Angle-Independent Doppler Velocity Measurement

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When the angle between the target heading and the range vector is not known a priori, a Doppler velocity radar must estimate them simultaneously by utilizing a longer section of the track. The conventional least squares iterative solution is compared with a new explicit alogorithm which utilizes range-rate derivatives. It is shown that the explicit algorithm is biased but is less sensitive to noise. The bias, however, can be estimated and removed. Hence, the computationally simpler explicit velocity estimation method yields better performance. An analytical closed form expression for the resulting mean square estimation error and simulation results are given.

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I. INTRODUCTION

The well-known relation between the range-rate (Doppler) $R^{(1)}$ and the target velocity V requires knowledge of the angle θ between the velocity vector and the range vector (Fig. 1):

$$V = -R^{(1)} / \cos\theta . \qquad (1)$$

In applications such as police radars or muzzle velocity radars, a priori knowledge of θ is obtained by presetting a narrow beam radar at a predetermined angle relative to a known track. If the angle is not known a priori, then it has to be estimated simultaneously with the velocity. In the case of a point target moving on a straight line course at a constant velocity, three parameters must be estimated simultaneously. These could be (V, R_0, θ_0) or (V, x_0, m), which are defined in Fig. 1.

The measured parameters in a Doppler radar can be the Doppler frequency f or the Doppler count n. The Doppler frequency is given by

$$f = f_T - f_R = 2\, R^{(1)} / \lambda \qquad (2)$$

where f_T is the transmitted frequency, f_R is the received frequency, and λ is the wavelength of the transmitted frequency. The Doppler count is given by

$$n_i = \int_{t_0}^{t_i} f\, dt = 2(R_i - R_0)/\lambda = 2\Delta R_i/\lambda \qquad (3)$$

where $R_i = R(t_i)$. The Doppler count n_i is obtained by counting the number of Doppler frequency cycles occurring between time marks t_0 and t_i. The Doppler count is therefore equivalent to the phase, since

$$\phi(t_i) - \phi(t_0) = 2\pi\, n_i . \qquad (4)$$

The Doppler count is more difficult to measure, since it requires continuous monitoring of the signal between time marks. However, it is known to yield better results, e.g., in satellite Doppler navigation [1]. In the discussion to follow it is shown that the Doppler count (phase) approach is an optimal one.

Suppose our data consist of M range difference (phase difference) measurements $\Delta\hat{R}_i$, $i = 1, \ldots, M$. An estimation procedure widely regarded as good is given by

$$\min_{(V,\, R_0,\, \theta_0)} \left\{ \sum_{i=1}^{M} (\Delta\hat{R}_i - \Delta R_i)^2 \right\} \qquad (5)$$

where the ΔR_i are synthetic data points generated by assuming a particular choice of (V, R_0, θ_0). For sufficiently high signal-to-noise ratio (SNR) conditions, the method of least squares is known to yield a bias-free estimate of the vector parameters under consideration. Since ΔR_i is a nonlinear function of the target track parameters, the solution to (5) can only be obtained iteratively.

A computationally simpler procedure is suggested here, based on explicit relations between target track parameters and the range rate and its time derivatives [2]:

$$V = (R^{(1)2} - 3R^{(1)}R^{(2)2}/R^{(3)})^{1/2} \qquad (6)$$

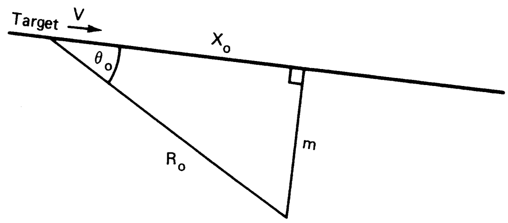

Fig. 1. Target–radar geometry.

$$R_0 = -3R^{(1)}R^{(2)}/R^{(3)} \quad (7)$$

$$\theta_0 = \arccos[(1-3R^{(2)2}/R^{(1)}R^{(3)})^{-1/2}]. \quad (8)$$

In this setting, $R^{(1)}$, $R^{(2)}$, and $R^{(3)}$ are first estimated by performing a linear operation on the measurement vector. The resulting estimates are then substituted into (6)–(8) to yield the target track estimates.

