DETECTION

Radar cell coordinates:

- Range (delay)
- Velocity (Doppler)

  From coherent processing of several pulses

- Direction (azimuth, elevation)

  From antenna pointing direction

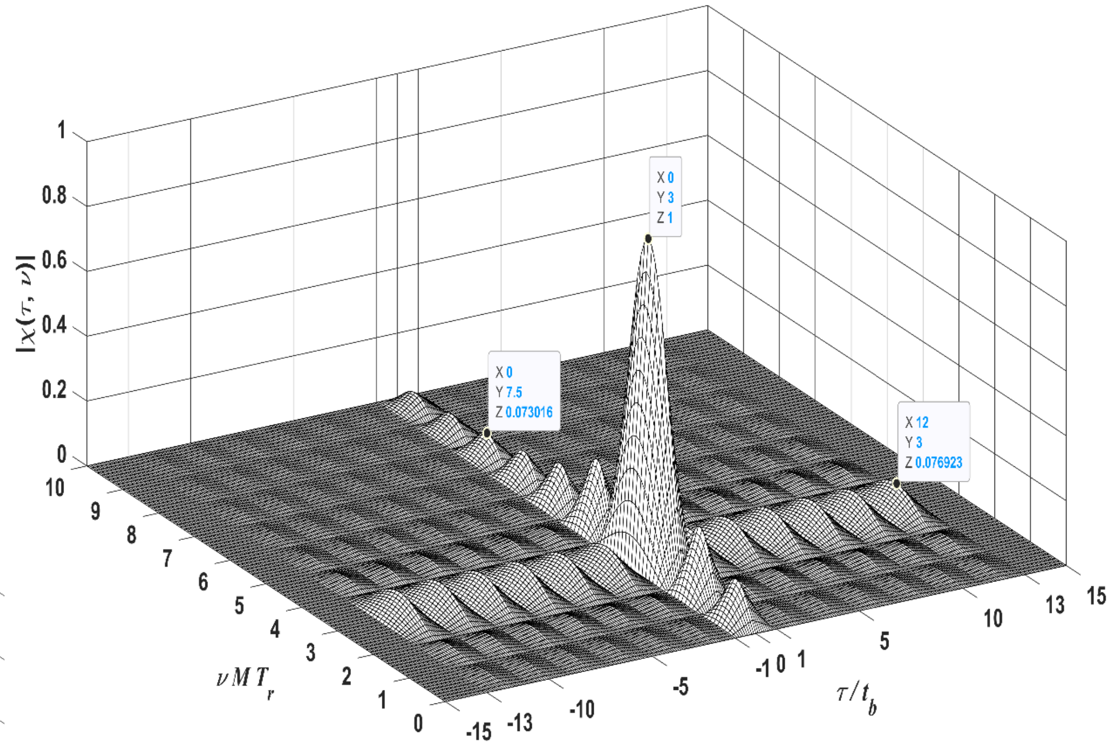
Once the processing that defines a cell is completed, we need to decide if the signal received in that cell represents "noise" (including background and interference) or target + noise.

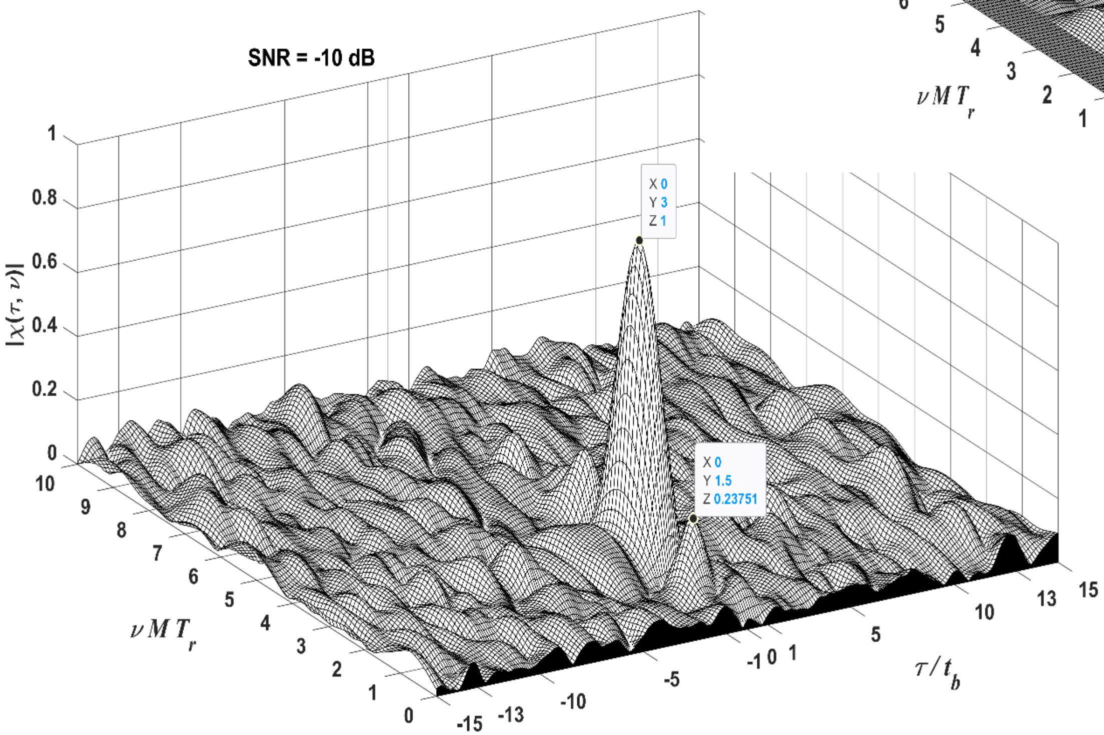
We may look at neighboring cells at the same CPI or at the same cell at different CPIs, or both, to help in that decision (CFAR, integration, track before detect)

CPI - coherent processing interval

Delay-Doppler response of a processor matched to a coherent train of 32 Barker-13 pulses, when the noise-free return is Doppler-shifted by $\nu = 3/(32T_r) = 3/CPI$

