Incoherent Compression of Complementary Code Pairs for Laser Ranging and Detection

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Abstract **— A scheme for the incoherent compression of complementary code pairs is proposed and demonstrated in a laser range-finding experiment. The off-peak aperiodic auto-correlations of complementary bipolar codes sum up to zero, hence they are attractive for radar and laser ranging systems. In incoherent compression, the two codes are converted to unipolar representations using a pulse position modulation algorithm prior to transmission. Following incoherent detection, the two received echoes are compressed through digital filtering, and added together to obtain strong sidelobe suppression. The scheme does not require the preservation of phase information in transmission or reception, and the length of the code pairs is scalable through simple procedures. Incoherent compression is particularly attractive for photonic implementations, in which direct detection is phase insensitive. A peak-to-sidelobe ratio of 42 dB is obtained in a laser range-finding experiment using 832 bits-long codes.**

Index Terms **— Complementary code pairs, LADAR, Matched filtering, Optical signal processing, Pulse compression, Ranging measurements.**

I. Introduction

Long coded sequences are widely employed in ranging, detection and imaging applications where both high resolution and large signal to noise ratios (SNRs) are necessary [1]. The auto-correlation, or matched-filtering, of carefully-constructed long sequences effectively compresses their entire energy into an intense and narrow virtual peak with low residual sidelobes [1]. Such sequences may therefore reproduce the high resolution and low background that are provided by a short and high-power single pulse, with significant added values: The instantaneous power of coded sequences can be orders-of-magnitude lower, making them safer and simpler to generate in a real-world system and more difficult to intercept by an adversary [2].

The most effective pulse compression is usually achieved using either frequency modulation or phase coding [1]. Among phase-coded sequences, binary-phase (or *bipolar*) codes draw much attention for the relative simplicity of their realization in a practical system. The employment of such codes requires coherent transmission and detection schemes, in which phase is maintained on transmit and recovered on receive. The processing of amplitude-based codes, on the other hand, can be based on simpler incoherent detection, however they generally provide inferior sidelobe suppression.

The overheads associated with coherent transmit/receive in radio-frequency (RF) and microwave-frequency systems are relatively modest. Coherent detection in the optical domain is more troublesome, since the current that is directly provided by a photo-detector is phase insensitive. Although coherent optical receivers using interferometric techniques are widely known, they come at the expense of significant complexity [3-6]. Strong incentive exists, therefore, to develop elaborate amplitude-based coding schemes for use in laser range-finders [7], lasers detection and ranging (LADAR) systems [6, 8-10], free-space optical communication [11] etc.

Recently, we have demonstrated the effective compression of a long and dense pulse sequence in an incoherently-operating laser range-finder experiment [12]. The binary unipolar sequence used was generated from a bipolar, minimum peak sidelobe (MPSL) code that was 1112 bits long, through a pulse position modulation algorithm [13]. The directly-detected laser echoes were digitally processed by a reference matched-filter sequence that was stored in the receiver [12, 13]. With the exception of the two time slots immediately adjacent to the main lobe, the sidelobes suppression in the incoherently compressed unipolar representation matched that of the original MPSL code [12, 13]. The ratio of main lobe peak power to the power of the highest sidelobe (PSLR) in the experiments was 33 dB, in agreement with predictions. A spatial accuracy of 2.5 cm was demonstrated. The range to the target could be measured in the presence of additive noise, at optical SNRs as low as -20 dB [12].

While the previous results bring together the simplicity of incoherent direct detection and the sidelobe suppression of coherent receivers in a laser range-finder setup, the scaling of performance obtained is rather difficult. The further suppression of the sidelobes may be pursued in one of two manners: First, since the PSLR scales with the length of the MPSL sequence, longer codes may be used. However, the search for such codes is a daunting task. Alternatively, cross-correlation with a second, specially constructed reference code, referred to as mismatched-filtering, may be used instead of auto-correlation in the compression of sequences [1]. Mismatched filters are several times longer than the original code, and they comprise precise analog values. Use of a mismatched filter improved the PSLR of the incoherently compressed laser range-finder setup mentioned above to 46 dB [12]. Nevertheless, binary-valued matched filters are much simpler to implement, and their performance is less susceptible to small-scale variations in their coefficients.

