Detection Loss Due to Interfering Targets in Ordered Statistics CFAR

NADAV LEVANON, Senior Member, IEEE
Tel-Aviv University

Ordered statistics (OS) constant false alarm rate CFAR is relatively immune to the presence of interfering targets among the reference cells used to determined the average background. OS CFAR performance in a multitarget environment was previously studied by simulation. Here we obtain analytic expressions for the added detection loss, assuming strong interfering targets. The real target is assumed to be a Rayleigh fluctuating target. Numerical examples are included.

Manuscript received August 3, 1987; revised December 18, 1987.

IEEE Log No. 24682.

Author's address: Dep't. of Electronic Systems, Tel-Aviv University, P.O. Box 39040, Ramat-Aviv 69978, Tel-Aviv, Israel.

0018-9251/88/1100-0678 $1.00 © 1988 IEEE

I. INTRODUCTION

Ordered statistics (OS) constant false alarm rate (CFAR) algorithm, introduced by Rohling [1], is a CFAR technique with special immunity to interfering targets. CFAR usually suffers some detection loss due to the adaptive threshold concept. Furthermore, the presence of strong returns among the cells used to determine the background noise or clutter (reference cells), results in an increase in the threshold, and therefore an increase in the required signal strength of the desired target. This is in effect an additional detection loss. In some CFAR methods, the presence of a strong return among the reference cells can cause a drastic reduction in the probability of detection. OS CFAR is a detection technique in which the threshold is just a scalar times one of the ranked reference cells. This concept provides inherent protection against a drastic drop in performance in the presence of interfering targets. In OS CFAR interfering targets cause only gradual detection loss. This loss can be analytically calculated when the interfering targets yield very strong returns, and when the desired target is a Rayleigh fluctuating one.

II. OS CFAR PERFORMANCE

Without loss of generality we normalize the signal in a reference cell with respect to the noise-plus-clutter rms value. The normalized cell input to the CFAR processor is the random variable z. There are M reference cells. In OS CFAR the reference cells are ranked according to their input level

$$z_1 \le z_2 \le \ldots \le z_i \le \ldots \le z_K \le \ldots \le z_M. \tag{1}$$

The variable K is the rank of the cell whose input is selected to determine the threshold (representative rank). The threshold level Z_T is obtained by multiplying the input from the Kth ranked cell by a scaling factor α

$$Z_T = \alpha z_K. \tag{2}$$

The factor α provides the mechanism by which the false alarm probability can be controlled.

It can be shown [1] that when z is a random variable with a probability density function (PDF) $p(z)$ and a distribution function $P(z)$, then the Kth ranked sample (out of a total of M samples) has a PDF

$$p_K(z) = K\binom{M}{K}[P(z)]^{K-1}[1 - P(z)]^{M-K}p(z). \tag{3}$$

For a Rayleigh clutter-plus-noise, and a square-law envelope detector, $p(z)$ and $P(z)$ are given by

$$p(z) = \exp(-z); \qquad P(z) = 1 - \exp(-z). \tag{4}$$

Using (4) in (3) we get the PDF of the Kth ranked sample

$$p_K(z) = K\binom{M}{K}[\exp(-z)]^{M-K+1}[1 - \exp(-z)]^{K-1}. \tag{5}$$

The probability of a noise-plus-clutter input from the cell-under-test, crossing a threshold Z_T, is

$$P(z \geq Z_T | Z_T) \int_{Z_T}^{\infty} \exp(-z)\, dz = \exp(-Z_T). \quad (6)$$

The threshold Z_T is a function of the random variable z_K (2). Thus, the probability of false alarm, (P_{FA}) will be given by averaging (6), with Z_T expressed as function of z_K, over all values of z_K

$$P_{\text{FA}} = \int_0^{\infty} \exp(-\alpha z_K)\, p(z_K)\, dz_K = \int_0^{\infty} \exp(-\alpha z)\, p_K(z)\, dz. \quad (7)$$

Using (5) in (7) we get

$$P_{\text{FA}} = K \binom{M}{K} \int_0^{\infty} \exp[-z(\alpha + M - K + 1)] [1 - \exp(-z)]^{K-1} dz. \quad (8)$$

For an integer α, (8) becomes

$$P_{\text{FA}} = K \binom{M}{K} \frac{(\alpha + M - K)!\,(K-1)!}{(\alpha + M)!}. \quad (9)$$

For a noninteger α the factorial should be replaced with the corresponding Gamma function.

