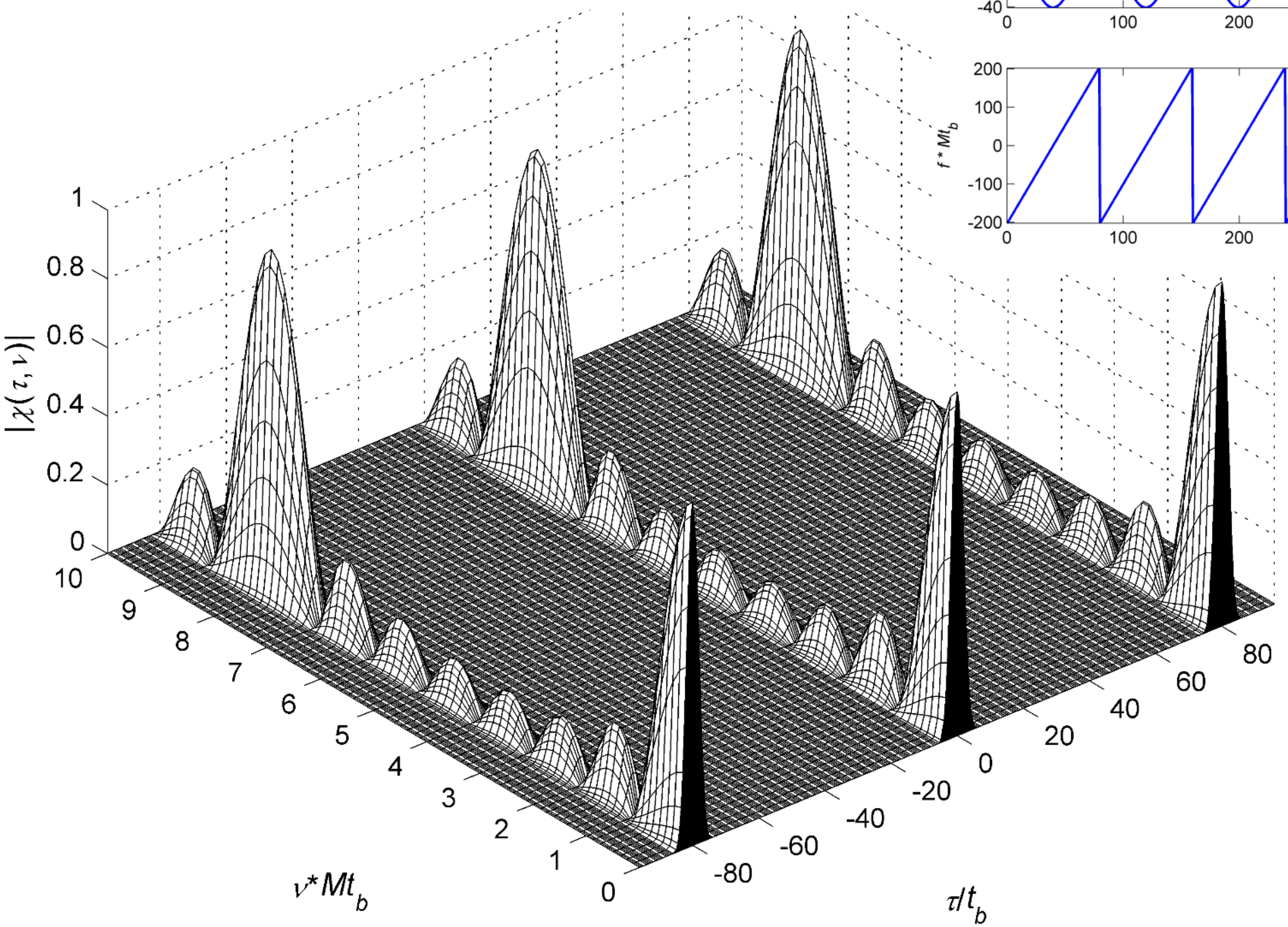
High PRF

No weighting

High PRF, BlackmanHarris intra-period weighting

Coherent processing of several periods yields Doppler resolution but is more complicated.





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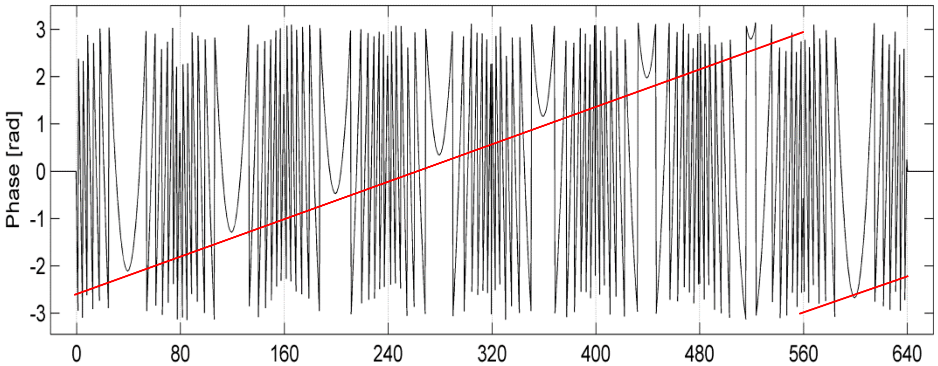
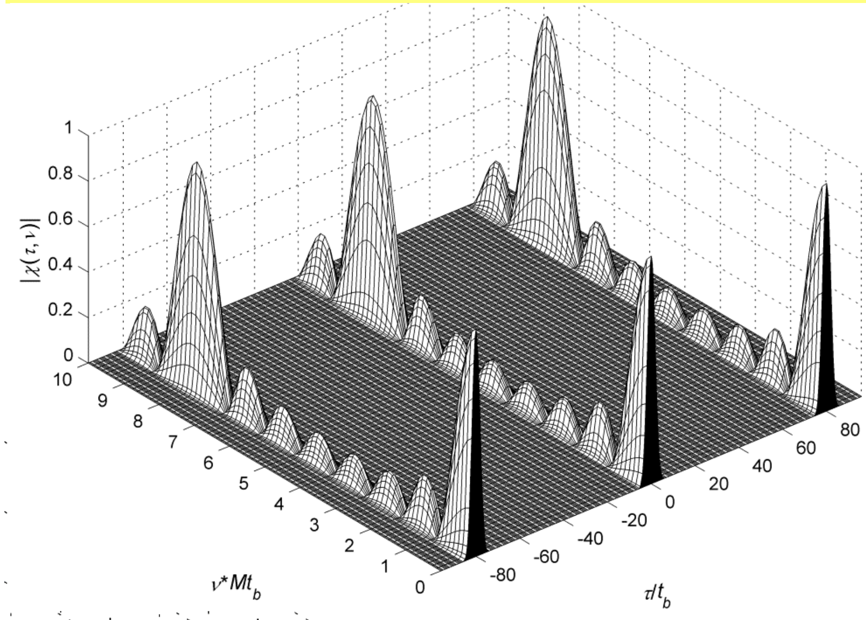
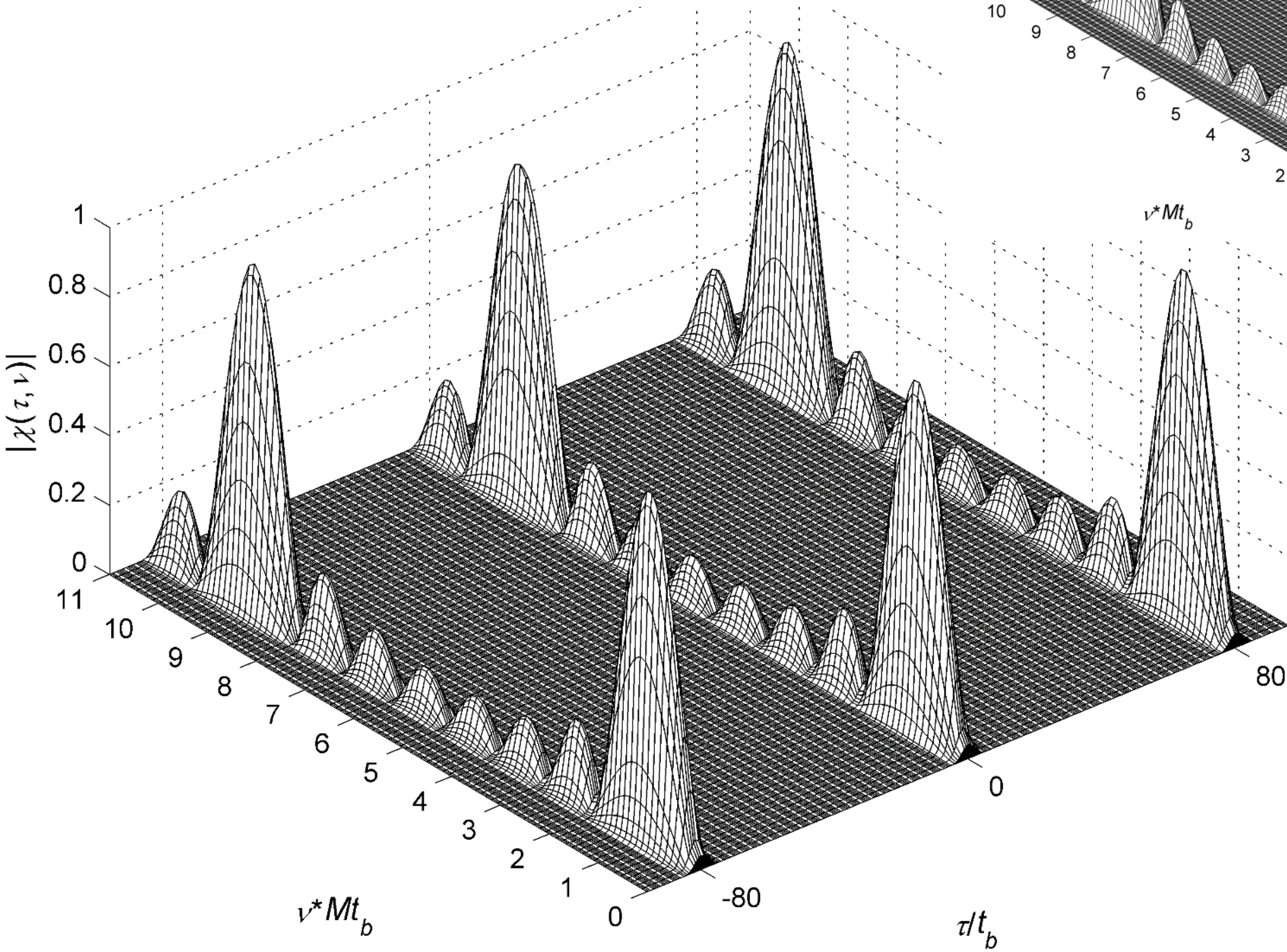
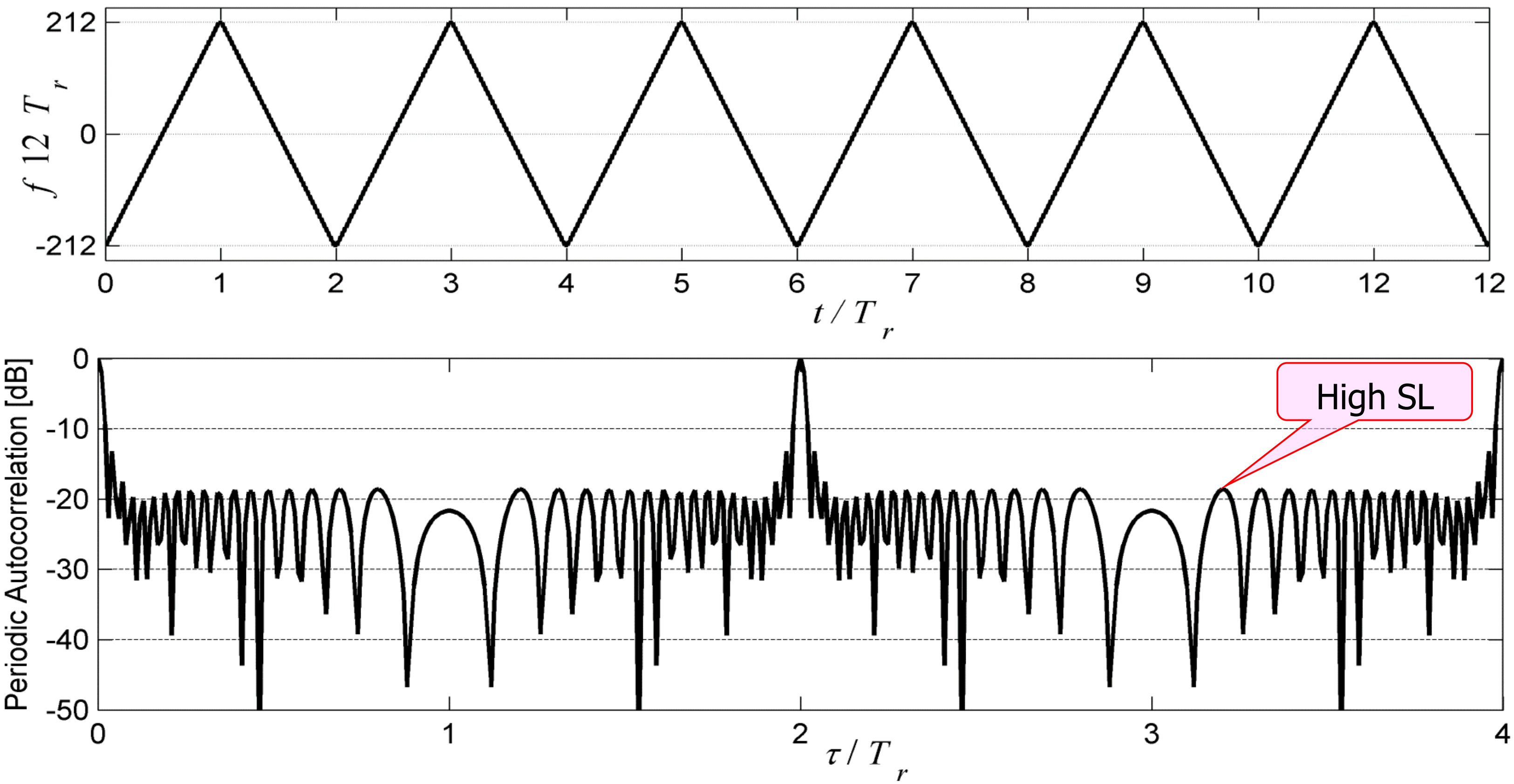