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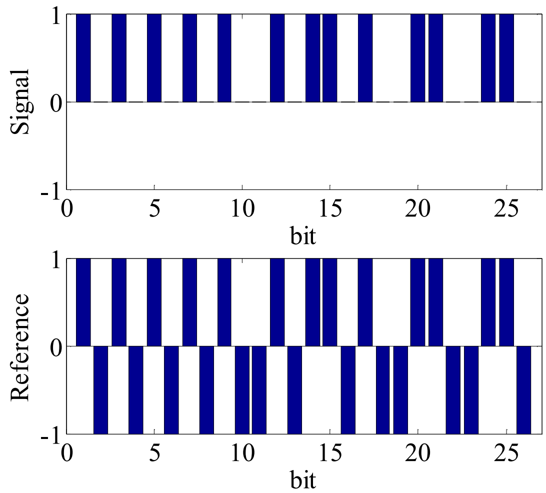

Fig. 1: Transmitted code T (top) and matched filtering code R (bottom), corresponding to the Barker 13 code: [+++++--++-+-+] [12].

In this work, the incoherent compression of a pair of complementary codes using simple matched filters is proposed and demonstrated. The pair of codes is characterized by the following useful property: the auto-correlation sidelobes of one code are equal in magnitude to those of the other code, albeit with an opposite sign [14]. Adding the matched-filtered forms of the two codes together, therefore, reduces the sidelobe power drastically [15]. The advantage of using complementary code pairs is two-fold: 1) unlike MPSL sequences, their length is scalable through several simple procedures [16], some of which are described in subsequent sections; and 2) The obtained sidelobe suppression is equivalent to, or better than, that of a mismatched-filtered MPSL sequence of similar length.

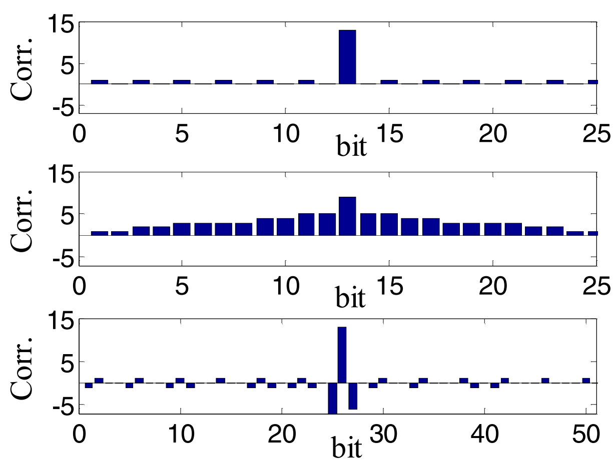

Fig. 2: Top – aperiodic auto-correlation of the Barker 13 bipolar code: [+++++--++-+-+]. The correlation peak is 13, whereas the maximal sidelobe equals unity. Center – aperiodic auto-correlation of a unipolar representation of the Barker 13 code: [1111100110101], showing a weaker central peak and inferior sidelobe suppression. Bottom – aperiodic cross-correlation between the transmitted code T and matched filtering code R corresponding to the Barker 13 bipolar code (see Fig. 1). With the exception of the two time slots in the immediate vicinity of the central peak, the suppression of sidelobes reaches that of the original bipolar sequence [12].

The matched-filtered, incoherent compression of complementary code pairs was demonstrated in a laser range-finder experiment. A PSLR of 42 dB was obtained using 832 bits-long codes. The sidelobe suppression was an order of magnitude better than that of a longer match-filtered MPSL sequence. The results illustrate the simple scaling of the proposed principle. They carry a promise for high performance in simple-architecture laser range-finders and other photonic systems.

The remainder of this paper is organized as follows. The principle of incoherent pulse compression is reiterated briefly in section II, for completeness [12, 13]. Sidelobe suppression using a complementary code pair is discussed in section 3. Finally, the combination of both methods in a laser range-finder experiment is reported in section 4. A concluding discussion is provided in section 5.