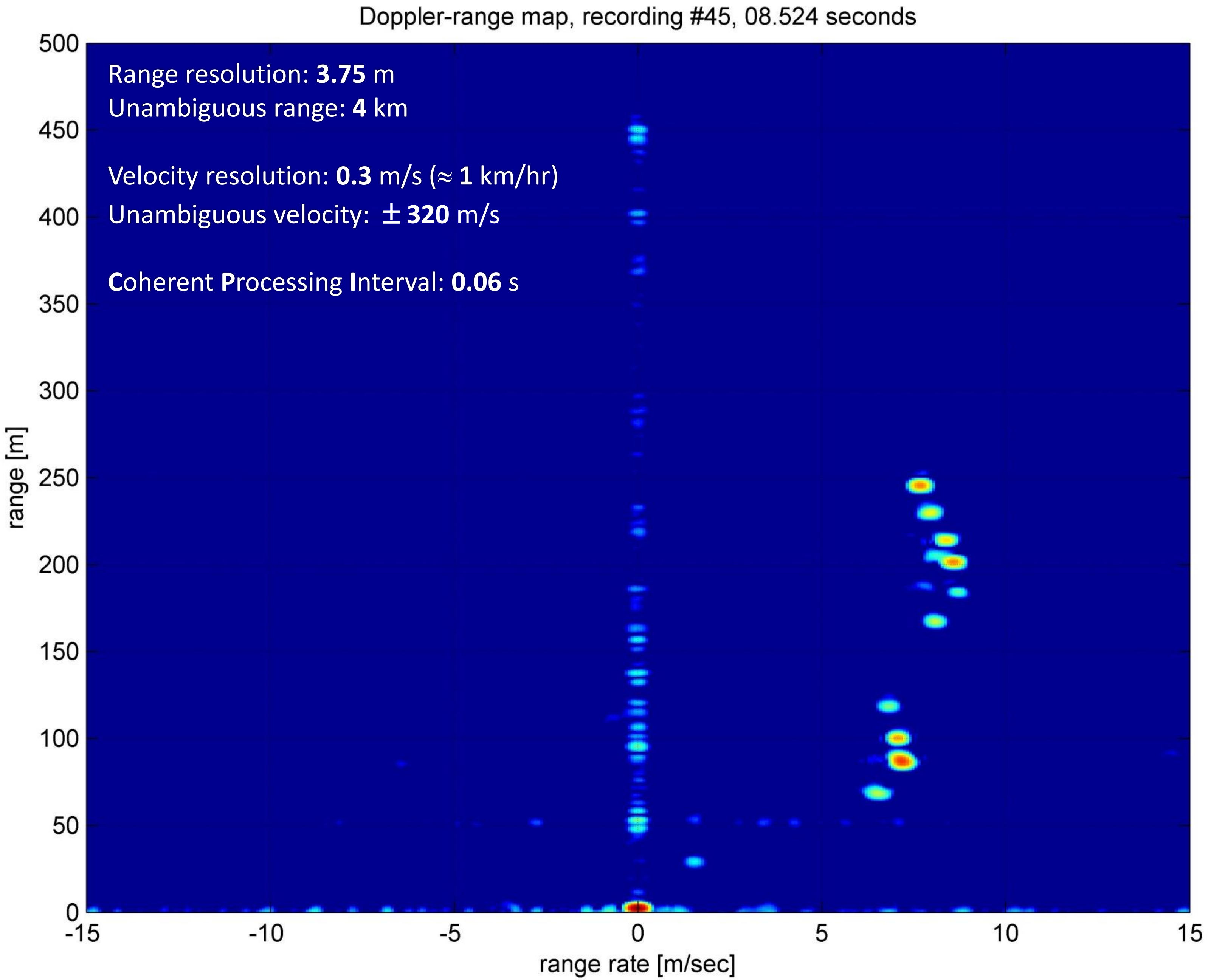
The avoid false-alarms the detection threshold needs to consider the sidelobes peaks



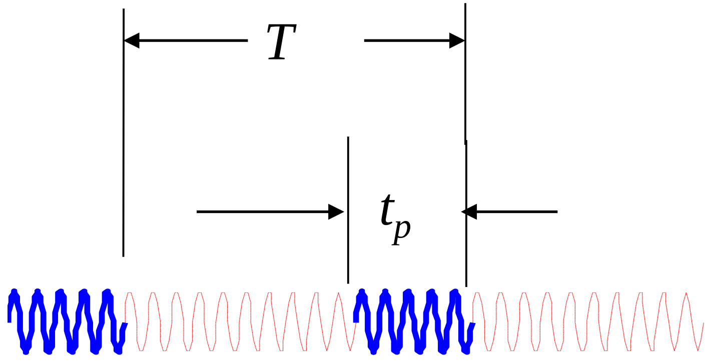


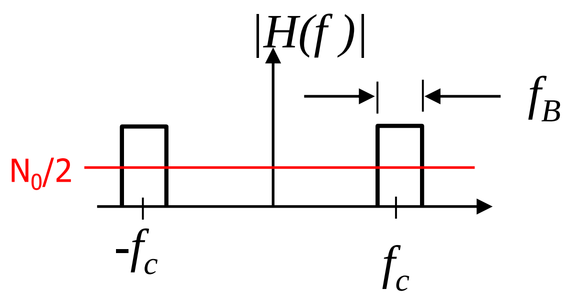
The response when the same Doppler-shifted return is received at low SNR

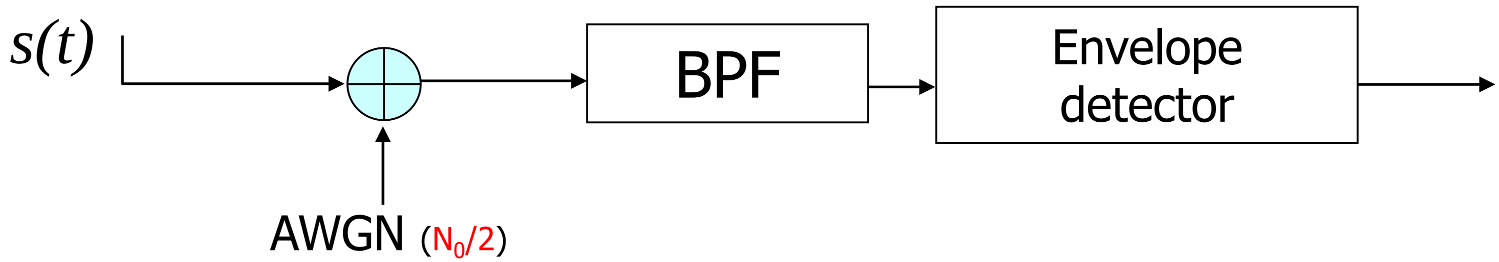
The avoid false-alarms the detection threshold needs to consider both the sidelobe peaks and the noise-induced peaks


Doppler-range map, recording #45, 08.524 seconds
Range resolution: **3.75** m
Unambiguous range: **4** km
Velocity resolution: **0.3** m/s (≈ **1** km/hr)
Unambiguous velocity: ±**320** m/s
Coherent **P**rocessing **I**nterval: **0.06** s
range [m]
range rate [m/sec]
500
450
400
350
300
250
200
150
100
50
0
-15
-10
-5
0
5
10
15

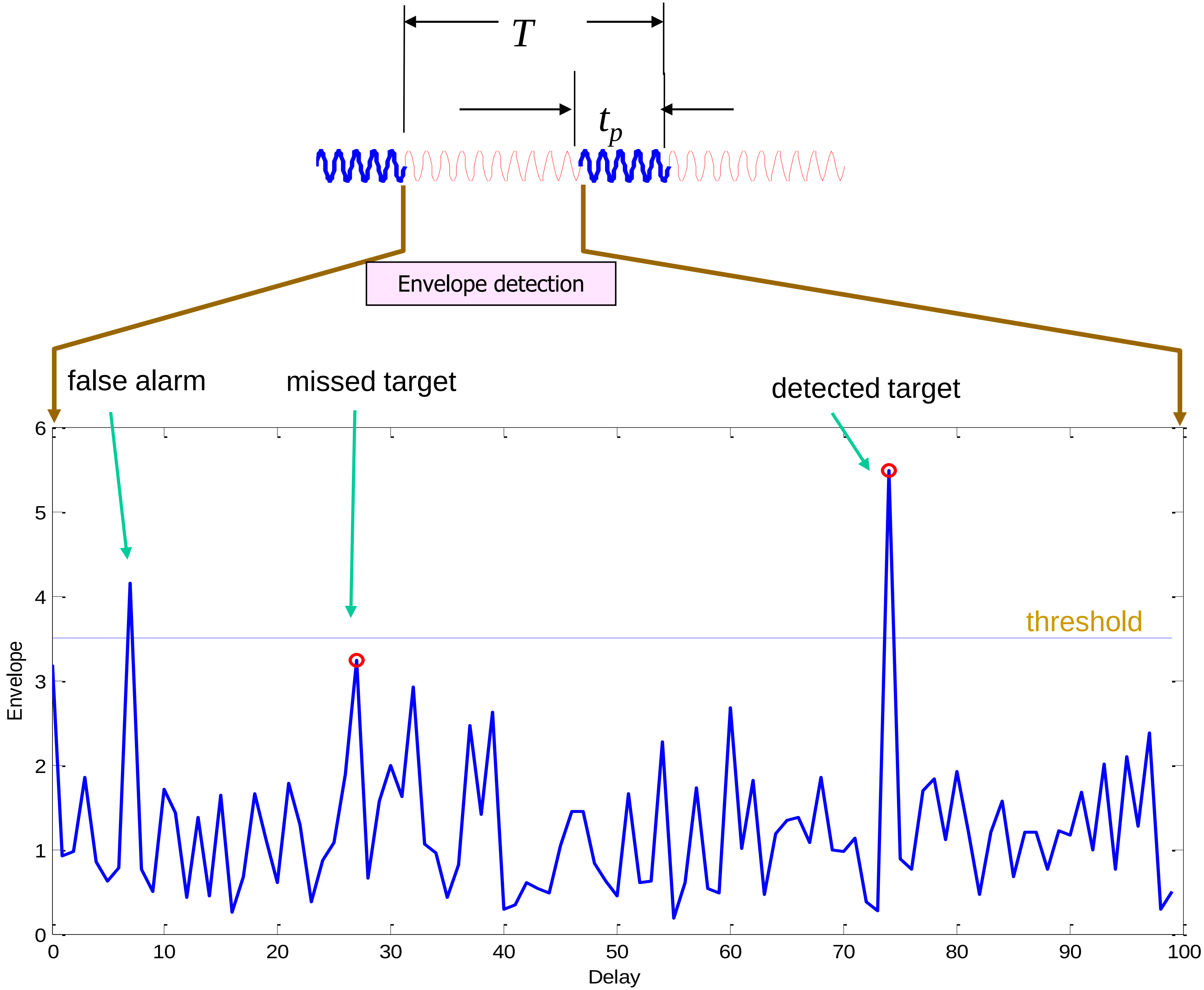
Simple detection example – envelope* detection of a single pulse