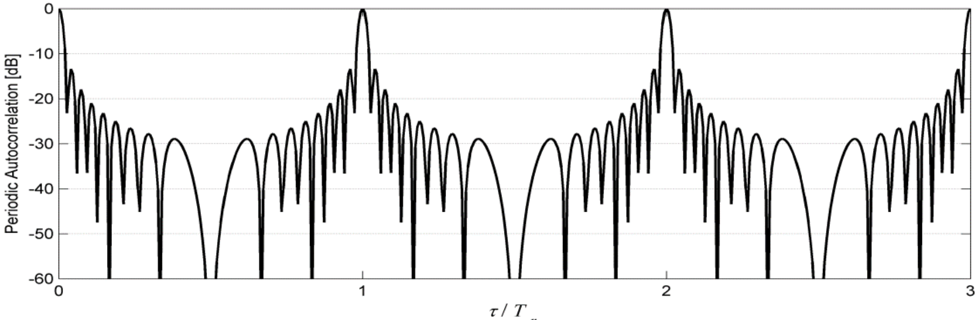


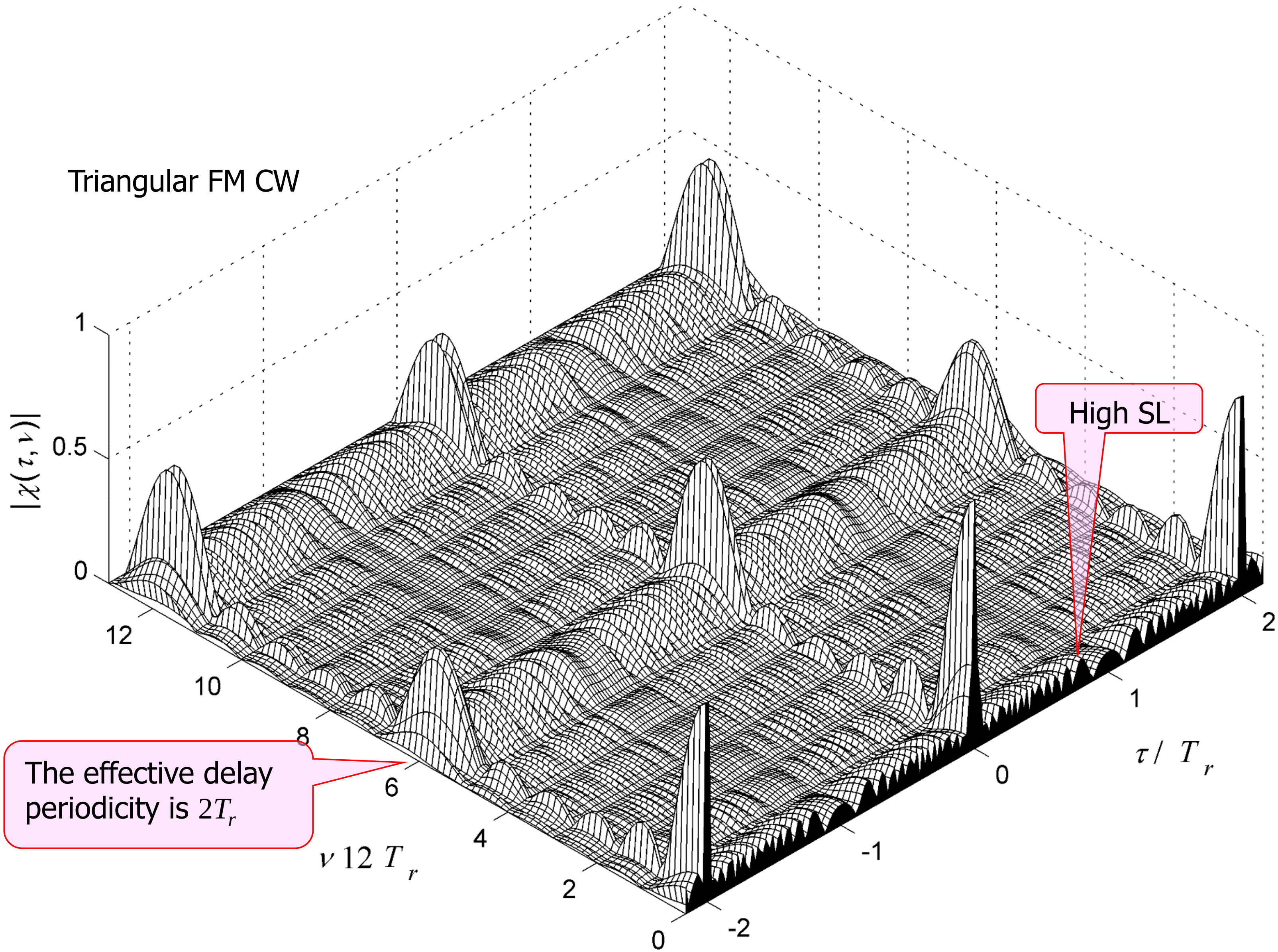


Triangular FM CW

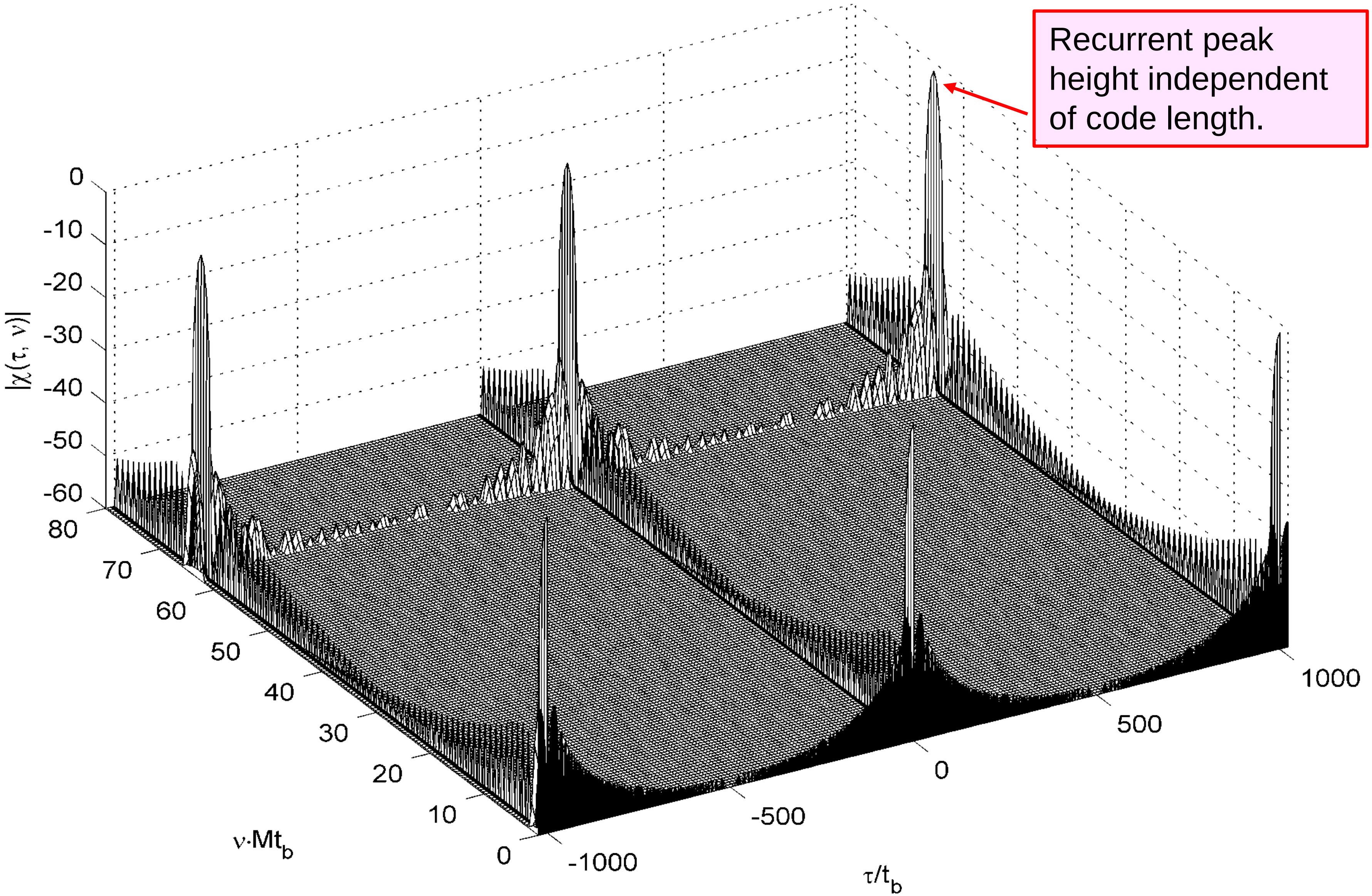


Saw-tooth FM CW





Judging from the PAF, triangular FM CW is inferior to saw-tooth FM CW.



Periodic cross-ambiguity function (in dB) of CW **LFM** waveform. **Inter**-period Hamming weighted reference (to reduce Doppler sidelobes). **Intra**-period weighting (to reduce delay sidelobes). (TBW=1000, 64 periods)

Lessons from the LFM-CW example

- The (periodic) ambiguity function predicts the expected delay-Doppler performances of a signal, when processed by a matched-filter.
- A matched-filter is preferred over other processors in the presence of noise and other targets (which is almost always the case).
- Other processors may be simpler to implement, but their performances are likely to be poorer, and difficult to predict.
- In the LFM-CW example the "other processor" estimated (measured) the beat frequency Δf between the transmitted and received signals.
- The coherent duration used for "measuring" the beat frequency was shorter than one modulation period (one saw tooth), which is too short to yield Doppler resolution.
- To overcome the problem of resolving Doppler and range, for several targets, simultaneously, the signal was made much more complicated, and the processing involved non-coherent processing steps, which entails SNR loss.
- LFM-CW using beat-frequency measurements is useful in cases of a single target with little or no Doppler shift (e.g., an aircraft radar altimeter).

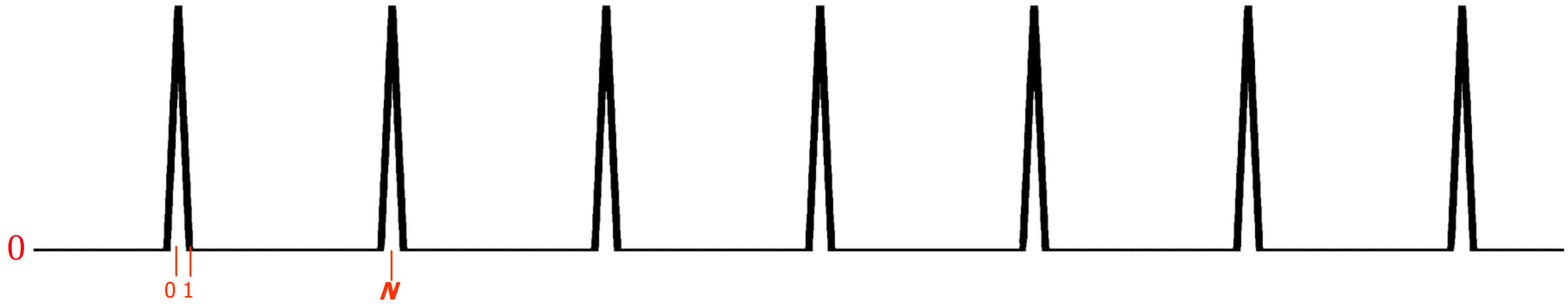
Another major issue in CW radar is the necessary isolation between transmitter and receiver:

- Use two different antennas
- Use circulator

Phase coded CW signals
with good periodic autocorrelation function

CW signals with ideal (or perfect) PACF

Perfect periodic autocorrelation function (PACF)



A relatively simple signal to transmit is a binary signal {±1}.

Of the binary signals only Barker 4 exhibits perfect PACF.

To get higher compression we have to compromise.

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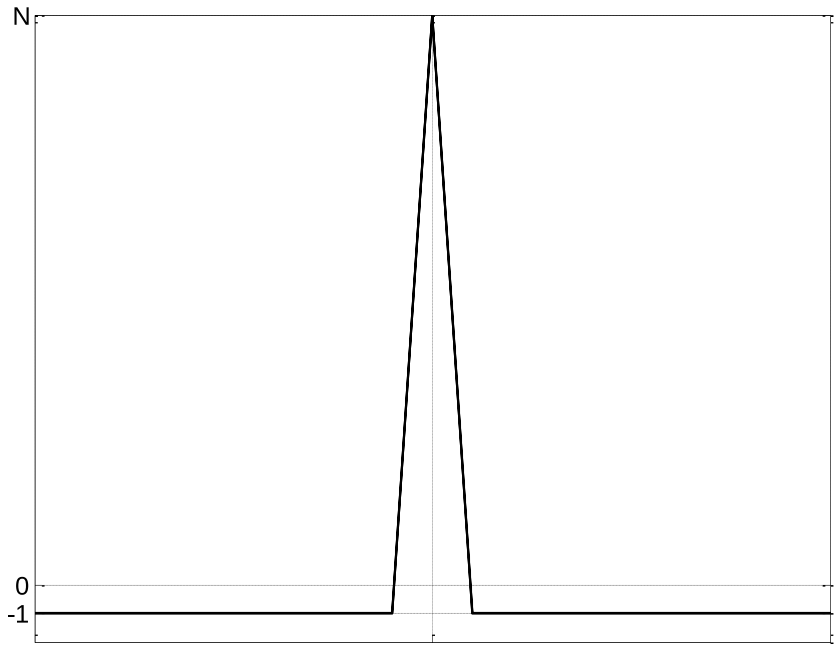
IDEAL vs. PERFECT CORRELATION CODES

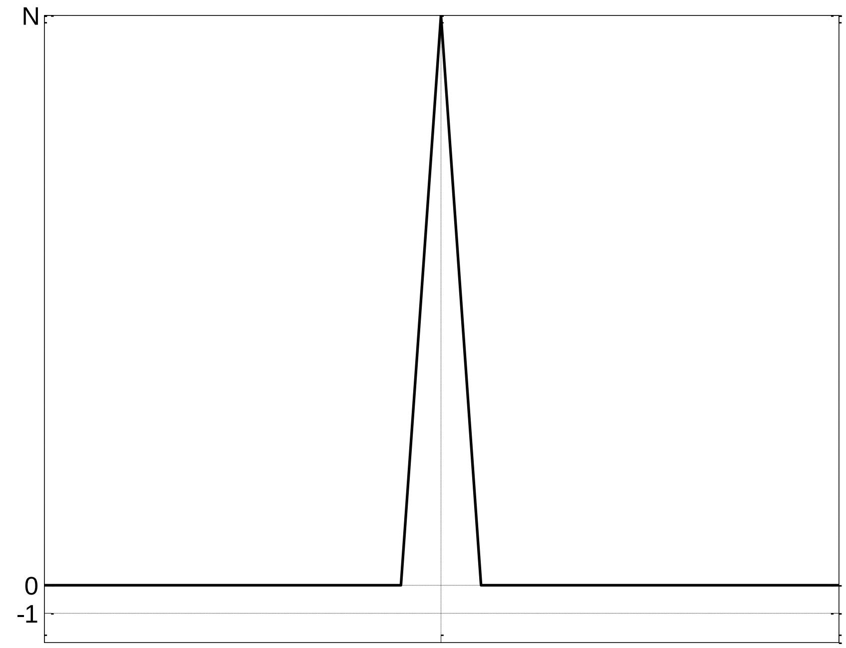
Ideal correlation:

$$C(k)=\begin{cases} N & k=0 \\ -1 & k\neq 0 \end{cases}$$



Perfect correlation:

$$C(k)=\begin{cases} N & k=0 \\ 0 & k\neq 0 \end{cases}$$



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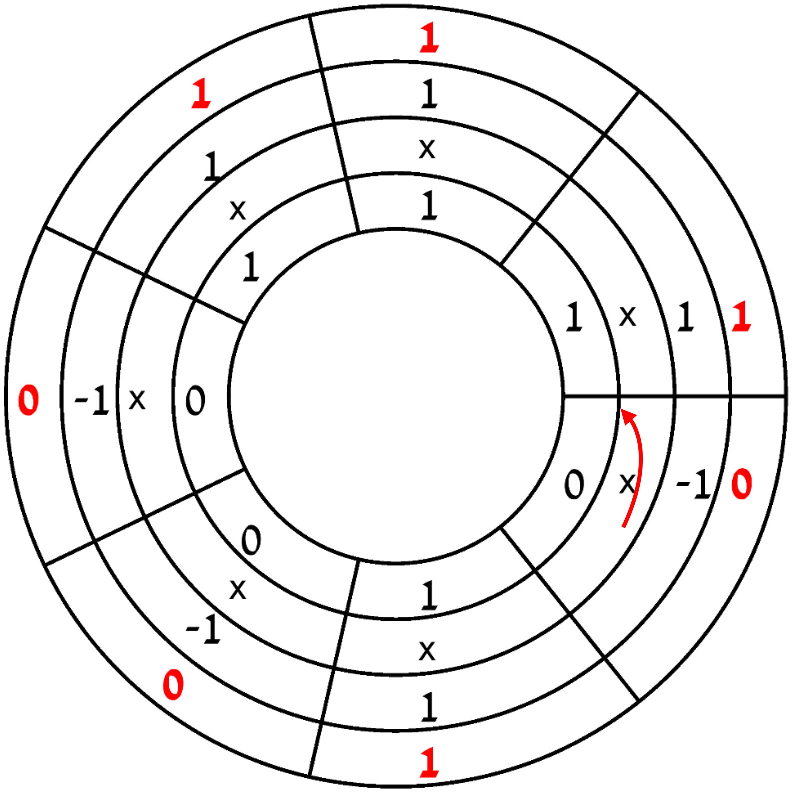
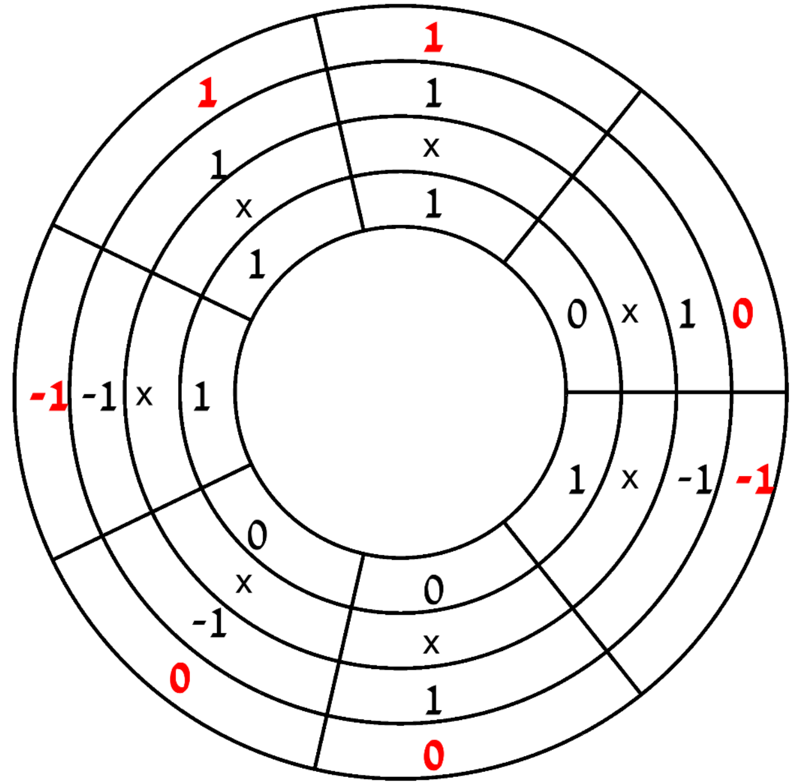
Perfect periodic cross-correlation (Demo using "ON-OFF" Barker 7)

Cyclic shift = 0



1+1+1+0+0+1+0=4

Cyclic shift = 1



0+1+1-1+0+0-1=0

MATLAB script for periodic cross-correlation of two signals of the same length

```
r=ifft(fft(s1).*conj(fft(s2)))
```

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Table 8.3 The Autocorrelation Sequence of a Barker Code of Length 7

$\{u_n\}$	+	+	+	−	−	+	−						
$\{u^*_{N-n+1}\}$													
−	−	−	−	+	+	−	+						
+		+	+	+	−	−	+	−					
−			−	−	−	+	+	−	+				
−				−	−	−	+	+	−	+			
+					+	+	+	−	−	+	−		
+						+	+	+	−	−	+	−	
+							+	+	+	−	−	+	−
Output sequence	−1	0	−1	0	−1	0	+7	0	−1	0	−1	0	−1

	+	+	−	+	+	−	+	+	−	+
−	−	−	+							
+		+	+	−						
+			+	+	−					
Σ	−1	0	+3	0	−1					

symmetric

a-periodic autocorrelation (Barker 3)

	+	+	−	+	+	−	+	+	−	+
−	−	−	+	−	−	+	−	−	+	−
+		+	+	−	+	+	−	+	+	−
+			+	+	−	+	+	−	+	+
Σ			+3	−1	−1	+3	−1	−1	+3	−1

periodic

periodic autocorrelation

	+	+	−	+	+	−	+	+	−	+
−	−	−	+	−	−	+	−	−	+	−
+		+	+	−	+	+	−	+	+	−
+			+	+	−	+	+	−	+	+
Σ			+3	−1	−1	+3	−1	−1	+3	−1

Ideal correlation

periodic autocorrelation

	+	+	−	+	+	−	+	+	−	+
0	0	0	0	0	0	0	0	0	0	0
+		+	+	−	+	+	−	+	+	−
+			+	+	−	+	+	−	+	+
Σ			+2	0	0	+2	0	0	+2	0

Perfect correlation

periodic cross-correlation

	+	+	0	+	+	0	+	+	0	+
−	−	−	0	−	−	0	−	−	0	−
+		+	+	0	+	+	0	+	+	0
+			+	+	0	+	+	0	+	+
Σ			+2	0	0	+2	0	0	+2	0

Perfect correlation

Switching the roles of signal and reference

Phase coded CW waveforms

Polyphase-coded: Frank, Zadoff-Chu, P3, P4 (Lewis & Kretschmer), etc.

