Multicarrier radar signals with low peak-to-mean envelope power ratio

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Abstract: A multicarrier phase-coded (MCPC) pulse consists of N sequences of M bits each. The N sequences modulate N carriers that are transmitted simultaneously. The carriers are equally spaced. The carrier separation Δf equals the inverse of the bit duration t_b forming orthogonal frequency division multiplexing (OFDM). A family of MCPC signals is described, based on modulating all carriers with the same sequence while also phase-modulating the carriers to lower the peak-to-mean envelope power ratio (PMEPR). The new MCPC signals have a mainlobe width of $\sim t_b/N$ and near sidelobes extending only as far as t_b, followed by a zero sidelobe gap. The frequency spectrum of the new MCPC signals is compared with other low PMEPR MCPC signals and with single-carrier (fix envelope) phase-coded waveforms exhibiting the same compression ratio.

1 Introduction

A multicarrier phase-coded (MCPC) pulse consists of N carriers transmitted simultaneously. Each carrier is phase modulated using a sequence of M bits. Some design principles of single MCPC pulse, MCPC CW signals and MCPC pulse train yielding favourable correlation properties were described in [1–3]. An important advantage of the MCPC signal is the limited frequency spectrum. The effective bandwidth is N/t_b (without weighting) and the spectral sidelobes are very low compared to the sinc-squared spectrum of single-carrier phase-coded radar signals (where, for the same compression ratio, the null-to-null spectral mainlobe width is twice as wide, and the first sidelobe is at -13 dB).

A major drawback of the multicarrier signal is its varying envelope. If the signal generator contains a power amplifier it becomes desirable to reduce the peak-to-mean envelope power ratio (PMEPR) as much as possible. A method for designing MCPC signals with low PMEPR was introduced in [1–2]. The method is based on using consecutive ordered cyclic shifts (COCS) of an ideal sequence with quadratic phase (an ideal sequence is a sequence with zero periodic autocorrelation for all non-zero shifts). Zadoff [4] first described ideal sequences with quadratic phase for any length. The ideal sequences with quadratic phase will be referred to for brevity as 'chirp-like ideal sequences'. The PMEPR of the COCS of chirp-like ideal sequence MCPC signals is low (less than two) for a large span of N and M, and the signal yields a correlation function with low sidelobes.

The problem of the minimising the PMEPR of a multicarrier (or multitone) signal has been addressed in the context of instrumentation and measurement. In that field, it is common to test a system by injecting a multicarrier signal into the systems input. Looking at the output teaches about the system transfer function. It is often required that the PMEPR of the input signal be as low as possible (e.g. to avoid saturations or other nonlinearities of the tested system) [5–7].

This paper describes a family of MCPC signals where the sequences modulating all carriers are identical. When using identical sequences, the magnitude of the complex envelope is a function of only the carriers' initial phase and amplitude. This allows using the carriers' phases to lower the PMEPR. For brevity we will refer to MCPC signals where all carriers are modulated by the same sequences as 'identical sequence MCPC signals' or 'IS MCPC signals'.

2 Complex envelope of an IS MCPC

The complex envelope of a single MCPC pulse where all N carriers are modulated by the same sequence with length M is given by

$$g(t) = \sum_{n=1}^{N}\sum_{m=1}^{M} w_n a_m s[t-(m-1)t_b] \times \exp\left[2\pi j\left(n-\frac{N+1}{2}\right)\frac{t}{t_b}\right] \quad 0 \le t < Mt_b \quad (1)$$

where a_m is the mth element of the sequence modulating all carriers ($|a_m| \equiv 1\ \forall 1 \le m \le M$), w_n is the complex weight associated with the nth carrier and $s(t) \equiv 1$ for $0 \le t < t_b$ and zero elsewhere.

The real envelope (magnitude of the complex envelope) of the IS MCPC signal mth bit is given by

$$|g(t)| = |a_m|\left|\sum_{n=1}^{N} w_n \exp\left(j2\pi n\frac{t}{t_b}\right)\right| = \left|W\left(\frac{t}{t_b}\right)\right| \quad (m-1)t_b \le t < mt_b \quad (2)$$

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where $W(t)$ is the inverse discrete Fourier transform of the complex carrier weights w_n and is periodic with period 1. Note that the real envelope is identical for all bits, independent of the sequence used to modulate all the carriers, and is a function of only the complex carrier weights. The problem of minimising the peak-to-mean envelope power ratio (PMEPR) becomes a problem of minimising the PMEPR of a multicarrier bit or a multitone signal.

Several phasing schemes, giving the carrier initial phases in closed form such that the PMEPR of a multicarrier signal is low, have been described ([7] and the references therein). Their advantage is that the closed-form construction rule allows us to easily generate multicarrier signals with a varying number of carriers. Their common disadvantage is that while the resulting PMEPR is low, it is not close enough to the lowest achievable value. Note that some of the methods are applicable for any carrier amplitude weighting [5] while others are derived only for equal carrier amplitude weights [8, 9].

When it is desired to further reduce the PMEPR, the carrier phasing can be optimised using iterative numerical methods. One such method is an iterative time-Fourier domain clipping approach [6]. Other iterative methods and a comparison between iterative and closed form methods can be found in [7]. For a flat spectrum multicarrier signal, PMEPR values as low as 1.17, 1.15 and 1.11 were reported in [7] for $N=50$, 100 and 500 respectively. Optimised carrier phases for $2 \le N \le 15$ were also given.

