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CW Alternatives to the Coherent Pulse Train—Signals and Processors

We discuss CW signals, phase modulated by a periodic waveform, and their corresponding receivers. The combined response in delay and Doppler is almost identical to the (ideal) response of the coherent pulse train. The receivers are matched to an integral number N of modulation periods of the transmitted signal. CW implies a duty cycle of 100%. However, the signal duration need not be longer than $N + 2$ periods. The CW signals have the advantage that their peak power is equal to the average power. Their disadvantage is a more complicated receiver processing and the need for two antennas.

I. INTRODUCTION

The coherent pulse train (CPT) is one of the most important radar signals. It provides independent control of both delay and Doppler resolution. It also exhibits a range window which is inherently free of sidelobes.

The ambiguity function [1] represents the magnitude of the matched receiver output, in the delay-Doppler domain. The ambiguity fuction of the CPT indicates that the Doppler resolution is the inverse of the total duration of the signal—NT, while the delay resolution is the pulse duration $t_P = T/M$. (T is the pulse interval, N is the number of pulses processed coherently, and M is the inverse of the duty cycle.) The Doppler sidelobes can be reduced by utilizing nonuniform weights (e.g., Hamming) in the receiver. Such a receiver is not matched any more, and the penalties are a signal-to-noise ratio (SNR) loss (1.36 dB for Hamming), and a broadening of the mainlobe in the Doppler dimension. The (central part of the) response of a Hamming weighted receiver to the CPT signal discussed before, is shown in Fig. 1. The figure applies to a CPT signal with $N = 16$ and $M = 16$. In Fig. 1 and all the following three-dimensional (3-D) figures, the delay axis extends over $-3t_P \leq \tau \leq 2T + 3t_P$, and the Doppler axis over $-2/(NT) \leq \nu \leq (2N + 5)/(NT)$. The vertical axis is linear in voltage units.

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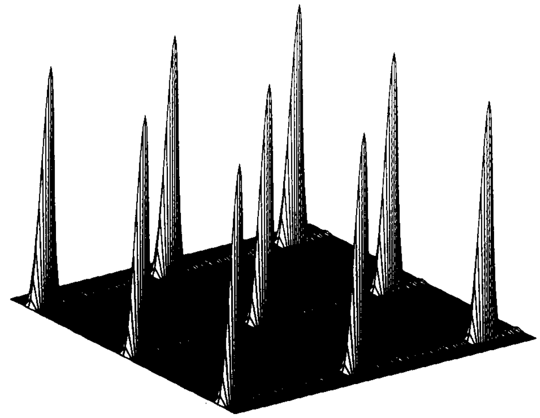

Fig. 1. Delay-Doppler response of Hamming weighted processor to CPT with $N = 16$ pulses and $M = T/t_P = 16$.

Ignoring the unavoidable ambiguity of T in delay and $1/T$ in Doppler, the receiver response in Fig. 1 approaches an ideal response. It is this ideal response that makes the CPT such an important radar signal.

The main drawback of the CPT signal is the high ratio of peak to average power. The average power is what determines the detection performances and the estimation accuracy of the parameters of the target. In order to maintain sufficient average power, the CPT signal usually requires high peak power, with all its implications: vacuum tubes, high voltages, heavy transmission lines, etc.

In angle modulated CW signals the peak-to-average power ratio is unit, which allows small solid-state transmitters. On the other hand, CW signals do not readily provide the desired range and Doppler resolution, as depicted in Fig. 1. We show that it is possible to find CW signals and corresponding receivers, whose combined response closely approach the ideal response shown in Fig. 1.

By CW signals we mean periodic signals with 100% duty cycle, i.e., uninterrupted transmission. We do not imply infinitely long signals. As we show later, the receiver processes N periods of the signal. In order to obtain the desired sidelobe-free response, the received signal has to fill up the processor. This is achieved by a dwell time of $N + 2$ periods, or longer.