- Yield perfect PACF, but are more difficult to generate and process

Two-valued phase-coded (not binary): Golomb, Legendre

- Yield perfect PACF, but are slightly more difficult to generate and process
- Limited lengths

Binary, mismatched receiver: Ipatov

- Ideal periodic cross correlation
- Simple to transmit and relatively simple to process
- Entails small SNR loss
- Limited lengths

Binary, near ideal PACF

- Near sidelobes are small but not zero
- Simple to transmit and process

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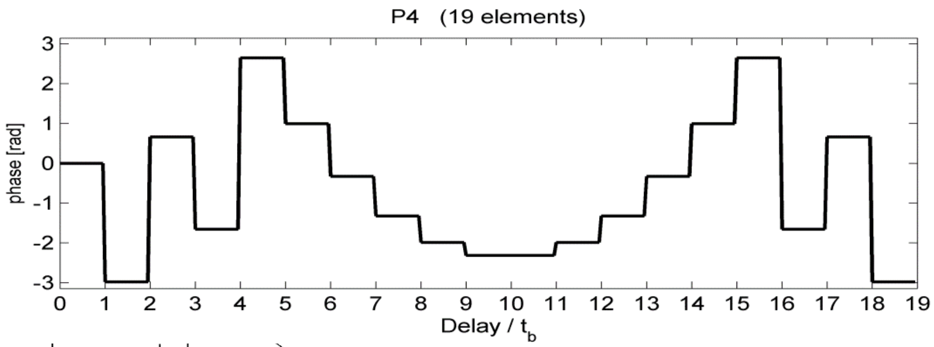
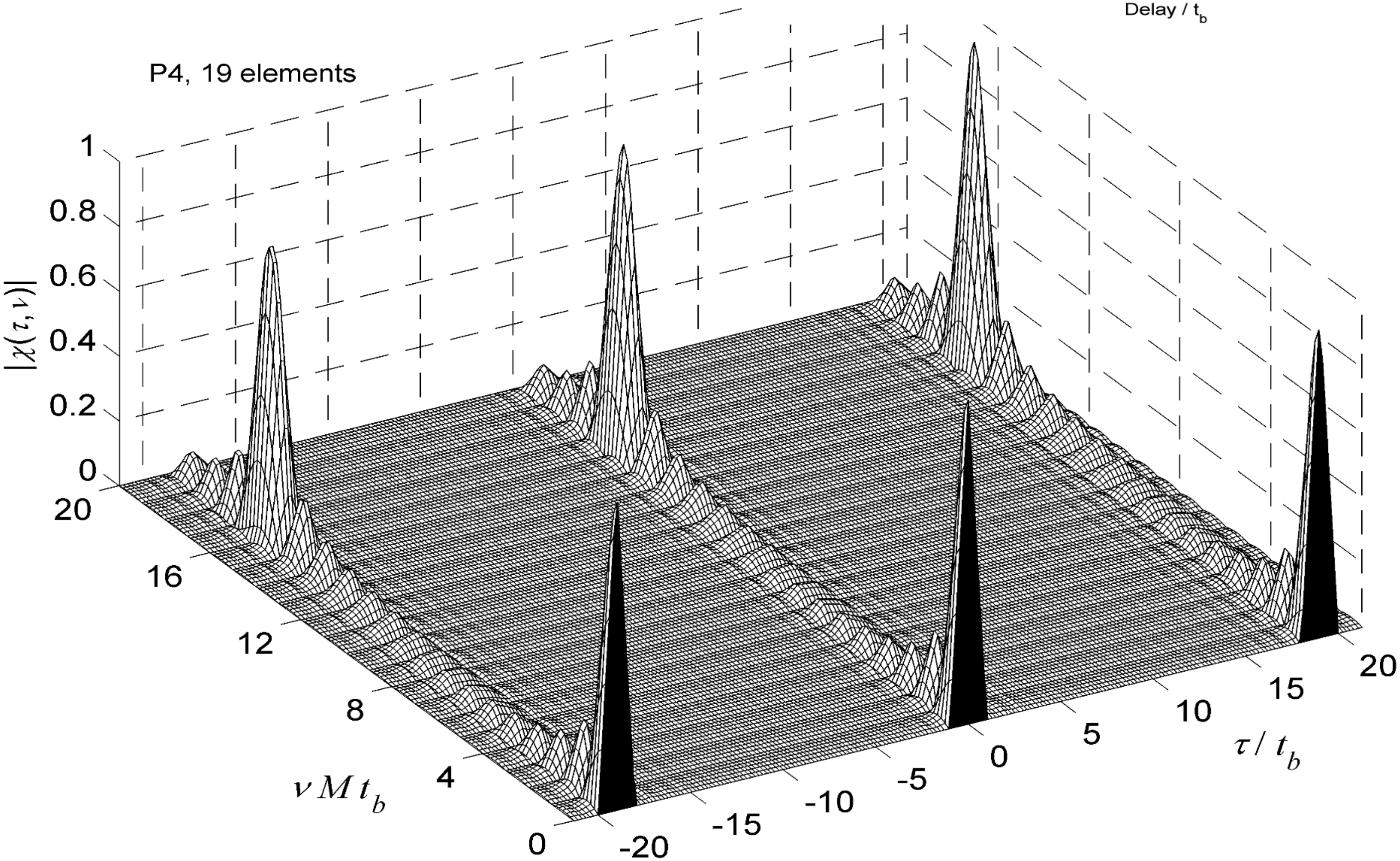


Periodic ambiguity function of 16 periods of P4 19 element (no amplitude weighting)



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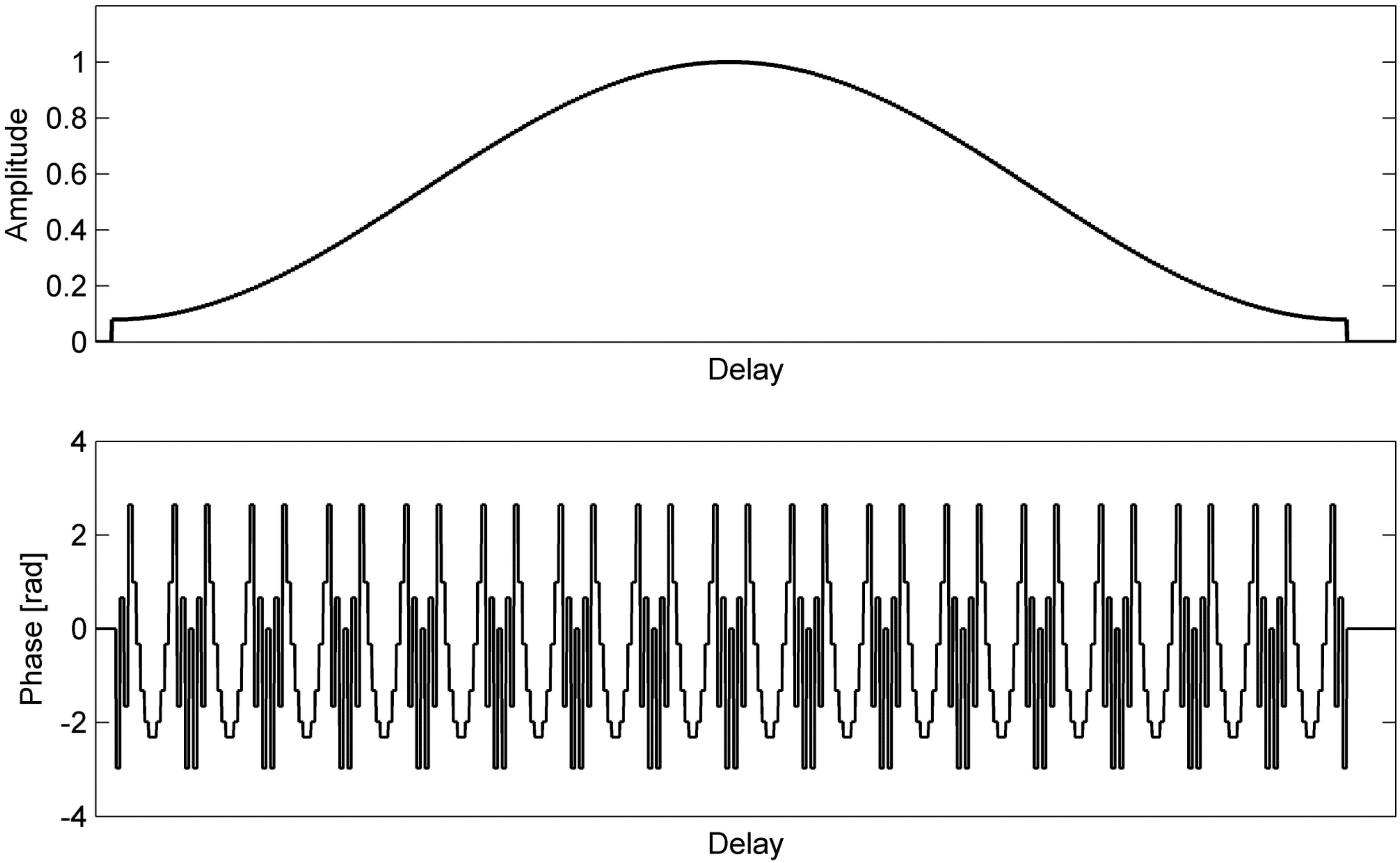
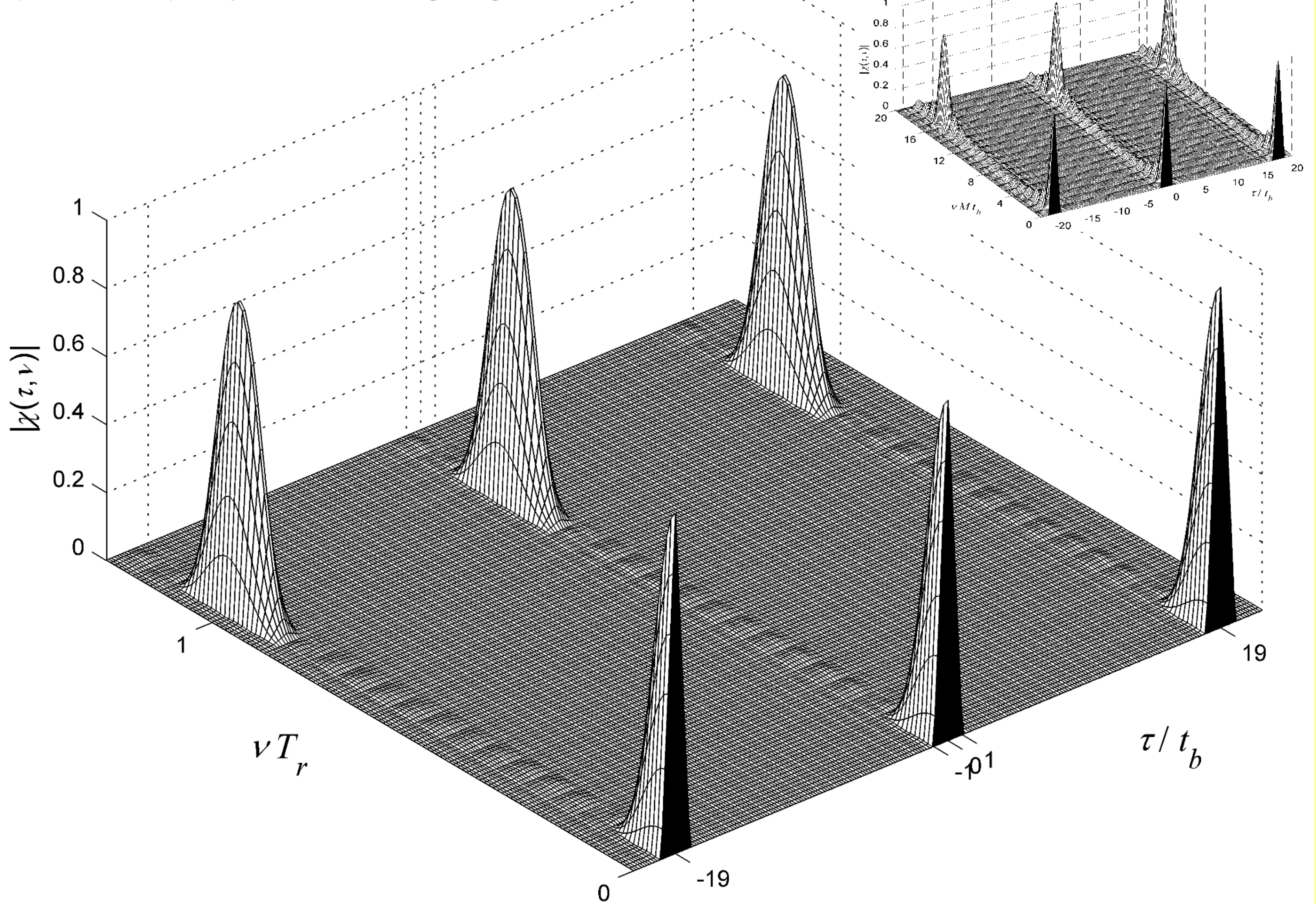
Transmitted: Periodic P4 (19 elements) , Reference: 16 periods, Hamming weighted

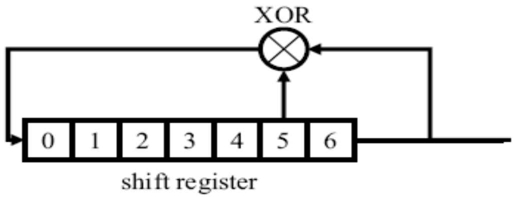


P4 (19 elements), 16 periods Hamming weighted reference



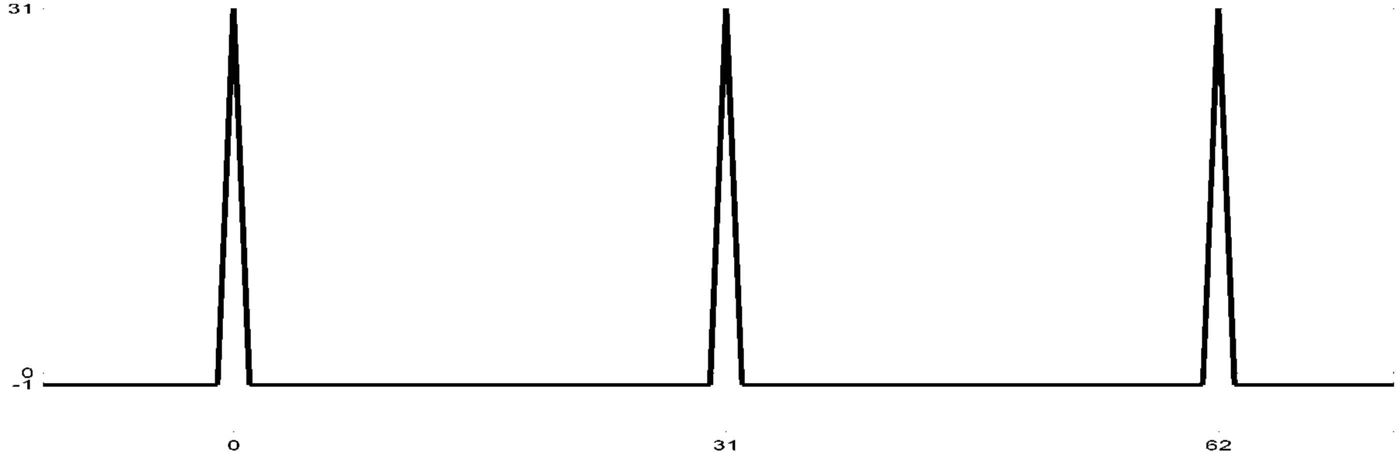
Maximum-length linear feedback sequences

