Two-Valued Frequency-Coded Waveforms with Favorable Periodic Autocorrelation

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There are known phase-coded (two-valued or polyphase) CW radar signals that exhibit perfect periodic autocorrelation function (PACF). A PACF is perfect when all its out-of-phase autocorrelation values are identically equal to zero. This paper investigates periodic, two-valued, frequency-coded signals. While none could be found with perfect PACF, we present examples with nearly perfect PACF. Their relationship to binary phase-coded signals is also considered. These signals should be attractive for CW radars because of their simple implementation, clean spectrum, and the favorable range response of their matched receiver.

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I. INTRODUCTION

Constant amplitude signals with periodic modulation waveform are used in CW radars. When they exhibit perfect periodic autocorrelation function (PACF), their range response is free of sidelobes and resembles the range response of a pulse train. Two-valued signals are easier to generate than multi-valued signal, and among two-valued phase-coded signals, those with binary envelope $\{+1,-1\}$ are the simplest to transmit. However, binary phase-coded signal suffer from two limitations: 1) there are no known binary signals longer than 4 that exhibit perfect PACF, and 2) their spectrum extends much beyond the inverse of the bit duration, with spectral sidelobes that decay slowly at a rate of 6 dB/octave.

At least two approaches are known for circumventing the first limitation: 1a) replace the binary phases $\{0^\circ, 180^\circ\}$ with two phase values with different spacing [1, 2], and 1b) transmit a binary signal, but at the receiver, cross correlate it with a mismatched signal [3, 2, 4]. The penalty for the first approach is the need to transmit a more complex signal. The penalty of the second approach is an SNR loss. We offer a third approach, in which we relax the requirement for perfect PACF, and allow small sidelobes at the vicinity of the mainlobe. The phase-coded signal remains binary, and the receiver remains matched.

In radar there are also at least two known methods to approximate phase-coded signals by frequency-coded signals: 2a) quadriphase coding [5, 2], and 2b) derivative phase modulation (DPM) [6, 7]. DPM resembles minimum shift keying (MSK) used in communications. Both transformations are used in pulse compression radar signals (not necessarily periodic), to reduce spectrum sidelobes. They are briefly described in Appendix A. We offer a different transformation here from phase coding to frequency coding, which is simpler, and yet works well for periodic signals.

The complex envelope of a phase-coded signal is defined by the duration t_b of a phase element (called bit) and by a sequence of complex numbers $\{c_n\}$, $n = 1,2,\ldots,N$. The PACF of a periodic repetition of such a signal [2] is straight lines, in the complex plane, connecting the PACF values at integer multiples of t_b. These values are given by

$$R(pt_b) = \frac{1}{N}\sum_{n=1}^{N} c_n c_{n+p}^{*}. \tag{1}$$

A PACF is considered perfect if it yields

$$R(pt_b) = \frac{1}{N}\sum_{n=1}^{N} c_n c_{n+p}^{*} = \begin{cases} 1 & p = 0 \ (\mathrm{mod}\ N) \\ 0 & p \neq 0 \ (\mathrm{mod}\ N) \end{cases}. \tag{2}$$

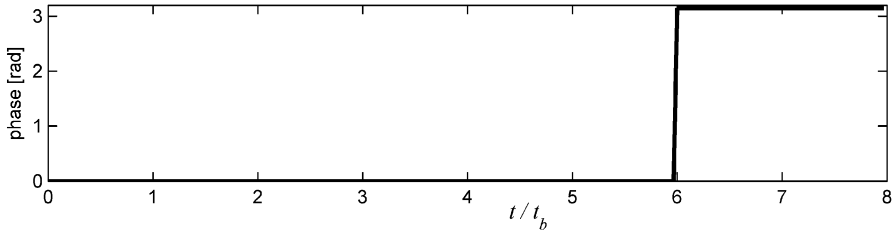

Fig. 1. Phase evolution of Barker 4 sequence (each element stretches over 2 bits).

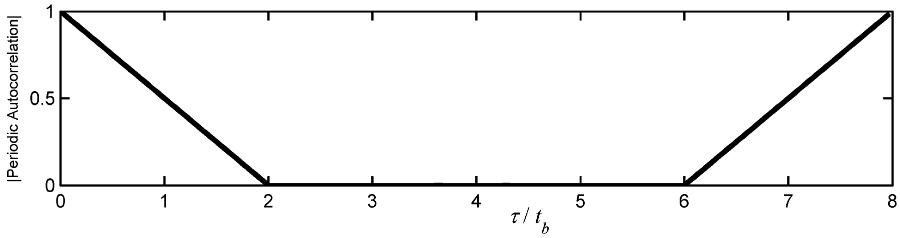

Fig. 2. PACF of Barker 4 sequence in Fig. 1.

The only known binary signal with perfect PACF is the Barker signal of length 4. It can be described by any cyclic shift of the sequence $\{c_n\} = \{1\,1\,1\,-1\}$ corresponding to the phase sequence $\{\phi_n\} = \pi\{0\,0\,0\,1\}$ through

$$c_n = \exp(j\phi_n). \tag{3}$$

There are other two-valued phase-coded signals, but not binary, that yield a perfect PACF [1, 2]. There are many polyphase signals that yield perfect PACF, e.g., Frank code [8], Lewis-Kretschmer P3 and P4 codes [9], and variations of them [10, 11]. An example of a quaternary code that yields a perfect PACF is described by the phase sequence

$$\{\phi_n\} = \frac{\pi}{2}\{0\,3\,1\,3\,0\,1\,1\,1\}. \tag{4}$$

Because we could not find long binary sequences or two-valued frequency-coded sequences that yield perfect PACF, in this paper we search for sequences that yield nearly perfect PACF. For a periodic complex envelope $u(t)$ with unit magnitude $|u(t)| = 1$ and period Nt_b, "nearly perfect PACF" implies

$$|R(\tau)| = \left|\frac{1}{Nt_b}\int_0^{Nt_b} u(t)u^*(t-\tau)dt\right| = \begin{cases} 1 & \tau = 0 \\ a(\tau) \ll 1 & k_0 t_b \le |\tau| \le kt_b \\ 0 & kt_b \le |\tau| \le (N-k)t_b \end{cases} \tag{5}$$

where τ is the delay modulo the period Nt_b, $\max[a(\tau)]$ is much smaller than 1, and $k_0 \le k < N/2$. We hope to find sequences where k is much closer to 1 than to $N/2$. In a perfect PACF $k = k_0 = 1$, and the gap, where the PACF is identically zero, extends over $t_b \le |\tau| \le (N-1)t_b$. In a nearly perfect PACF we would like the PACF to be identically zero over as large a gap as possible. Our "nearly perfect PACF" should not be confused with the PACF obtained with "almost perfect autocorrelation sequences" [12, 13]. That PACF exhibits a single, large, non-zero sidelobe at $\tau = t_b N/2$.