It will be shown that the explicit derivatives approach can be designed to yield a smaller error variance at the expense of introducing a bias error. The indicated bias, however, can be estimated and subtracted to yield a nearly unbiased velocity estimate. The computationally attractive explicit derivatives approach is therefore less sensitive to random measurements noise than the iterative method.

II. MEASUREMENT SCHEME

The phase difference data can be obtained from counting Doppler frequency cycles, as described in (4), or by a large number of phase observations of the received signal relative to the transmitted (reference) signal. The second approach is applicable only in high SNR cases. It should be pointed out that each phase measurement can be completed in less than one cycle of the carrier (or intermediate) frequency by measuring the time elapsed between the positive going zero-crossing of the received signal waveform and the next nearest positive going zero-crossing of the reference signal. Such a phase measuring technique is utilized extensively in very low frequency (VLF) navigation [3].

A typical phase history, recorded with an acoustic radar simulation, is shown in Fig. 2. The phase history

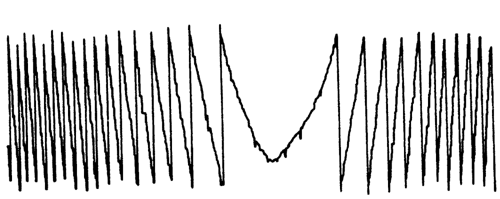

Fig. 2. Typical phase history.

contains the point of closest approach ($\theta = 90°$), but that does not imply that this point has to be included in the observation period. As seen from Fig. 2, the measured phase is given modulo 2π. For high SNR conditions there is no difficulty in identifying the 2π steps and correcting them to obtain the original unambiguous phase.

A key feature of the derivatives approach is to parameterize the received phase by using the Taylor series expansion,

$$\phi_r(t) \approx b_0 + b_1 + b_2t^2 + b_3t^3, \qquad -T/2 \le t \le T/2. \quad (9)$$

Clearly there is a simple relation between b_1, b_2, and b_3 and the range rate and its two derivatives,

$$R^{(n)} = \lambda\, n!\, b_n/4\pi, \qquad n=1,2,\ldots . \quad (10)$$

For $n=0$, (10) is true if the left side is $R-R_0$.

The parameter set to be estimated is therefore

$$(b_0, b_1, b_2, b_3). \quad (11)$$

The last three entries constitute sufficient statistics for the velocity estimation problem. Here b_0 is some unknown constant phase which, however, must be included in the estimation process in order to eliminate an otherwise almost certainly unacceptable bias error.

The least squares estimate of (11) is considerably simplified if the data set consists of an odd number of equally spaced phase difference measurements,

$$\phi_r(t_i) - \phi_r(0) = \phi_r(i\Delta t) - \phi_r(0), \qquad i = 0, \pm 1, \pm 2, \ldots, \pm N. \quad (12)$$

In that case, the estimated parameters are given by

$$\begin{bmatrix} \hat{b}_0 \\ (\Delta t)^2\hat{b}_2 \end{bmatrix} = (P_0P_4-P_2^2)^{-1}\begin{bmatrix} P_4 & -P_2 \\ -P_2 & P_0 \end{bmatrix} \begin{bmatrix} \sum_{i=-N}^{N} [\phi_r(i\Delta t) - \phi_r(0)] \\ \sum_{i=-N}^{N} i^2[\phi_r(i\Delta t) - \phi_r(0)] \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} (\Delta t)\,\hat{b}_1 \\ (\Delta t)^3\hat{b}_3 \end{bmatrix} = (P_2P_6-P_4^2)^{-1}\begin{bmatrix} P_6 & -P_4 \\ -P_4 & P_2 \end{bmatrix} \begin{bmatrix} \sum_{i=-N}^{N} i[\phi_r(i\Delta t) - \phi_r(0)] \\ \sum_{i=-N}^{N} i^3[\phi_r(i\Delta t) - \phi_r(0)] \end{bmatrix} \quad (14)$$