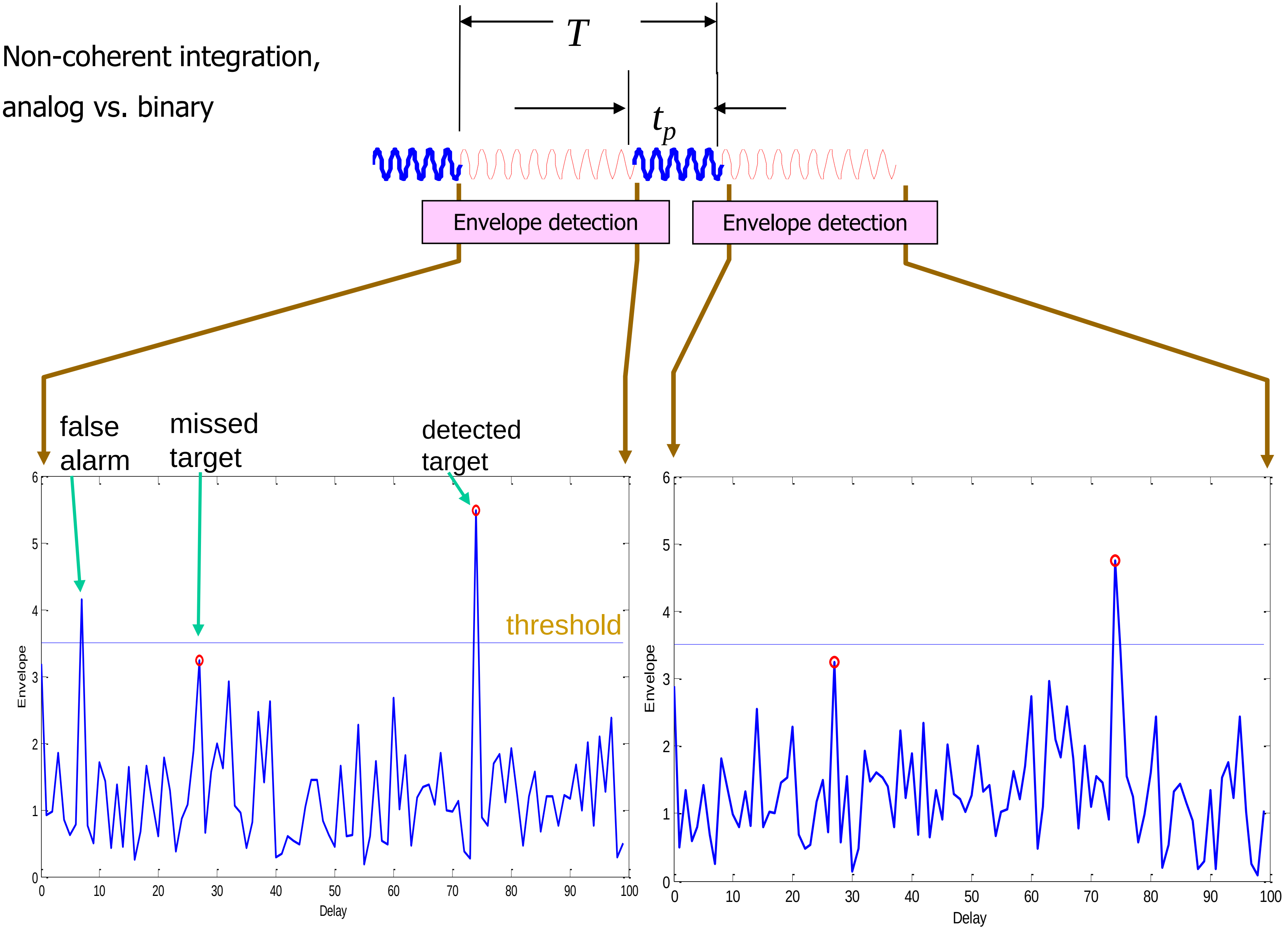


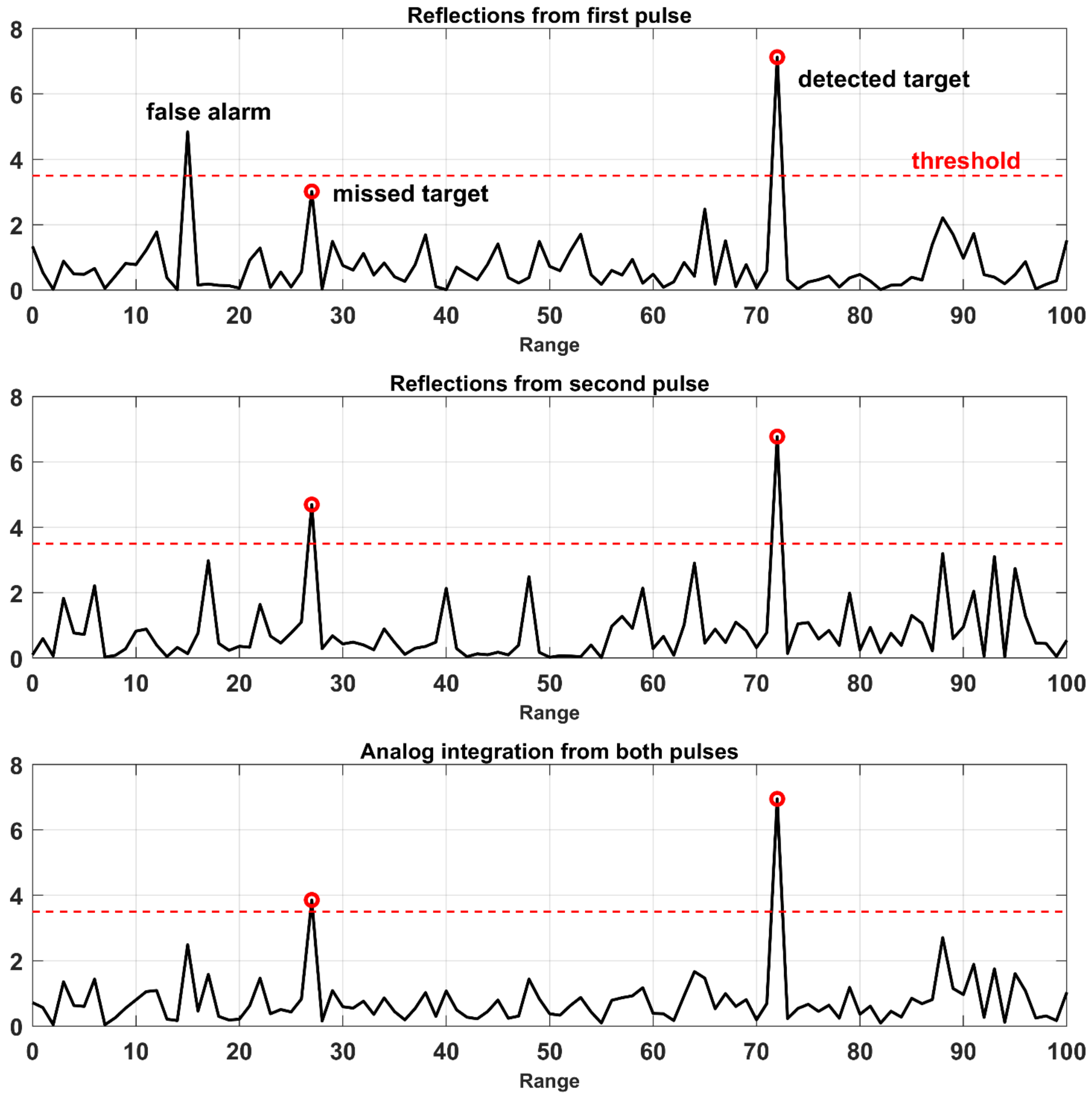
* envelope detection ➔ non-coherent detection ➔ no Doppler information