Note that any carrier phasing used is not unique. Multiplying w_n by $\exp(j2\pi\lambda n/N)$ adds a linear carrier phase term (λ acting as the slope parameter) with no change to the PMEPR (the complex envelope of each bit is cyclically rotated within the bit). However the correlation function and frequency spectrum will change when changing λ. Thus, the free parameter λ can be used to shape the correlation function of the IS MCPC signals.

3 IS MCPC pulse

It is possible to show that to have minimal peak sidelobe level (PSLL) at integer multiples of the bit duration, the modulating sequence should be ideal. For example a polyphase Barker sequence of length M (e.g. 0° −20° −58° −66° −86° 117° 27° −79° 75° 87° 158° −67° 165° −58° 55° for $M=15$) [10]. We found that the behaviour of the ACF between the peaks (non-integer multiples of t_b) exhibits lower sidelobes than in the single-carrier case (where it follows a straight line). The only sidelobes that remain to be lowered are the ACF sidelobes close to the mainlobe (first bit). We can use the free parameter λ to optimise the sidelobe level within the first bit.

An example of the autocorrelation function (top) and real envelope (bottom) of a 15-carrier 15-bit MCPC pulse is given in Fig. 1. Two types of a 15-carrier 15-bit MCPC pulse are shown. One is an MCPC pulse based on modulating all carriers with a 15-element polyphase Barker sequence. The second MCPC pulse is based on consecutive ordered cyclic shifts (COCS) of a chirp-like ideal

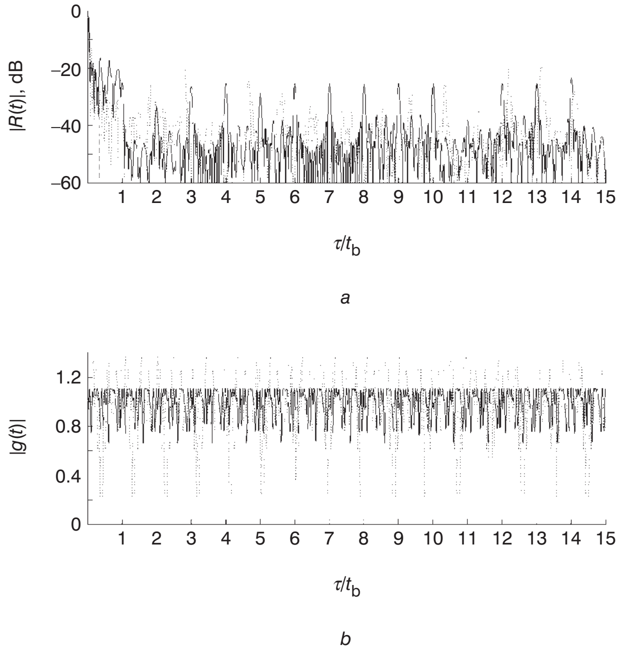

Fig. 1 *Autocorrelation and real envelope of a 15-carrier 15-bit MCPC pulse based on modulating all carriers with a polyphase Barker sequence of length 15 (solid) and a 15-carrier 15-bit MCPC pulse based on COCS of a CLS (dashed)*

No Carrier amplitude weighting
a Autocorrelation
b Real envelope

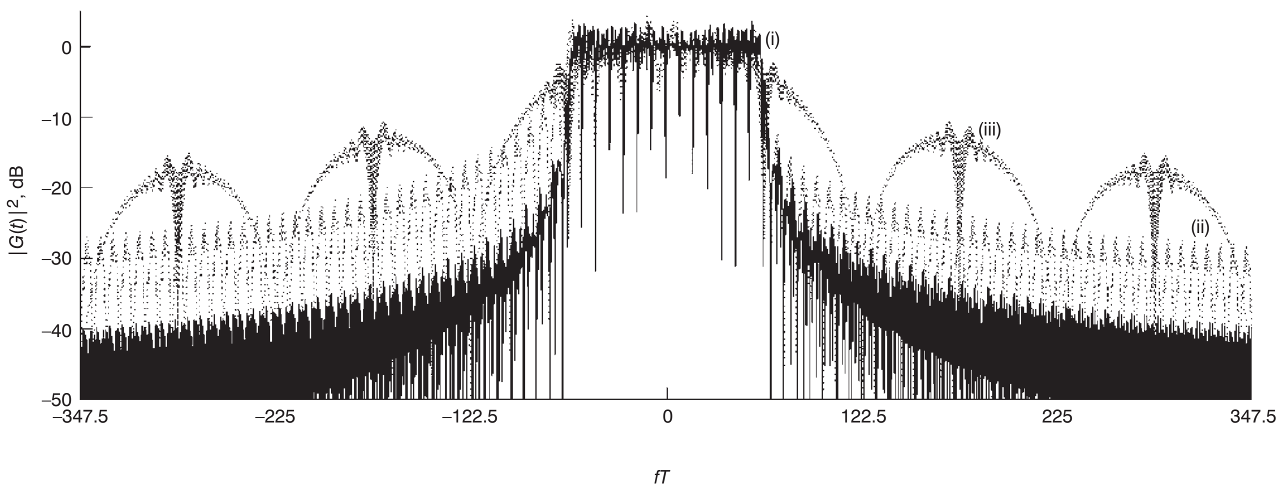

Fig. 2 *Frequency spectra*

(i) 15 × 15 MCPC based on COCS of a chirp-like ideal sequence
(ii) MCPC based on a polyphase Barker sequence
(iii) 225-element chirp-like phase-coded single-carrier pulse with the same energy and pulse compression-ratio (255)