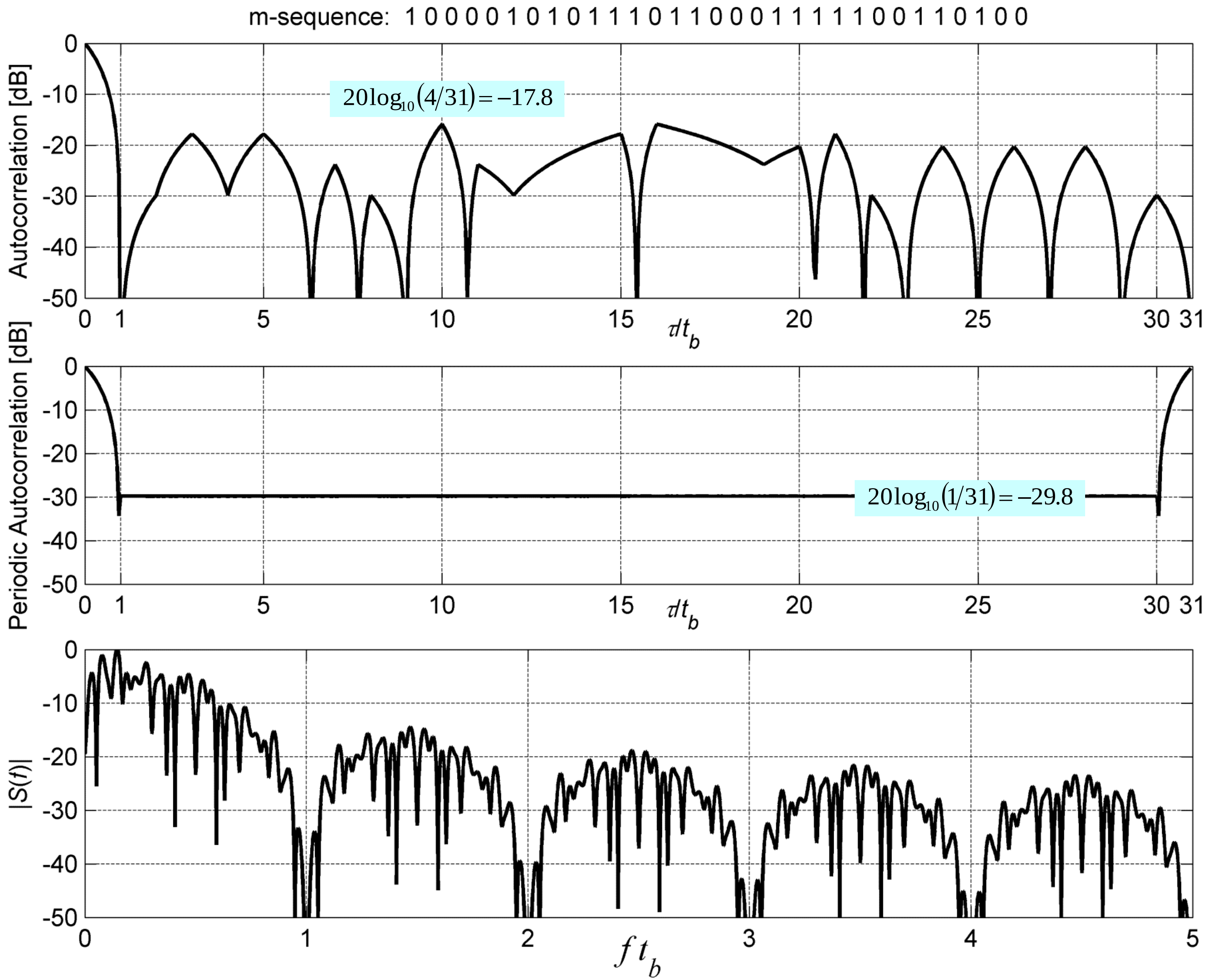
(*m*-sequences, pseudo-random sequences, shift-register sequences)



Length of shift register	Maximum length	Number of sequences with maximum length	Example: Back looped shift register elements for sequence with maximum length
3	7	2	1,2
4	15	2	2,3
5	31	6	2,4
6	63	6	4,5
7	127	18	5,6
8	255	16	3,4,5,7
9	511	48	4,8
10	1023	60	6,9
11	2047	176	8,10

Periodic autocorrelation function




m-sequence: 1 0 0 0 0 1 0 1 0 1 1 1 0 1 1 0 0 0 1 1 1 1 1 0 0 1 1 0 1 0 0
Autocorrelation [dB]
20log10(4/31) = −17.8
τ/tb
Periodic Autocorrelation [dB]
20log10(1/31) = −29.8
τ/tb
|S(f)|
f tb

m-sequences: conclusions

- Poor a-periodic autocorrelation (larger sidelobes than MPSL)
- Few available signals (only for lengths $N=2^n-1$)
- Non-perfect periodic autocorrelation (two-valued PACF of {-1,N})

Binary version (0^0,180^0) not attractive for pulse and for CW radar

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Ideal correlation binary codes

- BPSK codes generated by difference sets of type $(N, \frac{N+1}{2} - 1, \frac{N+1}{4} - 1)$
- Types of difference sets by length:
  - $N = 2^m - 1$ (m-sequence over GF(2^m))
  - $N = 4k - 1$, N is prime (quadratic residue) (Legendre)
  - $N = 4u^2 + 27$ (Hall's sextic residue)
  - $N = p(p+2)$, $p\&(p+2)$ are primes (twin primes)

N = 4***k***-3, ***N*** is prime, ***k*** positive integer, Legendre (3 phase)

Ideal ➔ perfect correlation codes

- Perfect periodic correlation (zero sidelobes)
  - 2-phase codes (Golomb 1992):
    - Phase value:

$$\theta = \cos^{-1}\left(-\frac{N-1}{N+1}\right)$$

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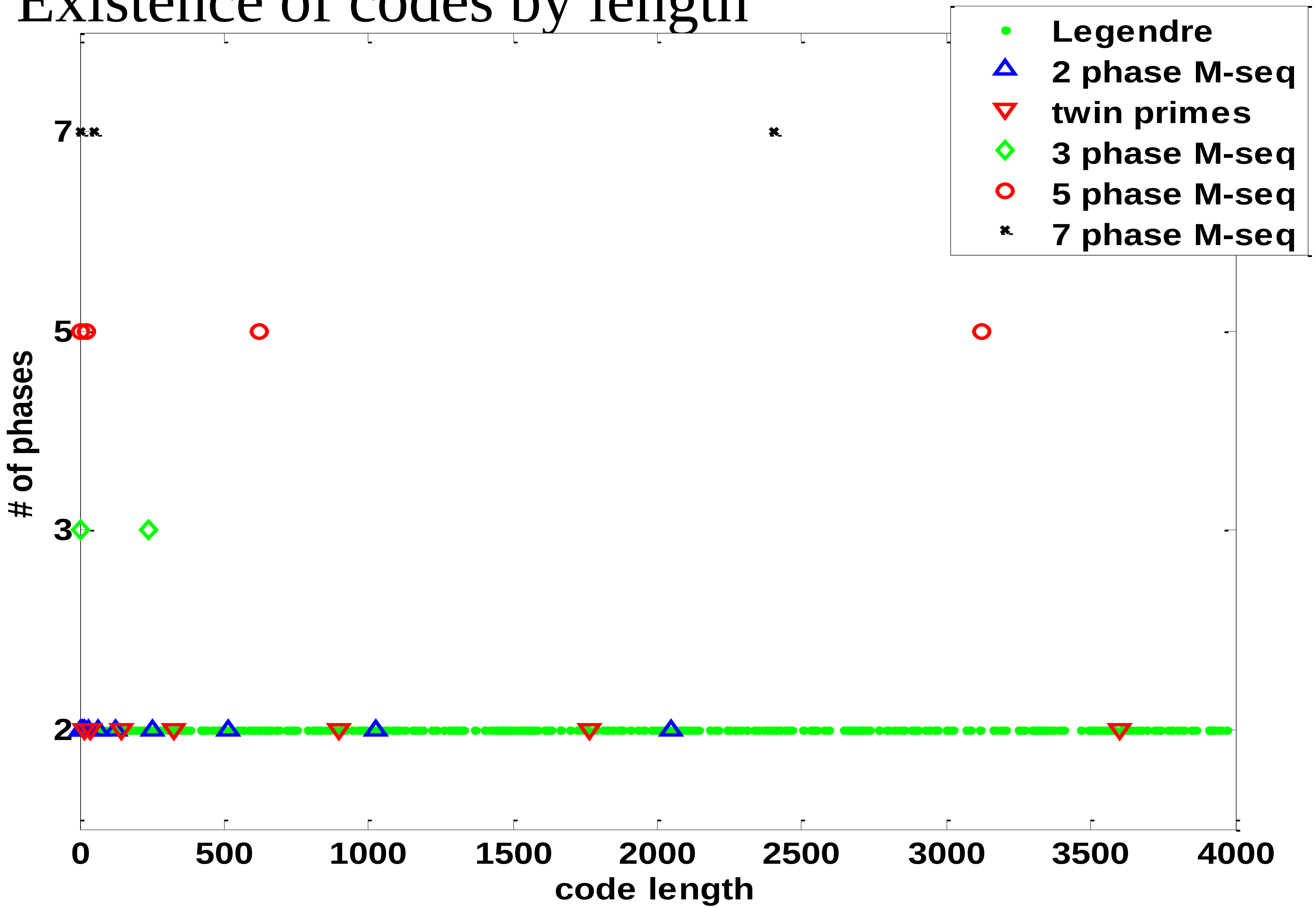
Existence of codes by length

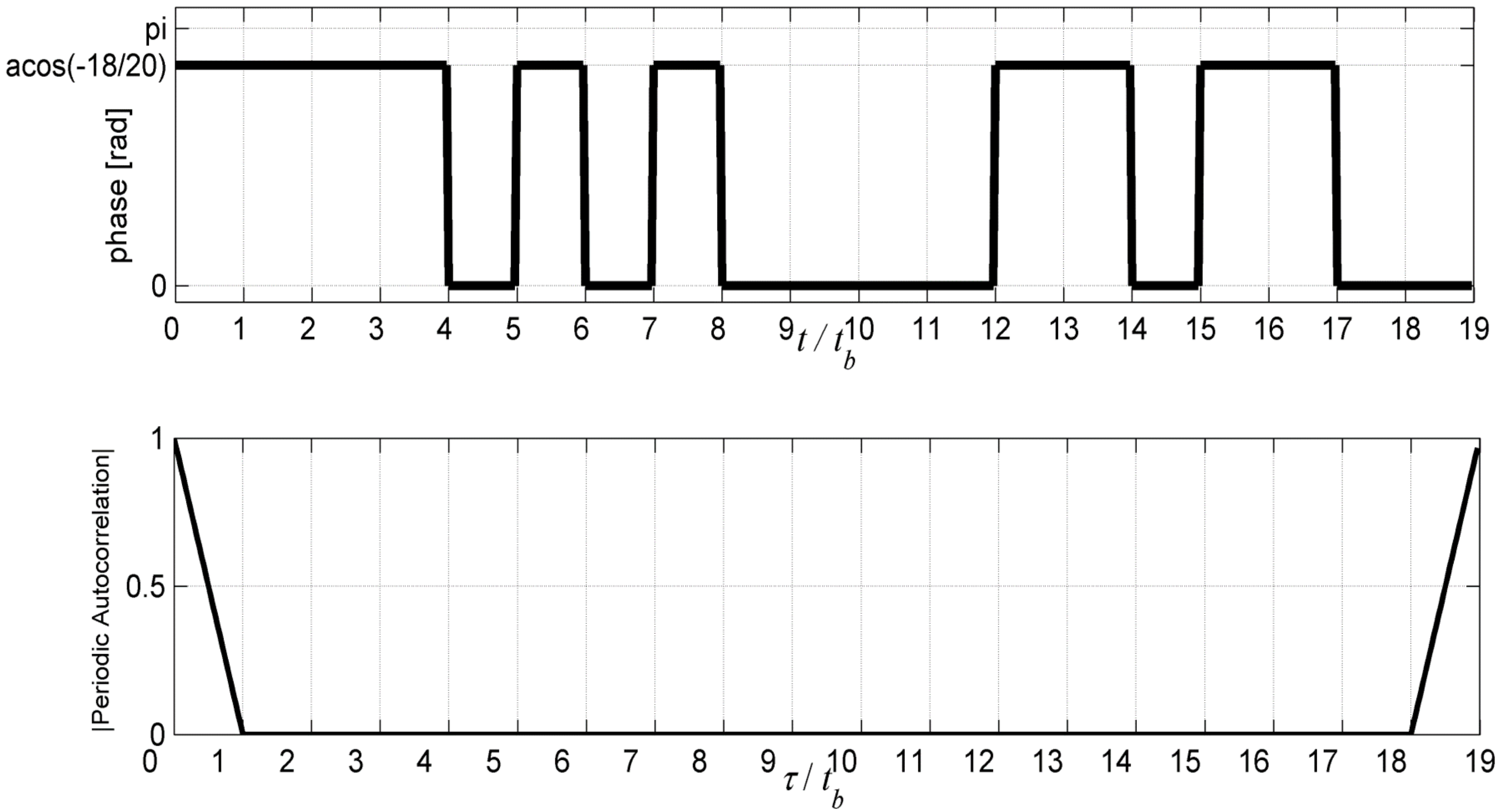


MATLAB function for constructing a phase-coded periodic waveform of any odd-prime length, based on **Legendre** sequences.

```
function [ s ] = perfect_periodic_Legendre_waveform( N )
% Generates a periodic coded signal using 2 or 3 phases
% The signal exhibits perfect periodic autocorrelation
% N is any odd prime
Nspt=sprintf('%g element phase-coded waveform ',N);
if isprime(N)==0
    disp('Not a prime')
return
end
s=ones(1,N);
if rem((N+3)/4,1)==0
    c=0.25*(N-1);
    c1=2-1/c-1/(2*c^2);
    c2=1-1/c-1/(4*c^2);
    arg2=acos(-c1/2-sqrt((c1/2)^2-c2));
    s(mod((1:N-1).^2,N)+1)=exp(1i*arg2);
    s(1)=exp(1i*arg2/2);
else
    arg3=acos(-(N-1)/(N+1));
    s(mod((1:N-1).^2,N)+1)=exp(1i*arg3);
end
d=abs(ifft(fft(s).*conj(fft(s))));
figure, plot(d, 'k')
title(['Periodic autocorrelation of ' Nspt ]);
end
```

I. Cohen, R. Elster and N. Levanon: "Good practical continuous waveform for active bistatic radar", *IET Radar Sonar Navigation.*, Vol. 10 (4), pp. 798-806, 2016.