T
t_p
Envelope detection
false alarm
missed target
detected target
threshold
Envelope
Delay
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100

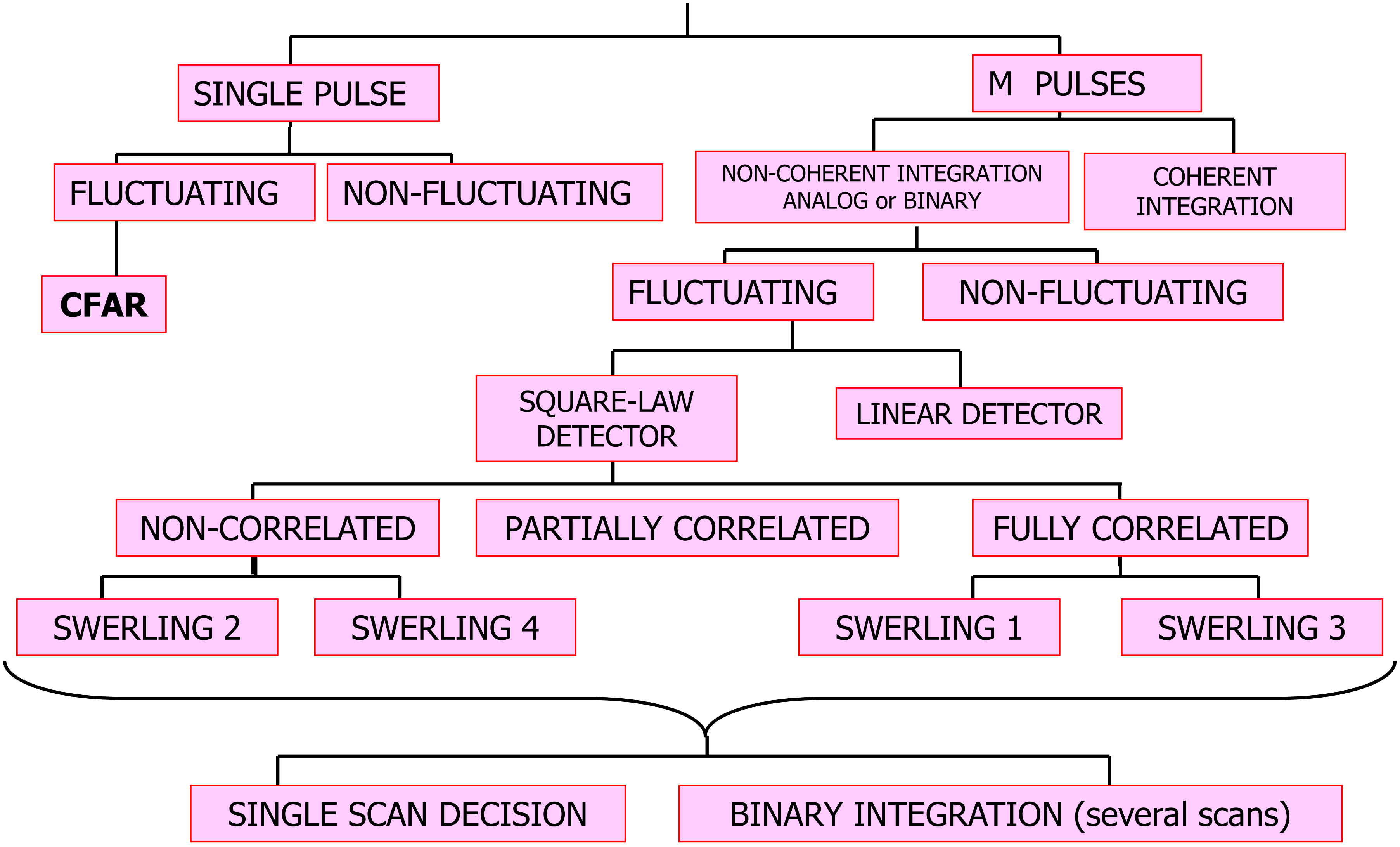
Non-coherent integration,

analog vs. binary




Reflections from first pulse
false alarm
missed target
detected target
threshold
Range
Reflections from second pulse
Range
Analog integration from both pulses
Range

DETECTION (discussion outline)



Probabilities of Interest

- Probability of Detection, P_D:
  - The probability that a target is declared (*i.e.*, we choose H_1) when a target is in fact present.
- Probability of False Alarm, P_{FA}:
  - The probability that a target is declared (*i.e.*, we choose H_1) when a target is in fact not present.
- Probability of Miss, P_M:
  - The probability that a target is not declared (*i.e.*, we choose H_0) when a target is in fact present. Note that $P_M = 1\text{-}P_D$. Thus, we need only P_D and P_{FA}.

P_{FA} and P_M remind us that sometimes we will make the wrong decision!

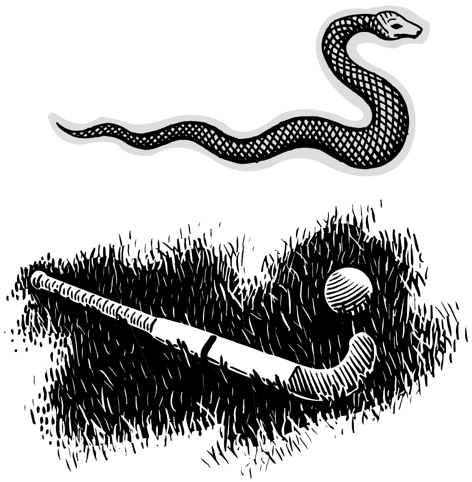
How bad that is depends on what happens when we:

-declare a false target

-fail to detect a real target

"From a point of view of survival … the cost of treating a stick as a snake is less, in the long run, than the cost of treating a snake as a stick."