sequence. No carrier amplitude weighing was used. The carrier phases used for the IS MCPC were the optimised phases given in [7] (0° 0° 5° 294° 268° 11° 97° 124° 310° 233° 130° 301° 37° 215° 56°) with optimised $\lambda = 0$ such that the sidelobe level within the first bit are minimised. The PSLL of the IS MCPC within the first bit is −16.5 dB and the main lobe width is $0.89t_b/N$ (measured at the point where the mainlobe slope is equal to the PSLL). The IS MCPC sidelobe level at integer multiples of t_b is −23.5 dB. In the COCS MCPC pulse zero initial phases were used. The PSLL within the first bit is −13 dB (first sidelobe), mainlobe width is $0.8t_b/N$ and the PSLL excluding the first bit is −18.9 dB. The PMEPR was 1.23 for the IS MCPC pulse and 1.87 for the COCS MCPC pulse.

The MCPC pulse spectrum is characterised by a mainlobe bandwidth of N/t_b and relatively low sidelobes with very little energy (almost all the signal energy is in the mainlobe). In the signal examples given in Fig. 1 more than 99% of the signals energy is in the mainlobe. In Fig. 2 we compare the frequency spectra of the single-carrier phase-coded pulse and the MCPC pulses having the same mainlobe width and energy. Note that both MCPC pulses have lower effective bandwidth than the single-carrier phase-coded pulse. The frequency spectrum was calculated by computing the discrete Fourier transform of the pulse zero-padded to 16 times its length.

4 Multicarrier phase-coded CW signal based on an ideal sequence

In the continuous-wave (CW) case the MCPC signal defined in (1) is transmitted continuously. In continuous-wave radar applications, the matched-filter delay response is determined by the periodic autocorrelation function (PACF). We have shown [11] that when N is odd or M is even and the sequence modulating all carriers is an ideal sequence, the PACF is independent of the type of sequence used. For brevity we will name the new design 'IIS' (identical ideal sequence) and will refer to a multicarrier phase-coded CW signal when all carriers are phase modulated by the same ideal sequence as an 'IIS MCPC CW signal'.

As we did for the MCPC pulse, we can still add a linear phase term (λ acting as the slope parameter) with no effect on the PMEPR. The PACF for the optimal $\lambda = 4.525$ (minimal PSLL) and no carrier amplitude weighting (flat spectrum) is shown in Fig. 3 for $N = 25$. Schroeder [5] carrier phasing was used. The PMEPR was 1.83, the PSLL is −16.35 dB and the mainlobe width (measured at the

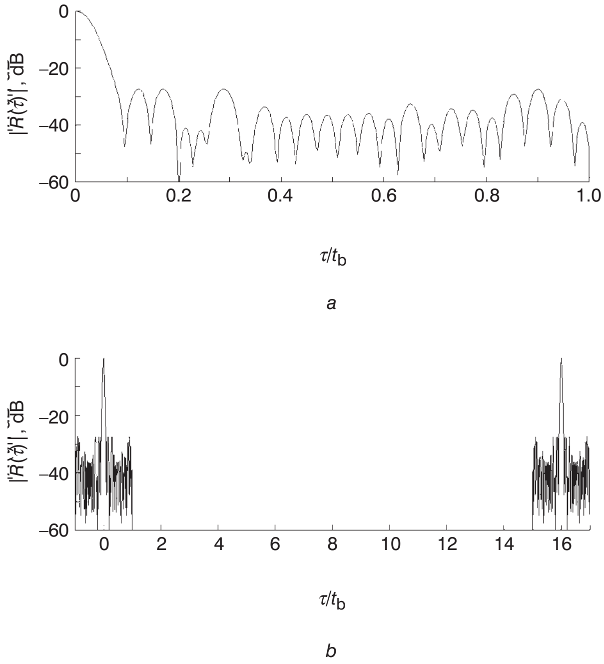

Fig. 4 *PACF first bit and full PACF for IIS MCPC CW signal with carrier amplitude weight optimised for minimum PSLL and mainlobe width of $2t_b/N$*

Schroeder carrier phasing with optimal λ. The number of carriers was $N = 25$ and the number of bits in each carrier $M = 16$. PMEPR = 1.73
a PACF first bit
b Full PACF