II. INCOHERENT PULSE COMPRESSION

Consider a bipolar code of length N: $c[n]$, where $n = 1 \ldots N$. A unipolar code of length $2N$ is generated based on $c[n]$ through pulse position modulation: if $c[n]=1$, then $T[2n-1]=1$ and $T[2n]=0$. For $c[n]=-1$, $T[2n-1]=0$ and $T[2n]=1$ are chosen instead. This procedure resembles the Manchester coding known in telecommunications. The code T can be used, for example, to modulate the intensity of a laser range-finder light source. A bipolar matched filtering sequence R of length $2N$ is constructed in a similar manner: $R[k]$ is set to 1 if $T[k]=1$ and equals -1 if $T[k]=0$, $k = 1 \ldots 2N$. The code R is digitally stored at the receiver for post-detection processing.

As an example, the construction of the T and R codes corresponding to the Barker 13 bipolar sequence is illustrated in Fig. 1 [12]. The transmitted signal (top) is a dense batch of narrow pulses. In the absence of noise, directly detected reflections from targets would be scaled and delayed replicas of the transmitted signal. The reference signal (bottom) is a dense batch of bipolar pulses, stored numerically in the receiver. The aperiodic cross-correlation between these two codes is shown in Fig. 2 (bottom), alongside the aperiodic auto-correlation of the original Barker 13 sequence itself (top). With the exception of the two sidelobes immediately adjacent to the main correlation peak, the cross-correlation replicates the sidelobe suppression of the original bipolar code [12, 13]. In contrast, the cross-correlation of a unipolar representation $\tilde{c}[n]$ of the Barker code itself, in which -1 symbols are replaced by 0, exhibits inferior sidelobe suppression performance (Fig. 2, center).

III. ADVANTAGES OF USING COMPLEMENTARY PAIRS

A complementary pair of sequences satisfies the property that their out-of-phase a-periodic autocorrelation coefficients sum to zero [14]. For our application of incoherent compression, the relevant complementary pairs must be binary, in order to allow for Manchester encoding. In conventional coherent radars the two sequences are usually modulated on consecutive pulses, which are then coherently processed jointly, using matched filters. The pulse repetition interval (PRI) needs to be large enough to avoid range ambiguity. The main drawback for radar use is the sensitivity to Doppler shift. If during the PRI the range to the target has changed by a meaningful fraction of a wavelength, the second pulse will exhibit an additional phase shift that will degrade or destroy the complementary property of the pair. Direct-detection laser applications of complementary pair coding are insensitive to phase. Thus our use of complementary pairs is immune to their main drawback.

The ideal zero-sidelobes correlation property of the complementary pair is nearly preserved by the Manchester encoding, with a PSLR of $1/(2N)$ instead of zero, where N is the length of the each code in the pair [15]. The benefit of using long codes is therefore two-fold: increased signal energy and suppressed sidelobes.

TABLE I
PRIMITIVE COMPLEMENTARY PAIRS

N	sequence **a**	sequence **b**
2	++	+-
10	++-+-+--++	++-+++++--
10	+++++-+--+	++--+++-+-
20	++++-+---++--++-+--+	++++-+++++---+-+-++-
26	++++-++--+-+-+--+- +++--+++	++++-++--+-+++++-+--- ++---

In contrast to MPSL sequences, the finding of long complementary pairs is relatively simple. The generation of a new complementary pair starts with one or two of the primitive pairs listed in Table I, followed by the application of one of several construction rules. The procedure can be repeated as needed. A recent detailed description of the presently known construction rules, including proofs, appears in section 7.3 of [16]. The most basic construction rule [14], which creates the pair $\{\mathbf{c},\mathbf{d}\}$ based on a pair $\{\mathbf{a},\mathbf{b}\}$, is:

$$\{\mathbf{c},\mathbf{d}\} = \{\mathrm{cat}(\mathbf{a},\mathbf{b}), \mathrm{cat}(\mathbf{a},-\mathbf{b})\}, \qquad (1)$$

where $\mathrm{cat}(\mathbf{a},\mathbf{b})$ stands for concatenation of the two sequences **a** and **b**. The expression **–b** implies polarity reversal of the elements of sequence **b**. This basic construction rule doubles the length of the sequences in the new pair. The same rule can be used to create a different pair from the same original pair:

$$\{\mathbf{e},\mathbf{f}\} = \{\mathrm{cat}(\mathbf{b},\mathbf{a}), \mathrm{cat}(\mathbf{b},-\mathbf{a})\}. \qquad (2)$$

TABLE II
NUMBER OF PAIRS FOR LENGTH $N < 100$

N	1	2	4	8	10	16	20
pairs	4	8	32	192	128	1536	1088
N	26	32	40	52	64	80	
pairs	64	15360	9728	512	184320	102912	

Thus, the number of possible pairs having a longer length is larger. Table II lists the number of complementary Golay pairs for all lengths up to 100 [17].

