Orthogonal Train of Modified Costas Pulses

Nadav Levanon and Eli Mozeson
Dept. of Electrical Engineering – Systems, Tel Aviv University
P.O. Box 39040
Tel Aviv 69978 Israel

***Abstract*- Two recent results are combined to create a radar signal with improved performances. The signal is created initially from a coherent train of *N* identical modified Costas pulses. An orthogonal set of *N* phase codes is then overlayed on the *N* pulses.**

I. INTRODUCTION

The time-bandwidth product (TBW) of a Costas pulse [1] is determined solely by its duration T and the number of its elements M. Since the delay resolution of such a signal is inversely related to the TBW, improving delay resolution requires increasing M. In recent papers [2,3] we show how the bandwidth of a modified Costas pulse can be increased without increasing M, and yet avoiding grating lobes.

The Costas signal, its modification and any other pulse compression signal usually exhibit sidelobes in their autocorrelation function (ACF). In another recent paper [4] we show that in a coherent train of N identical pulses (any type), the ACF sidelobes can be completely removed from most of the single pulse duration T, where by most we mean $(N-1)/N$ of T. The sidelobe removal is achieved by overlaying an orthogonal phase coding on the N pulses.

In the present paper both techniques are combined to yield an orthogonal train of modified Costas pulses. The new signal achieves both high TBW, hence high delay resolution; low ACF sidelobes in the remaining $1/N$ part of T that is not identically zero; and low recurrent lobes at multiples of the pulse repetition interval.

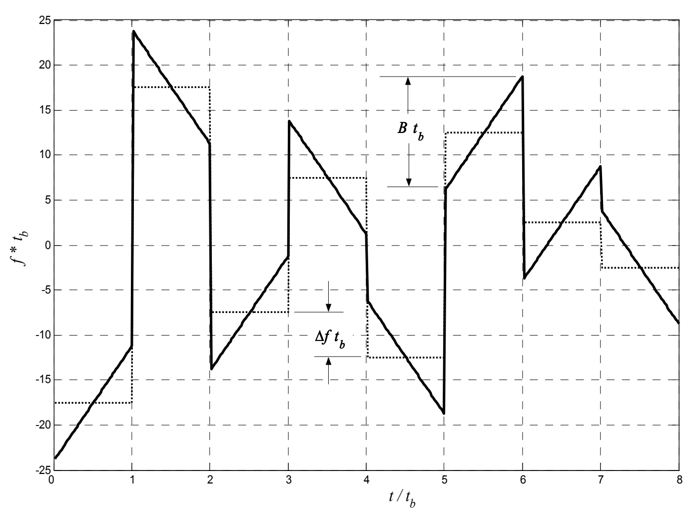

Fig. 1. Frequency evolution of a modified Costas pulse,
$\Delta f\, t_b = 5,\ B t_b = 12.5$

Sections II and III describe briefly the modified Costas pulse and the orthogonal overlay. Section IV gives an example of an 8 pulse signal. In section V it is compared to an orthogonal train of LFM pulses with the same TBW. Finally, in section VI we show that despite the diversity between the pulses in the coherent train, creating filters matched to non-zero Doppler shifts can still be implemented effectively using discrete Fourier transform (DFT), as is usually done when the pulses are identical.

II. MODIFIED COSTAS PULSE

A modified Costas pulse differs from a conventional Costas pulse [1] by increasing the subcarriers spacing Δf beyond the nominal spacing $\Delta f = 1/t_b$, where t_b is the transmission duration of each subcarrier. Normaly, when $\Delta f > 1/t_b$, the autocorrelation function (ACF) exhibits grating lobes at delay multiples of $1/\Delta f$. As shown in [2,3], replacing the fixed frequency during t_b by linear-FM with frequency deviation B, can nullify the grating lobes, when one of several particular relationships exist between $t_b\Delta f$ and $t_b B$. The advantage of the modified signal is the increased bandwidth, hence improved delay resolution, without an increase in the number of elements in the Costas array. A modified Costas pulse with M elements (bits) achieves the same pulse compression as a conventional Costas with $M\sqrt{t_b\Delta f}$ elements and equal total pulse width. For example, the delay resolution of a modified Costas pulse with M = 8 and $t_b\Delta f = 5$ is equal to the delay resolution of a conventional Costas pulse, of the same duration, but with M = 18 elements.