Joseph Ledoux, *The Emotional Brain*, 1996, Simon & Schuster, p. 163.



Desired values of P_{FA}

False alarm rate (FAR) - Rate of appearance of FA on the display

False alarm time (t_{FA}) - Average time between false alarms

$$t_{FA} = \frac{1}{FAR}$$

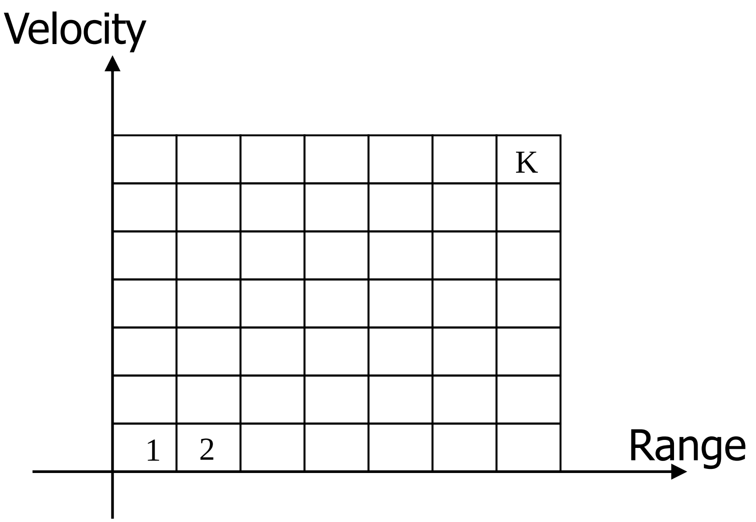
$t_{FA} = 1$ hour, will be unnoticed

$t_{FA} = 1$ sec, will be unacceptable

K - number of cells on display

t_{int}- Integration time - the time devoted to the signal before checking threshold and making a decision (in all the cells simultaneously)

$$t_{FA} = \frac{t_{\text{int}}}{P_{FA}\,K}$$



Lucky 8 - a casino example


1


2


3


4


5

A casino has 5 roulette tables. If in any one of them the wheel stops on 8, a bell rings, activity stops, and all the gamblers at that table get a glass of Champaign. How often (on the average) will the bell ring?

Parameters:

The roulette wheel is turned every 3 minutes (t_{int}=3min) .

The wheel has 38 slots (P_{FA}=1/38).

There are 5 tables (K=5)

Calculation:

On the average, number 8 will win once every 38 rounds, which takes 38x3min=114min. Since there are 5 roulette tables, the bell will ring, on the average, once every 114/5 ≈23min.

$$t_{FA} = \frac{t_{\text{int}}}{P_{FA}K} = \frac{3}{\frac{1}{38}5} = \frac{3\cdot 38}{5} = \frac{114}{5} \approx 23\,\text{min}$$

Radar example

PRF=1000 pulses/sec $\Rightarrow T_r = 0.001$sec

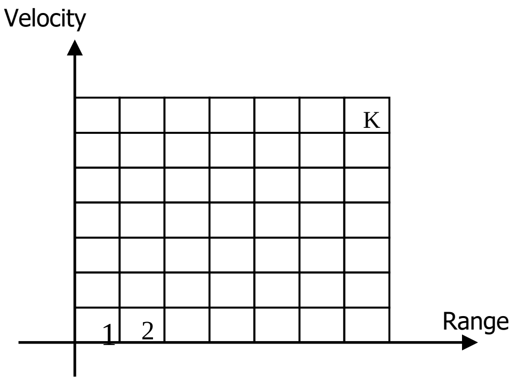
$M = 10$ pulses are integrated before threshold decision

$\therefore\ t_{int} = M\,T_r = 0.01$ sec

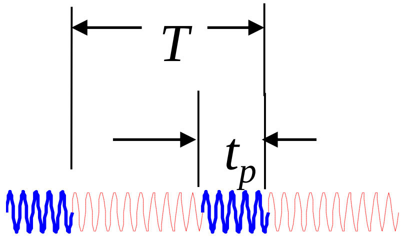
K= 200 range columns x 16 Doppler rows = 3200

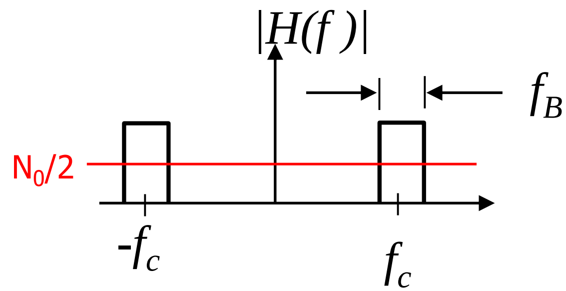
The desired average time between false alarms, $t_{FA} = 30$ sec

$$P_{FA} = \frac{t_{\text{int}}}{t_{FA} K} = \frac{0.01}{30 \cdot 3200} \approx 10^{-7}$$

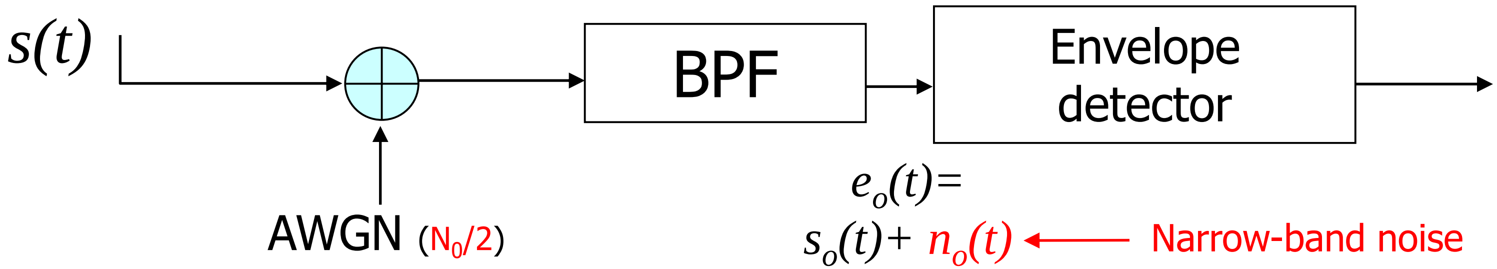


Calculating P_{FA} and P_D for a single-pulse return from a non-fluctuating target in the presence of AWGN





Single pulse non-fluctuating



$$n_o(t) = n_I(t)\cos\omega_c t + n_Q(t)\sin\omega_c t$$

$$\overline{n_I^2(t)} = \overline{n_Q^2(t)} = \overline{n_o^2(t)} = N_0 f_B$$

$n_I(t)$ and $n_Q(t)$ are two independent r.v. with Gaussian distribution and zero mean.

$$s_o(t) = A\cos(\omega_c t - \phi_s) = a\cos\omega_c t + b\sin\omega_c t$$

$$A^2 = a^2 + b^2 \; , \; \phi_s = \tan^{-1}\left(\tfrac{b}{a}\right)$$

$$e_o(t) = s_o(t) + n_o(t)$$
$$= [a + n_I(t)]\cos\omega_c t + [b + n_Q(t)]\sin\omega_c t$$
$$= r(t)\cos[\omega_c t + \phi(t)]$$

Single pulse
non-fluctuating

$$\phi(t) = \tan^{-1}\frac{b+n_Q(t)}{a+n_I(t)} = \tan^{-1}\frac{Y(t)}{X(t)}$$

$X(t)$ and $Y(t)$ are independent Gaussian r.v. with averages a and b

$$r(t) = \sqrt{X^2(t)+Y^2(t)}$$

$$p(X) = \frac{1}{\beta\sqrt{2\pi}}\exp\frac{-(X-a)^2}{2\beta^2}$$

$$p(Y) = \frac{1}{\beta\sqrt{2\pi}}\exp\frac{-(Y-b)^2}{2\beta^2}$$

$$\beta = \sqrt{\overline{n_o^2(t)}} = \sqrt{N_0 f_B}$$

$$p(X,Y) = p(X)p(Y) = \frac{1}{\beta^2 2\pi}\exp\frac{-(X-a)^2-(Y-b)^2}{2\beta^2}$$