We can conclude that for a Rayleigh noise-plus-clutter, OS-CFAR yields a false alarm probability which is a function of the number of reference cells M, the rank of the representative cell K, and the scaling factor α.

Reference [1] contains a table which lists the required α to obtain $P_{\text{FA}} = 10^{-6}$, with M and K as parameters. From that table we extracted the relationship between α and K for $M = 16$ (see Table I).

For the detection probability P_{D} of a target return in the cell-under-test, we assume a fluctuating target with a Rayleigh amplitude PDF, and an average signal-to-noise ratio ($\overline{\text{SNR}}$). For such a target, P_{D} as a function of the scaling factor α is given by the same expression as P_{FA}, but with α replaced by α_D, where

$$\alpha_D = \frac{\alpha}{1 + \overline{\text{SNR}}}. \quad (10)$$

We therefore get

$$P_{\text{D}} = K \binom{M}{K} \frac{(\alpha_D + M - K)!\,(K-1)!}{(\alpha_D + M)!}. \quad (11)$$

The triple relationship between P_{D}, P_{FA}, and $\overline{\text{SNR}}$ is hidden in (9)–(11). The average SNR cannot be easily pulled out. However, a simpler relationships can be obtained by using Stirling's formula. We first note that (9) can be written as

$$P_{\text{FA}} = \frac{(\alpha + M - K)!}{(\alpha + M)!} \frac{M!}{(M-K)!}. \quad (12)$$

Equation (11) can also be rewritten in the above form. We now define the function

$$f(\alpha, M, K) = \ln \frac{(\alpha + M - K)!}{(\alpha + M)!}. \quad (13)$$

Using Stirling's formula

$$\ln(n!) \approx \tfrac{1}{2} \ln(2\pi) + (n + \tfrac{1}{2}) \ln(n) - n + \frac{1}{12n} \quad (14)$$

(13) can be rewritten as

$$f(\alpha, M, K) \approx (\alpha + M - K + \tfrac{1}{2}) \ln(\alpha + M - K) - (\alpha + M + \tfrac{1}{2}) \ln(\alpha + M) + \frac{K}{12(\alpha + M - K)(\alpha + M)} + K. \quad (15)$$

Equation (15) can accept a noninteger α as well.

Using (13) in (12) we can write

$$\ln P_{\text{FA}} = f(\alpha, M, K) - f(0, M, K). \quad (16)$$

Similarly we can write

$$\ln P_{\text{D}} = f(\alpha_D, M, K) - f(0, M, K). \quad (17)$$

Equations (16) and (17) together with (10) provide a somewhat simpler relationship between P_{FA}, P_{D}, and $\overline{\text{SNR}}$.

We now use these three equations to obtain OS-CFAR performance without interfering targets. We demonstrate the calculations on a specific case where $M = 16$, $K = 10$, $P_{\text{FA}} = 10^{-6}$, and $P_{\text{D}} = 0.5$.

For the given M and K we first construct a table of $f(\alpha, M = 16, K = 10)$ for all relevant αs. (For $0 \leq \alpha \leq 2$, in steps of 0.01; and for $2 < \alpha \leq 200$ in steps of 0.1). Using that table and (16) we get

$$\ln P_{\text{FA}} + f(0, M, K) = f(\alpha, M, K) \rightarrow \alpha = 32.9.$$

Similarly

$$\ln P_{\text{D}} + f(0, M, K) = f(\alpha_D, M, K) \rightarrow \alpha_D = 0.78.$$

Using α and α_D in (10) will yield $\overline{\text{SNR}} = 41.21$ (= 16.15 dB).