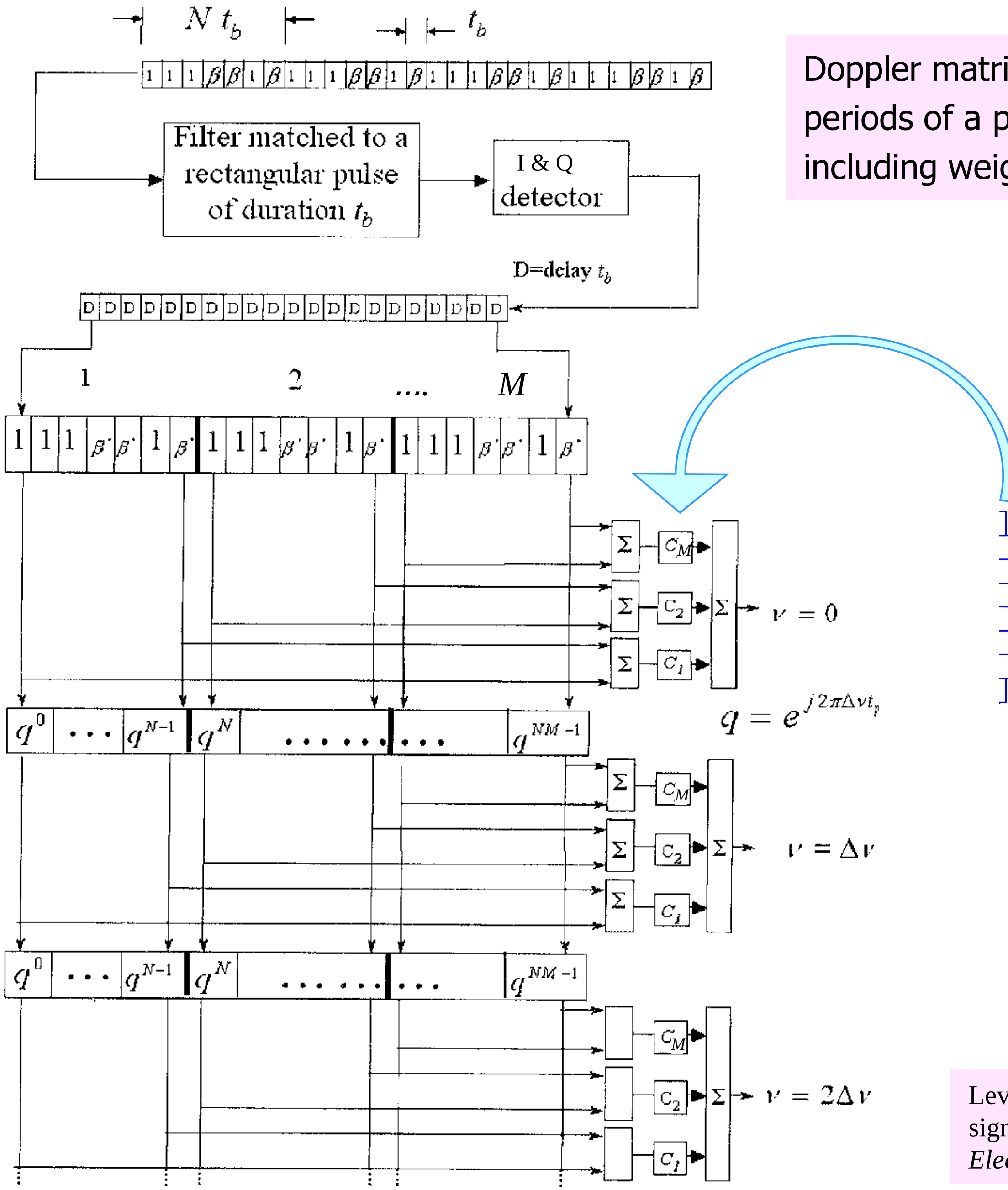
Golomb, N = 19

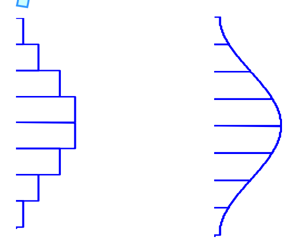


$$\text{code}\{1, \beta\}, \quad \beta = \exp\left[j \ \text{acos}\left(-18/20\right)\right]$$

Nadav Levanon, Tel-Aviv University

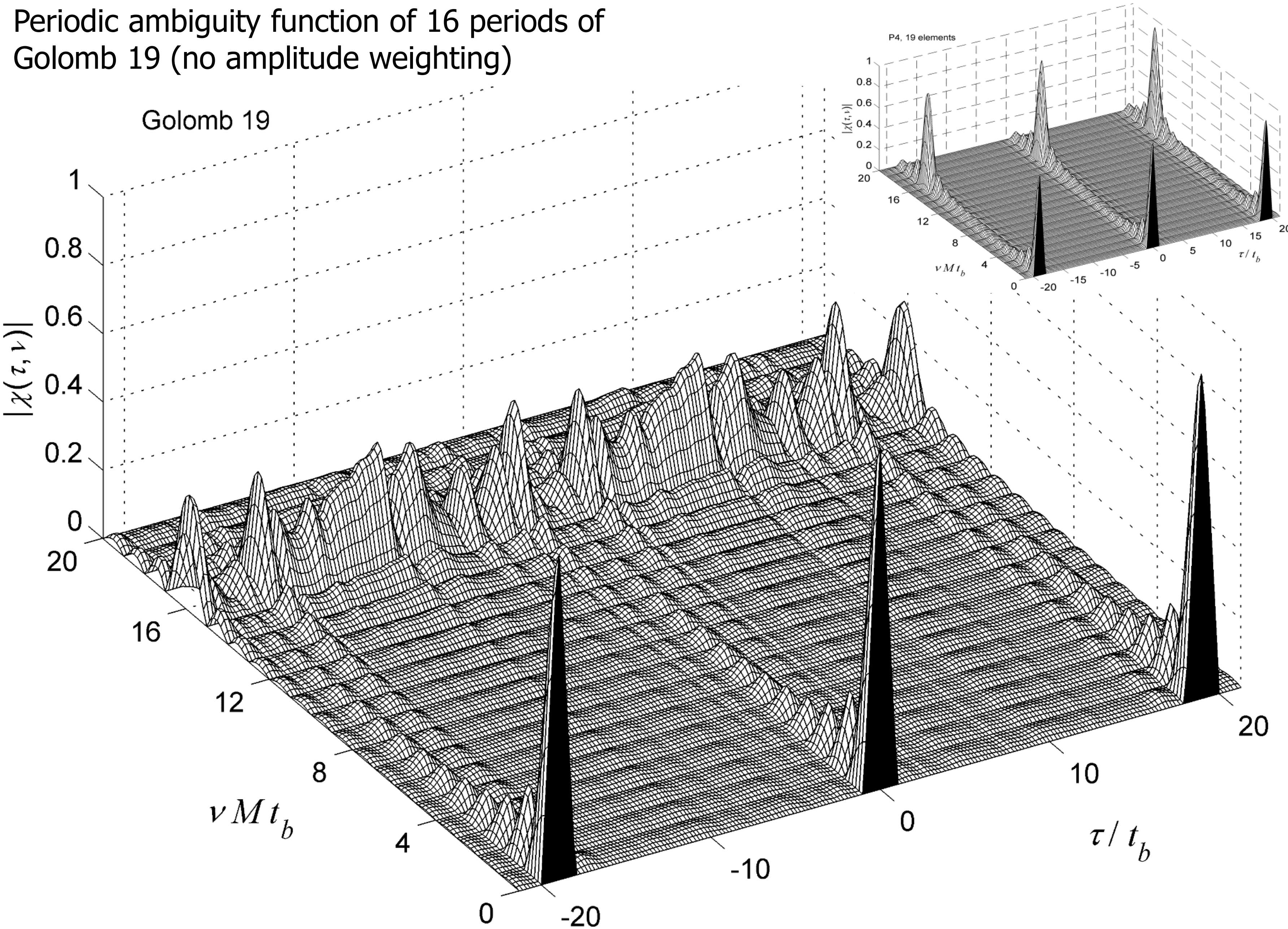
Doppler matrix correlation receiver matched to M periods of a phase-coded signal of length N=7, including weighting C_n for Doppler sidelobe reduction.





Levanon, N., "CW alternatives to the coherent pulse train – signals and processors," *IEEE Trans. on Aerospace and Electronic Systems*, Vol 29, No. 1, 250-254, Jan. 1993.

Periodic ambiguity function of 16 periods of Golomb 19 (no amplitude weighting)



Golomb biphase sequences

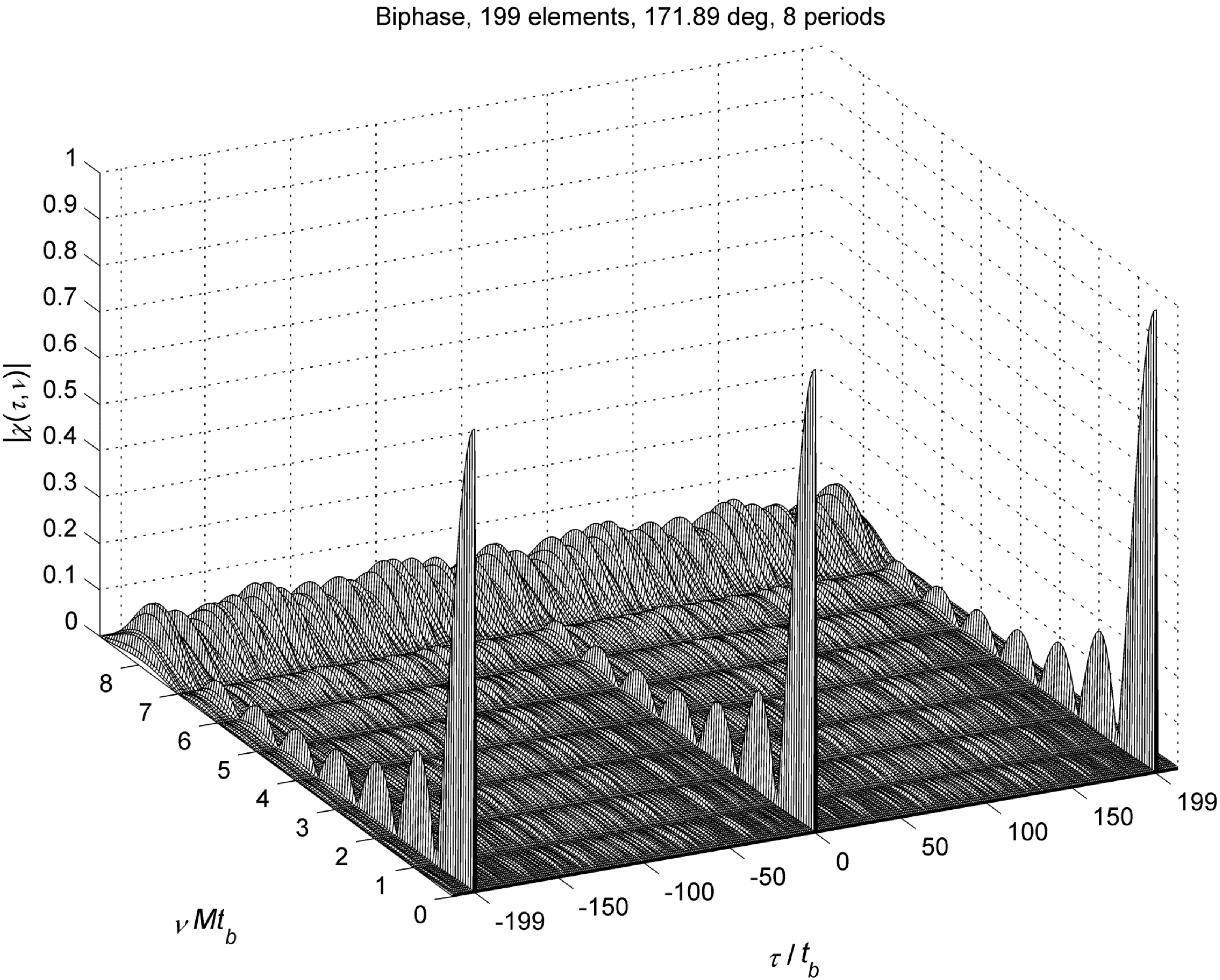
M	Phase shift		*D*
2	90º		{1}
3	120º		{1 2}
4	180º		{1 2 4}
7	138.6º		{1 2 3 5}
11	146.4º		{1 2 3 5 6 8}
15	151.0º		{1 2 3 4 6 8 9 12} or {1 4 5 7 9 10 11 12}
19	154.2º		{1 2 3 4 6 8 13 14 16 17}
23	156.4º		{1 2 3 4 5 7 9 10 13 14 17 19}
31	159.6º		{1 2 3 4 6 8 12 15 16 17 23 24 27 29 30}
35	160.8º		{1 2 4 5 8 10 12 13 14 15 17 18 22 28 29 30 34}
43	162.7º		{1 2 3 4 5 8 11 12 16 19 20 21 22 27 32 33 35 37 39 41 42}
63	165.6º		{1 2 3 4 5 7 8 9 10 13 14 15 17 19 20 25 27 28 29 33 34 36 37 39 42 46 49 50 53 55 57} {1 2 3 4 5 6 7 9 10 11 13 17 18 19 21 24 25 28 30 33 34 35 37 41 44 46 47 49 54 55 59}

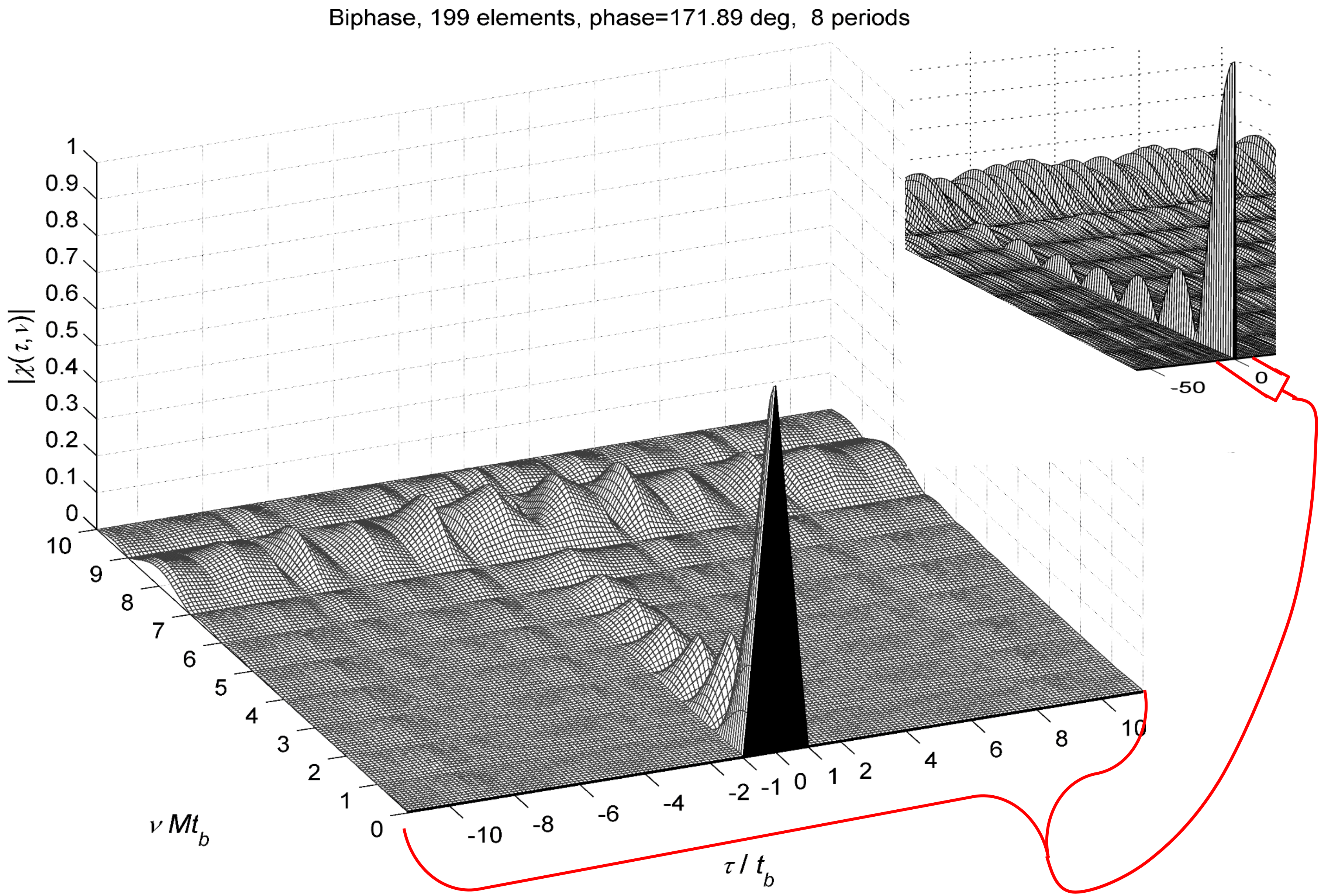
Longer list of Golomb codes of length 63

1	2	3	4	5	7	8	9	10	13	14	15	17	19	20	25	27	28	29	33	34	36	37	39	42	46	49	50	53	55	57
1	2	3	4	5	6	7	9	10	11	13	17	18	19	21	24	25	28	30	33	34	35	37	41	44	46	47	49	54	55	59
1	2	3	4	5	7	14	15	17	19	21	22	23	26	27	32	36	38	41	43	44	47	50	51	52	54	55	57	58	59	60
1	2	3	4	5	12	14	16	19	20	23	27	30	32	33	35	36	40	41	42	44	49	50	52	54	55	56	59	60	61	62
1	2	3	4	5	10	11	13	14	16	18	20	21	24	26	27	28	29	31	32	33	35	38	45	47	51	52	53	56	57	61
1	2	3	4	5	9	10	11	12	14	15	17	18	19	22	25	26	28	31	33	37	42	43	46	47	48	50	52	54	55	62
1	2	3	4	5	8	12	13	16	17	18	22	24	31	34	36	37	38	40	41	42	43	45	48	49	51	53	55	56	58	59