Change of variables from $X(t)$ and $Y(t)$ to r and ϕ

$$p(r,\phi) = \frac{r}{2\pi\beta^2}\exp\frac{-\left(r^2+a^2+b^2-2ra\cos\phi-2rb\sin\phi\right)}{2\beta^2}$$

Single pulse
non-fluctuating

$$p(r) = \int_0^{2\pi} p(r,\phi)\,d\phi$$

Modified Bessel function

$$p(r) = \frac{r}{\beta^2} \exp\frac{-(r^2 + A^2)}{2\beta^2}\, \mathrm{I}_0\left(\frac{rA}{\beta^2}\right)$$

Rician distribution
(after S.O. Rice)

$$\frac{A^2}{2\beta^2} = SNR$$

No signal, noise only , $A=0$

$$p_{\text{noise}}(r) = p(r)\Big|_{A=0} = \frac{r}{\beta^2}\exp\frac{-r^2}{2\beta^2}$$

Rayleigh distribution

It is sometimes useful to normalize r :

$$z = \frac{r^2}{2\beta^2}$$

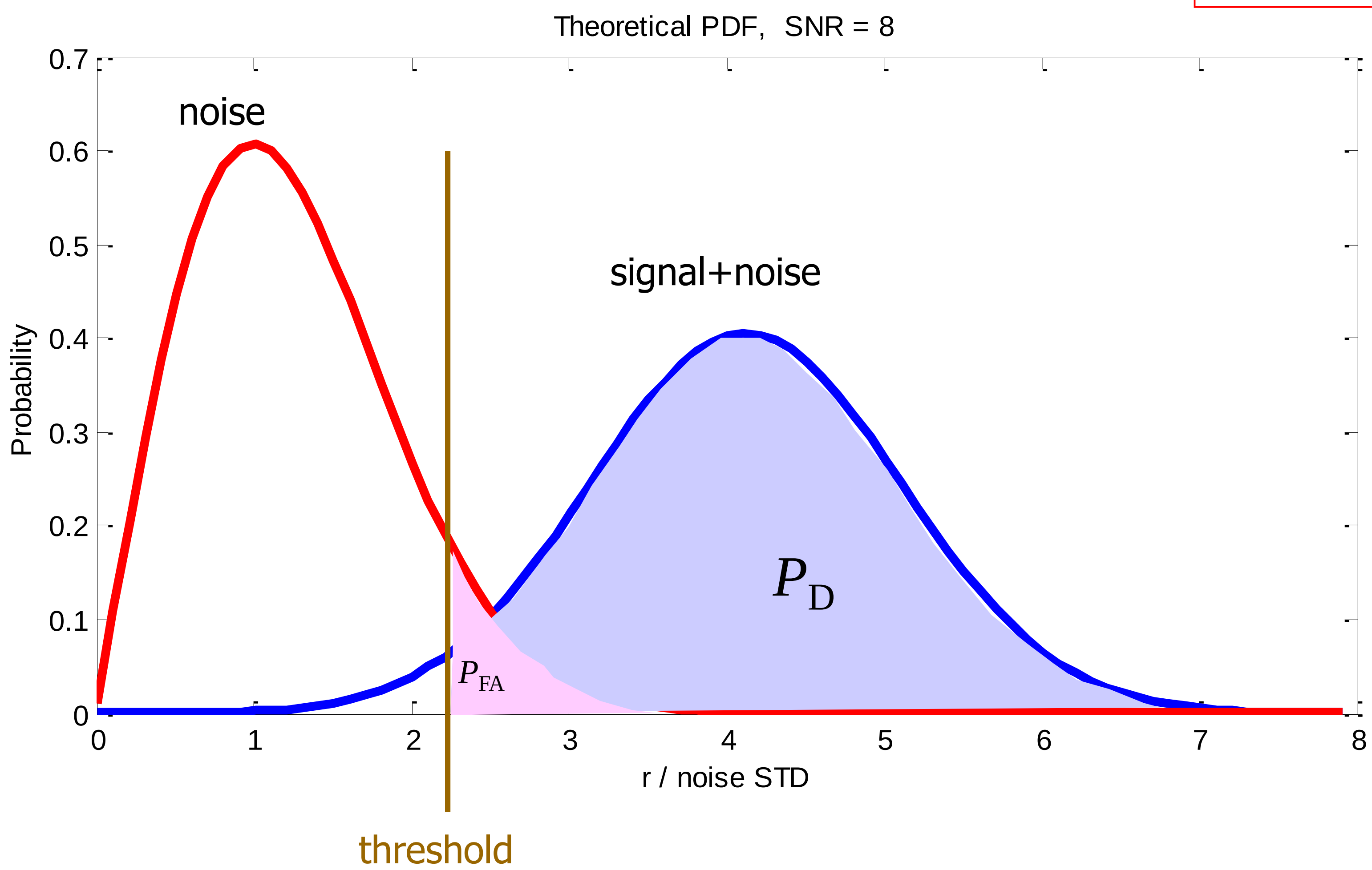
This is a mathematical tool because in practice we don't know β

$$p_{\text{noise}}(z) = p(z)\Big|_{A=0} = \exp(-z)$$

Exponential distribution

Nadav Levanon, Tel-Aviv University

Single pulse
non-fluctuating



Single pulse
non-fluctuating

P_{FA}, P_{D} as function of the threshold level V_{T}

$$P_{FA} = \int_{V_T}^{\infty} \frac{r}{\beta^2} \exp\frac{-r^2}{2\beta^2}\, dr = \exp\frac{-V_T^2}{2\beta^2}$$