II. PERIODIC AMBIGUITY FUNCTION

In a recent paper [2] we discussed the Doppler behavior of CW periodic coded sequences, when the receiver processor is matched to an integral number N of periods of the transmitted signal. We termed the response of such a receiver the "periodic ambiguity function" (PAF), and gave it the symbol $|\chi_{NT}(\tau,\nu)|$. (τ is delay and ν is Doppler frequency shift.) We showed in [2] that the PAF of N periods is obtained from the PAF of a single period by multiplying it with a universal function of the Doppler frequency ν:

$$|\chi_{NT}(\tau,\nu)| = g(\nu)|\chi_T(\tau,\nu)| \tag{1}$$

where

$$\chi_T(\tau,\nu) = \frac{1}{T}\int_0^T u\left(t+\frac{\tau}{2}\right)u^*\left(t-\frac{\tau}{2}\right)\exp(j2\pi\nu t)\,dt \tag{2}$$

$$g(\nu) = \left|\frac{\sin(N\pi\nu T)}{N\sin(\pi\nu T)}\right| \tag{3}$$

and $u(t)$ is the complex envelope of the signal and T is the duration of one sequence, namely, the period. The nature of the function $g(\nu)$ is such that for large N, the PAF is quenched to zero for all ν, except near $\nu = n/T$, $n = 0,\pm1,\pm2,\ldots$. For an infinitely large N, the function $g(\nu)$ becomes a train of impulses [3]. It follows from the behavior of $g(\nu)$ that, for large N, the PAF of a sequence exhibiting perfect periodic autocorrelation will strongly resemble the ambiguity function of a CPT. Here the term "perfect periodic autocorrelation" is used in the same context as in [2] to mean an autocorrelation of 1 when $\tau = 0$ (mod T) and zero everywhere else.

Note that $g(\nu)$ resembles a function of both τ and ν, found in the (nonperiodic) ambiguity function of a repetitive signal (see [1, sec. 6.4])

$$g(pT,\nu) = \left|\frac{\sin[(N-|p|)\pi\nu T]}{N\sin(\pi\nu T)}\right|. \tag{4}$$

There are many phase-coded sequences that exhibit perfect periodic autocorrelation. If such a sequence is M bits long, and the duration of each bit is $t_P = T/M$ then the zero-Doppler cut of its PAF will be identical to the ideal zero-Doppler cut of the ambiguity function of a CPT signal, whose pulse duration is t_P. Namely, zero everywhere except for triangles with base duration of $2t_P$ centered at delays which are multiples of T. The major difference between the ambiguity function of a CPT and the PAF of the CW signals is in the cuts at $\nu = n/T$, where n is an integer *different* from zero. In the CPT, these cuts are nearly identical to the zero Doppler cuts, namely a parallel sequence of "nails". In the PAF of the CW signal, they can get different forms: "nails", ridges, etc., depending on the particular sequence.

An important family of phase-coded periodic signals with perfect periodic autocorrelation was studied in [4] and [2]. Its importance stems from the fact that the signal utilizes only two phase values: 0 and ϕ. Hence, the two complex envelope values are 1 and β where

$$\beta = \exp(j\phi). \tag{5}$$

A subgroup of these sequences can be obtained from the maximal length shift register sequences [5]. As shown in [4], in these sequences the required phase is

$$\phi = \arccos\frac{1-M}{1+M}. \tag{6}$$

If more than two phases are permitted, then there are many more phase-coded signals that yield perfect periodic autocorrelation. A polyphase code which resembles the CPT signal more closely than the previous code, is the Lewis-Kretschmer P4 code [6]. In a P4 code of length M the phase sequence is given by

$$\phi_m = \frac{\pi}{M}(m-1)^2 + \pi(m-1), \qquad m = 1,2,\ldots,M. \tag{7}$$

III. MATCHED RECEIVER

A Doppler matrix receiver [7], matched to a signal of this kind, is demonstrated in Fig. 2. The particular signal is of length $M = 7$. The receiver is matched to N periods. The receiver contains a bandpass filter matched to a rectangular RF (or IF) pulse of duration t_P. In practice, a flat response filter of bandwidth $\approx 1/t_P$, will suffice. I and Q sampling at intervals of t_P, will produce digital representation of the complex envelope, with one sample per bit. Convolution with a complex conjugate replica of N periods of the signal is performed with the help of an MNt_P long delay line. For an output matched to a zero-Doppler signal, the MN products can be simply added together. If, however, weighting is desired, the M samples of each sequence must first be added. Then, each one of the N sums should be multiplied by its corresponding weight C_n, and the N results added again, to produce the output marked $\nu = 0$.

Note that with uniform weights the top output corresponds to the output of a receiver matched to the signal without Doppler shift. Hence, the output signal is identical to the ideal autocorrelation function, only when there is no Doppler shift. This is why the top output in Fig. 2 was marked as $\nu = 0$. When the received signal is Doppler shifted, the output waveform behaves like the corresponding cut of the PAF. In other words, with uniform weights the response of the output marked $\nu = 0$, in the delay-Doppler plane, is identical to the PAF. Doppler sidelobes can be reduced when the C_n are not uniform. A response, with Hamming weights, is shown in Fig. 3, for a two-valued sequence of $M = 15$ and when the receiver is matched to $N = 16$ consecutive sequences. We see in Fig. 3 two sidelobe-free periods. From that we can deduce that the received signal contained at least $N + 2$ periods.