Thanks to the experience gained from quadriphase and DPM coding (see Appendix A), our main search was limited to the case in which the two frequency values were $\pm\Delta f = \pm 1/4t_b$, or 0 and $+2\Delta f = 1/2t_b$. Both choices are identical as far as the PACF is concerned. They differ in their effective carrier frequency. Forays into other frequency spacings were futile.

II. SIMPLE RELEVANT ANALOGIES BETWEEN PHASE AND FREQUENCY CODING

We begin our example by showing frequency-coded signals derived from the Barker 4 signal. We first show the Barker 4 signal (Fig. 1) and its PACF (Fig. 2), with the small modification in which each phase element stretches over 2 bits. Namely, the phase sequence of our "stretched" Barker is: $\{\phi_n\} = \pi\{0\,0\,0\,0\,0\,0\,1\,1\}$. The Barker 4 signal is compared with a frequency-coded sequence $\{f_n\} = 1/2t_b\{0\,0\,0\,0\,0\,1\,0\,1\}$ (Fig. 3) and its PACF (Fig. 4). Choosing the specific frequency separation of $1/2t_b$ causes a phase accumulation during one bit equal to

$$\Delta\phi = 2\pi\frac{1}{2t_b}t_b = \pi. \tag{6}$$

That specific phase accumulation creates the similarity to the "stretched" Barker 4 signal. Indeed, comparing the PACFs in Figs. 2 and 4 (top subplot) shows the similarity. The main difference is that while in the "stretched" Barker 4 the PACF value becomes identically equal to zero at $\tau/t_b = 2$, the PACF of the corresponding frequency-coded signal becomes

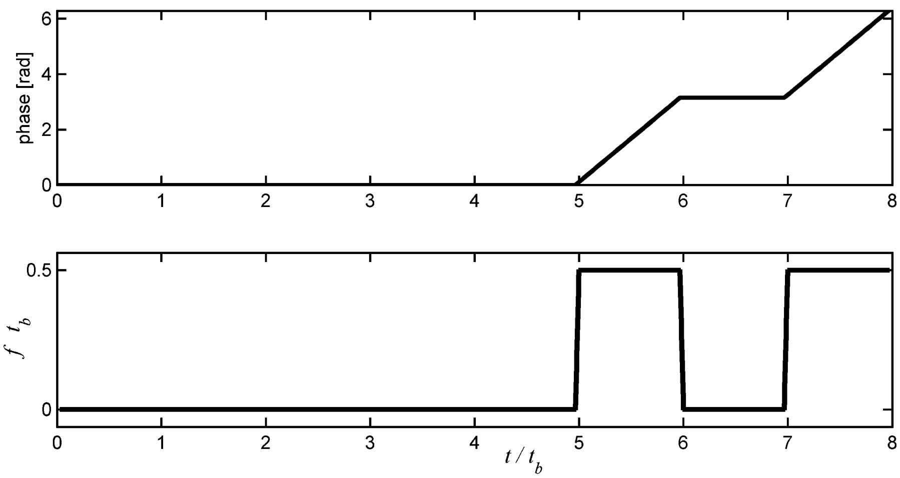

Fig. 3. Phase (top) and frequency (bottom) evolution of 8 element frequency-coded signal.

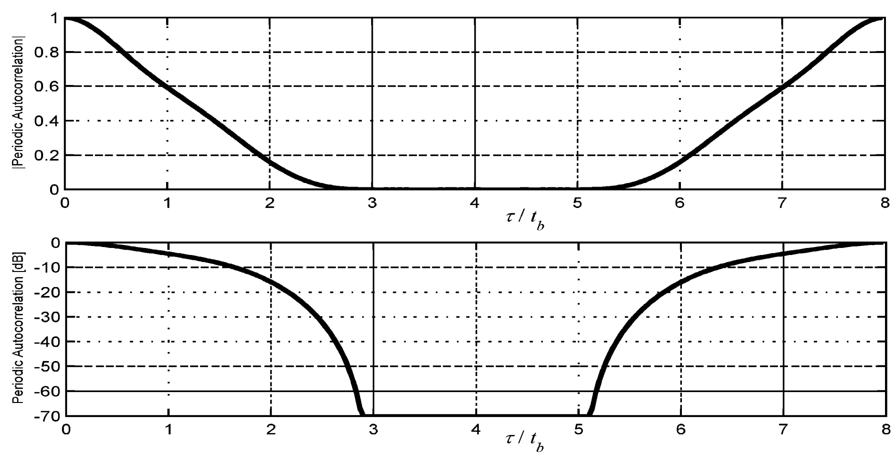

Fig. 4. PACF of frequency-coded sequence in Fig. 3. Top: linear scale. Bottom: dB scale.

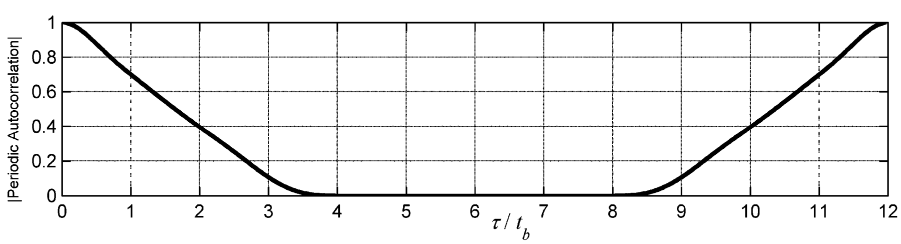

Fig. 5. PACF of frequency-coded sequence $1/2t_b\{000000001001\}$.

identically equal to zero at $\tau/t_b = 3$. The Barker 4 signal can be "stretched" by factors higher than 2. For example, using a stretch of 3 will result a frequency coded sequence $\{f_n\} = 1/2t_b\{000000001001\}$. Its PACF is shown in Fig. 5. Of course any cyclic shift of the sequence will yield an identical PACF.