Golomb bi-phase code 199 elements

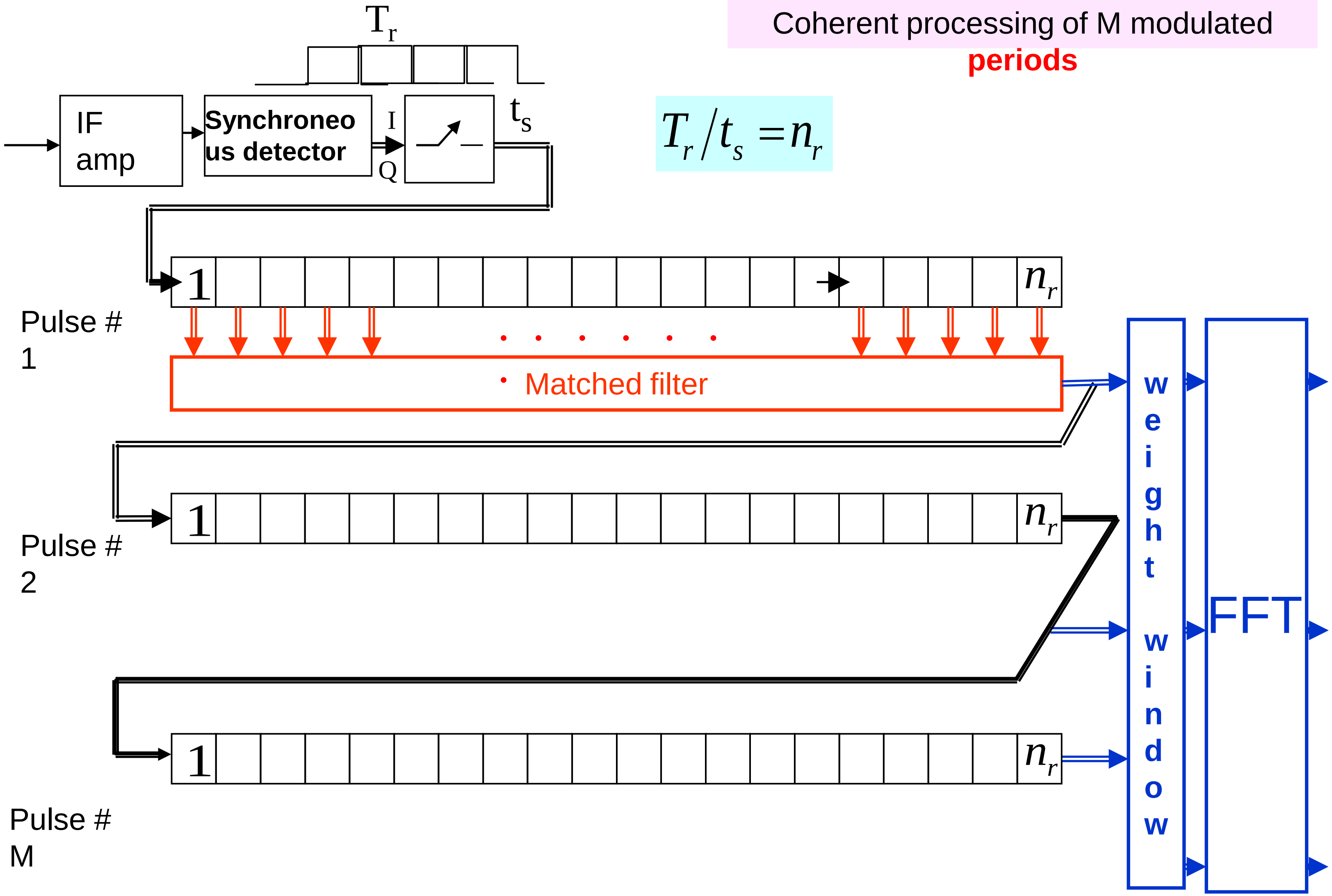
```
nn=199; % code length
ll=99; lam=49;
arg=-(nn-2*ll+2*lam)/2/(ll-lam)
phase=acos(arg);
phase_deg=phase/pi*180;
a=exp(1i*phase);
code=ones(1,nn);
code([1   2    4    5    7    8    9   10   13   14   16   18 ...
   20   23   25   26   28   29   31   32   33   35   36   40 ...
   43   45   46   47   49   50   51   52   53   56   57   58 ...
   61   62   63   64   65   66   70   72   79   80   81   86 ...
   89   90   91   92   94   98  100  102  103  104  106  111 ...
  112  114  115  116  117  121  122  123  124  125  126  128 ...
  130  131  132  139  140  144  145  151  155  157  158  160 ...
  161  162  165  169  172  175  177  178  180  182  184  187 ...
  188  193  196  ])=a;
```


Biphase, 199 elements, 171.89 deg, 8 periods
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
|χ(τ,ν)|
8
7
6
5
4
3
2
1
0
ν Mt_b
-199
-150
-100
-50
0
50
100
150
199
τ / t_b

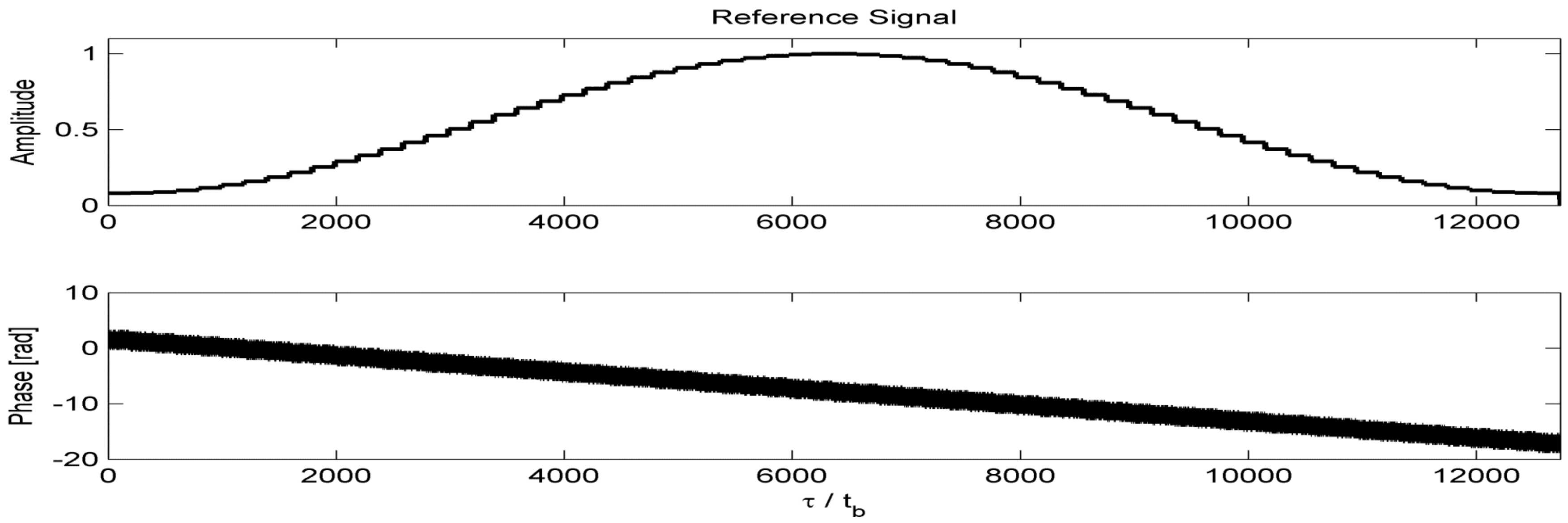

Biphase, 199 elements, phase=171.89 deg, 8 periods
$|\chi(\tau,\nu)|$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\nu\, Mt_b$
10
9
8
7
6
5
4
3
2
1
0
$\tau\, /\, t_b$
-10
-8
-6
-4
-2
-1
0
1
2
4
6
8
10
-50
0

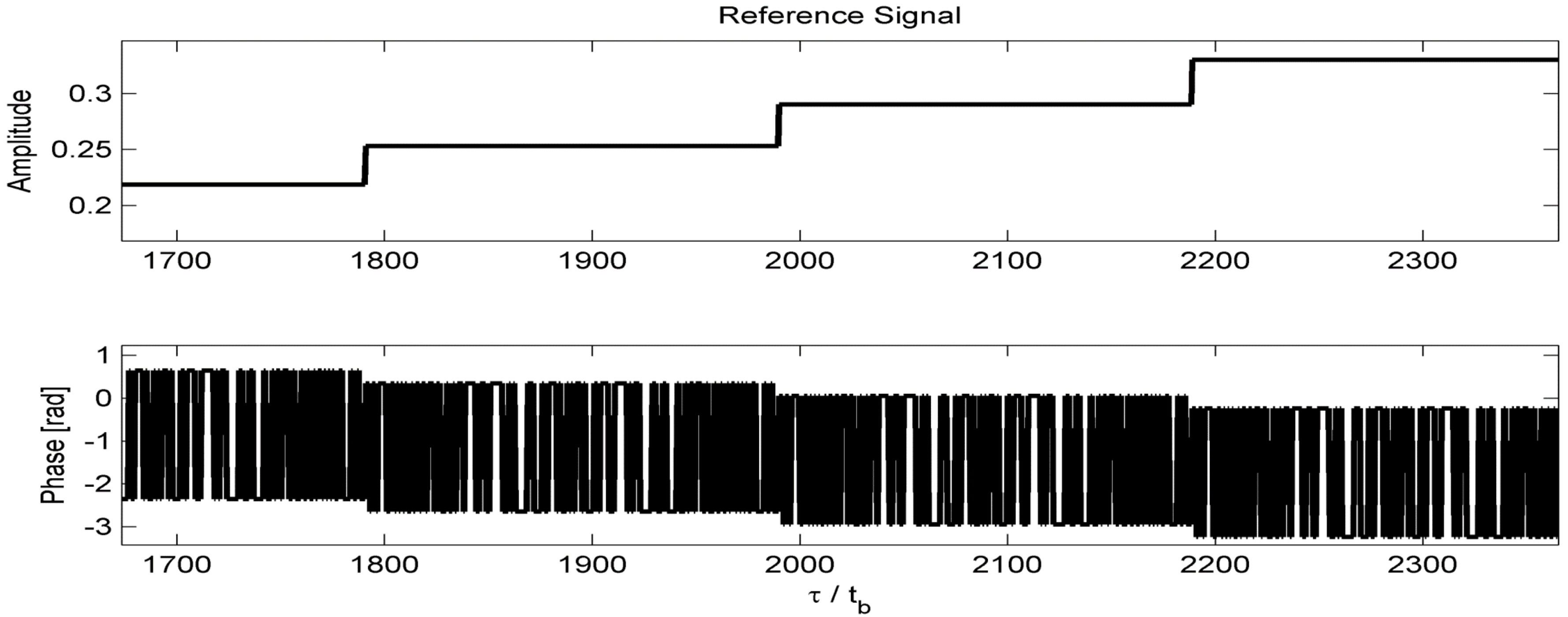
Coherent processing of M modulated **periods**

$$T_r / t_s = n_r$$

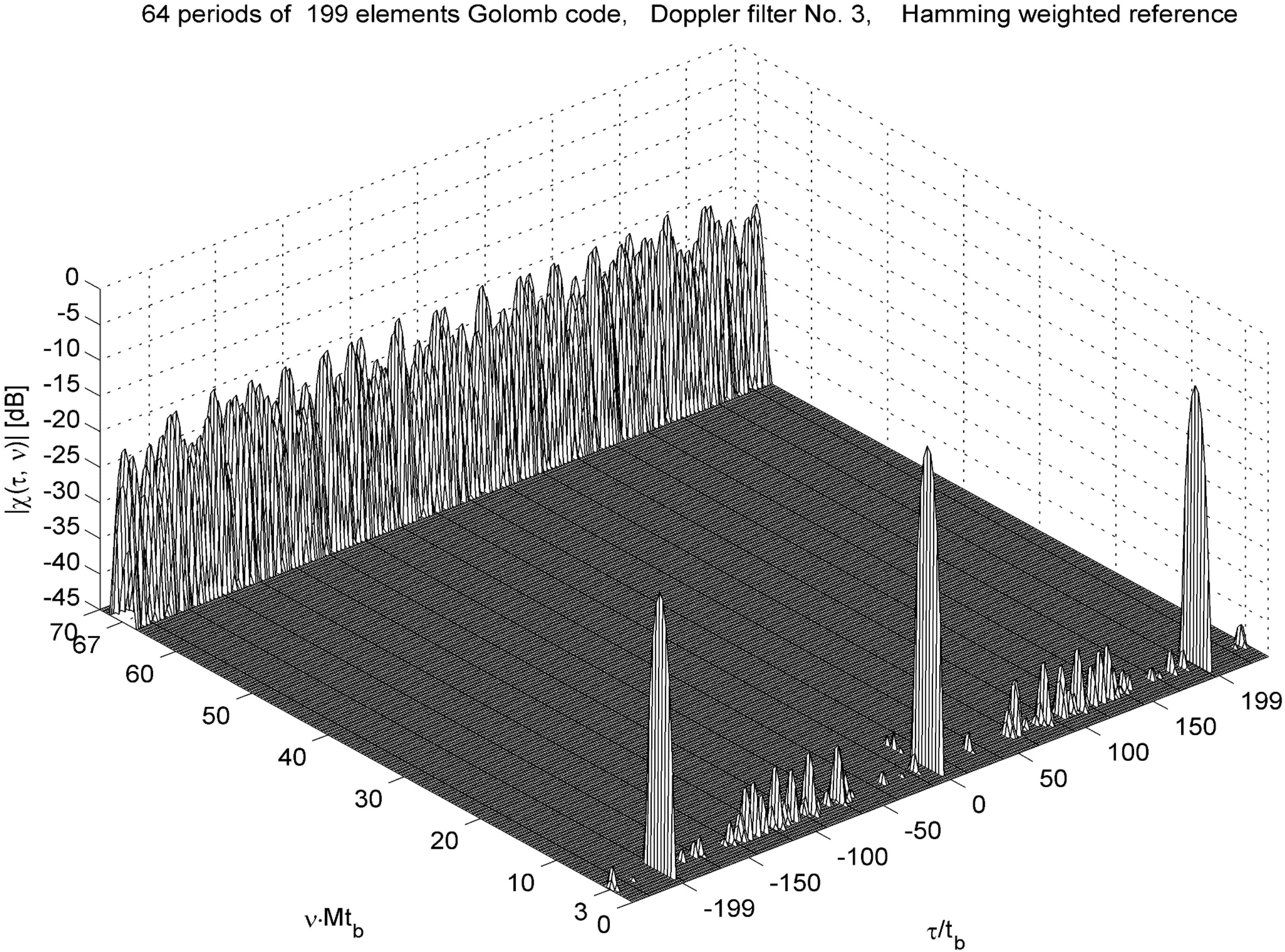


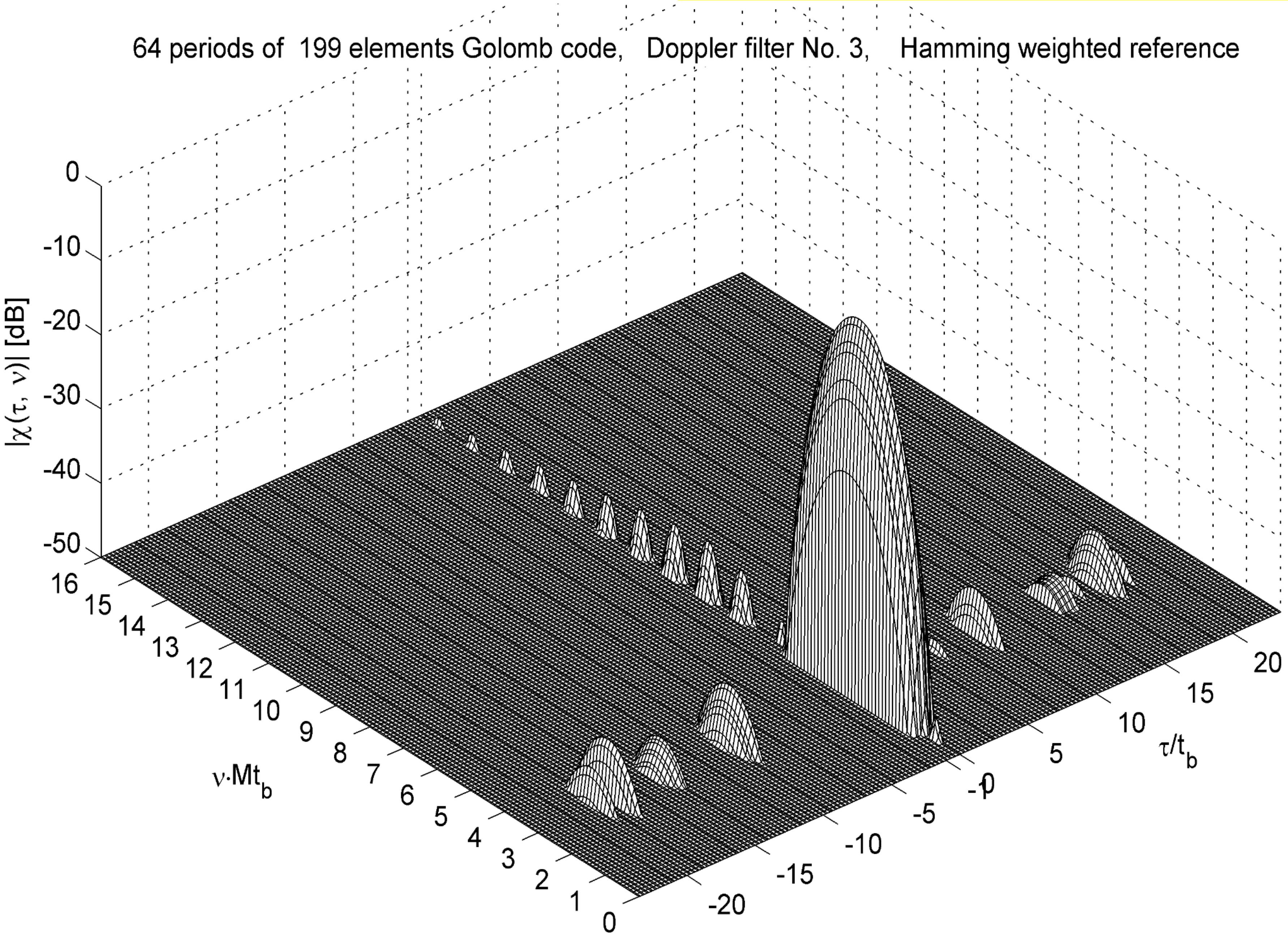
Integration of M pulses


Reference Signal
Amplitude
1
0.5
0
Phase [rad]
10
0
-10
-20
0
2000
4000
6000
8000
10000
12000
τ / t_b