A comparison with cell-averaging CFAR (CA-CFAR) can be obtained, if we use Nitzberg's [2] results for CA-CFAR and Rayleigh targets

TABLE I
Scaling Factor α As Function Of K ($M = 16$; $P_{\text{FA}} = 10^{-6}$)

$K =$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\alpha =$	15476	1483	443	207	120	79.4	56.6	42.4	32.9	26.1	20.9	16.9	13.7	10.9	8.3

$$\overline{\mathrm{SNR}}_M = \frac{\left[\frac{P_D}{P_{FA}}\right]^{1/M} - 1}{1 - P_D^{1/M}}. \tag{18}$$

Using the same values of P_{FA}, P_D, and M, in (18) yielded, for CA-CFAR, an $\overline{\mathrm{SNR}} = 29.97$ (= 14.77 dB). Hence, in this case, OS-CFAR suffers an additional loss of 1.38 dB. Choosing a higher value of K (which effects OS-CFAR only) reduces the loss slightly.

The non-CFAR SNR for a Rayleigh fluctuating target is given by

$$\overline{\mathrm{SNR}}_\infty = \frac{\log \frac{P_{FA}}{P_D}}{\log P_D}. \tag{19}$$

(The subscript ∞ is assigned because (19) can be obtained from CA-CFAR when $M = \infty$.)

For the same detection and false alarm probabilities as above, the required non-CFAR SNR is 18.93 (= 12.77 dB). We can conclude that for this particular example CA-CFAR exhibits a CFAR loss of 2 dB, and OS-CFAR has a CFAR loss of 3.38 dB.

III. OS-CFAR PERFORMANCE WITH INTERFERING TARGETS

In the presence of interfering target returns in the reference cells, OS-CFAR obviously performs better than CA-CFAR, since it practically ignores the top ranking reference cells. The effect of interfering targets on OS-CFAR detection probability can be easily evaluated if we accept the following argument. A strong, unexpected target return in one of the M reference cells, effectively reduces the number of reference cells to $M - 1$. The representative rank K and the scaling factor α remain unchanged. The threshold set by the Kth ranking cell out of $M - 1$ cells (and the original α) is higher than a threshold set by the Kth ranking cell out of M cells. A higher threshold implies lower P_D and therefore additional CFAR loss. The higher threshold also results in (unasked for) lower P_{FA}.

Calculating the additional CFAR loss due to J interfering targets, involves the following procedure.

Step 1):

$$\ln P_{FAnom} + f(0,M,K) = f(\alpha,M,K) \rightarrow \alpha. \tag{20}$$

Step 1) determines the nominal α selected with the assumption that there is no interfering targets and all M reference cells have the same input statistics.

Step 2):

$$\ln P_{Dnom} + f(0,M-J,K) = f(\alpha_D,M-J,K) \rightarrow \alpha_D. \tag{21}$$

Step 2) determines the α_D that would have yielded the nominal P_D. Since J cells have strong returns from interfering targets, the adaptive threshold is effectively deduced from $M - J$ reference cells.

Step 3):

$$\overline{\mathrm{SNR}} = \frac{\alpha}{\alpha_D} - 1. \tag{22}$$

Step 3) results the required average SNR in the cell-under-test.

The actual higher P_{FA} obtained because of the higher threshold, can also be calculated, using Step 4).

Step 4):

$$\ln P_{FA} = f(\alpha,M-J,K) - f(0,M-J,K). \tag{23}$$

It should be emphasized again that the entire analysis is valid only if the target in the cell-under-test is a fluctuating target with a Rayleigh amplitude PDF.

The results of calculating the additional CFAR loss in OS-CFAR, caused by interfering targets, are presented in Table II.