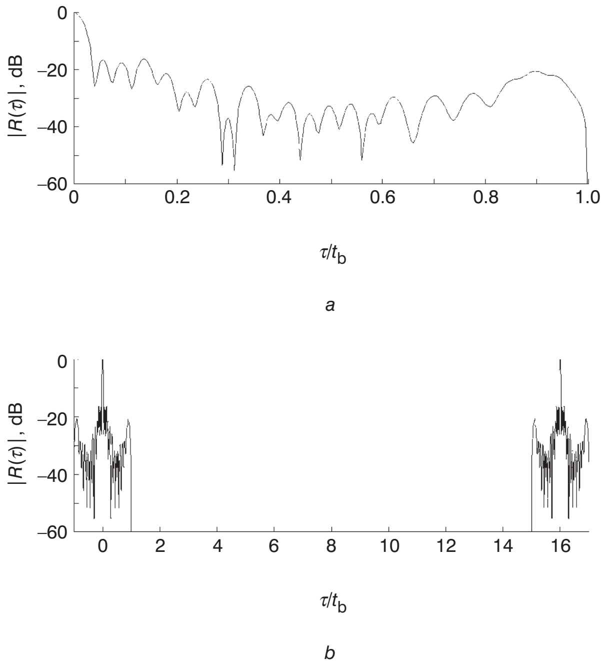

Fig. 3 *PACF first bit and full PACF for IIS MCPC CW signal*

No carrier amplitude weighting. Schroeder carrier phasing with minimal PSLL $\lambda = 4.525$. The number of carriers was $N = 25$ and the number of bits modulating each carrier is $M = 16$. PMEPR = 1.83
a PACF first bit
b Full PACF

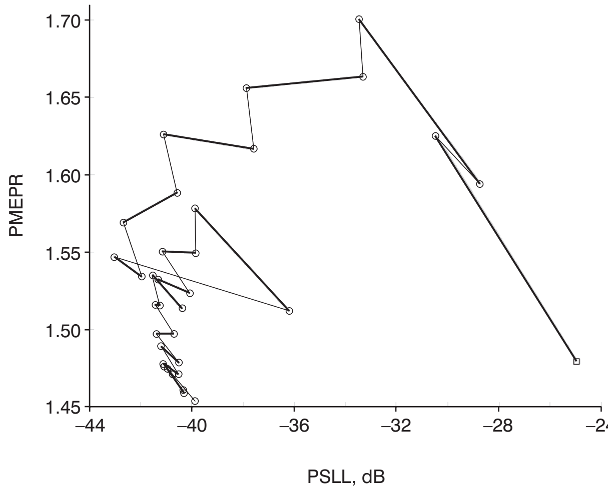

Fig. 5 *PSLL and PMEPR iterative optimisation steps for IIS MCPC CW signal*

PSLL is minimised using gradient optimisation of the carrier complex weights (thick). PMEPR is minimised by using a single clipping step (thin). The number of carriers is $N = 25$. Designed mainlobe width is $2t_b/N$

point on the mainlobe slopes where the main lobe equals the PSLL) is $0.88t_b/N$. $M=16$ was used to plot one complete period of the PACF.

Amplitude modulating the carriers widens the mainlobe width but can help lower the sidelobe level for large N. Some numerical methods of designing optimal PSLL, PMEPR and mainlobe width were considered. The weight function that yields the best results with relatively low computational effort for various values of N, and a prescribed mainlobe width, is the Taylor weight function raised to a power α given by

$$|w_n| = \left| \left\{ 1 + \sum_{k=0}^{K-1} c_{k+1} \cos\left[\frac{2\pi}{N} \left(n - \frac{N+1}{2} \right) k \right] \right\}^{\alpha} \right| \quad n = 1, 2, \ldots, N \qquad (3)$$

where c_k $(1 \le k \le K)$ are the optimised weight coefficients, α is the weight power (also optimised) and K is the number of the phase weight coefficients. Note that the optimisation is on $K+2$ free parameters instead of N parameters.

The PACF using carrier amplitude weights for $N=25$ and $M=16$, when the designed mainlobe width is $2t_b/N$, is shown in Fig. 4. The value of K that was used for the plot was 8. Note that increasing the number of carriers (N) for fixed mainlobe width results in lowering the PSLL. The carrier phase was calculated using Schroeder's method and optimising λ for minimum PSLL. The PSLL is −27.4 dB and the PMEPR is 1.73 (−16.35 dB and 1.83 were obtained without amplitude weights).

Lower PMEPR and PSLL can be obtained using various iterative optimisation methods. For example, we use a two-step iterative scheme where the first step is to use a numerical gradient method to lower the PSLL by changing carrier complex weights and the second step is to lower the PMEPR using a single clipping step. The optimisation steps are repeated until a stop criterion is met.

An example of the optimisation process is shown in Fig. 5 for $N=25$ and designed mainlobe width of $2t_b/N$. We start from the initial weight yielding a PSLL of −24.9 dB and PMEPR of 1.48. An optimisation step where PSLL is minimised (moving to the left on Fig. 5) is marked with thick lines. The optimisation step where the carrier phasing is calculated (a single clipping step and optimising λ) is marked with the thin line (moving down on Fig. 5). Note that it is possible to choose to stop after any iteration. Note also that the PSLL or PMEPR optimisation methods are the same for all iterations; alternatively one could try to adapt the type of optimisation to the