Reference Signal
Amplitude
0.3
0.25
0.2
Phase [rad]
1
0
-1
-2
-3
1700
1800
1900
2000
2100
2200
2300
τ / t_b

Nadav Levanon, Tel-Aviv University



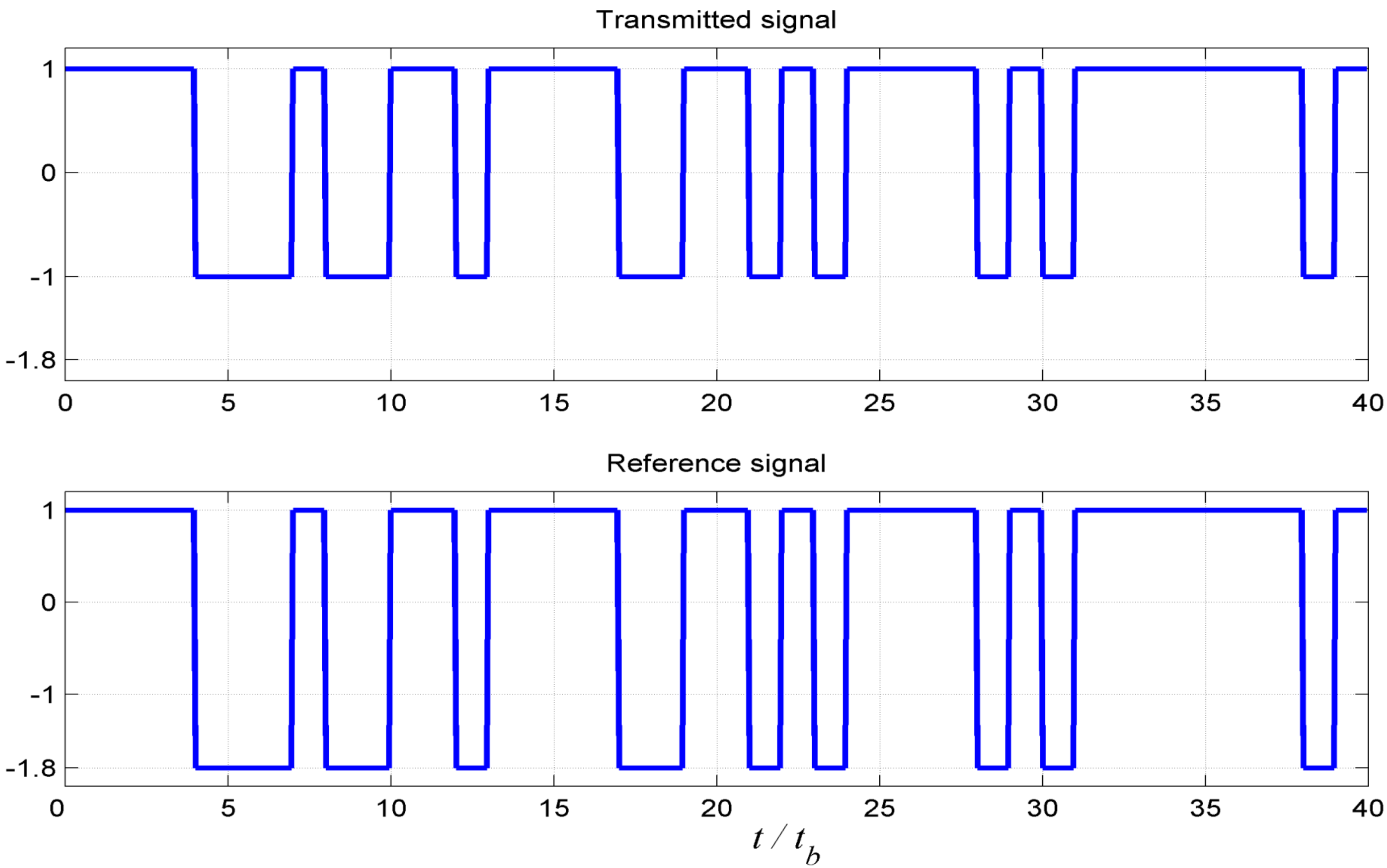

64 periods of 199 elements Golomb code, Doppler filter No. 3, Hamming weighted reference
|χ(τ, ν)| [dB]
0
-10
-20
-30
-40
-50
ν·Mt_b
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
τ/t_b
-20
-15
-10
-5
0
5
10
15
20

Solomon W. Golomb, TAU, February 2012

Ipatov, N = 40

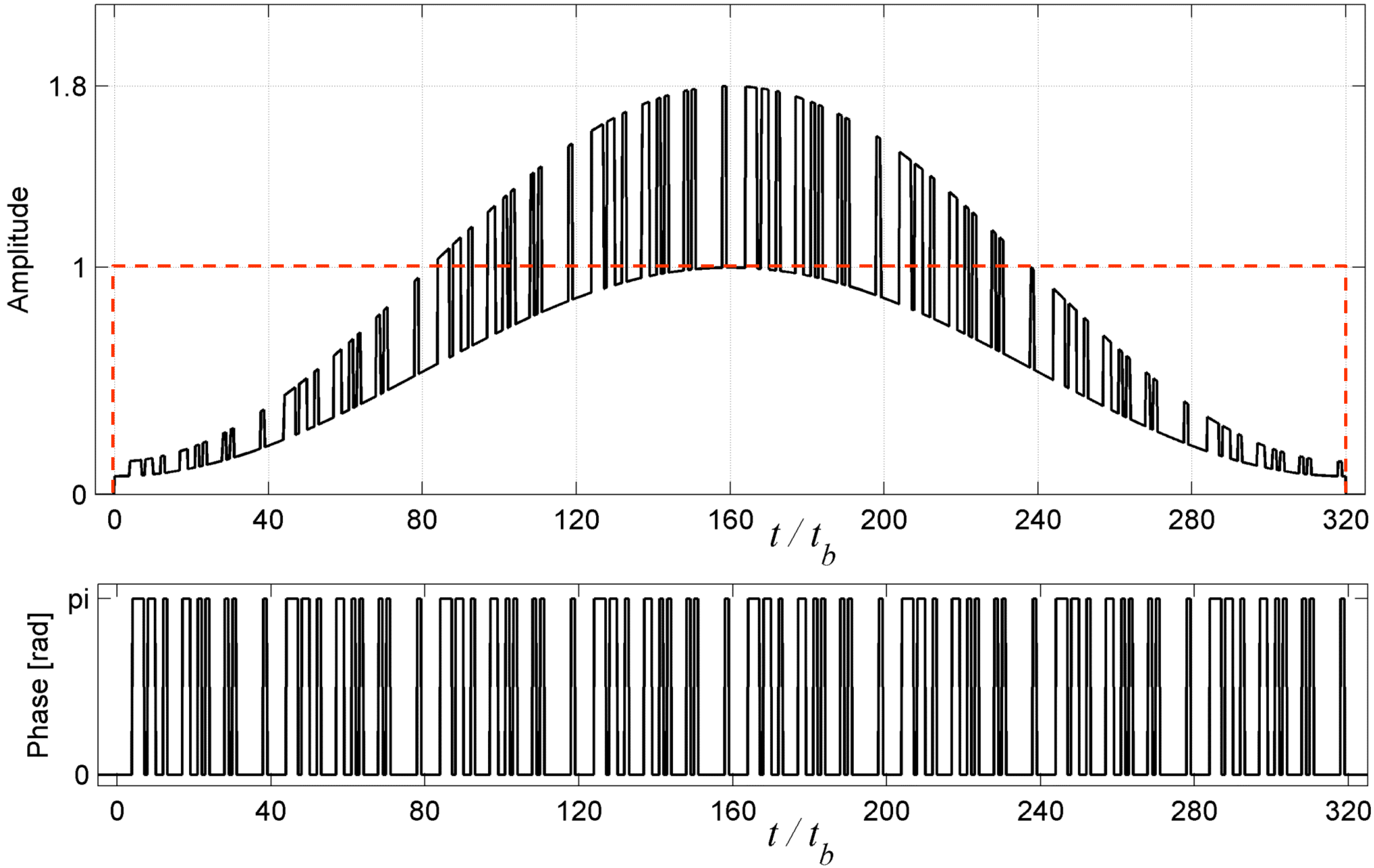
Binary transmitted signal {+1, -1} , Two-valued reference signal {+1, -1.8}

This can be considered a mismatched periodic reference (must be of identical length)

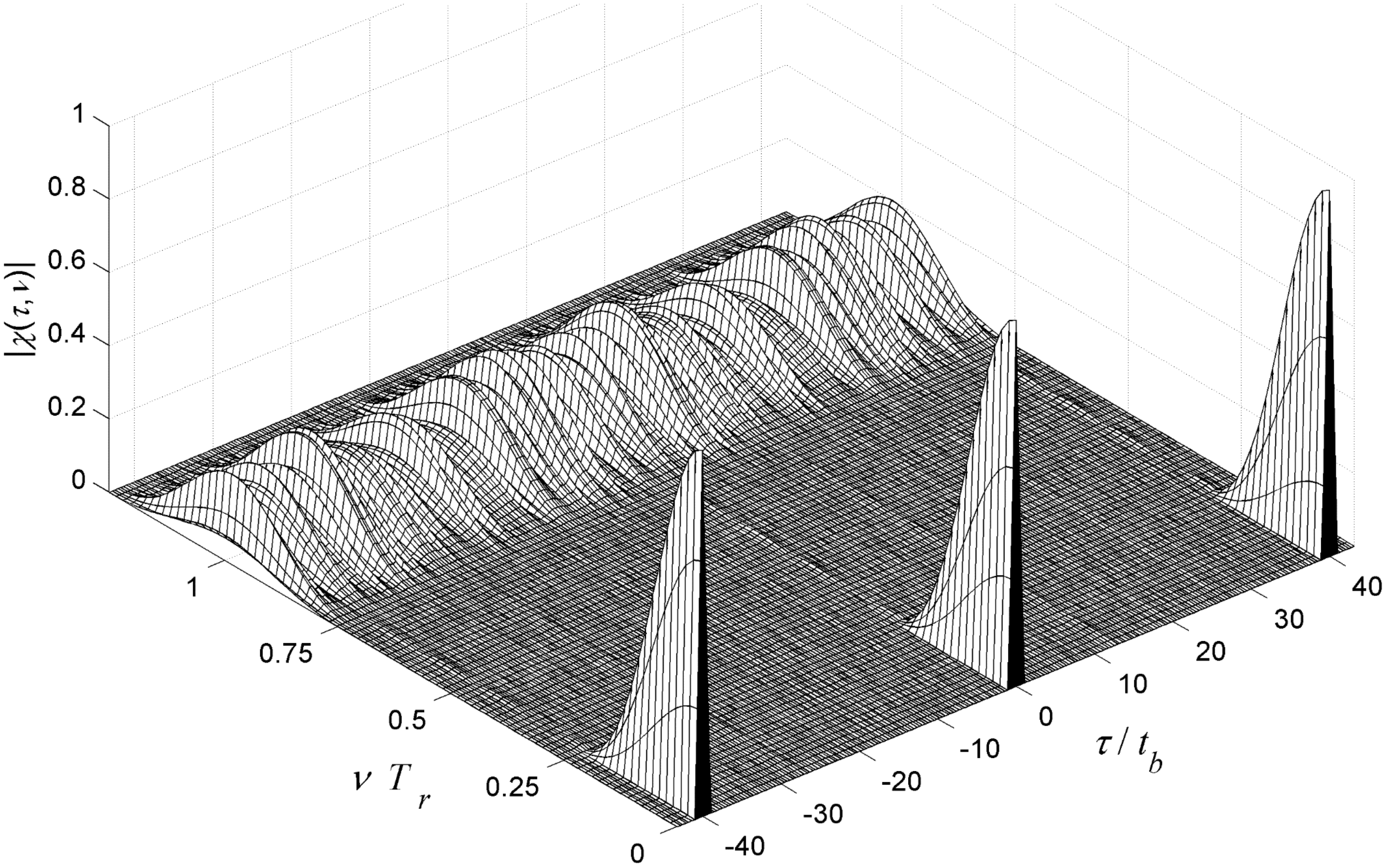


Nadav Levanon, Tel-Aviv University

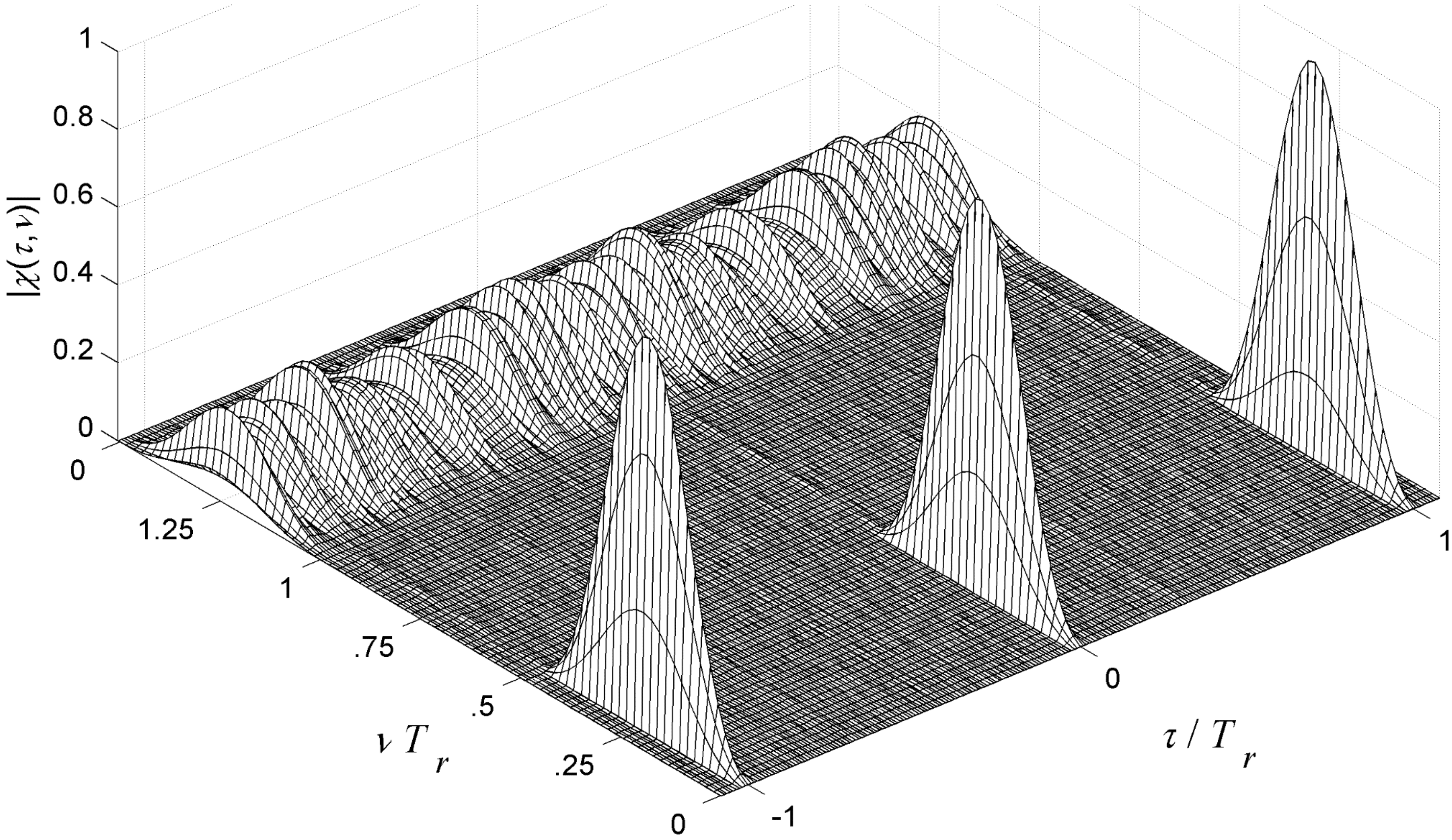
Periodic Ipatov-40, Reference signal = 8 periods, Hamming weighted



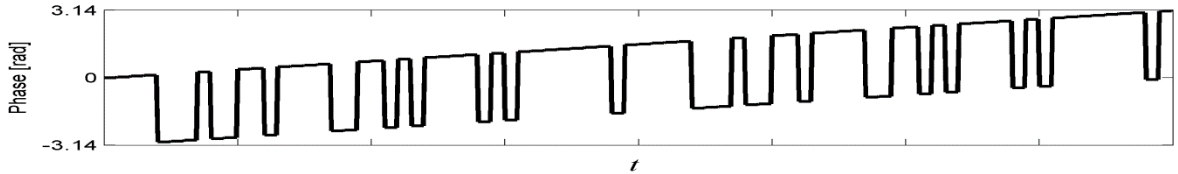
Periodic Ipatov-40, Reference signal = 8 periods, Hamming weighted



Periodic Ipatov-40, Ref. = 8 periods, Hamming weighted, Doppler compensated



$$d\phi/dt = \pi/2T_r$$



Ipatov sequences *N* = 13, 24, 40 (Loss = 0.17, 0.28, 0.37dB)