We can generalize and state that following the Barker 4 signal, and its stretched versions, we can create a frequency-coded signal of length N that is a multiple of 4, in which the positive frequency bits will be located at $\{n, n+N/4\}\mathrm{mod}\ N$. The PACF of such a signal will reach a value of zero at a normalized delay of

$$\frac{\tau}{t_b} = \frac{N}{4} + 1. \quad (7)$$

The resulted PACF obeys the criteria outlined in (2) with the parameters $k = k_0 = 1 + N/4$. Having $k = k_0$ implies that there is only a wide mainlobe and no sidelobe pedestal.

Stretching the Barker 4 code is not going to produce signals that are significantly better than Barker 4 itself. A search for $N = 12$ found a phase-coded sequence $\{\phi_n\} = \pi\{000111001001\}$

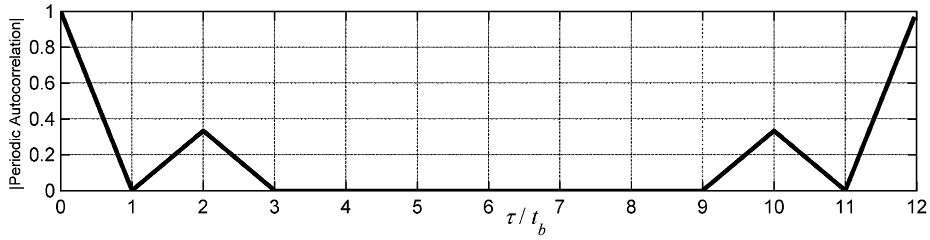

Fig. 6. PACF of phase-coded sequence $\pi\{001110010010\}$.

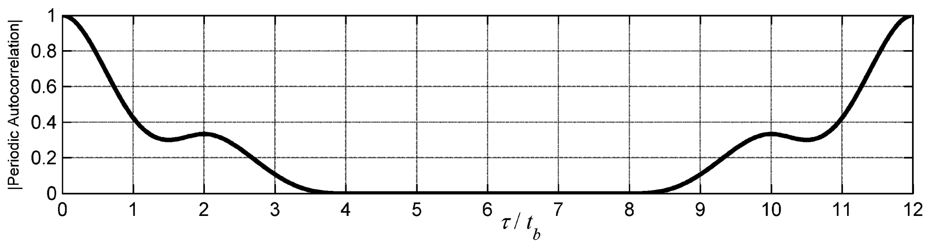

Fig. 7. PACF of frequency-coded sequence $1/2t_b\{001001011011\}$.

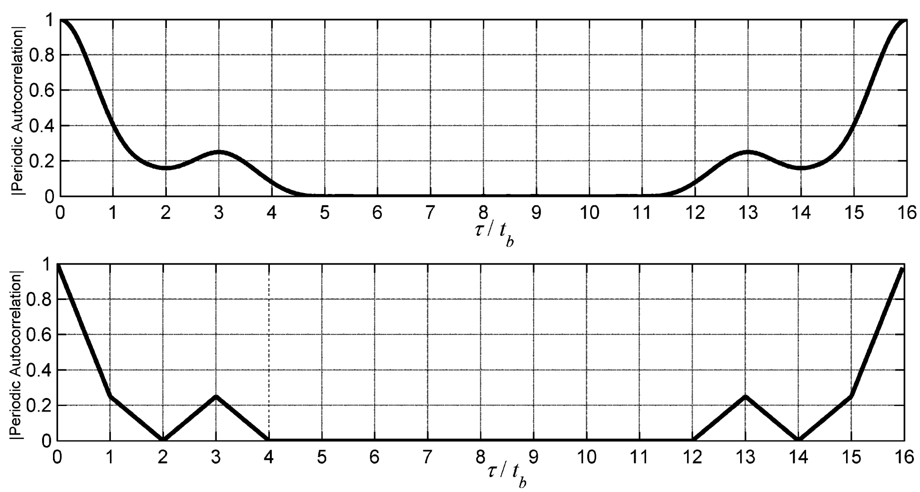

Fig. 8. Top: PACF of frequency-coded sequence $1/2t_b\{0010010100000111\}$.
Bottom: PACF of phase-coded sequence $\pi\{0011100111111010\}$.

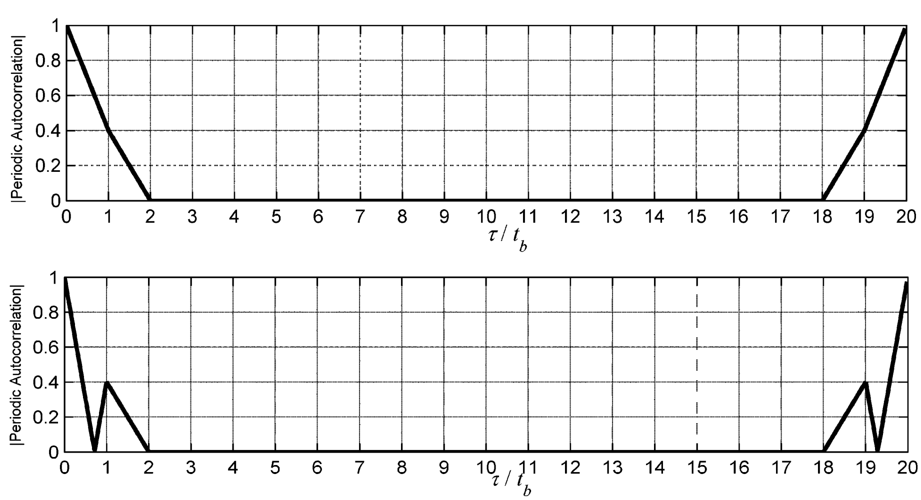

Fig. 9. PACF of phase-coded sequences. Top: $\{\phi_n\} = \pi\{00000000011100011011\}$.
Bottom: $\{\phi_n\} = \pi\{00011001001011010101\}$.

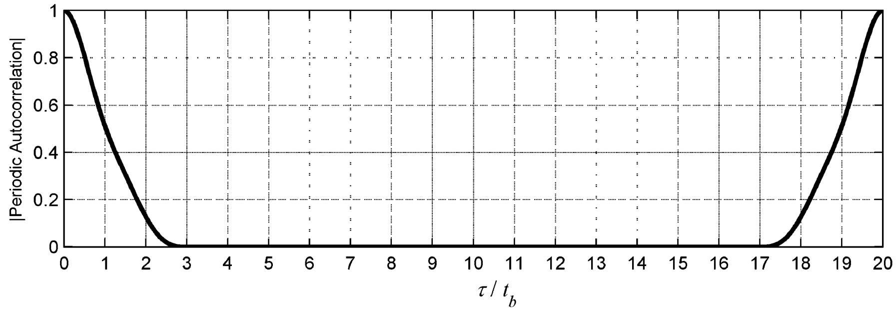

Fig. 10. PACF of 20 element two-valued frequency-coded sequence $\{2t_b f_n\} = \{00000000100100101101\}$.