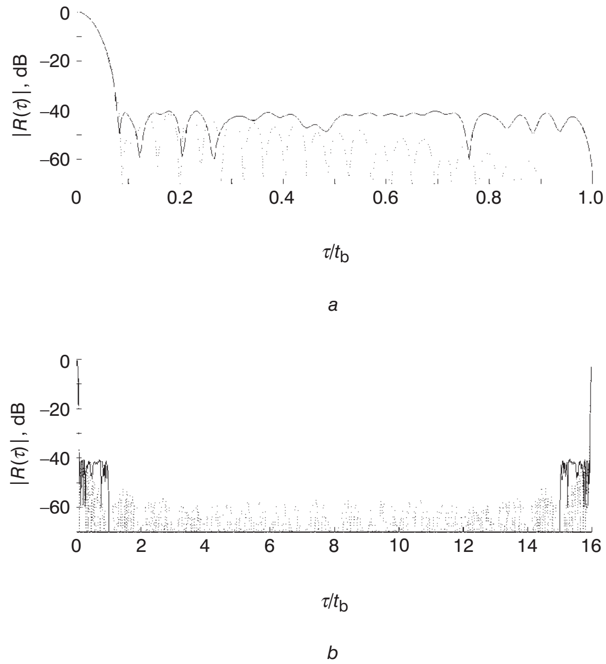

Fig. 6 *PACF of a 25-carrier 16-bit MCPC CW signal based on identical shifts of any ideal sequence (solid line) or based on COCS of a chirp-like ideal sequence (dashed line)*

Carrier weighting used for the IIS MCPC was calculated using the iterative method described in Fig. 5. The carriers of the COCS MCPC CW signal were amplitude weighted using a square root of Hamming window

a PACF first bit

b Full PACF

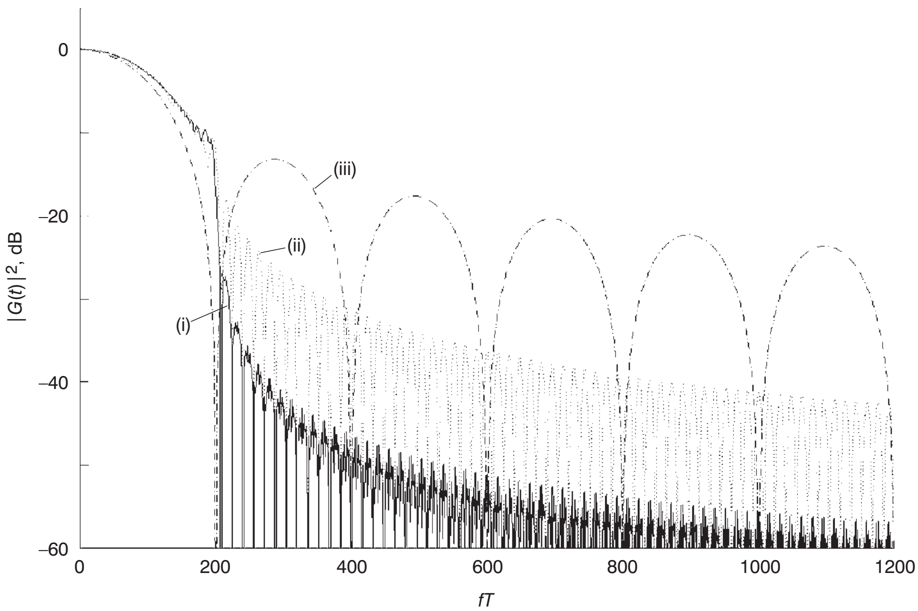

Fig. 7 *Frequency spectra*

(i) 25-carrier 16-bit MCPC CW signal based on COCS of an ideal sequence

(ii) 25-carrier 16-bit IIS MCPC CW signal

(iii) Single-carrier signal phase-coded using 200-element ideal sequence

All signals have the same ACF mainlobe width and energy resulting PSLL and PMEPR. Fig. 6 gives the PACF when the iterations stopped after the PMEPR reached 1.46 and the PSLL was −40.5 dB. $M = 16$ was used for the plot.

The PACF for consecutive ordered cyclic shifts (COCS) of an ideal sequence MCPC is also shown in Fig. 6 (dotted). The Figure shows the PACF of an MCPC pulse based on a consecutive arrangement of any ideal sequence. Sequence length (M) is 16 and the number of carriers (N) is 25. Carriers are amplitude modulated using the square root of a Hamming window. The peak-to-mean envelope power ratio when using the optimal chirp-like ideal sequence is 1.67. The PSLL was −41.5 dB.

The frequency spectrum of the periodic MCPC pulse is given by samples of the frequency spectrum of the pulsed version at multiples of $1/Mt_b$. The frequency spectra of the 25-carrier 16-bit MCPC CW signals are compared with the frequency spectrum of a single-carrier phase-coded CW signal in Fig. 7. Note that the resulting spectrum is not a function of the ideal sequence used for the calculation (since the PACF is also not a function of the specific ideal sequence).

5 Diverse MCPC pulse train

A coherent pulse train extends the coherently processed signal duration and hence provides improved Doppler resolution. In this section we will discuss three different designs of diverse pulse trains labelled ICS, COCS and MOCS.