The frequency evolution of a modified Costas pulse with $M = 8,\ t_b\Delta f = 5$ and $t_b B = 12.5$ is shown in Fig. 1. Using up and down LFM slopes [3] minimizes the overlap between neighboring subcarriers, hence minimizes autocorrelation sidelobes.

III. ORTHOGONAL OVERLAY

Match processing a coherent train of N identical pulses, of any kind, yields the same delay resolution as a single pulse. As a matter of fact, over the delay span $|\tau| \le T$, where T is the pulse duration, there is no difference between the ACF of a single pulse and that of a train of pulses. Using a train of identical pulses improves the Doppler resolution, which now drops from $1/T$ to $1/(NT_r)$, where T_r is the pulse repetition interval. However, as shown in [4], when the pulses are not identical, but overlaid by an orthogonal phase-coded set, the ACF of the train can be improved in two ways: (a) the ACF sidelobes within most of the pulse duration can be reduced to zero, and (b) the recurrent ACF lobes (at multiples of the repetition interval) are drastically reduced.

Fig. 2 presents an example of overlaying the 8 rows of a binary orthonormal 8x8 matrix on N = 8 costas pulses, each pulse is constructed from M = 8 elements (bits). The + and – symbols indicate the overlayed binary phase coding of the bits in each pulse. It makes no difference if during the bits the frequency remains constant (conventional Costas) or shifts linearly (modified Costas). Orthogonal overlay works as well for any other pulse signal, not necessarily devided into bits, but arbitrarily sliced into P slices. If the signal is constructed from M bits, it can still be sliced into P slices, and it is not required that $M = P$.

An N-by-P matrix **A** is said to be *orthogonal* when the dot product between any two columns of **A** is zero ($\mathbf{A}^T\mathbf{A}$ is diagonal). Note that orthogonal N-by-P matrices exist only for $P \le N$. An important case is where all elements in the matrix have the same absolute value (normalized to unity) and differ only in their phase denoted by $\varphi_{n,p}$ (in this case we will refer to the matrix **A** as an *orthonormal* phase coding matrix). For an orthonormal phase coding N-by-P matrix $\mathbf{A}=\{a_{n,p}\}=\{\exp(j\varphi_{n,p})\}$ the signal maintains its envelope power properties and we can write that $\mathbf{A}^T\mathbf{A}=N\mathbf{I}$ where **I** is a P-by-P identity matrix. In the example given in this paper $M = N = P = 8$.

Note that the n'th Costas pulse in Fig. 2 was overlaid by the n'th row of $\mathbf{A} = \{\exp(j\varphi_{n,p})\}$, where $\varphi_{n,p}$ is the binary phase matrix in (1). An example of a polyphase orthonormal matrix is given in (2). Its rows are all the cyclic shifts of a P4 signal of length 8.

$$\varphi_{\text{binary}} = \pi\begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&1&1&1&1&0&0\\ 0&1&1&0&0&1&1&0\\ 0&0&0&0&1&1&1&1\\ 0&1&0&1&0&1&0&1\\ 0&0&1&1&0&0&1&1\\ 0&1&0&1&1&0&1&0\\ 0&1&1&0&1&0&0&1 \end{bmatrix} \quad (1)$$