The major difference between Fig. 3 and Fig. 1 is that the parallel sets of nails at $\nu = 1/T, 2/T$, etc. (found in the case of a CPT) are replaced by ridges. As a matter of fact, at the exact locations (off $\nu = 0$) where the ambiguity function of the CPT has its peaks, the PAF is inherently zero. It was proved in [2] that the

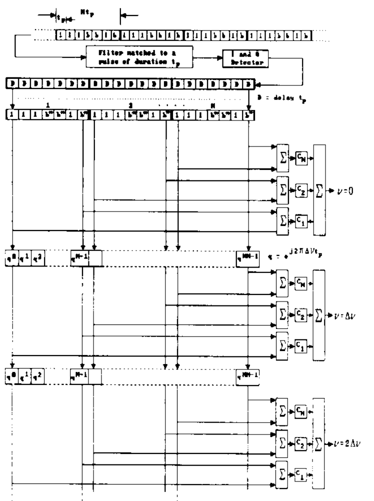

Fig. 2. Doppler matrix receiver matched to N periods of two-valued sequence of length $M = 7$, including weighting for Doppler sidelobe reduction.

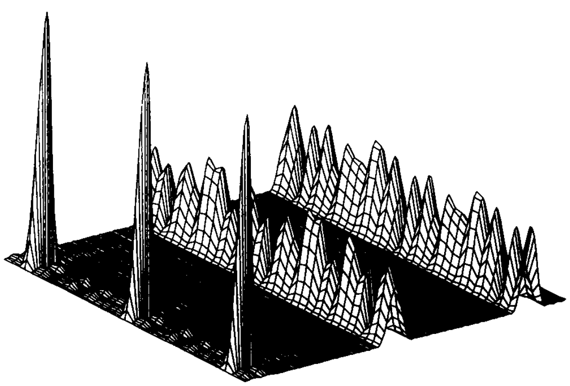

Fig. 3. Delay-Doppler response of Fig. 2 type receiver with Hamming weights, to a two-phase coded CW signal with perfect periodic autocorrelation ($M = 15, N = 16$).

cuts of the PAF at $\tau = nT$ are universal:

$$|\chi_{NT}(nT,\nu)| = \left|\frac{\sin(\pi\nu NT)}{\pi\nu NT}\right|. \qquad (8)$$

Equation (8) implies that the PAF gets a zero value at all the grid points $\tau = nT, \nu = m/T$ where $n = 0, \pm1, \pm2, \ldots, m = \pm1, \pm2, \ldots$.

The response of a receiver like the one in Fig. 2, with $N = 16$ and Hamming weights, to a P4 signal with $M = 16$, is shown in Fig. 4. The cut at $\nu = 1/T$ is almost identical to the cut at zero Doppler, but with a delay shift of one bit. The next cut is shifted one more bit, etc.

The two remaining outputs of the processor in Fig. 2, represent receivers matched to the signal when it is Doppler shifted by $\Delta\nu$ and $2\Delta\nu$. It is important to note that Doppler compensation must be performed for each bit, prior to adding them up. Hence, it is necessary to deal with MN samples. A processor for the corresponding CPT, has to deal with only N samples. The resulting additional processing is a major penalty for using a CW signal.

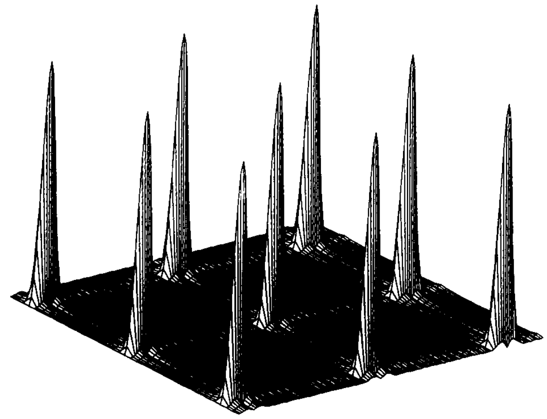

Fig. 4. Delay-Doppler response of Fig. 2 type receiver with Hamming weights, to P4 polyphase-coded periodic signal ($M = 16, N = 16$).