The ICS design is based on using an identical complementary set to modulate all carriers. (In a complementary set of sequences the sum of the autocorrelation functions is zero for all non-zero shifts [12]). In other words, one sequence out of the complementary set modulates all the carriers in one, and only one, pulse. The initial carrier phase is set such that the PMEPR is low. The COCS design is based on using consecutive ordered cyclic shifts of a chirp-like ideal sequence to modulate all carriers in all pulses. The pulse diversity is obtained by using different cyclic carrier order in each pulse such that for any carrier the sequences modulating all pulses are different. The PMEPR is low due the inherent COCS structure of the pulse (no additional carrier phasing is used). The MOCS design is based on using (different) mutually orthogonal complementary sets to modulate the different carriers (two complementary sets are mutually orthogonal when the sum of the cross correlation between the sequences in one complementary set with the corresponding sequences in the second set is zero for all shifts). The initial phase is set such that the PMEPR is low. Unlike the ICS design where calculating the initial carrier phase is independent of the complementary set used, the carrier initial phase calculation for the MOCS design is a function of the MOCSs used.

In Fig. 8 we present an example showing the periodic correlation function of such a diverse ICS-MCPC pulse train, designed for a mainlobe width of $2t_b/N$ with $N = 16$. Note the relatively high correlation sidelobes in the first bit (−23.5 dB). The PMEPR is 1.59. The complementary set used to modulate all carriers is based on all cyclic shifts of a chirp-like ideal sequence of length eight [13].

Increasing the number of carriers N can lower the PSLL in the first bit. This however will increase the bandwidth (and reduce the ACF mainlobe width). To keep those unchanged an increase in N will be accompanied by an identical increase of t_b (increasing t_b increases the sidelobe area). For example increasing N from 16 to 25 and increasing t_b by the same factor will result a reduction of the PSLL from −23.5 dB to −30 dB.

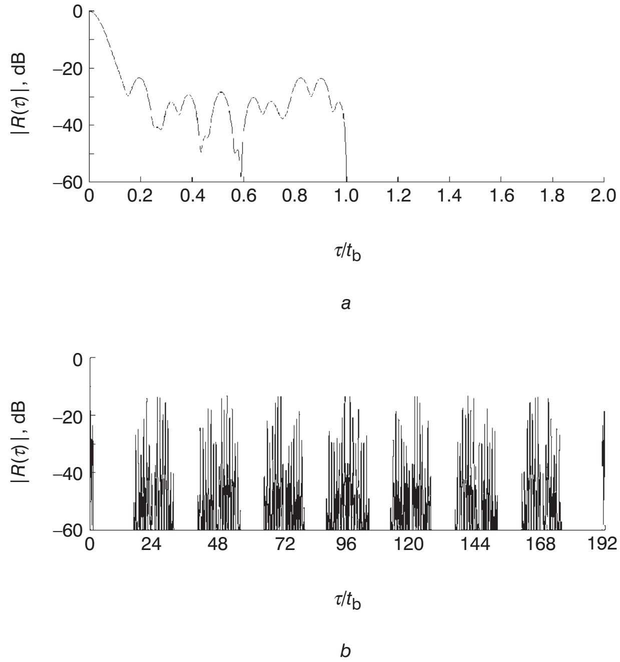

Fig. 8 *Periodic correlation function of a $P = 8$ pulse, $N = 16$ carrier, $M = 8$ bit ICS MCPC pulse train based on a complementary set formed from all cyclic shifts of a chirp-like ideal sequence*
{5 6 2 7 4 8 1 3} pulse permutations. $T_r = 3Mt_b$. Carrier weight is optimised to give a mainlobe width of $2t_b/N$. PMEPR = 1.59
a PACF first bit
b Full PACF

The PACF for consecutive ordered cyclic shifts (COCS) of an ideal sequence MCPC pulse train is shown in Fig 9. The number of pulses in the train (P) is 8. The sequence length (M) is 8 bits and the number of carriers (N) is 16. The square root of Hamming was used for carrier

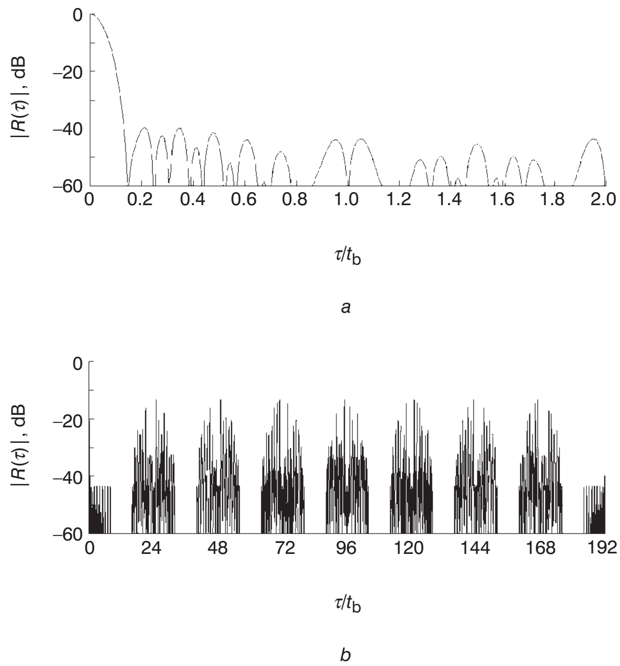

Fig. 9 *Periodic correlation function of a $P = 8$ pulse, $N = 16$ carrier, $M = 8$ bit COCS MCPC pulse train based on a chirp-like ideal sequence with length eight*
Square root of Hamming carrier amplitude weighting. {5 6 2 7 4 8 1 3} pulse permutations. $T_r = 3Mt_b$. PMEPR = 1.9
a PACF first bit
b Full PACF