P_{D} for $SNR >> 1$

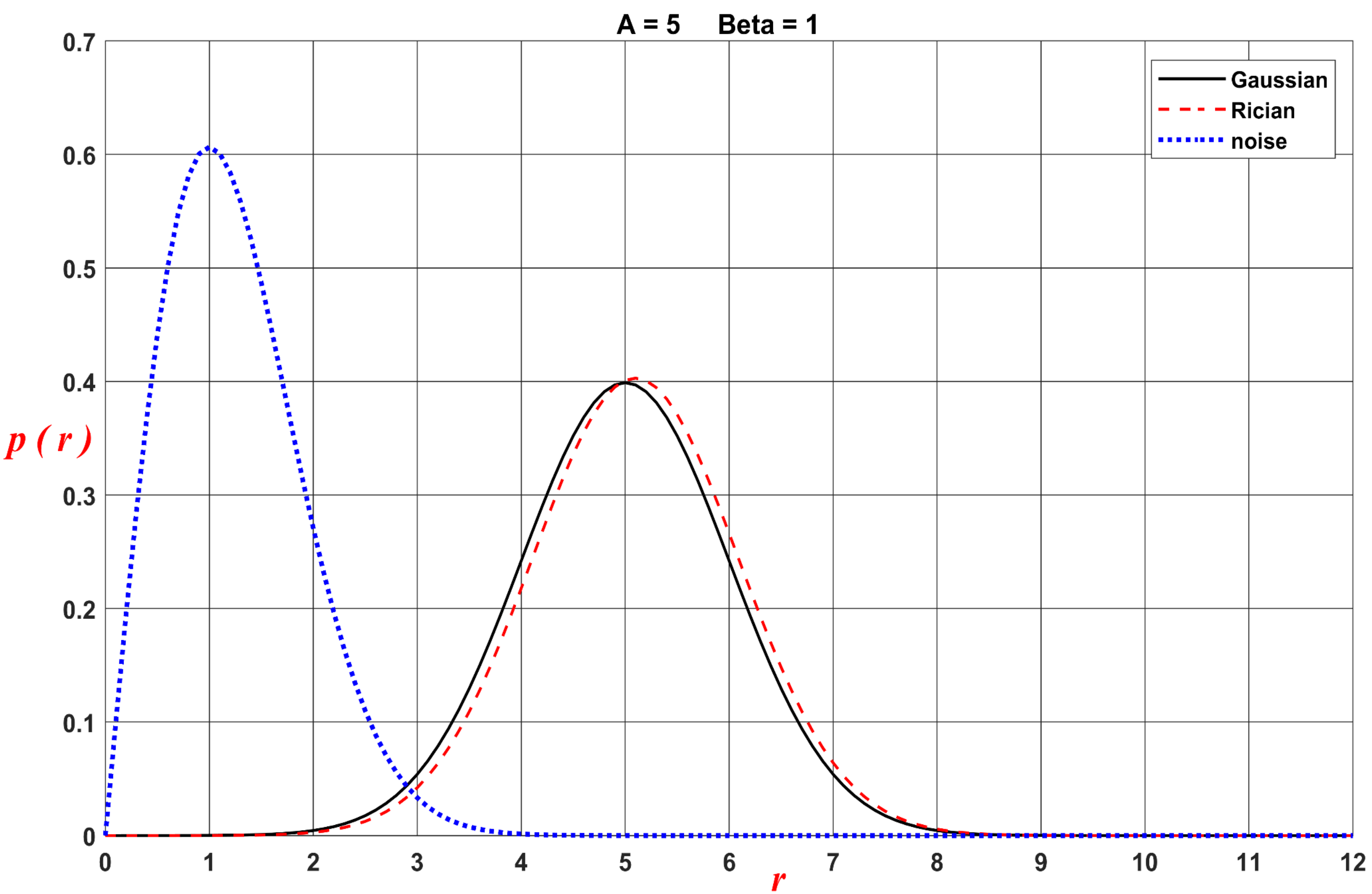
$$SNR = \frac{A^2}{2\beta^2} >> 1$$

$$r \approx A \Rightarrow rA \approx A^2$$

$$I_0(x) \approx \frac{\exp(x)}{\sqrt{2\pi x}} \quad , \quad x >> 1$$

$$p(r) = \frac{r}{\beta^2} \exp\frac{-\left(r^2 + A^2\right)}{2\beta^2}\, \mathrm{I}_0\left(\frac{rA}{\beta^2}\right) \approx \frac{1}{\beta\sqrt{2\pi}} \exp\frac{-(r-A)^2}{2\beta^2}$$

$$P_D = P\left(V_T < r < \infty\right) \approx \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{V_T}{\beta\sqrt{2}} - \sqrt{SNR}\right)\right]$$


A = 5 Beta = 1
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
p (r)
0 1 2 3 4 5 6 7 8 9 10 11 12
r
Gaussian
Rician
noise

$\mathrm{erf}(x)$ and $\mathrm{erfc}(x)$

- Integral of a Gaussian pdf means we need the *error function* $\mathrm{erf}(x)$ and *complementary error function* $\mathrm{erfc}(x)$:

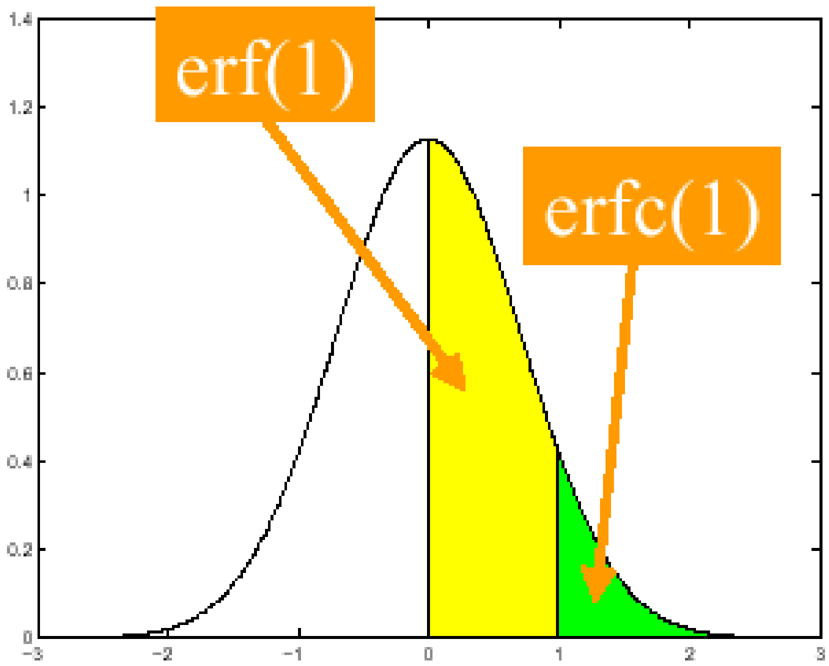
$$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\mathrm{erfc}(x) \equiv \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-t^2} dt = 1 - \mathrm{erf}(x)$$

$$\mathrm{erf}^{-1}(z) = \mathrm{erfc}^{-1}(1-z), \quad \mathrm{erfc}^{-1}(z) = \mathrm{erf}^{-1}(1-z)$$

Visual Definition of erf(x) and erfc(x)

- erf(x) is similar to the integral of the pdf of a Gaussian with zero mean and variance $1/\sqrt{2}$



$$p(x)=\frac{1}{\sqrt{\pi}}\exp\left\{-x^2\right\}=\frac{1}{2}\mathrm{erf}(x)$$

Albersheim's Equation

- Empirical, easily computable approximation to the Swerling 0/5 case
  - nonfluctuating target in white noise
  - linear detector
  - noncoherent integration
- Error in the estimate of SNR_1 is claimed to be less than 0.2 dB (0.4 dB if used for square law) for
  - $10^{-7} \le P_{FA} \le 10^{-3}$
  - $0.1 \le P_D \le 0.9$
  - $1 \le N \le 8096$

$$A = \ln\left(\frac{0.62}{P_{FA}}\right), \quad B = \ln\left(\frac{P_D}{1-P_D}\right)$$

$$SNR_1 = -5\log_{10} N + \left(6.2 + \left(\frac{4.54}{\sqrt{N+0.44}}\right)\right) \bullet \log_{10}\left(A + 0.12AB + 1.7B\right) \quad \text{dB}$$

