CENSORED VIDEO INTEGRATION IN RADAR DETECTION

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ABSTRACT

Video integration, used extensively in radar detection, is an efficient noncoherent integration method. However, integrating the signal amplitude directly makes it sensitive to interference spikes. Binary integration (M-out-of-N) has inherent immunity to as many as N-M spikes, but exhibits added detection loss. The proposed censored video integrator (CVI), is also immune to spikes, yet suffers a smaller loss than the binary integrator. The CVI follows Ritcey's estimator which combines sorting and censoring with averaging of the remaining samples with a non-uniform weighting low.

INTRODUCTION

For a Swerling II target in Gaussian noise, we compare the performances of the classical video integrator, and the well known binary integrator (which can also be called an order statistics integrator), with a new integrator which we termed censored video integrator (CVI). CVI performes sorting, censoring and non-uniform weignted averaging, which for Swerling II targegts in Gaussian noise, can be described by

$$y = (N-K)x_{(K)} + \sum_{j=1}^{K} x_{(j)} \tag{1}$$

where

$$x_{(1)} \le x_{(2)} \le .... \le x_{(K)} \le .... \le x_{(N)} \tag{2}$$

N is the total number of samples, and K is the highest order used. Choosing K = N-M+1 provides the CVI with immunity against the same number of spikes as the M-out-of-N binary integrator. We show that for a Swerling II target, the added loss of the CVI is always lower than the added loss of the corresponding binary integrator, and that CVI is also less susceptible to strong interferences.

Before proceeding with the analysis, we recall that at the output of a square-law detector, the normalized (with respect to the RMS noise) signal magnitude has a probability density function (PDF) and a distribution function given by

$$p(x) = D\exp(-Dx) \quad ; \quad P(x) = 1 - \exp(-Dx) \tag{3}$$

where

$$D = \frac{1}{1 + SNR} \tag{4}$$

This simple PDF allows for simple analytic expressions of the detection and false alarm probabilities, of the three integrators.

VIDEO INTEGRATION

The video integrator sums all the N signal magnitude samples

$$y = \sum_{j=1}^{N} x_j \tag{5}$$

The PDF of y is well known,

$$p(y) = \frac{D^N}{(N-1)!} y^{N-1} \exp(-Dy) \tag{6}$$

Target detection is assumed when y exceeds a predetermined (normalized) threshold T. The probability of detection is therefore

$$P_D = \frac{D^N}{(N-1)!} \int_T^{\infty} y^{N-1} \exp(-Dy)\, dy \tag{7}$$

which yields

$$P_D = \exp(-TD) \sum_{r=0}^{N-1} \frac{(TD)^r}{r!} \tag{8}$$

The probability of false alarm is obtained from (8) in the absence of signal, namely,

$$P_{FA} = P_D \Big|_{D=1} \tag{9}$$

For a given P_{FA} the normalized threshold T is obtained iteratively from (9). For $P_{FA} = 10^{-5}$, the threshold as a function of N is given in Table 1.

Of the three integration methods, video integration is the most efficient. Therefore, the two other methods will be compared to it. A major drawback of video integration is its sensitivity to interference. It is easy to see that one or more very strong spikes will result in the sum y exceeding the threshold, even though the remaining samples are thermal noise samples. The next integration method overcomes this problem.

Table 1. Normalized threshold in video integration ($P_{FA}=10^{-5}$)

N	10	9	8	7	6	5	4	3	2
T	29.522	27.842	26.123	24.358	22.538	20.648	18.666	16.554	14.237

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BINARY INTEGRATION

In binary integration [1,2] each sample x_j; $j = 1, \ldots, N$; is compared to a threshold T. The analog threshold is then followed by a binary one, which requires that at least M out of the total of N samples, have exceeded the analog threshold.

Binary integration could also be called order statistics (OS) integration. The requirement for at least M-out-of-N threshold crossings, is exactly the same as the requirement that the Kth ordered sample be larger than the threshold, where K = N-M+1.