Note that complementary pairs are not found in all lengths. It can be shown that the possible lengths must comply with:

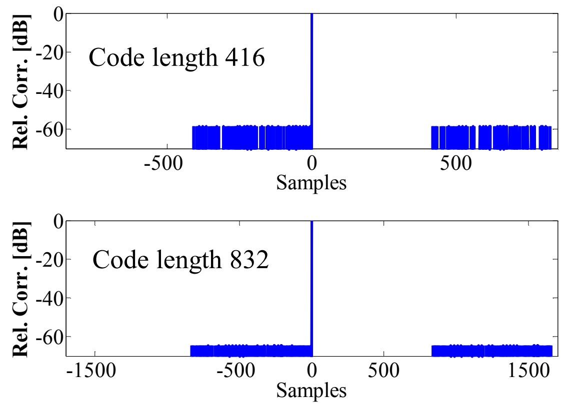

Fig. 3: Calculated, incoherently compressed forms of complementary code pairs, added together to obtain a narrow virtual peak with strongly suppressed sidelobes. The lengths of each code in the pair were 416 bits (top) and 832 bits (bottom).

$$N = 2^{\alpha}\, 10^{\beta}\, 26^{\gamma}, \qquad \alpha,\beta,\gamma \geq 0\,, \qquad (3)$$

where α, β and γ are integers. The large choice of codes, shown in Table II and equation (3), helps reduce the probability of intercept (LPI) of complementary pair-encoded transmission.

Fig. 3 shows the calculated incoherently compressed forms of pairs of complementary codes added together, for N of 416 bits (top) and 832 bits (bottom). The two codes were obtained by applying (1) to the 26 element primitive pair of Table I, recursively 4 and 5 times. The PSLRs for the two code lengths are 58 dB and 64 dB, respectively, in agreement with expectation [15].

IV. EXPERIMENTAL RESULTS

The setup for laser ranging measurements using incoherent pulse compression of complementary code pairs is shown in Fig. 4. Light from a laser diode at 1550 nm wavelength passed through a Mach-Zehnder electro-optic intensity modulator (MZM), driven by an arbitrary waveform generator programmed to the transmission of the code pair. The coding symbol duration was 200 ps. The specific codes used in the experiment were the same as those of the simulations in Fig. 3, having lengths of 416 and 832 bits. The codes were repeatedly transmitted every 1 µs-long

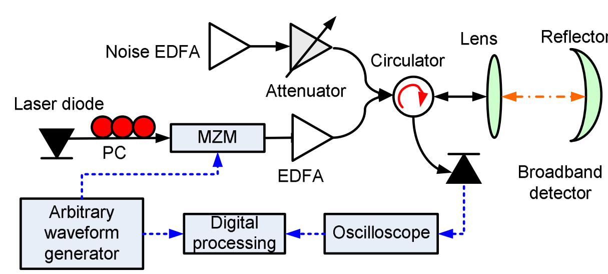

Fig. 4: Experimental setup for laser ranging measurements using incoherent pulse compression. MZM: Mach-Zehnder modulator. PC: polarization controller. EDFA: erbium-doped fiber amplifier. Black solid lines denote fiber connections, blue dashed lines represent electrical cables, and red dash-dotted lines describe free-space propagation [12].