where

$$P_k = \sum_{i=-N}^{N} i^k \quad (15)$$

or specifically

$$P_0 = 2N + 1$$

$$P_2 = (2N^3 + 3N^2 + N)/3$$

$$P_4 = P_2(3N^2 + 3N - 1)/5,$$

$$P_6 = P_2(3N^4 + 6N^3 - 3N + 1)/7. \quad (16)$$

It should be pointed out that the reference phase does not have to be located at the center of the observation period, i.e., $\phi_r(0)$, but could be any other point, e.g., $\phi_r(-N)$, without affecting the results. The estimates $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ are now used, through (10), in (6) to yield the velocity estimate.

III. ANALYSIS OF THE BIAS ERROR DUE TO THE ALGORITHM

The form of (9) assumes that $\phi_r(t)$ is specified by a third-order polynomial in t. The existence of higher order Taylor coefficients is the source of a bias error. In its most general form, the observed phase may be characterized by

$$\phi_r(t) = \phi_s(t) + \phi_n(t) \tag{17}$$

where $\phi_n(t)$ is a zero-mean phase component resulted from the additive noise at the receiver output, and $\phi_s(t)$ is a deterministic (nonrandom) phase component of the signal. It may generally be represented as

$$\phi_s(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + \phi_e(t) \tag{18}$$

where $\phi_e(t)$ is the remainder in the Taylor series expansion.

Let us denote by Δb_j the difference between the estimated value $\hat{b}_j$ and the true value b_j. Then the vector of estimation errors is given by

$$\boldsymbol{\Delta b} = (\Delta b_0, \Delta b_2, \Delta b_1, \Delta b_3)^{\mathrm{T}}. \tag{19}$$

If $E\{\ \}$ denotes the statistical expectation of the bracketed quantity, and if the sums in (13) and (14) are replaced by integrals, then we get

$$E\{\boldsymbol{\Delta b}\} = \begin{bmatrix} 2.25/T & -15/T^3 & 0 & 0 \\ -15/T^3 & 180/T^5 & 0 & 0 \\ 0 & 0 & 75/T^3 & -420/T^5 \\ 0 & 0 & -420/T^4 & 2800/T^7 \end{bmatrix} \begin{bmatrix} \int_{-T/2}^{T/2} \phi_e(t)dt \\ \int_{-T/2}^{T/2} t^2 \phi_e(t)dt \\ \int_{-T/2}^{T/2} t\phi_e(t)dt \\ \int_{-T/2}^{T/2} t^3 \phi_e(t)dt \end{bmatrix}. \tag{20}$$

For moderate observation intervals the Taylor series is likely to converge rapidly so that $\phi_e(t)$ can be approximated closely by the next two terms of the series. Thus

$$\phi_e(t) \approx b_4 t^4 + b_5 t^5. \tag{21}$$

Substituting (21) into (20) and performing the indicated matrix multiplication, one obtains

$$E\{\boldsymbol{\Delta b}\} = (-3b_4T^4/560,\ 3b_4T^2/14,\ -5b_5T^4/336,\ 5b_5T^2/18)^{\mathrm{T}}. \tag{22}$$

The coefficients b_4 and b_5 are related through (10) to $R^{(4)}$ and $R^{(5)}$, respectively. In terms of the target track parameters, these are given by

$$R^{(4)} = (3V^4/R_0^3)\sin^2\theta_0(5\cos^2\theta_0 - 1) \tag{23}$$

$$R^{(5)} = (15V^5/R_0^4)\sin^2\theta_0\cos\theta_0(4 - 7\sin^2\theta_0). \tag{24}$$