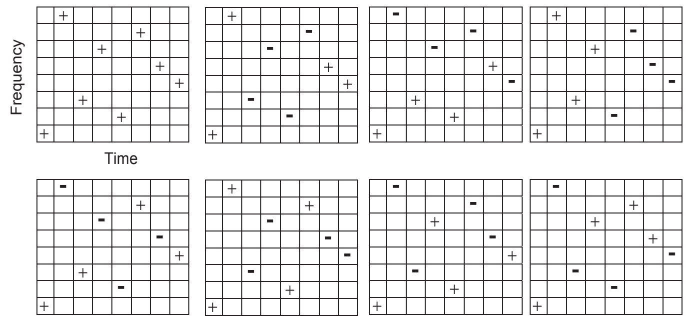

Fig. 2. Binary orthogonal overlay on 8 Costas pulses

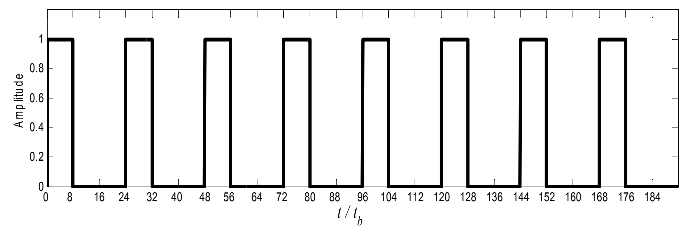

Fig. 3. Real envelope of 8 Costas pulses

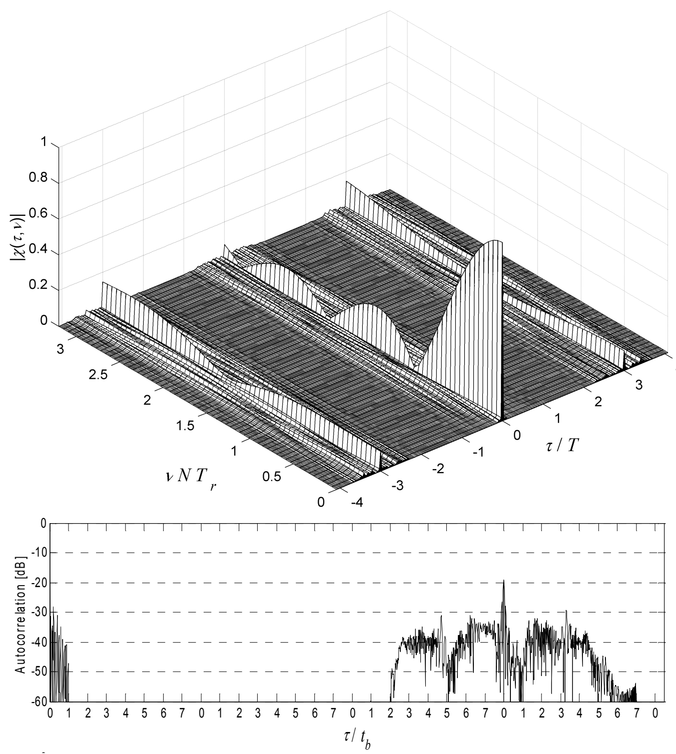

Fig. 4. AF and ACF (zoom on a repetition period)

$$\varphi_{\text{P4}} = \frac{\pi}{8}\begin{bmatrix} -1&-4&7&0&7&-4&-1&0\\ -4&7&0&7&-4&-1&0&-1\\ 7&0&7&-4&-1&0&-1&-4\\ 0&7&-4&-1&0&-1&-4&7\\ 7&-4&-1&0&-1&-4&7&0\\ -4&-1&0&-1&-4&7&0&7\\ -1&0&-1&-4&7&0&7&-4\\ 0&-1&-4&7&0&7&-4&-1 \end{bmatrix} \quad (2)$$

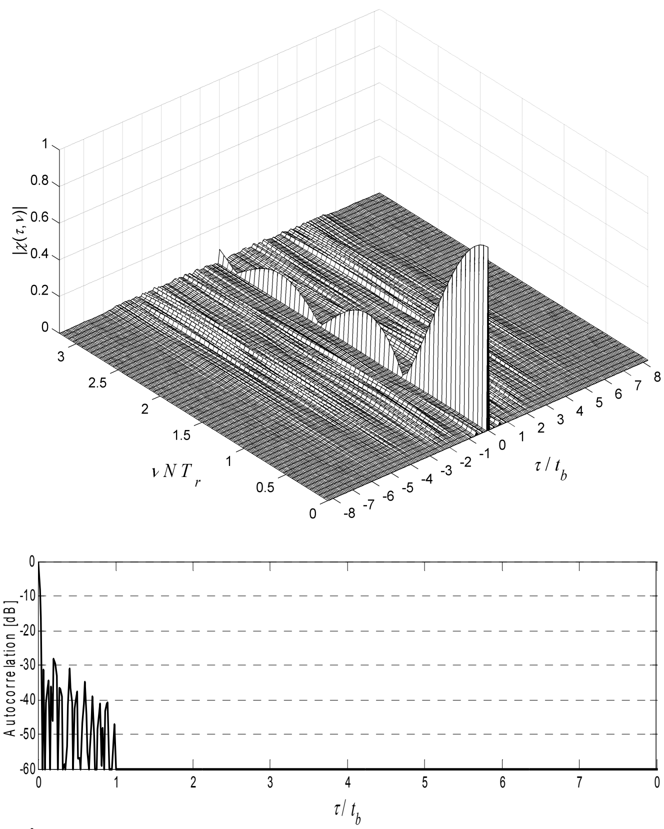

Fig. 5. AF and ACF (zoom on a single pulse)