The analysis of binary integration can therefore follow OS analysis or cumulative probability analysis, and the end result will be the same. Following the cumulative probability approach we recall that the probability P_C of at least M hits out of N tries, when the probability of a hit is p, is given by

$$P_C = \sum_{r=M}^{N} \binom{N}{r} p^r (1-p)^{N-r} \tag{10}$$

Given the cumulative probability of false alarm P_{FA}, the single sample probability of false alarm P_{FA1} should be extracted from

$$P_{FA} = \sum_{r=M}^{N} \binom{N}{r} P_{FA1}^{r} (1-P_{FA1})^{N-r} \tag{11}$$

Once the single sample P_{FA1} is obtained, the single sample probability of detection is found from

$$P_{D1} = (P_{FA1})^{D} = \exp(-TD) \tag{12}$$

The cumulative probability of detection is then found from the single sample probability of detection, using (10), namely

$$P_D = \sum_{r=M}^{N} \binom{N}{r} P_{D1}^{r} (1-P_{D1})^{N-r} \tag{13}$$

In order to include the normalized threshold T in the computation, we will use the r.h.s. of (12) in (13) and get

$$P_D = [1-\exp(-TD)]^N \sum_{r=M}^{N} \binom{N}{r} [\exp(TD)-1]^{-r} \tag{14}$$

In order to follow the order statistics approach, we need the distribution function of the Kth ordered sample, which we will term $P_K(x)$. It is given by

$$P_K(x) = \sum_{r=K}^{N} \binom{N}{r} [P(x)]^r [1-P(x)]^{N-r} \tag{15}$$

Using (3) in (15) we get

$$P_K(x) = \sum_{r=K}^{N} \binom{N}{r} [1-\exp(-Dx)]^r [\exp(-Dx)]^{N-r} \tag{16}$$

The probability of detection is obtained from

$$P_D = 1-P_K(T) \tag{17}$$

yielding

$$P_D = \exp(-TDN) \sum_{r=0}^{K-1} \binom{N}{r} [\exp(TD)-1]^r \tag{18}$$

The threshold T is found iteratively from (18) after setting D = 1. For $P_{FA} = 10^{-5}$ and N = 10, the normalized threshold as a function of K is given in Table 2.

We point out again that the order statistics approach and the binary integration approach yield exactly the same results when M = N - K + 1. As a matter of fact, it can be shown that (14) is identical to (18).

CENSORED VIDEO INTEGRATION (CVI)

CVI is effectively a compromise between video integration and order statistics (binary) integration. In order to censor the high ordered samples, it is first necessary to sort the samples according to magnitude. After sorting, the lowest K samples are integrated. However, not by simple addition. While the lowest K-1 ordered samples are each given a unit weight, the Kth ordered sample is given an additional weight. Ritcey [3] has shown that when the samples are exponentially distributed, the optimal additional weight is N-K+1. Such a weight yields (after multiplying by N/K) an unbiased, minimum variance estimate of the original sum of the N samples. With this weighting the integrator output is given by

$$y = (N-K)x_{(K)} + \sum_{j=1}^{K} x_{(j)} \tag{19}$$

Ritcey also found that the PDF of y is the same as that of the sum of only K samples of the exponentially distributed x_j, without sorting. Hence, all the results obtained for video integration, apply to censored video integration, after replacing N with K. Thus, using (8) we get

Table 2. Normalized threshold in OS integration ($P_{FA}=10^{-5}$)

K	2	3	4	5	6	7	8	9
T	1.51	1.8785	2.2885	2.773	3.38	4.197	5.425	7.658

$$P_D = \exp(-TD) \sum_{r=0}^{K-1} \frac{(TD)^r}{r!} \quad (20)$$

Again, the probability of false alarm is obtained from (20) by setting D = 1, and Table 1 still applies to the case of $P_{FA} = 10^{-5}$, after replacing N with K.