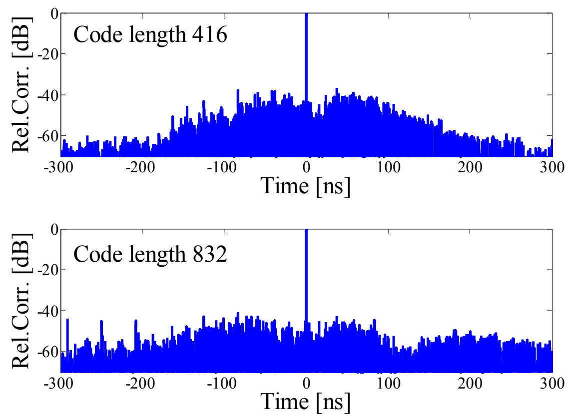

Fig. 5: Experimental incoherently compressed forms of complementary code pairs, added together to obtain a narrow virtual peak with strongly suppressed sidelobes. The optical SNR was 30 dB. The lengths of each code in the pair were 416 bits (top) and 832 bits (bottom).

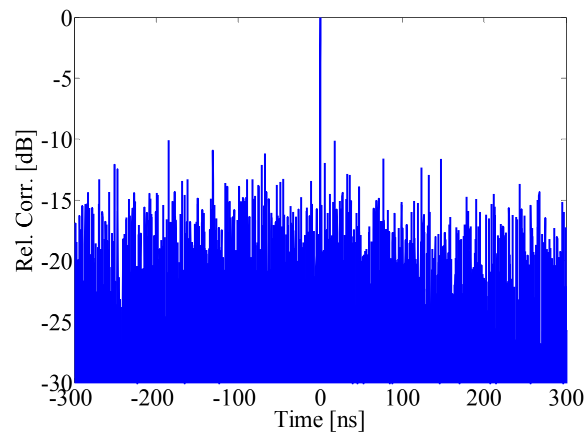

Fig. 6: Experimental incoherently compressed form of a 832 bits-long complementary code pair, added together to obtain a narrow virtual peak with strongly suppressed sidelobes. The optical SNR was -20 dB.

intervals.

The modulated waveform was amplified by an erbium-doped fiber amplifier (EDFA) and launched towards a movable retro-reflector via a fiber circulator and a collimating lens. In a proof of concept experiment, the reflector was placed tens of cm away from the lens. Reflections were partially collected by the lens, directly detected by a photo-diode with 12 GHz bandwidth, and sampled by a digitizing oscilloscope of 6 GHz bandwidth. The detected sequences were incoherently compressed through digital match-filtering of both codes, using the corresponding bipolar reference sequences R described in section II. The cross-correlation of the two codes were then added together to obtain a ranging measurement with low sidelobes.

The measurement optical SNR was controlled by the addition of amplified spontaneous emission (ASE) of variable power from a second EDFA. In this manner the optical SNR could be quantified by switching the ASE on and off, while the power of collected reflections remained above the thermal noise floor of the photo-detector.

Fig. 5 shows the experimentally obtained, incoherently compressed forms of complementary code pairs, of length 416 (top) and 832 bits (bottom). The PSLRs of the two curves are 36 dB and 42 dB, respectively. The experimental compression is restricted by the noise and distortion of the RF amplifiers used to match the output voltage of the waveform generator to the levels necessary for the MZM electrical input port. The full width at half maximum of the main correlation lobe is 200 ps as expected, signifying a spatial resolution of 3 cm. Lastly, Fig. 6 shows the incoherently compressed 832 bits-long pair, acquired in the presence of strong additive noise (optical SNR of -20 dB). The PSLR is degraded to 10 dB, but the main correlation peak remains discernible.

V. Conclusion

In this work, the incoherent compression of pulse sequences was extended to accommodate complementary code pairs. The compression relies on the transmission and direct detection of Manchester coded, unipolar representations of each of the two codes, and their matched-filtering by stored bipolar references on receive. The addition of the two complementary correlation functions reduces the residual sidelobes drastically.

The principle is of particular consequence for photonic applications, in which coherent detection is more difficult to implement, and was experimentally demonstrated in a simple laser range-finder setup. A PSLR of 42 dB was achieved using a pair of 832 bits-long codes. For comparison, the PSLR of an incoherently compressed, matched-filtered 1112 bits long MPSL sequence in an earlier experiment was only 33 dB [12]. The comparable incoherent compression of the MPSL sequence required a precise mismatched filter of 3336 coefficients [12]. Furthermore, the length of the complementary codes is arbitrarily scalable using simple procedures. The method is applicable to a variety of optical ranging and detection, imaging, and communication systems. Ongoing work is being dedicated to extended ranging measurements.

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