#	sig	ref
1.00	-1.00	-2.00
2.00	1.00	3.00
3.00	1.00	3.00
4.00	-1.00	-2.00
5.00	1.00	3.00
6.00	-1.00	-2.00
7.00	-1.00	-2.00
8.00	-1.00	-2.00
9.00	-1.00	-2.00
10.00	-1.00	-2.00
11.00	1.00	3.00
12.00	-1.00	-2.00
13.00	-1.00	-2.00

#	sig	ref
1.00	1.00	11.00
2.00	1.00	5.00
3.00	-1.00	-7.00
4.00	-1.00	-7.00
5.00	-1.00	-7.00
6.00	-1.00	-7.00
7.00	-1.00	-7.00
8.00	1.00	5.00
9.00	1.00	11.00
10.00	-1.00	-7.00
11.00	-1.00	-7.00
12.00	-1.00	-7.00
13.00	-1.00	-7.00
14.00	1.00	5.00
15.00	-1.00	-7.00
16.00	-1.00	-7.00
17.00	-1.00	-7.00
18.00	1.00	11.00
19.00	-1.00	-7.00
20.00	1.00	5.00
21.00	-1.00	-7.00
22.00	1.00	11.00
23.00	1.00	11.00
24.00	-1.00	-7.00

#	sig	ref
1.00	-1.00	-5.00
2.00	1.00	9.00
3.00	1.00	9.00
4.00	1.00	9.00
5.00	-1.00	-5.00
6.00	1.00	9.00
7.00	1.00	9.00
8.00	-1.00	-5.00
9.00	-1.00	-5.00
10.00	1.00	9.00
11.00	-1.00	-5.00
12.00	-1.00	-5.00
13.00	-1.00	-5.00
14.00	-1.00	-5.00
15.00	1.00	9.00
16.00	1.00	9.00
17.00	-1.00	-5.00
18.00	-1.00	-5.00
19.00	1.00	9.00
20.00	-1.00	-5.00
21.00	1.00	9.00
22.00	-1.00	-5.00
23.00	-1.00	-5.00
24.00	-1.00	-5.00
25.00	-1.00	-5.00
26.00	1.00	9.00
27.00	-1.00	-5.00
28.00	1.00	9.00
29.00	-1.00	-5.00
30.00	-1.00	-5.00
31.00	-1.00	-5.00
32.00	-1.00	-5.00
33.00	-1.00	-5.00
34.00	-1.00	-5.00
35.00	-1.00	-5.00
36.00	1.00	9.00
37.00	-1.00	-5.00
38.00	-1.00	-5.00
39.00	-1.00	-5.00
40.00	-1.00	-5.00

Ipatov sequences N = 21, 56, 63 (Loss = 1.0, 0.69, 0.3 dB)

#	sig	ref
1	-1	-3
2	1	8
3	1	8
4	-1	-3
5	-1	-3
6	1	8
7	-1	-3
8	-1	-3
9	-1	-3
10	-1	-3
11	-1	-3
12	-1	-3
13	-1	-3
14	-1	-3
15	-1	-3
16	1	8
17	-1	-3
18	1	8
19	-1	-3
20	-1	-3
21	-1	-3

#	sig	ref	#	sig	ref
1	1	43	29	-1	-23
2	1	65	30	1	65
3	-1	-23	31	-1	-23
4	-1	-23	32	-1	-23
5	-1	-23	33	-1	-23
6	-1	-23	34	-1	-23
7	-1	-23	35	-1	-23
8	-1	-23	36	1	43
9	-1	-23	37	1	43
10	-1	-23	38	-1	-23
11	-1	-23	39	-1	-23
12	-1	-23	40	1	43
13	-1	-23	41	-1	-23
14	-1	-23	42	1	43
15	-1	-23	43	-1	-23
16	1	65	44	1	65
17	1	43	45	-1	-23
18	-1	-23	46	1	43
19	-1	-23	47	1	43
20	1	43	48	-1	-23
21	-1	-23	49	1	43
22	-1	-23	50	-1	-23
23	-1	-23	51	-1	-23
24	1	43	52	-1	-23
25	1	43	53	-1	-23
26	-1	-23	54	-1	-23
27	-1	-23	55	1	43
28	-1	-23	**56**	-1	-23

#	sig	ref	#	sig	ref
1	1	17	32	1	17
2	1	32	33	1	17
3	1	17	34	1	17
4	1	17	35	1	17
5	-1	-28	36	-1	-28
6	1	17	37	1	17
7	-1	-28	38	1	32
8	1	17	39	-1	-28
9	1	17	40	-1	-28
10	1	17	41	-1	-28
11	1	32	42	-1	-28
12	1	17	43	1	17
13	1	17	44	1	17
14	1	17	45	1	17
15	1	17	46	-1	-28
16	-1	-28	47	1	32
17	-1	-28	48	-1	-28
18	1	17	49	-1	-28
19	1	17	50	1	17
20	1	32	51	-1	-28
21	-1	-28	52	-1	-28
22	-1	-28	53	1	17
23	1	17	54	-1	-28
24	-1	-28	55	-1	-28
25	1	17	56	1	32
26	-1	-28	57	1	17
27	1	17	58	1	17
28	-1	-28	59	-1	-28
29	1	32	60	1	17
30	1	17	61	1	17
31	1	17	62	-1	-28
			63	-1	-28

Nadav Levanon, Tel-Aviv University

Ipatov, N = 121

Loss = 0.46 dB

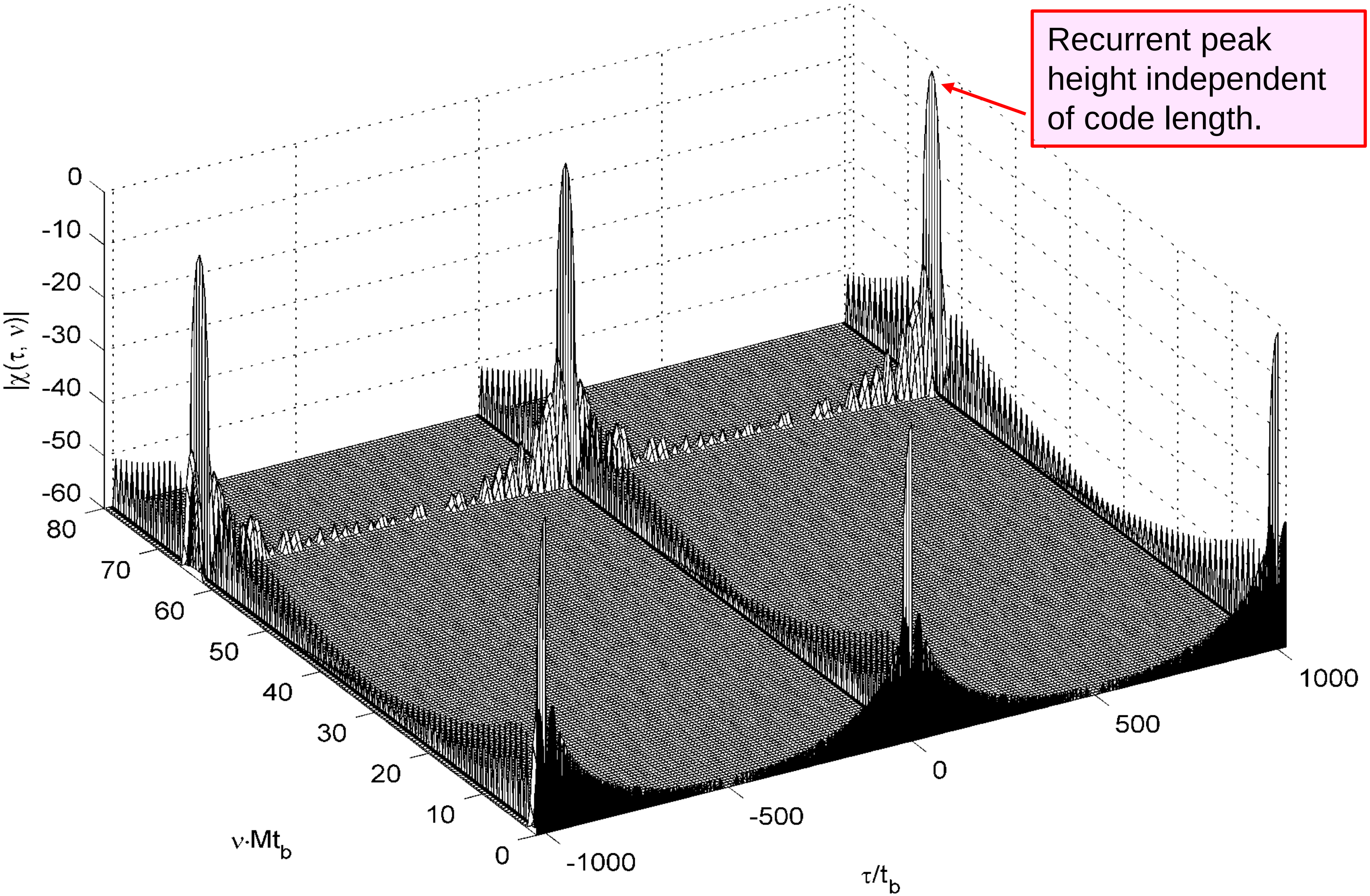
1	0	0	0	0	0	1	1	1	1	0	1	1	1	0	0	1	1	0	0
0	1	0	1	1	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0
0	0	1	0	0	1	0	0	0	0	0	0	1	1	0	1	0	1	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0
1	1	0	1	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	1
0	1	0	0	0	0	0	1	1	0	0	0	1	0	0	1	0	0	1	0
0																			

Binary transmitted signal {+1, -1} , Two-valued reference signal {+27, -14}

Ipatov, N = 5

#	sig	ref
1.00	1.00	1.00
2.00	1.00	1.00
3.00	1.00	1.00
4.00	-1.00	-2.00
5.00	1.00	1.00

Valery Ipatov, St. Petersburg State Electrotechnical University “LETI”, October 2013



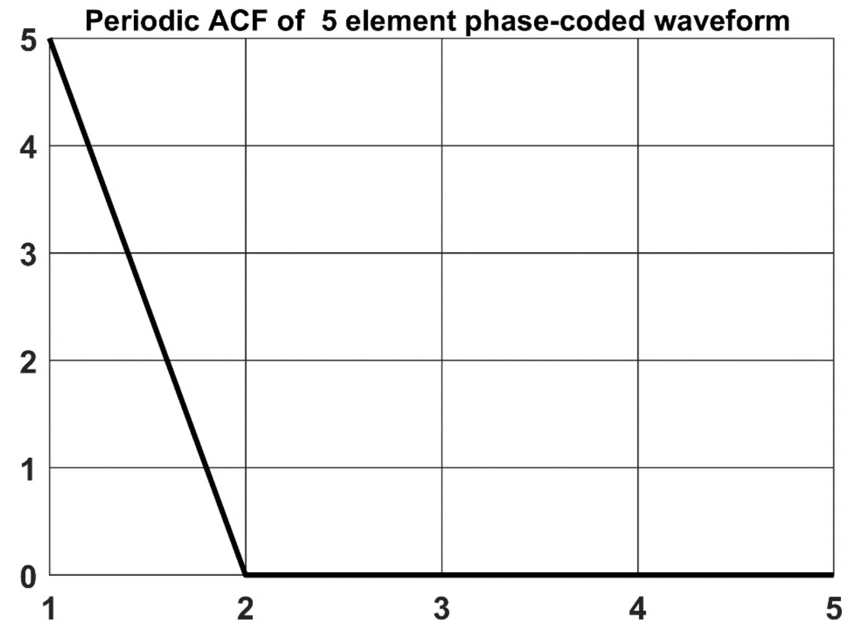
Periodic cross-ambiguity function (in dB) of CW **LFM** waveform. **Inter**-period Hamming weighted reference (to reduce Doppler sidelobes). **Intra**-period weighting (to reduce delay sidelobes). (TBW=1000, 64 periods)

Legendre two*-valued sequences

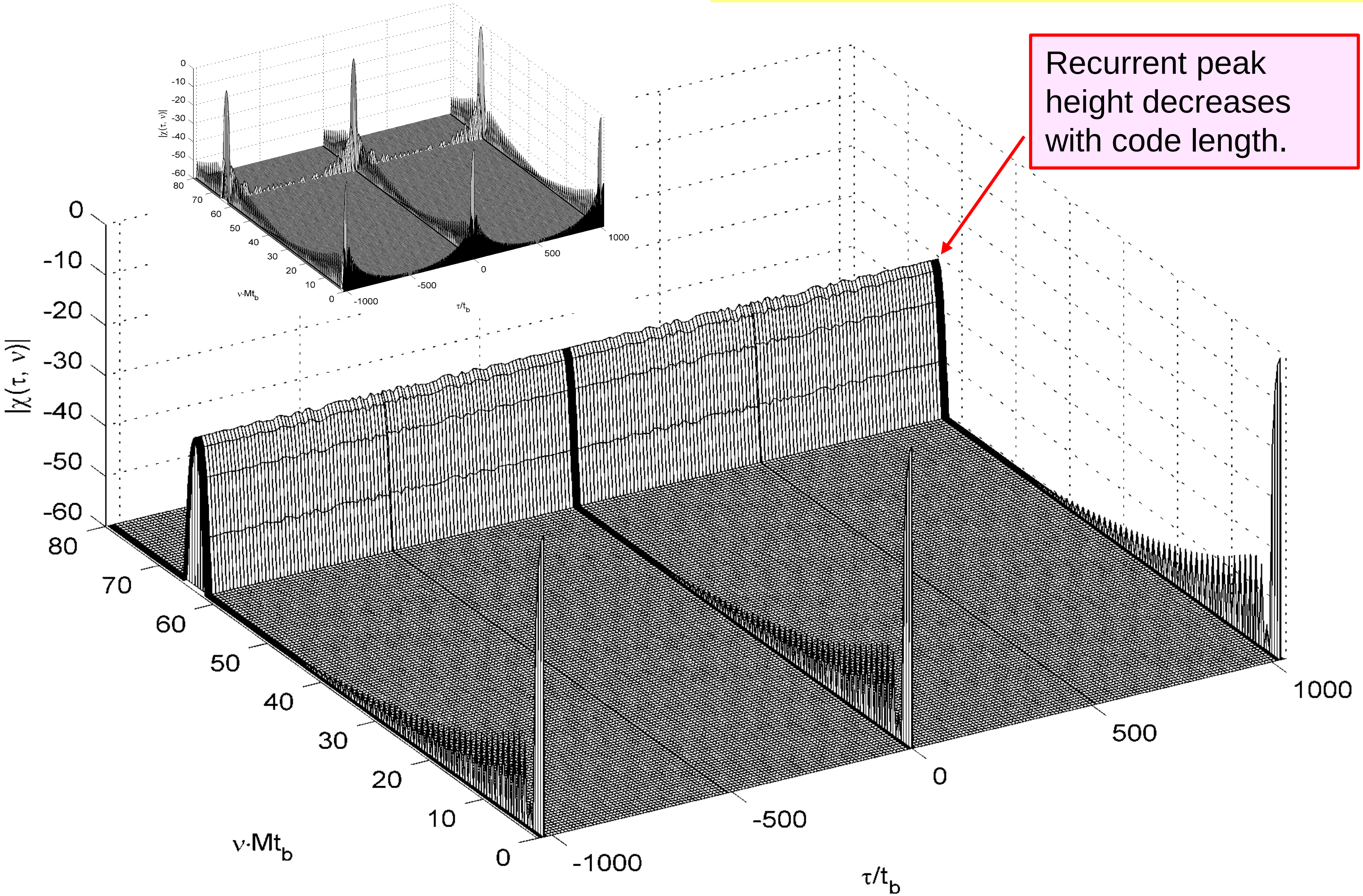
Adrien Legendre
1752-1833

```
function[s]=perfect_periodic_legendre_waveform(M)
% Generates periodic coded signaal using 2 or 3 phases
% The signal xhibits perfect periodic ACF
% M is any odd prime
mspt=sprintf(' %g element phase-coded waveform ',M);
if isprime(M)==0
    disp('Not a prime')
    return
end
s=ones(1,M);
if rem((M+3)/4,1)==0
    c=0.25*(M-1);
    c1=2-1/c-1/(2*c^2);
    c2=1-1/c-1/(4*c^2);
    arg2=acos(-c1/2-sqrt((c1/2)^2-c2));
    s(mod((1:M-1).^2,M)+1)=exp(1i*arg2);
    s(1)=exp(1i*arg2/2);
else
    arg3=acos(-(M-1)/(M+1));
    s(mod((1:M-1).^2,M)+1)=exp(1i*arg3);
end
% periodic autocorrelation
d=abs(ifft(fft(s).*conj(fft(s))));
figure, plot(d, 'k')
title(['Periodic ACF of ' mspt ]);
end
```

Legendre 5: **72^0 144^0 0^0 0^0 144^0**



Any cyclic shift of the sequence also yields perfect periodic autocorrelation



Periodic cross-ambiguity function (in dB) of CW **bi-phase** waveform, inter-period Hamming weighted reference (**Legendre** 1023 elements, 64 periods)

Legendre 5: **72**^{0} **144**^{0} **0**^{0} **0**^{0} **144**^{0}

Ipatov, $N = 5$

#	sig	ref
1.00	1.00	1.00
2.00	1.00	1.00
3.00	1.00	1.00
4.00	-1.00	-2.00
5.00	1.00	1.00