An attempt to reduce the processor complexity, by first adding the M samples of each sequence (thus skipping Doppler compensation within the sequence), then performing Doppler compensation only on the N sums, yields poor response. Doppler effectively multiplies each bit by a different (complex) weight. Summing an uncompensated sequence violates the rule requiring summation with uniform weight within the sequence, and does not yield the ideal zero sidelobe response. The nonideal response worsens in filters which correspond to higher Dopplers. The response of such imperfect processors is demonstrated in Fig. 5. It contains the response of the 0th, 2nd, and 4th Doppler processors to signals with their corresponding Doppler shifts, namely $\nu = 0$, $2/NT$ and $4/NT$. The 0th filter, with a zero Doppler shift input, does not require any Doppler compensation and maintains the ideal sidelobe-free response. Not so the higher filters. If they had Doppler compensation within the sequences they would have yielded the ideal response. Without that compensation, the sidelobe level increases with the order of the filter. The particular plot refers to a P4 signal. The P4 signal is rather Doppler tolerant, hence the loss of the mainlobe is small, and cannot be noticed in the figure. The exact losses are 0.22 and 0.91 dB for the 2nd and 4th filter, respectively.

The Doppler compensation can be implemented by a fast Fourier transform (FFT). The conclusion from the above discussion is that the FFT must operate on MN rather than N samples. Sidelobe reduction can be accomplished by weighting and summing consecutive Doppler outputs.

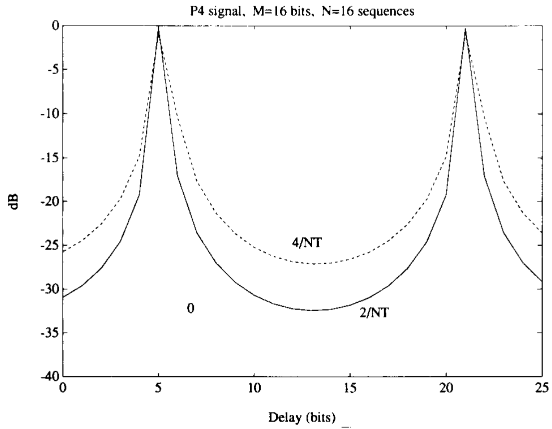

Fig. 5. Response of 0th, 2nd, and 4th imperfect filters, each to a signal shifted by respective Doppler frequency. (Imperfect filters lack Doppler compensation within the sequences.)

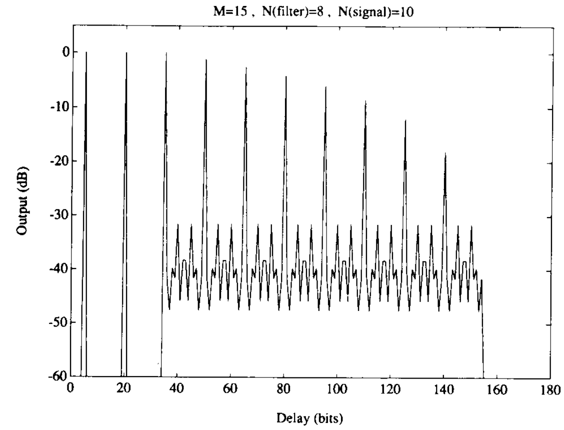

Fig. 6. Output response of filter matched to $N = 8$ periods of $M = 15$ bit two-valued sequence, to a corresponding *finite* signal of length $N = 10$ periods.

IV. DWELL TIME AND RANGE AMBIGUITY

In this section we discuss the minimum signal duration required in order to maintain the sidelobe-free response of the CW signals. Assume that the antenna beam dwells on a direction where the closest target is at a delay $0 < \tau_1 < T$, and the furthest target is at $pT < \tau_2 < (p+1)T$. An integer, p, is the order of the range ambiguity. In order to maintain the sidelobe-free response, a CW signal reflected from each target must fill the entire receiver delay line. Note also that the processor dumps its information about all the M range bins (including ambiguous range) during one full period, namely between $kT \leq t \leq (k+1)T$, where k is an integer.

Let transmission toward a given antenna direction begin at $t = 0$. The first complete period, during which the delay line is full with reflected signal from the closest target, is during $(N+1)T \leq t \leq (N+2)T$. In other words, with regard to the closest target, it suffices to transmit $N+2$ periods (sequences), and observe the processor output only during the last period, namely during $(N+1)T \leq t \leq (N+2)T$.