For the sake of completeness the lower order derivatives are also given:

$$R^{(1)} = -V\cos\theta_0 \tag{25}$$

$$R^{(2)} = V^2\sin^2\theta_0/R_0 \tag{26}$$

$$R^{(3)} = 3V^3\sin^2\theta_0\cos\theta_0/R_0^2. \tag{27}$$

The transition from (22) to the bias error associated with the velocity estimate is not straightforward because of the nonlinear nature of (6). However, by ignoring terms on the order of $(\Delta b_j/b_j)^2$, one finds that the normalized velocity bias error is given by

$$E\{\Delta V\}/V = \boldsymbol{L}^{\mathrm{T}} E\{\boldsymbol{\Delta b}\} \tag{28}$$

where $\boldsymbol{L}$ is the vector

$$\boldsymbol{L}^{\mathrm{T}} = (0,\ -2/b_2,\ b_3/b_2^2 - 1/b_1,\ 1/b_3)/(b_1b_3/b_2^2 - 2). \tag{29}$$

Performing the inner product between (29) and (22) and using (10) and (23)–(27) one obtains

$$E\{\Delta V\}/V = 0.025(VT\sin\theta_0/R_0)^2 (2 + \cos^2\theta_0). \tag{30}$$

The most critical feature in the normalized bias error is $VT\sin\theta/R$, the ratio of the target displacement, projected in the radial direction to the target range. For $VT/R < 1$ the bias error is likely to be acceptable, being only a small fraction of the actual velocity. However, even that bias error can essentially be removed by updating the velocity estimate as follows:

$$\hat{V}_{\text{unbiased}} = \hat{V}[1 - 0.025(\hat{V}T\sin\hat{\theta}_0/\hat{R}_0)^2 \cdot (2 + \cos^2\hat{\theta}_0)] \tag{31}$$

where $\hat{V}$, $\hat{R}_0$, and $\hat{\theta}_0$ are estimated using (6)–(8), (10), (13), (14), and (16).

Equation (31), together with the equations listed below it, are a major practical result of this work. We therefore pause at this point to present simulation results which confirm it. Random error analysis will follow the simulation results.

IV. SIMULATION RESULTS

Computer simulation was performed by generating phase data with additive Gaussian noise and from it estimating the velocity in three ways. One was the iterative least squares algorithm which is summarized in (5). The two others were the biased (6) and the unbiased (31) derivatives approaches.

The simulated data were generated using the following fixed parameters: $V = 1\ \text{ms}^{-1}$, $m = 1\text{m}$, $\Delta t = 10^{-3}$ s, and $N = 400$. The phase noise was introduced as a Gaussian range noise, with zero mean and a changeable standard deviation σ_R.

The simulation results are given in Fig. 3. The right side was obtained using $\sigma_R = 10^{-4}$ m. The left side used practically noiseless data ($\sigma_R = 10^{-17}$ m). The units scale is arbitrary; hence meters could be replaced by kilometers, as long as it is done consistently in all the relevant parameters. The horizontal axis in Fig. 3 is time or distance along the track, k being the serial number of the data sample. The velocity estimation (vertical axis) was calculated repeatedly every 20 samples (i.e., every 0.02 m along the track). Each velocity estimate is based on $2N+1 = 801$ consecutive samples. The point of closest approach occurred at $k = 3000$.

The upper pair of velocity curves was obtained using the iterative algorithm. The noiseless left side demonstrates the complete lack of bias error in this approach. The right side demonstrates its sensitivity to measurement noise. In particular, note the increased sensitivity near the point of closest approach (0.01 $k = 30$). Obviously it is difficult to measure velocity, using Doppler radar, when the target motion is perpendicular to the range vector.

The bottom pair of velocity curves was obtained using the derivatives algorithm without a bias correction (6). The bias error is clearly seen, reaching a maximum at the point of closest approach, as predicted by (30).