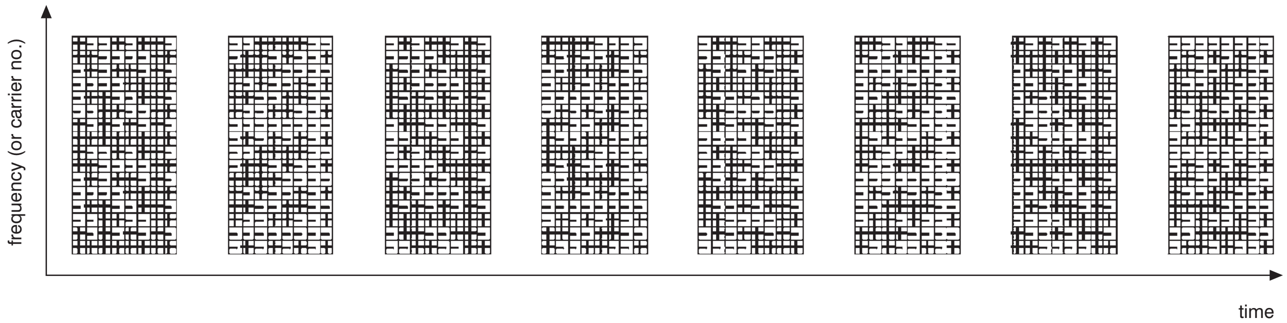

Fig. 10 *Structure of a 16 carrier 8 pulse MOCS MCPC based on two blocks of 8 complementary sets 8-pulse 8-bit binary MOCS*
The initial phase of each carrier, used to reduce PMEPR, is not shown

amplitude weighting. The resulting PMEPR is 1.9. The same complementary set pulse permutation and duty cycle as for the ICS MCPC pulse train were used. Note in Fig. 9 that we lost the property of zero ACF for $t_b \le |\tau| \le Mt_b$.

In the COCS MCPC pulse train, the number of carriers N cannot be increased without limit while keeping low PMEPR. Specific values of N (such as $N \cong M$ with no carrier amplitude weighting or $N \cong 2M$ with carrier amplitude weighting) give considerably lower PMEPR than other selections of N.

An example of a MCPC pulse train design using mutually orthogonal complementary sets (MOCS) is shown in Fig. 10. The MCPC signal consists of $N = 16$ carriers. In each pulse, the same block of 8 binary mutually orthogonal complementary sets are used to phase modulate the lower and upper eight carriers. Note that without additional carrier phasing (identical for all bits in all pulses) the PMEPR is maximised since the 7th bit in all pulses exhibits maximal amplitude. The correlation function of the MOCS MCPC diverse pulse train is shown in Fig. 11. The carrier amplitudes were weighted using a square root of Hamming window and initial carrier phase was optimised to minimise PMEPR ($= 1.99$). Note the low sidelobes within the first bit and lower and narrower recurrent lobes.

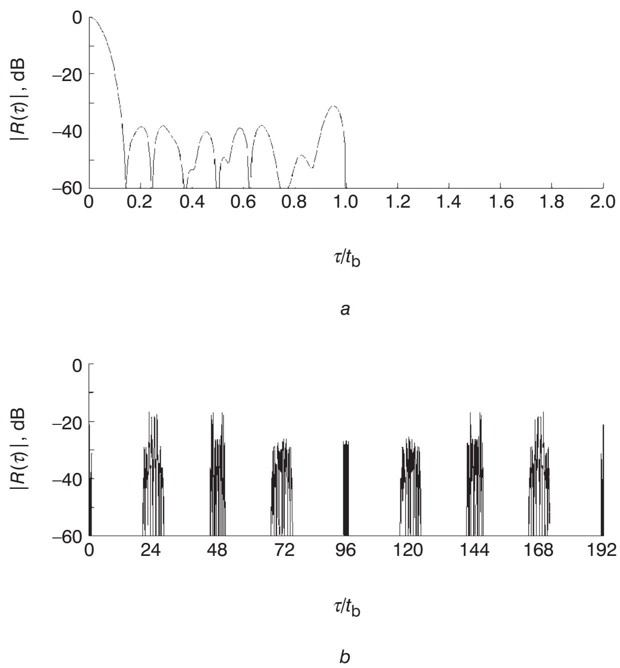

Fig. 11 *Partial correlation function of a $P = 8$ pulse, $N = 16$ carrier, $M = 8$ bit MOCS MCPC pulse train based on two blocks of binary $8 \times 8 \times 8$ MOCS*
Square root of Hamming carrier amplitude weighting. Carrier phasing given by {0° −30° −161° −19° −107° 151° 36° −169° 30° 140° −119° −144° 36° 86° −51° −73°}. $T_r = 3Mt_b$. PMEPR = 1.99
a PACF first bit
b Full PACF