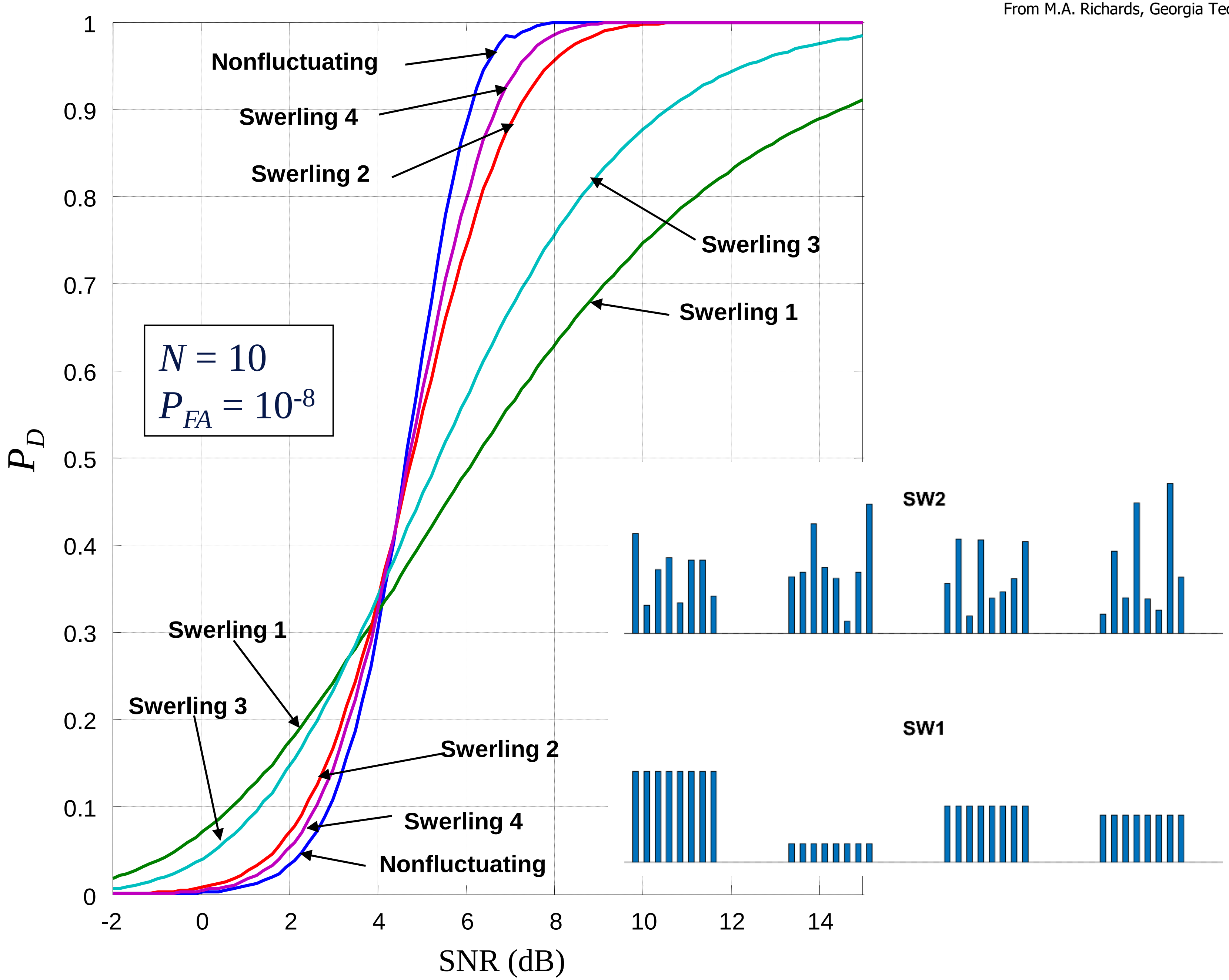
Albersheim, W. J. "A closed form approximation to Robertson's detection characteristics", *Proc. IEEE*, July 1981, p.839

A Better Approximation

- A newer approximation to required SNR that applies to:
  - square law detector
  - nonfluctuating target and all 4 Swerling models
  - additional chi-square target models (*e.g.* Weinstock)
- Accuracy claimed to be better than 1 dB for
  - $0.1 \leq P_D \leq 0.99$
  - $10^{-9} \leq P_{FA} \leq 10^{-3}$
  - $1 \leq N \leq 100$
  - better than 0.5 dB except near P_D extremes for Swerling 1

- D. A. Shnidman, "Determination of Required SNR Values," IEEE *Trans. Aerospace & Electronic Systems*, vol. 38(3), pp. 1059-1064, July 2002.
- D. A. Shnidman, "Update on Radar Detection Probabilities and Their Calculation," IEEE *Trans. Aerospace & Electronic Systems*, vol. 44(1), Jan. 2008, pp. 380 – 383.

Nadav Levanon, Tel-Aviv University
From M.A. Richards, Georgia Tech



Pioneering papers in detection:

J.I. Marcum “A statistical theory of target detection by pulsed radar”, RAND Research Memo, RM-754, December 1, 1947. + “Mathematical appendix” RAND RM-753, July 1, 1948.

Reprinted *IRE Trans. Information Theory*, vol. 6, no. 5, 1960, pp 59-267.

P. Swerling “Probability of detection for fluctuating targets”, RAND RM-1217, March 17, 1954.

Reprinted *IRE Trans. Information Theory*, vol. 6, no. 5, 1960, pp. 269-308.

P. Swerling “Radar probability of detection for some additional fluctuating target cases”, *IEEE Trans. AES*, vol. 33, no. 2, 1997, pp. 698-709.

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Simple and exact detection analysis is available for the case of:

Non-coherent integration of M pulses reflected from a Swerling 2 fluctuating target, using square-law detector.

M can also be =1. In that case the result applies to both SW 1 and SW 2.

Jess Ira Marcum

Jess Ira Marcum (originally Marcovitch) was born in Knoxville, Tennessee on December 30, 1919. His father had emigrated from Russia in 1904 at the age of 14 and was an entomologist and professor at the University of Tennessee. His mother, a librarian, had come from Austria in 1902. Both were fluent in Yiddish.

In 1947 he joined the highly respected Rand Corporation think-tank located at Santa Monica, California. He was not a founder of Rand, as reported in some articles, but by solving extremely difficult problems that no one else could, he became Rand's preeminent mathematician with an international reputation. This was an astonishing accomplishment in view of the fact he had only a BS degree and Rand was loaded with brainpower and Ph.D. mathematicians. His genius had emerged.

His principal interest at Rand was statistical physics with emphasis on nuclear radiation propagation, nuclear effects and the processing of electromagnetic wave data. His premiere accomplishment was formulating the original equations for calculating the radar backscatter from a steady target in a noisy background, the so-called signal-to-noise ratio on which all radar design is based. Since the government classified this work as secret, it was many years before the scientific literature could acknowledge it as the Marcum equation.

Jess became enchanted with the mathematical analysis of casino gambling around 1949. A few years later he left California for the casino world and lived in that environment for the next 15 to 20 years. Although he still consulted on scientific matters, gambling activities consumed most of his time

The equations that he developed for blackjack produced both a strategy for betting and algorithms for recording the running card count and translating the results into an actionable assessment. To my knowledge his notes have not survived, but it is safe to say that his feat has never been duplicated, Jess determined that his overall advantage against the casinos was about 3%, a number within the range of later results quoted for computer simulations with similar systems.

He won steadily. He tried to be as inconspicuous as he could, but before too long he began to attract the attention of casino management. The pit bosses hovered over him continuously,

Nine months passed and they were still baffled. Finally in desperation the Las Vegas casinos joined together and banned him from further play. He might be the first person ever banned anywhere for simply being too good at gambling. *No one* knows how much money he won during this, or any of his subsequent, blackjack ventures.

It was principally in this role as consultant to the industry that Jess continued his association with gambling long after his active blackjack days were over. Very late in his life he gained national notoriety from a consulting assignment that he undertook for Donald Trump and his associates in Atlantic City, as I will discuss briefly next.

On June 28, 1990 Jess called me from his hotel in Reno and mentioned that there was an article concerning him on that day's front page of *The Wall Street Journal*. It was entitled "Tale of a Whale: Mysterious Gambler Wins, Loses Millions" and it describes how Jess saved the Trump Plaza Casino from a mysterious gambler who, in his previous visit there, had won $6 million at baccarat.

Jess Marcum, Mathematical Genius, and the History of Card Counting

By Allan Schaffer, Ph.D.

(From *Blackjack Forum* Volume XXIV #3, Summer 2005)

http://www.blacjackformonline.com/content/JessMarcumEarlyDaysofCardCounting.htm