The furthest target, however, will fill the delay line only later. The first complete period, in which that target is present in the entire delay line, is during $(N + p+1)T \leq t \leq (N+p+2)T$. In order to guarantee that both targets fill the delay line, the radar must transmit $N+p+2$ sequences, and observe the processor output during $(N+p+1)T \leq t \leq (N+p+2)T$.

To sum up. At any given antenna beam dwell direction, when there is no range ambiguity, the transmitted signal duration should be $(N+2)T$, and the processsor output should be observed during the last period, namely during $(N+1)T \leq t \leq (N + 2)T$. For each range ambiguity, the signal should be prolonged by one period, and the processor observation time postponed by one period. Clearly, the signal durations mentioned above are lower limits.

What happens if the signal is shorter and does not fill up the delay line? A demonstration is given in Fig. 6. It was generated by sliding a signal containing 10 sequences of $M = 15$ bits each (plus at least 5 extra bits), through a processor matched to $N = 8$ sequences. At $t = 5$ the first complete 8 signal sequences are aligned with the processor delay line. While sliding the first $2M + 5$ bits, the output exhibits the mod T peaks, with no sidelobes in between. This is because the delay line remains full. While sliding the remaining 8 sequences, a certain sidelobe pattern repeats itself at the output, and the mod T peaks become gradually smaller. The signal used in generating Fig. 6 was a 15 bit long, two-valued signal. The sidelobe pattern is the nonperiodic autocorrelation of a single sequence, its absolute level (relative to the main peak) is a function of the filter length N.

V. CONCLUDING REMARKS

We showed that phase-coded periodic CW signals can yield a delay–Doppler response similar to the response of a CPT, within the Doppler range $|\nu| < 1/T$, where T is the period. As in CPT, the response is free of range sidelobes, and the Doppler resolution narrows as the number of periods (corresponding to the number of pulses in the CPT) processed coherently is increased. As in CPT, weight functions can be used to reduce Doppler sidelobes.

The delay-Doppler responses presented in this paper assumed ideal linear behavior of the receiver. Practical receivers do not maintain linearity over very large dynamic range. Another property of a CW radar is that at any given time, relatively strong signals are received. They can be the result of reflections from the antenna itself (if the same antenna is used for both transmitting and receiving), or through direct coupling between the transmitting and receiving antennas, or from close-range clutter. To get the response predicted by the PAF, the intensity of these signals must remain within the linear range of the receiver. In most cases, this could be achieved only by using two well-isolated antennas.

In conclusion, the main disadvantages of the CW signal are a more complicated receiver processor and a requirement for two antennas. The main advantage is the low peak RF power.

A note added at proof: The small sidelobe ridges off the zero Doppler cut will disappear if a smooth (changing from bit to bit) rather than staircase (changing from sequence to sequence) weight window will be implemented.

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On the Performance of Serial Networks in Distributed Detection

Distributed detection on the serial (or tandem) topology is considered with the probability of error performance criterion. Previously published efforts, while presenting probability of detection versus false alarm results, limited the number of array elements to two or three. Here, for the detection of known, equally likely signals in additive, symmetric noise, we present simple recursive expressions for the threshold values and the performance of the system. Examples for known signals in Gaussian and Laplacian noise show the degradation in performance due to the array structure.

I. INTRODUCTION

The classical problem in signal detection assumes that all of the data (with infinite resolution) is available at one central processing station; the classical solution is a threshold test of the likelihood ratio of the data. A more recent refinement of the problem, so-called decentralized or distributed detection, assumes that the observations occur at spatially separated sensors with the sensors connected in some network configuration with finite capacity communications channels. A substantial literature has recently developed on the design and performance of such systems (see e.g., Tsitsiklis [7]).

A few of the previous efforts in distributed detection have considered the so called serial, or tandem, network with binary communication between sensors [5, 6, 8, 9]. Fig. 1 shows such an array of n sensor stages. Each stage receives its own observation x_i, along with the output of the previous stage u_{i-1}, processing them to make its own decision u_i. The final decision is the output of the nth stage, u_n. Under the assumption of conditionally independent observations at each sensor, it has been shown that the optimum distributed detector consists of a likelihood ratio test, at each sensor, of the local data with a threshold dependent upon the output of the stage upstream. Letting Λ_i be the likelihood ratio of data x_i, the test of the ith stage is

$$\Lambda_i(x_i) \underset{u_i=0}{\overset{u_i=1}{\gtrless}} \alpha_i^{(u_{i-1})} \qquad (1)$$

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