The simulation demonstrates that the random velocity error in the derivatives approach is smaller than in the iterative approach. The reason is simple. In effect, the derivative approach first fits a smooth third-order polynomial to the noisy data. It then obtains explicitly what the iterative approach would have converged to, had it used the smooth data. As a matter of fact, the combined approach was tried of first fitting a third-order polynomial to the data and then applying the iterative

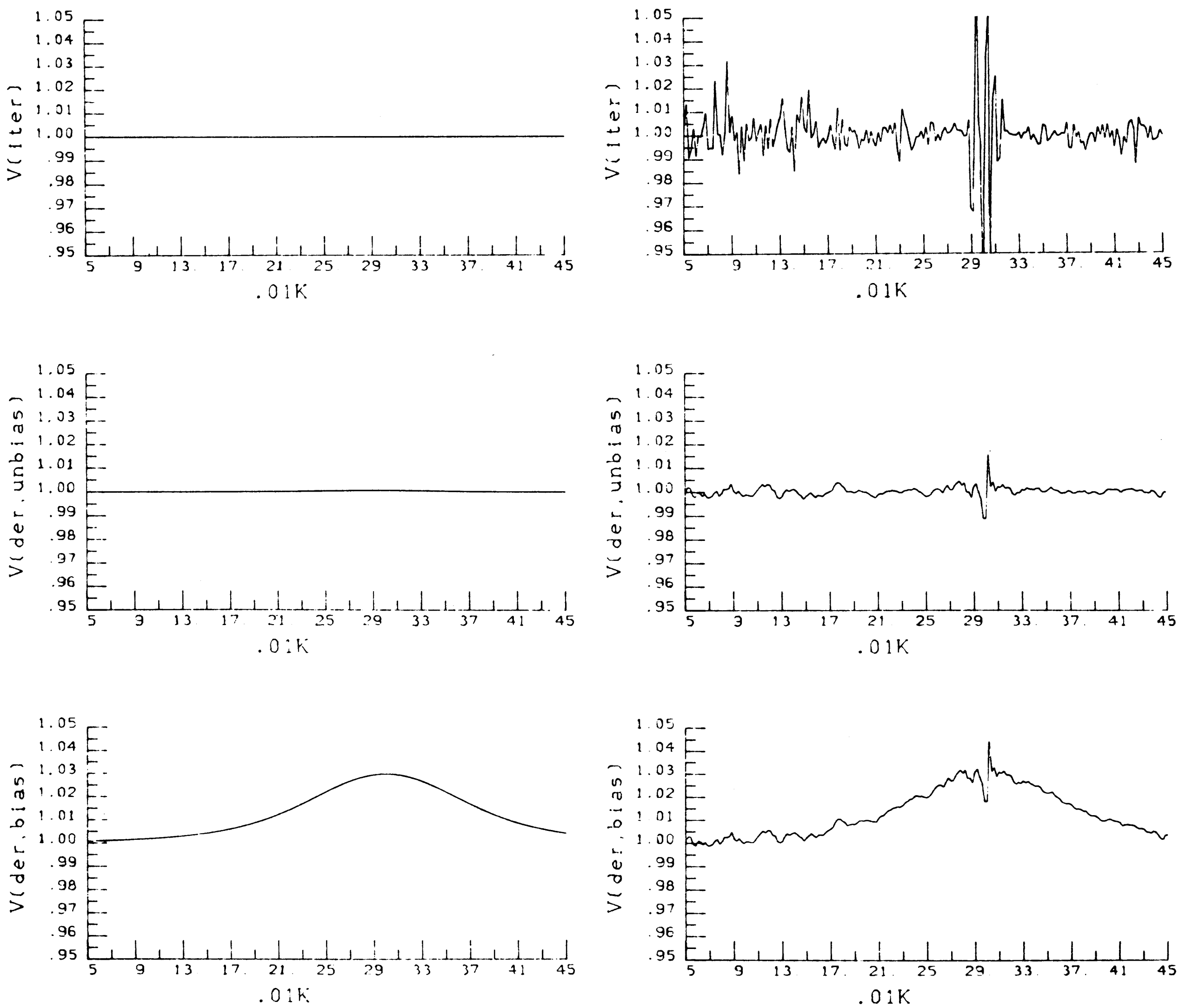

Fig. 3. Simulation results.

algorithm. Except near the singular point of closest approach it yielded very similar results in bias and random error to the results obtained with the uncorrected derivatives approach.