DETECTION LOSS COMPARISON (without interference)

For N = 10 samples and no interference, the added loss (over video integration) of both the binary integrator and the censored video integrator, are presented in Fig. 1, as a function of K (or M, where M = N-K+1). In binary integration, the smallest additional loss was obtained with K=8,(M=3). However, the loss dependence on K is rather flat for K=6,7,8 and 9 (when N=10). A comparison of the performances of the 3-out-of-10 binary integrator and a censored video integrator (CVI) with K = 8, is shown in Fig. 2. Both integrators can tolerate up to two very strong interfering spikes. But, as Fig. 1 or 2 demonstrate, without interfering spikes the added loss of the CVI (at $P_D = 0.5$) is 0.7 dB while the added loss of the binary integrator is 1.7 dB.

PERFORMANCES WITH STRONG INTERFERENCE

The major effect of strong spike interference is to increase the probability of false alarm. The analysis of the effect of J strong spikes on the binary integrator (M-out-of-N) is straightforward. We can assume that the J strong spikes cause J threshold crossing. The new probability of false alarm reduces to the cumulative probability that at least M-J noise samples, out of N-J trials, will exceed the original threshold. Modifying (11) we get

$$P_{FA}(J) = \sum_{r=M-J}^{N-J} \binom{N-J}{r} P_{FA1}^{r} (1-P_{FA1})^{N-J-r} \quad (21)$$

where P_{FA1} is the single sample false alarm probability calculated from the cumulative P_{FA}, using (11).

The analysis of the effect of strong interfering spikes on the censored video integrator is not straightforward, and we have resorted to Monte-Carlo simulation. We have also checked, in that simulation, the analytic result for binary integration, as expressed in (21). The simulation was conducted for the case of N=10 samples, K=8,(M=3), and $P_{FA} = 10^{-5}$. A 3-out-of-10 integrator can tolerate up to two strong interfering spikes, without a total collapse. The Monte-Carlo simulation used one million experiments. The results are summarized in Table 4. The theoretical results appear in parentheses. Table 4 shows clearly that CVI is considerably less sensitive to interference than binary integration. Combining this with its smaller detection loss, leads to the conclusion that from the point of view of performances, for the Swerling II case, CVI is superior to binary integration.

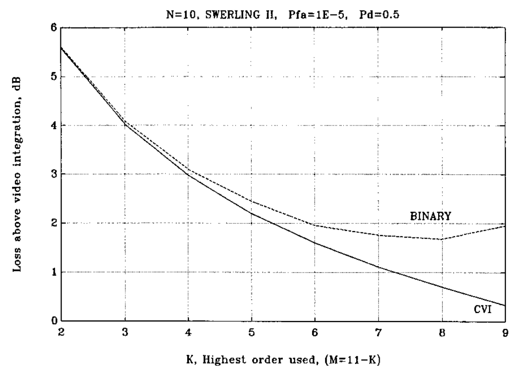

Fig.1 Additional integration loss of censored video integration (CVI) and M-out-of-N binary integration.

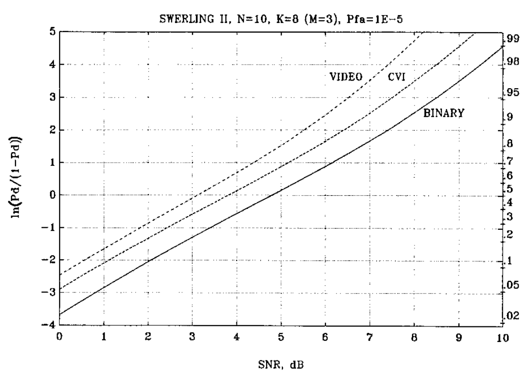

Fig.2 Performances of the three integration methods.

Table 4. The effect of J strong interfering spikes on the probability of false alarm. [N=10, K=8,(M=3)]

	J	0	1	2
$P_{FA}(J)$	Binary	0.9E-5 (1.0E-5)	6.68E-4 (6.84E-4)	3.46E-2 (3.47E-2)
	CVI	0.7E-5 (1.0E-5)	3.51E-4 -	1.79E-2 -

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