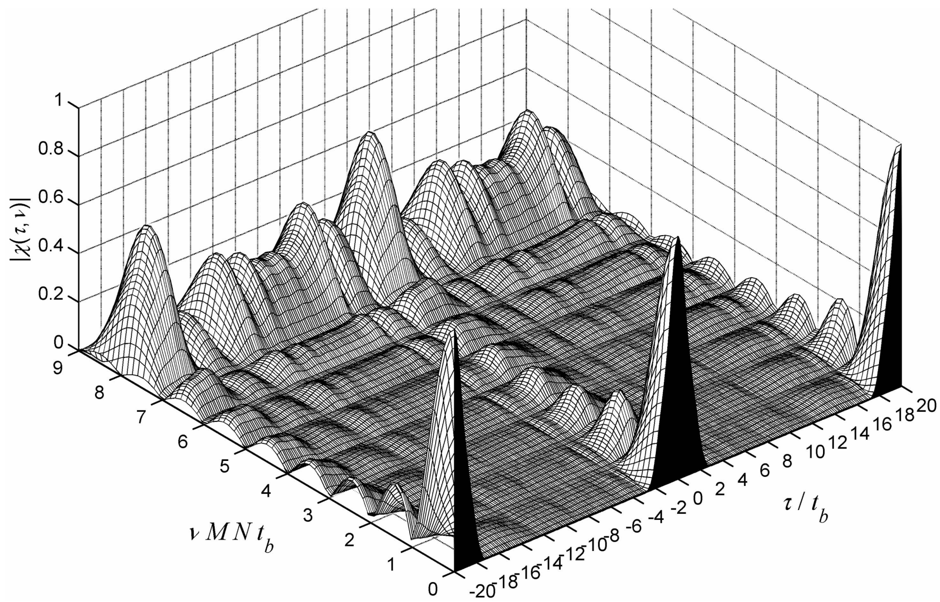

Fig. 11. Periodic ambiguity function of 20 element two-valued frequency-coded sequence $\{2t_b f_n\} = \{00000000100100101101\}$, with filter matched to $M = 8$ periods of the signal.

and its corresponding frequency-coded sequence $\{f_n\} = 1/2t_b\{001001011011\}$ that yield PACFs with a narrower mainlobe and a sidelobe pedestal. The PACFs of these sequences are shown in Figs. 6. and 7. The PACF in Fig. 7 is better than the PACF in Fig. 5, because its narrower mainlobe implies better range resolution.

While it is obvious that in both frequency and phase coding, all cyclic shifts, left/right flips, and bit reversals yield the same PACF, for longer phase-coded signals we often find two or more signals that yield exactly the same PACF, yet the signals do not poses any of the invariant permutations mentioned.

Stretching Barker 4 by a factor of 4 will yield an $N = 16$ frequency-coded signal with the expected PACF near-triangular mainlobe with no sidelobe pedestal. However a better PACF is obtained by many sequences, one of which is $\{f_n\} = 1/2t_b\{0010010100000111\}$. The corresponding binary phase sequence is $\{\phi_n\} = \pi\{0001110011111101\}$. Their PACFs appear in Fig. 8.

The relationship between binary phase-coded signals with phase sequence $\{\phi_n\}$ and frequency-coded signals with frequency sequence $\{f_n\}$, that yield similar PACF, is summarized by the following equations ($\oplus$ is xor or sum modulo 2) :

$$2t_b f_n = \frac{1}{\pi}\phi_n \oplus \frac{1}{\pi}\phi_{(n+1)\mathrm{mod}\ N} \tag{8}$$

or any cyclic shift of it. The inverse of (8) is

$$\frac{1}{\pi}\phi_{(n+1)} = \frac{1}{\pi}\phi_n \oplus 2t_b f_n. \tag{9}$$

So far the lengths of the given sequences were multiples of 4. There is a very simple explanation for that. The unnormalized PACF of any binary $\{+1, -1\}$ sequence of length N has a peak value N. A simple inspection will reveal that at non-zero shifts the PACF can only have values that are $N-4$, $N-8$, etc. (try: $++--++$). Since we look for PACF values that are mostly 0, this implies that N must be a multiple of 4. The next candidate is therefore length 20. Here we found a very favorable PACF, which is closer to the perfect than in the shorter lengths.

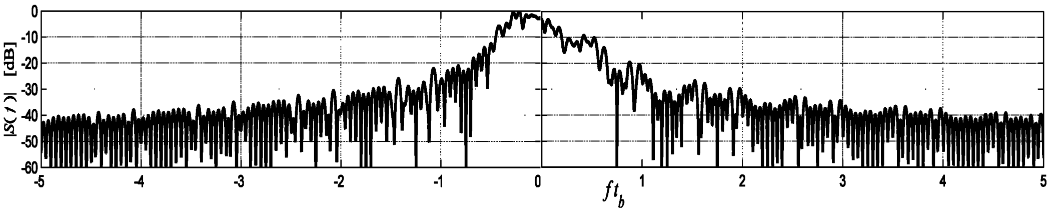

Fig. 12. Spectrum of complex envelope of 20 element two-valued frequency-coded sequence $\{1/2 + 2t_b f_n\} = \{00000000100100101101\}$.

TABLE I
Phase and Frequency Sequences that Yield Near Perfect PACF

N	$\{\phi_n/\pi\}$	$\{2t_b f_n\}$
12	000111001001	001001011011
16	0001110011111101	0010010100000111
20a	00000000011100011011	00000000100100101101
20b	00011001001011010101	00101011011101111111
24	000001000110011010010111	000011001010101110111001
28	0010011011111110101011100101	0110101100000111111000101111
32	00000000000111110001011001100011	00000000001000010011101010100101
36	000000001110010110110011000101010011	000000010010111011010101001111110101
40a	0000001111000011000011001111001100110011	0000010001000101000101010001010101010101
40b	0000000001010110010100100110111010010101	0000000011111010111101101011001110111111
40c	0000000000111101101100110000111000000111	0000000001000110110101010001001000001001
44a	00010001001010101010100100101110110100101001	00110011011111111111101101110011011101111011
44b	00000000000001110000100011110000011101110111	00000000000010010001100100010000100110011001
48a	000000011100100001110011111001110000001000100111	000000100101100010010100001010010000011001101001
48b	000000001001011010111001110010101000101000010111	000000011011101111001010010111111001111000111001
52a	0000000000000011100011001101100001111001010010101111	0000000000000100100101010110100010001011110111110001
52b	0000001001111110001101110000110010000101000101100111	0000011010000010010110010001010110001111001110101001

III. PACF FOR PHASE AND FREQUENCY CODED SIGNALS OF LENGTH $N = 20$

In the case of $N = 20$ we found two very interesting binary phase-coded signals with different PACFs that are equal only at the grid points. These are: $\{\phi_n\} = \pi\{00000000011100011011\}$ and $\{\phi_n\} = \pi\{00011001001011010101\}$. The two PACFs are plotted in Fig. 9.