Finally, the middle pair of velocity curves in Fig. 3 was obtained with the derivatives algorithm including bias estimation and correction (31). It shows that the bias error is almost completely removed, while the random error remains unchanged. This result confirms the validity of (31) and the assumptions leading to it. In particular, it confirms that the bias error is due mainly to the neglected fourth- and fifth-order coefficients of the Taylor expansion. Had we included these coefficients in the estimation process, the bias error would have disappeared, but the random error would have increased almost to the level of the random error in the iterative approach.

Comparing the top and middle parts in Fig. 3 clearly demonstrates the superiority of the derivatives approach. It should be added that the explicit derivatives approach is computationally simpler and faster than the iterative algorithm.

V. RANDOM ERROR ANALYSIS

In order to analyze estimator stability in the presence of the random phase component, one must make some assumptions concerning the noise statistics. Suppose the receiver additive noise is a sample function from a zero-mean Gaussian random process whose power is equally distributed over a frequency band of W hertz centered about the signal center frequency.

Consider all Nyquist rate samples ($\Delta t = 1/W$ s) of the observed phase difference. For high SNR conditions the $\phi_n(t_i)$ are statistically uncorrelated zero mean random variables whose variance is given by

$$E\{\phi_n^2(t_i)\} = WN_0/A^2. \tag{32}$$

Here $N_0/2$ is the (two-sided) noise spectral level and A is the amplitude of the radar return signal. The quantity WN_0/A^2 is the ratio of the average noise power to the signal power prior to the phase sampling operation.

If the observation time is large compared with the noise correlation time (i.e., $WT >> 1$) and if $\Delta t = 1/W$ then, in a similar way to the one that led to (20), it can be shown that

$$\operatorname{cov}(\boldsymbol{\Delta b}) = \frac{N_0}{A^2} \begin{bmatrix} 2.25/T & -15/T^3 & 0 & 0 \\ -15/T^3 & 180/T^5 & 0 & 0 \\ 0 & 0 & 75/T^3 & -420/T^5 \\ 0 & 0 & -420/T^5 & 2800/T^7 \end{bmatrix}. \tag{33}$$

It is interesting to note that the Fisher information matrix of the parameters set (b_0, b_2, b_1, b_3), calculated without assuming any prior processing of the received data, yielded the inverse of (33). This implies that in the high SNR situation, the proposed measurement procedure yields an estimate of $\boldsymbol{b}$ whose error variance is the minimum attainable. Furthermore, (33) was realized using Nyquist samples of the observed phase. A higher sampling rate will not reduce the variance error.

The transition from $\operatorname{cov}(\boldsymbol{\Delta b})$ to the velocity variance follows the same elimination of higher order terms, as was done to obtain (28), and yield

$$\operatorname{var}(\Delta V)/V^2 = \boldsymbol{L}^{\mathrm{T}} \operatorname{cov}(\boldsymbol{\Delta b})\, \boldsymbol{L}. \tag{34}$$

In terms of target track parameters, (34) becomes

$$\begin{aligned} \operatorname{var}(\Delta V)/V^2 = {} & (5/16)\,(1/2\pi)^2\,(N_0/A^2T) \\ & (\lambda/VT\cos\theta)^2\,[15(1+\cos^2\theta) \\ & + 48(5\cos^2\theta - 7)\,(R/VT)^2 \\ & + 2240(R/VT)^4]. \end{aligned} \tag{35}$$

The last of the three terms on the right side of (35) represents the contribution to the velocity error variance due to the random error in the b_3 estimate. For moderate changes in target–radar geometry ($VT/R < 1$), this last term becomes dominant, which indicates that the velocity estimate depends on the quality of the highest derivative. Ignoring the other two terms, and taking the square root of (35), yields

$$\sigma_v/V = (7^{1/2}\,5/\pi)\,(N_0/A^2T)^{1/2}\,(R/VT)^2\,(\lambda/VT\cos\theta) \tag{36}$$

where σ_v is the rms of the velocity random error.