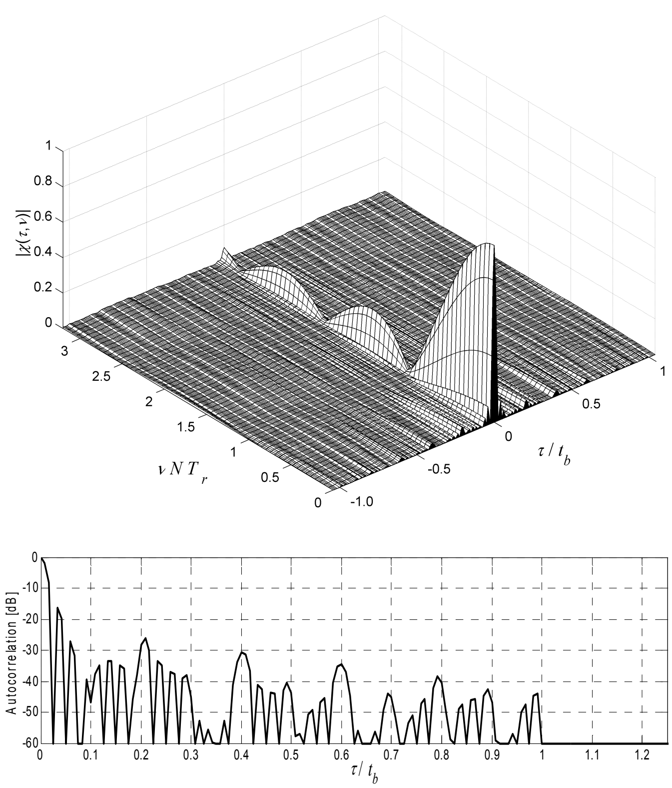

Fig. 6. AF and ACF (zoom on a single bit)

IV. Orthogonal Train of Modified Costas Pulses

In our example the pulse repetion period is $T_r = 3T = 24t_b$. The relatively large duty cycle was selected to simplify the drawings. The real envelope (amplitude) is plotted in Fig. 3. Because the number of slices P is equal to the number of bits M, the slice duration t_s equals the bit duration $t_s = t_b$. It was shown in [4] that due to the orthonormal overlay, the ACF sidelobes are identically zero for $t_s \le |\tau| \le T_r - T$. In our example this implies zero sidelobes for $t_b \le |\tau| \le 16t_b$. At $|\tau| = T_r - T = 16t_b$ the 1'st recurrent lobe begins. While not cancelled, the diversity introduced through the overlay has reduced the recurrent lobe peak to approximately -20 dB. The ambiguity function (AF) and the ACF, plotted in Fig. 4, extend in delay as far as the end of the 1'st recurrent lobe. They demonstrate the sidelobe and recurrent lobe behaviour outlined above. The AF (Fig. 4, top) shows the first Doppler null (of the zero-delay cut) at $\nu = 1/(NT_r)$. This (and the entire shape of the zero-delay cut) is a universal property of a pulse train in which the real amplitudes of the different pulses are identical. While not shown, the AF will exhibit a recurrent Doppler peak at $\tau = 0$, $\nu = 1/T_r$. The AF recurrent Doppler peak value will be only slightly less than one. Fig. 5 zooms in on the pulse duration of both the AF (top) and ACF (bottom). Note that for $t_b \le |\tau| \le 8t_b$ the ACF sidelobes are indeed zero, but at higher Doppler they slowly build up. Fig. 6 zooms in even further, and shows only the first bit. Only here the ACF sidelobes are not zero, but decrease toward zero as Doppler increases. From the ACF in Fig. 6, we note that the mainlobe width (1'st null) ocurs at $\tau_{\text{1st null}} \approx t_b/40$, implying a signal bandwidth of $BW \approx 1/\tau_{\text{1st null}} \approx 40/t_b$. Indeed, since $M = 8$, $t_b\Delta f = 5$, $t_bB = 12.5$, we get from Fig. 1

$$t_b(f_{\max} - f_{\min}) = (M-1)t_b\Delta f + t_bB = 47.5 \qquad (3)$$

The total TBW of one pulse is therefore approximately

$$T(f_{\max} - f_{\min}) = M\, t_b(f_{\max} - f_{\min}) = 8 \cdot 47.5 = 380 \qquad (4)$$