The corresponding frequency-coded sequences are: $\{2t_b f_n\} = \{00000000100100101101\}$ and $\{2t_b f_n\} = \{00101011011101111111\}$. Despite the fact that these are quite different frequency codes, their PACFs are identical. One of them is shown in Fig. 10. For this special signal we also plotted (Fig. 11) the periodic ambiguity function (PAF) [2], when the receiver is matched to 8 periods of the signal. The performances of this signal are very close to a perfect PACF.

Fig. 12 shows the spectrum of the complex envelope of the 20 element signal whose PACF was plotted in Fig. 10. To avoid a large frequency bias the frequency coding was $\pm\Delta f = \pm 1/4t_b$. Note that the spectrum is still slightly shifted toward negative frequencies (a shift of about 0.1 in units of ft_b). This shift happens because the sequence has 14 elements of $-\Delta f$ and only 6 elements of $+\Delta f$.

IV. SIGNALS WITH $N > 20$

As N increases the search becomes more computational intensive. So far we have done exhaustive searches for N = 24, 28, 32, 36, 40, and 44, and partial searches for 48 and 52. In all these lengths we did not find frequency-coded sequences that yielded a PACF whose zero sidelobes begin after the third frequency bit. This is not a proof that $N = 20$ was the only length that yielded such a "perfect" PACF. For less perfect PACF, what is a good PACF is less obvious, allowing several interpretations.

Table I summarizes the results obtained for the various lengths. When more than one signal appears for a given length, it means that several good PACF were found and are worth presenting. The following plots demonstrate the different PACFs. We present PACF plots of the phase-coded sequences, because they are simpler (straight lines) and easier to interpret. Note, for example, that at delays equal to integer multiples of the bit duration, the lowest non-zero

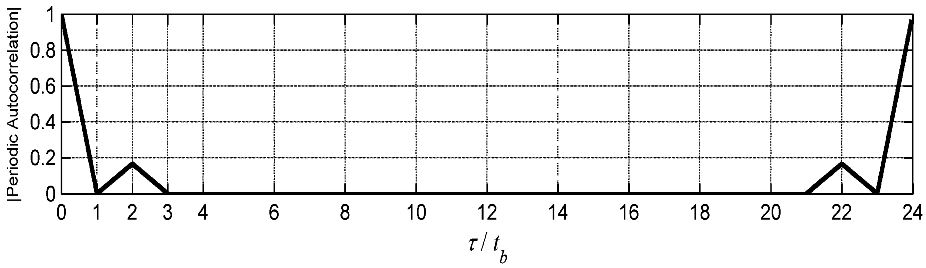

Fig. 13. PACF of 24 element binary phase-coded signal $\pi\{000001000110011010010111\}$.

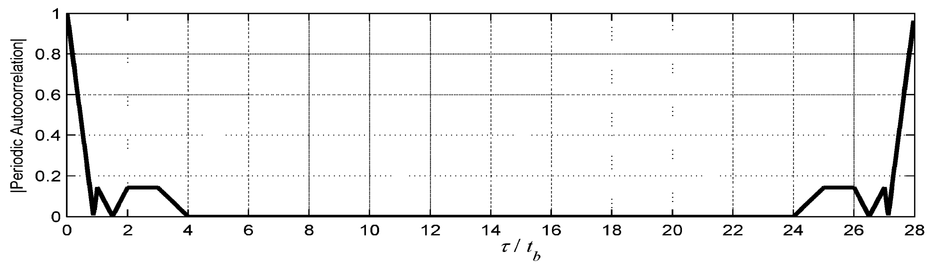

Fig. 14. PACF of 28 element phase-coded signal $\pi\{0100110111111010101111001010\}$.

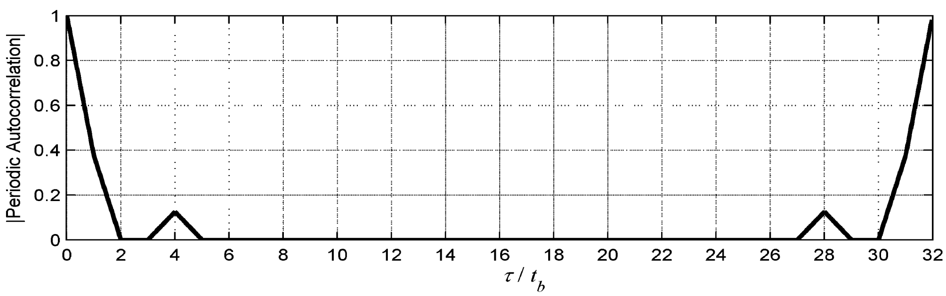

Fig. 15. PACF of 32 element phase-coded signal $\pi\{00000000000111110001011001100011\}$.

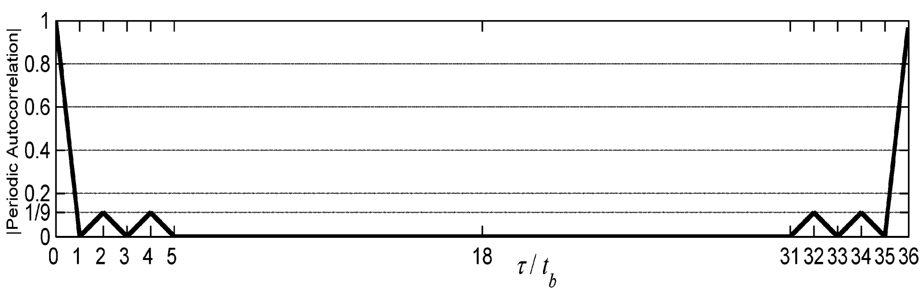

Fig. 16. PACF of 36 element signal $\pi\{000000001110010110110011000101010011\}$.

sidelobe must be $4/N$ and $4/N$ is also the smallest spacing between sidelobe values. PACFs of signals of length 24, 28, 32, and 36 appear in Figs. 13–16.