Finally, it is interesting to express σ_v/V as a function of σ_R, which was used in the simulation. Using (10) and (32), note that

$$\begin{aligned} (N_0/A^2T)^{1/2} &= (N_0W/A^2)^{1/2}\,(WT)^{-1/2} \\ &= (4\pi\sigma_R/\lambda)\,(2N+1)^{-1/2}. \end{aligned} \tag{37}$$

Inserting (37) in (36) yields the desired expression

$$\sigma_v/V = 7^{1/2}\,20\,\sigma_R\,(R/VT)^2 (1/VT\cos\theta)\,(2N+1)^{-1/2}. \tag{38}$$

When the numerical values used in the simulation are inserted in (38), the resulting σ_v is in very good agreement with the simulation velocity output noise.

VI. BIAS ERROR DUE TO ACCELERATION

Inclusion of an acceleration in the iterative algorithm (5) is straightforward (but costly in terms of sensitivity to noise). The explicit approach, however, depends on the form of (6)–(8), which were developed assuming constant velocity. An acceleration component results in an additional bias error. In this section only a fixed acceleration a, in the direction of the existing velocity, is treated. Let

$$V = V_0 + at \tag{39}$$

where V_0 is the velocity at $t=0$. Then the range-rate derivatives at $t=0$ become

$$R^{(2)} = V_0^2 \sin^2 \theta_0 / R_0 - 2a \cos \theta_0 \quad (40)$$

$$R^{(3)} = 3V_0^2 \sin^2 \theta_0 \cos \theta_0 / R_0^2 + 6 V_0\, a \sin^2 \theta_0 / R_0. \quad (41)$$

Using (40), (41), (10), and (19) we get

$$E\{\Delta \boldsymbol{b}\} = (4\pi\, a/\lambda)\, (0, -\cos\theta_0, 0, V_0 \sin^2\theta_0/R_0)^{\mathrm{T}}. \quad (42)$$

Using (42) and (29) in (28), the normalized bias error due to acceleration is obtained.

$$E\{\Delta V_0\}/V_0 = -(aR_0/V_0^2)\,(\cos\theta_0 + 1/\cos\theta_0), \quad aR_0/V_0^2 < 1. \quad (43)$$

The dominant feature of (43) is the dependence on aR_0/V_0^2. This can be explained intuitively as follows: at long range the acceleration will cause significant change in V before a significant change in target–radar geometry occurs, which is necessary for the velocity estimation. Note that this bias cannot be removed unless the acceleration coefficient a is estimated.

VII. CONCLUSIONS

We have described an explicit algorithm for angle-independent velocity measurement utilizing the range rate and its first two time derivatives. Closed form expressions for the bias and error variance (under high SNR conditions) were obtained. Using simulation, the explicit derivatives approach was compared with the iterative nonlinear least squares solution. One finds that the proposed method is less sensitive to random errors incurred by the additive noise component.

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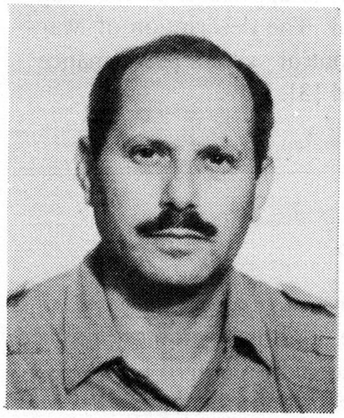

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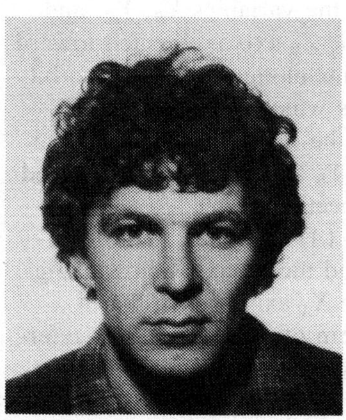

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