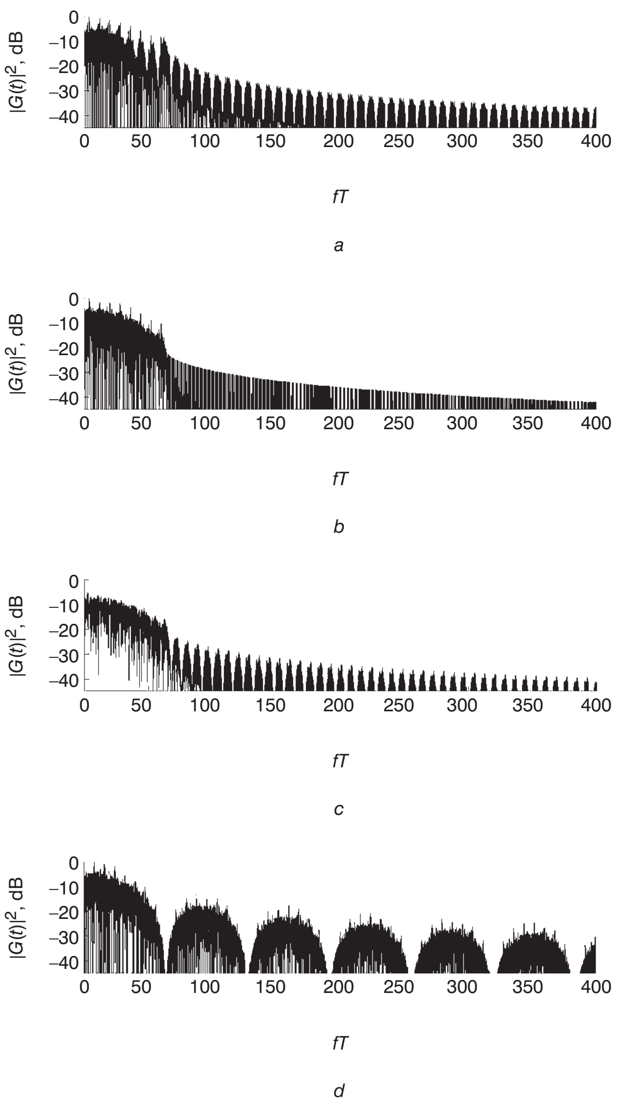

Fig. 12 *Frequency spectra*
a 16-carrier 8-pulse 8-bit ICS MCPC pulse train
b 16-carrier 8-pulse 8-bit MCPC pulse train based on COCS of chirp-like ideal sequence
c 16-carrier 8-pulse 8-bit blocks of MOCS MCPC pulse train
d Single-carrier pulse-train phase-coded using 8 complementary sequences with length 64
All signals have the same mainlobe width

The mainlobe width is the same as for the ICS MCPC pulse train (Fig. 8) and the COCS MCPC pulse train (Fig. 9).

The frequency spectrum of the ICS MCPC pulse train, the COCS MCPC pulse train, the MOCS MCPC pulse train and a single-carrier phase-coded pulse train are compared in Fig. 12. The complementary set used for the single-carrier signal is based on eight cyclic shifts of a 64-element chirp-like ideal sequence (shifted at integer multiples of 8 elements) [14]. All signals are designed to exhibit approximately the same ACF mainlobe width ($T/64$). The single-carrier complementary set has zero sidelobe gap starting at $\tau = T/64$, the ICS and MOCS near sidelobes extend as far as $\tau = T/8$ and the COCS MCPC near sidelobes extend as far as $\tau = T$.

Note the MOCS MCPC pulse train and the COCS MCPC pulse train exhibit the lowest frequency spectrum sidelobes. Note also that all signals except the MOCS MCPC pulse train exhibit spikes in their frequency spectrum mainlobe area due to the ordered nature of the designs.

6 Summary and conclusion

A new design approach of a multicarrier phase-coded CW and a pulse train has been introduced. The design is based on the use of the same modulating sequence for all carriers. The carriers are phase and amplitude weighted to lower peak-to-mean envelope power ratio and reduce the peak sidelobe level in the area of the mainlobe peak. The main two advantages of the new design over the consecutive ordered cyclic shifts design described in an early paper are: (*a*) lower PMEPR and (*b*) autocorrelation sidelobes extending as far as t_b instead of Mt_b. The two main drawbacks of the new design are: (*a*) higher correlation sidelobes in the first bit and (*b*) less bandwidth-efficient spectrum. Both the consecutively ordered cyclic shifts and the new design are considerably more bandwidth-efficient than the single-carrier complementary phase-coded signal having the same ACF mainlobe width. For a diverse MCPC pulse train, a third design approach based on using mutually orthogonal complementary sets (MOCS) has been introduced. It has been shown that the ACF of an MOCS MCPC pulse train benefits both from the large zero sidelobe gap of the ICS MCPC pulse train and the low first bit sidelobes of the COCS MCPC pulse train. The frequency spectrum of the MOCS MCPC pulse train has low sidelobes and is free from some of the peaks that can be found in the frequency spectrum of the other MCPC pulse trains. The PMEPR of the MOCS MCPC pulse train is higher but still not too high. The single-carrier signal has two clear advantages over the multicarrier signals: (*a*) fix envelope and (*b*) zero correlation sidelobes.

The multicarrier signals can be compared to a triathlon athlete (running, swimming and bicycling). Though the MCPC signal does not have the lowest PMEPR, the lowest ACF sidelobes or the highest efficiency frequency spectrum, we could not find other signals that outperform it when all three aspects are considered.

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