We use the 36 element phase-coded signal to demonstrate that the property of near perfect response holds when the reference signal is not exactly matched, but contains amplitude weighting. We use a Hamming window extended over 16 periods of the signal, namely over a total of $16 \times 36 = 576$ bit. The reference signal is shown in Fig. 17 and the delay-Doppler response is given in Fig. 18.

Comparing the zero-Doppler cut in Fig. 18, with the PACF in Fig. 16, demonstrates that adding amplitude weighting did not alter the near perfect response at zero-Doppler. As shown in [2, ch. 10] this holds true as long as the weight window extends over an integer number of periods. Comparing Fig. 18 with Fig. 11 shows that the inter-period Hamming weighting clearly lowers Doppler sidelobes.

The PACFs of three different binary signal of length 40 (Fig. 19), demonstrate a typical trade-off between the width and height of the PACF sidelobe pedestal. The height of the peak sidelobe in the top subplot is $16/40 = 0.4$, but the sidelobe pedestal reaches zero at $\tau/t_b = 4$. In the bottom subplot the peak sidelobe is $4/40 = 0.1$, but the sidelobe pedestal

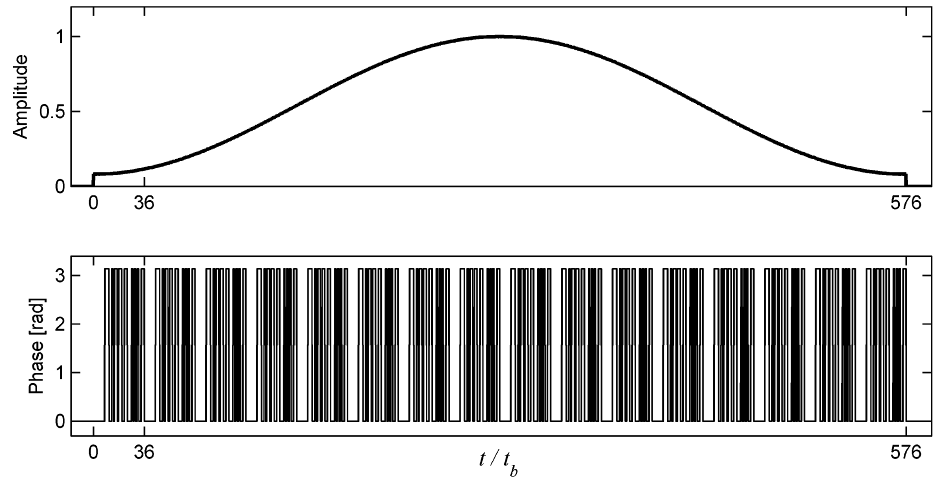

Fig. 17. Hamming weighted 16 periods of 36 element binary phase-coded signal.

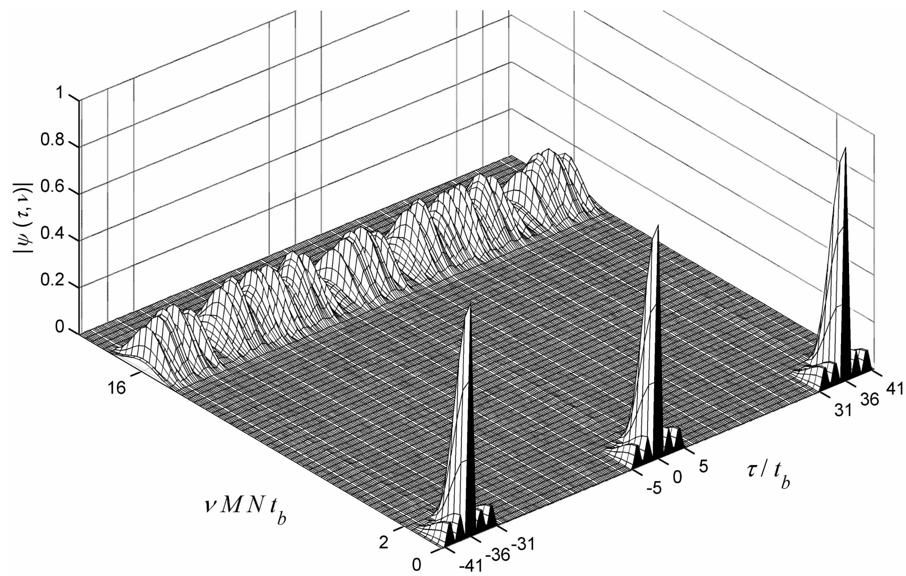

Fig. 18. Delay-Doppler response of periodic 36 element binary signal when mismatched reference signal extends over 16 periods and is Hamming weighted.

reaches zero at $\tau/t_b = 8$. The vertical scale of the bottom subplot uses ticks at 0.1 intervals in order to emphasize the sidelobe level of 0.1. Note in the signal whose PACF appears in the top subplot (signal 40a in Table I) that all the "1" or "0" runs in the sequence are of even length. This implies that this signal is a stretched version of a signal of length 20. Indeed, it is a stretched version of signal 20b in Table I. PACFs for signal lengths 44, 48, and 52 appear in Figs. 20–22.

V. SPECTRAL SHAPES

One motive for using frequency rather than phase modulation is the spectral shape. We use the $N = 36$ case to compare the spectrums of the binary phase-coded and its corresponding two-valued frequency-coded signals. The two spectrums are shown in Fig. 23. Indeed, the spectrum of the phase-coded signal (top) decays much slower than the frequency-coded signal (bottom). Around $fNt_b = 486$ or $ft_b = 13.5$ the spectral level of the frequency-coded signal is about 20 dB lower than the spectral level of the phase-coded signal. The better spectral shape of the frequency-coded signal is related to the smoother shape of its PACF.

