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Numerically Efficient Calculations of Clutter Map CFAR Performance

An alternative expression for the false alarm probability of clutter map constant false alarm rate (CFAR), as derived by Nitzberg, is suggested. The new expression converges more rapidly.

Performance analysis of clutter map constant false alarm rate (CFAR) with exponential smoothing appeared recently [1]. Nitzberg has reached a closed-form solution for the case of a fluctuating target with a Rayleigh (amplitude) probability density function. Exponential smoothing refers to a recursive background estimate at a given cell, which can be described by

$$\hat{p}_n = (1-w)\hat{p}_{n-1} + wq_n; \qquad 0 < w < 1 \tag{1}$$

where $\hat{p}_n$ is the present background estimate, $\hat{p}_{n-1}$ is the previous estimate, q_n is the present input, and w is a weight coefficient. Target-present decision is made by comparing the recent input with the previous background estimate multiplied by a scaling factor c,

$$q_n > c\hat{p}_{n-1}. \tag{2}$$

For this implementation and for a fluctuating target with an average signal-to-noise power ratio ($\overline{\text{SNR}}$), Nitzberg's results can be summarized as follows: The false alarm probability is given by

$$P_{\text{FA1}} = \frac{1}{\prod_{m=0}^{M} [1 + cw(1-w)^m]}; \qquad M \to \infty \tag{3}$$

and the detection probability is given by a similar expression,

$$P_D = \frac{1}{\prod_{m=0}^{M} [1 + c_D w(1-w)^m]}; \qquad M \to \infty \tag{4}$$

where

$$c_D = \frac{c}{1 + \overline{\text{SNR}}}. \tag{5}$$

In order to reach numerical results, Nitzberg approximated the infinite product by an $M = 1000$ term product. Convergence was checked by comparing the 1000 term product to a 500 term product.

Faster convergence can be obtained if we observe (see Appendix) that the infinite product can be expressed as an infinite sum of products. Thus, (3) can be rewritten as,

$$P_{\text{FA2}} = \frac{1}{1 + \sum_{m=0}^{M} \prod_{k=0}^{m} \frac{cw(1-w)^k}{1-(1-w)^{k+1}}}; \qquad M \to \infty. \tag{6}$$

Replacing c with c_D will yield the expression for P_D.

The rate of convergence of the two expressions is demonstrated in Tables I–III. The denominator of (3) is termed $1/P_{\text{FA1}}$ and the denominator of (6) is termed 1/

TABLE I

M	$1/P_{\text{FA1}}$	$1/P_{\text{FA2}}$
4	1834.56	48742.17
6	12273.00	285367.29
8	49882.46	708222.57
10	135921.54	995148.89
16	600187.33	1081201.47
20	845701.98	1081208.40
36	1073658.99	1081208.41
100	1081208.40	1081208.41

Note: $w = 0.2$, $c = 27$.

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TABLE II

M	$1/P_{FA1}$	$1/P_{FA2}$
4	168263.74	412583.39
6	593708.92	956782.72
8	888429.93	1028154.34
10	991150.32	1028884.28
16	1028280.59	1028884.78
20	1028847.01	1028884.78
36	1028884.78	1028884.78
100	1028884.78	1028884.78

Note: $w = 0.5$, $c = 77$.

TABLE III

M	$1/P_{FA1}$	$1/P_{FA2}$
4	906592.63	988952.00
6	1005238.31	1009730.95
8	1009551.85	1009732.25
10	1009725.04	1009732.25
16	1009732.25	1009732.25
20	1009732.25	1009732.25
36	1009732.25	1009732.25
100	1009732.25	1009732.25

Note: $w = 0.8$, $c = 349$.

P_{FA2}. The three tables correspond to three weights 0.2, 0.5, and 0.8. The corresponding scaling factors were selected to yield $1/P_{FA} \approx 10^6$.

Up to the least significant digit presented in the tables, the same results were obtained with single precision (60 bit) and double precision (120 bit) on the CDC-CYBER 180-990/NOS as well as with double precision (64 bit) of a VAX 750 computer. Single precision (32 bit) on the VAX yielded convergence to results which differed by a factor of $< 10^{-6}$.

The three tables demonstrate that $1/P_{FA2}$ converges at about one-half the number of terms M, at which $1/P_{FA1}$ converges. It may be argued that (3) involves a single M term product, while (6) involves a sum of M products of increasing length, the longest being an M term product. However, what tasks the computer most is the highest power into which $(1 - w)$ is raised. That power is M in (3) and $(M + 1)$ in (6).

It is therefore recommended to use (6) rather than (3) as the preferred expression for the false alarm probability. For the much higher typical detection probability, both expressions converge very rapidly.

The format of (6) has another advantage over that of (3). It yields an expression of the limit of P_{FA} when $w = 0$, which corresponds to the fixed threshold case. We note that

$$\lim_{w \to 0} \frac{cw(1-w)^k}{1 - (1-w)^{k+1}} = \frac{c}{K+1} \,. \tag{7}$$

Using (7) in (6) we get

$$\lim_{w \to 0} P_{FA} = \frac{1}{1 + c + \frac{c^2}{2!} + \frac{c^3}{3!} + \cdots} = \exp(-c) \tag{8}$$

which is the well known result for the non-CFAR case. Replacing c with c_D will yield the detection probability of the non-CFAR, single-pulse, Rayleigh fluctuating target.

APPENDIX

The denominator of (3) has the form

$$D = \prod_{m=0}^{\infty} (1 + ab^m) = (1 + ab^0)(1 + ab^1)(1 + ab^2)(1 + ab^3) \ldots . \tag{9}$$

Performing the multiplications we get

$$\begin{aligned} D = 1 &+ a(b^0 + b^1 + b^2 + \cdots) \\ &+ a^2[b^0(b^1 + b^2 + b^3 + \cdots) \\ &+ b^1[b^2 + b^3 + b^4 + \cdots) + \cdots] \\ &+ a^3[b^0 b^1(b^2 + b^3 + b^4 + \cdots) \\ &+ b^0 b^2(b^3 + b^4 + b^5 + \cdots) + \cdots \\ &+ b^1 b^2(b^3 + b^4 + b^5 + \cdots) \\ &+ b^1 b^3(b^4 + b^5 + b^6 + \cdots) + \cdots \\ &+ \cdots] \\ &+ a^4[b^0 b^1 b^2(b^3 + b^4 + b^5 + \cdots) + \cdots] \\ &+ \cdots . \end{aligned} \tag{10}$$

Making use of the equality

$$1 + b^k + b^{2k} + b^{3k} + \cdots = \frac{1}{1 - b^k} \tag{11}$$

we get after some manipulations

$$D = 1 + \frac{a}{1-b} + \frac{a}{1-b}\frac{ab}{1-b^2} + \frac{a}{1-b}\frac{ab}{1-b^2}\frac{ab^2}{1-b^3} + \cdots \tag{12}$$

which can be written as

$$D = 1 + \sum_{m=0}^{\infty} \prod_{k=0}^{m} \frac{ab^k}{1 - b^{k+1}} \,. \tag{13}$$

NADAV LEVANON

Dep't. of Electronic Systems
Tel-Aviv University
P.O. Box 39040
Tel-Aviv 69978
Israel

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