TABLE II
Additional OS-CFAR Loss Caused By J Interfering Targets
($M = 16$, $K = 10$, $\alpha = 32.9$, $P_D = 0.5$)

J	P_{FA}	$\overline{\mathrm{SNR}}$ [dB]	Additional CFAR loss [dB]
0	$1.00\ 10^{-6}$	16.15	0
1	$4.70\ 10^{-7}$	16.63	0.48
2	$1.98\ 10^{-7}$	17.17	1.02
3	$7.19\ 10^{-8}$	17.78	1.63
4	$2.12\ 10^{-8}$	18.48	2.33
5	$4.54\ 10^{-9}$	19.56	3.41
6	$5.35\ 10^{-10}$	20.66	4.51

The last row in the table was calculated using the fact that

$$f(0,K,K) = -\ln(K!). \tag{24}$$

The example summarized in Table II, shows that as long as the number of interfering targets is smaller than or equal to $M - K$, OS CFAR maintains its performance with only a small additional CFAR loss. No limit was set on the strength of the interfering targets. As a matter of fact they were assumed to be very strong, so that they rank their respective cells above any reference cell with clutter-plus-noise return. If weak interfering targets are included the additional CFAR loss in Table II becomes an upper limit.

Another example, extended over the full range of P_D is presented in Fig. 1. This example applies to a $P_{FA} = 10^{-5}$. The OS CFAR was obtained with $M = 16$ reference cells, and a representative rank $K = 10$ (the required α is 23.8). The three solid curves represent the OS CFAR performance in the presence of $J = 0$, 2, and 4 strong interfering targets. The broken curve represents the performance of a non-CFAR detector, as given by (19). Simulation results, performed on several points

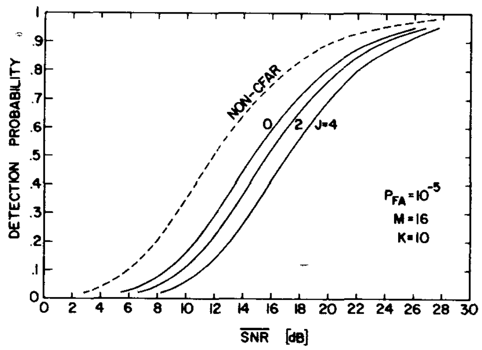


Fig. 1. Effect of J interfering targets on OS CFAR performances.

along the $J = 0$, 2, and 4 curves, confirmed the analytic results by yielding the calculated $P_D \pm 0.01$.

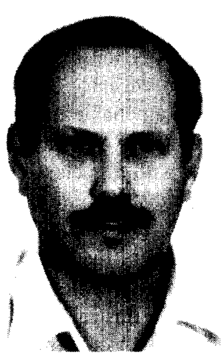
Simulation results of OS CFAR performance in the presence of two interfering targets, $M = 17$ and $P_{FA} = 10^{-5}$, appear also in [3, Fig. 9]. The representative rank K is not indicated. The results in [3] require between 2.5 to 3 dB higher SNR than our results. They could be duplicated with $M = 17$, $K = 5$, and $\alpha = 134.4$. A choice of such a low K, while allowing many interfering targets, yield low performance (by about 3 dB) in normal operation.

REFERENCES

[1] Rohling, H. (1983)
Radar CFAR thresholding in clutter and multiple target situations.
IEEE Transactions on Aerospace and Electronic Systems, *AES-19* (July 1983), 608–621.

[2] Nitzberg, R. (1978)
Analysis of the arithmetic mean CFAR normalizer for fluctuating targets.
IEEE Transactions on Aerospace and Electronic Systems, *AES-14* (Jan. 1978), 44–47.

[3] Barboy, B., Lomes, A., and Perkalski, E. (1986)
Cell-averaging CFAR for multiple-target situations.
IEE Proceedings, *133*, Pt. F (Apr. 1986), 176–186.



Nadav Levanon (S'67—M'69—SM'83) was born in Israel in 1940. He received the B.Sc. and M.Sc. in electrical engineering from the Technion-Israel Institute of Technology, Haifa, in 1961 and 1965, respectively, and the Ph.D. in electrical engineering from the University of Wisconsin, Madison, in 1969.

From 1961 to 1965 he served in the Israeli army. He has been a faculty member at Tel-Aviv University since 1970, first in the Department of Geophysics, and since 1977 in the Department of Electronic Systems, where he is an Associate Professor. He was Chairman of that department from 1983 to 1985. He was a Visiting Associate Professor at the University of Wisconsin, from 1972 to 1974, and a Visiting Scientist at The Johns Hopkins University, Applied Physics Laboratory, in the academic year 1982–1983.

He is the author of the book *Radar Principles* (Wiley, 1988).