VI. SUMMARY AND CONCLUSIONS

We showed that two-valued frequency-coded signals can yield near perfect PACF, when the code length N is a multiple of 4, and the two frequency values are $\pm\Delta f = \pm 1/4t_b$, or 0 and $2\Delta f = 1/2t_b$, where t_b is the duration of one element of the sequence. By near perfect PACF we mean a narrow mainlobe (2 bits or less) and a large gap in the center of the PACF, in which the sidelobes are identically

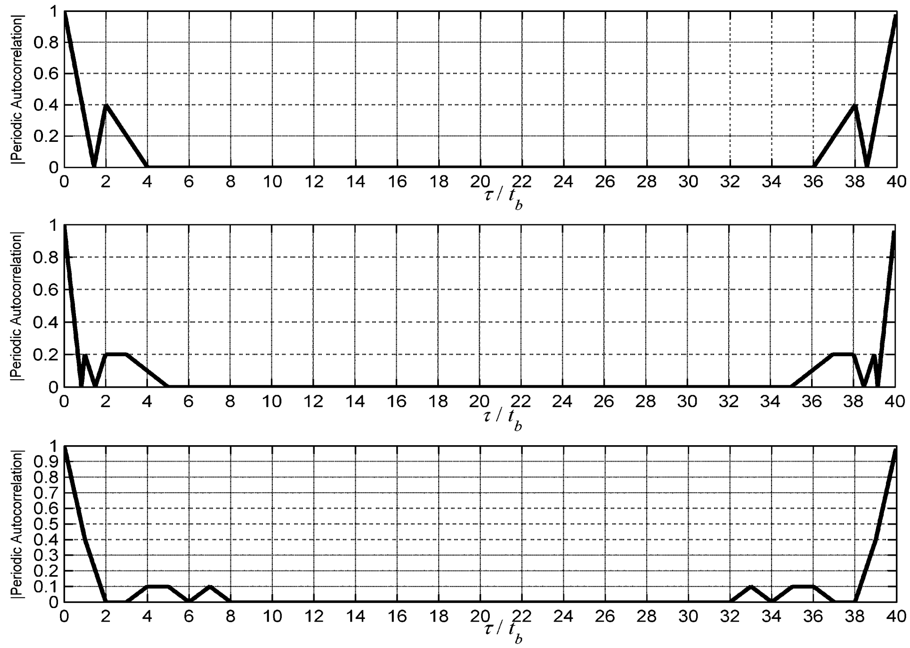

Fig. 19. PACF of the three 40 element binary signals in Table I. Top: $\pi\{0000001111000011000011001111001100110011\}$.
Middle: $\pi\{0000000001010110010100100110111010010101\}$.
Bottom: $\pi\{0000000000111101101100110000111000000111\}$.

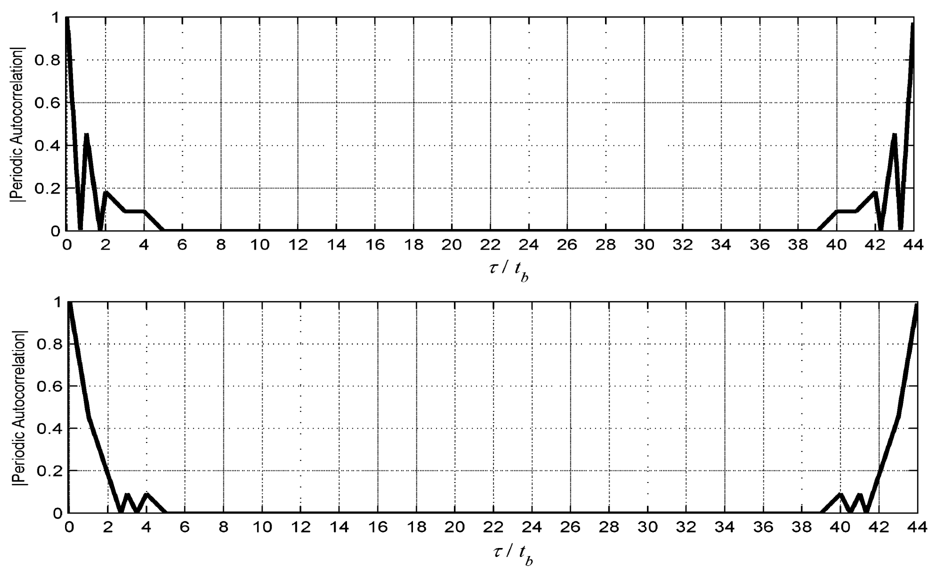

Fig. 20. PACF of the two 44 element binary signals in Table I.
Top: $\pi\{00010001001010101010100100101110110100101001\}$.
Bottom: $\pi\{00000000000011100001000111100000011101110111\}$.

zero. The gap duration is 2/3 of the sequence length or longer. Each frequency-coded signal of this type has a corresponding binary phase-coded signal, yielding a very similar PACF.

Exhaustive searches for such signals were conducted up to and including length 44. For lengths 48 and 52 the search was extensive but not exhaustive. The best signals found were listed in Table I, and their PACF were plotted. $N = 20$ yielded signals with PACFs that are the nearest to being perfect.

The PACF of the two types of signals (phase-coded and frequency-coded) are quite similar, with the PACF of the frequency-coded signal being smoother. That smoothness contributes to its improved spectral shape. The phase-coded signals should be attractive for CW radars because of their binary nature and because the near perfect range response is obtained with a matched receiver, with no SNR loss. The frequency-coded signals should be attractive because of their two-value

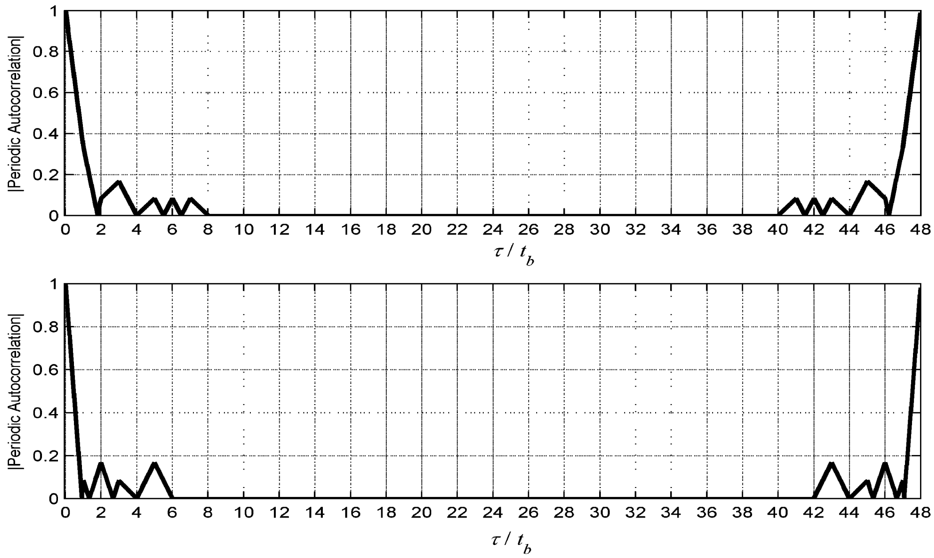

Fig. 21. PACF of the two 48 element binary signals in Table I.
Top: $\pi\{0000000111001000011100111110011100000001000100111\}$.
Bottom: $\pi\{000000001001011010111001110010101000101000010111\}$.

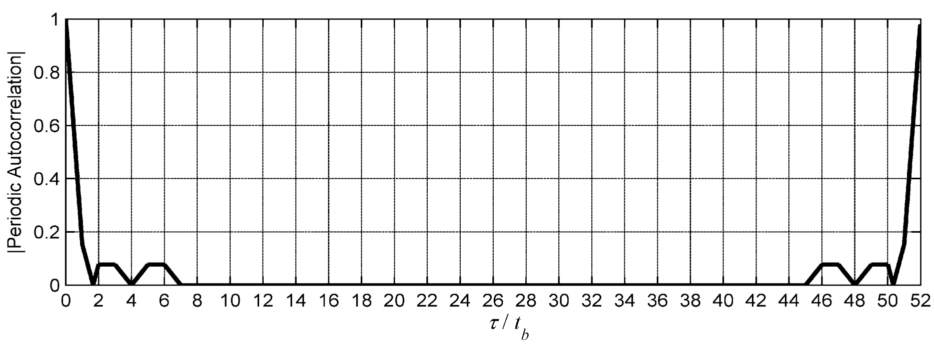

Fig. 22. PACF of second 52 element binary signal in Table I.
$\pi\{0000001001111110001101110000110010000101000101100111\}$.

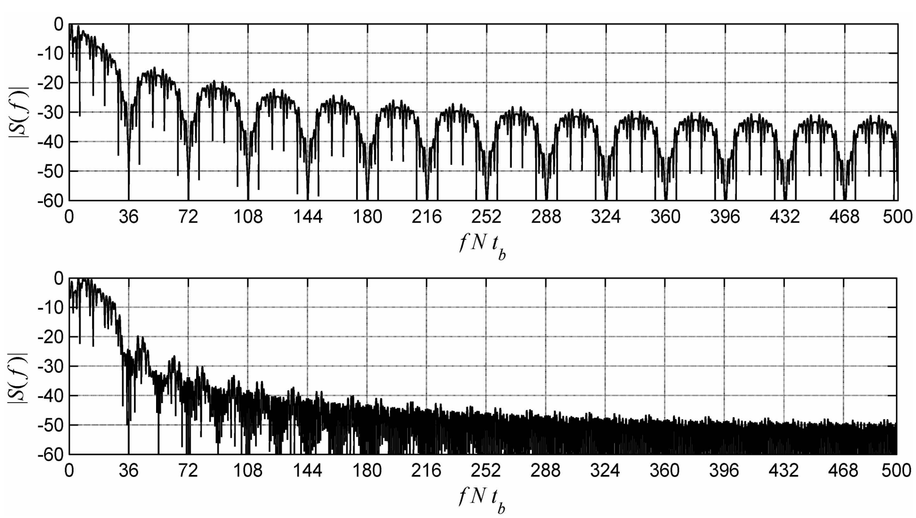

Fig. 23. Spectrums (in dB) of the 36 element signals. Top: Binary phase-coded signal. Bottom: Two-valued frequency-coded signal.

nature, the matched receiver, and their clean spectrum.

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APPENDIX A. QUADRIPHASE AND DPM

In constant-amplitude phase-coded radar pulse compression waveforms, there have been attempts to reduce bandwidth by replacing phase coding with frequency coding. Quadriphase [5,2] is one example of converting binary phase-coding into a two-valued frequency coding (although the first and last bit use a third frequency and are also amplitude modulated). DPM [6,7] is another approach in which binary phase coding is replaced by frequency coding. In DPM each element of the original binary code is divided into two bits, each of duration t_b, which are frequency shifted by either positive or negative value Δf given by $\Delta f = 1/4t_b$. Frequency coding in DPM is designed to achieve, at the end of each pair of bits, an accumulated phase change of 0 or π, corresponding to binary phase values. Zero phase accumulation is obtained when during the first bit of the pair the frequency step is $\Delta f = 1/4t_b$ yielding accumulated phase of $2\pi\Delta f t_b = \pi/2$, and during the second bit the frequency step is $-\Delta f$ yielding accumulated phase of $-\pi/2$; hence, zero total phase accumulation during a pair of bits. Phase accumulation of π (or $-\pi$) is achieved by maintaining the frequency step of $-\Delta f$ during both bits in the pair.

There are several variations to DPM and its relation to the original binary phase-coded sequence. In one variation an FM pair of $\{\Delta f, -\Delta f\}$ is used to represent the first element of a binary sequence, and whenever the current element is identical to the previous element. A $\{-\Delta f, -\Delta f\}$ FM pair is used when the current element is different from the previous element. The above rule implies that even bits are always coded by $-\Delta f$, while odd bits can have either negative or positive Δf shifts. It is interesting to note that in quadriphase coding the two frequency values are also $\pm\Delta f = \pm 1/4t_b$, but the relationship with respect to the original phase code is different than in DPM. The name quadriphase hints to the fact that during each bit the accumulated phase shift is $\pm\pi/2$, thus the phase (modulo 2π) at time instances corresponding to multiples of the bit duration can have four equally spaced values.

There is only limited analogy between two-valued frequency coding (with $\pm\Delta f = \pm 1/4t_b$) and quaternary phase coding (where the alphabet are the 4th roots of unity). In two-valued frequency coding the accumulated phase changes between time intervals equal to one bit duration must be $\pm\pi/2$. In quaternary phase coding there can also be contiguous identical phase values, as well as phase jumps of $\pm\pi$. For example, the quaternary phase-coding sequence $\pi/2\{0\,3\,1\,3\,0\,1\,1\,1\}$, which yields perfect PACF, cannot have a corresponding two-valued frequency-coded sequence.

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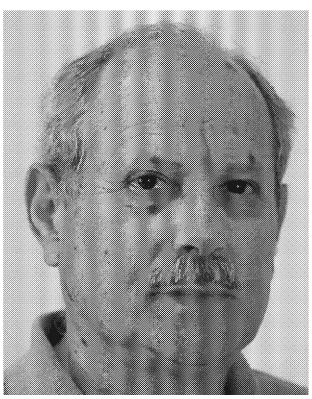

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