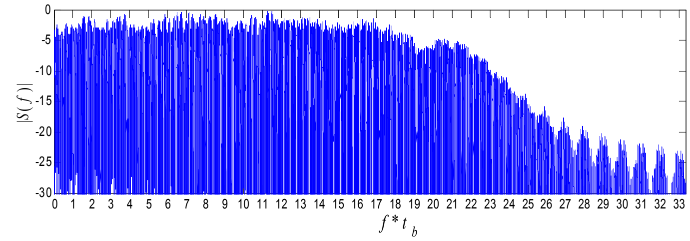

Fig. 7. Spectrum of 8 modified Costas pulses with binary orthogonal overlay

The frequency evolution, as indicated in Fig. 1, suggests less time spent at the edge frequencies than at center frequencies. In a conventional Costas signal, the frequency distribution is more uniform. The one sided spectrum of the complex envelope of our signal (Fig. 7) shows the tapering beginning at $f \approx 20/t_b$.

V. COMPARISON WITH ORTOGONAL TRAIN OF LFM PULSES

An expected question is how different are the performances of the signal discussed above, with a train of LFM pulses, overlaid with the same orthogonal phase coding. Using M = 8 bits, setting $t_b\Delta f = 6$ and $t_b B = 6$ and maintaining the same slope polarity in all 8 bits, will create a single LFM pulse with $T(f_{\max} - f_{\min}) = 380$. Then N = 8 such pulses with the same duty cycle $T_r = 3T$ are overlayed with the same binary code, creating the signal to be compared with. The resulted AF and ACF (zoom on a single bit) appear in Fig. 8. Comparing it with Fig. 6 indicates significantly lower level of near-sidelobes (within the first bit) when the train was constructed from modified Costas pulses. Outside the first bit the ACF sidelobes are inherently zero in both signals.

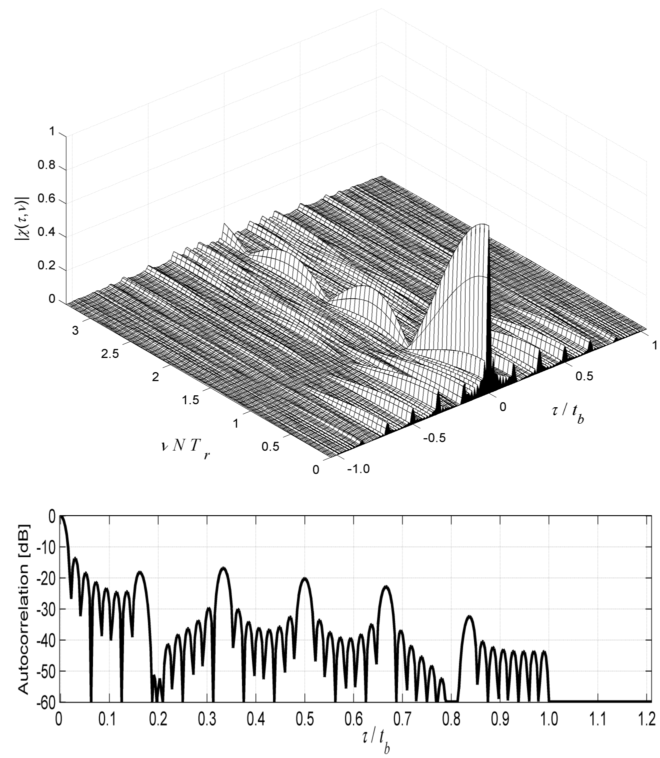

Fig. 8. AF and ACF (zoom on a single bit) of a train of LFM pulses with orthogonal overlay

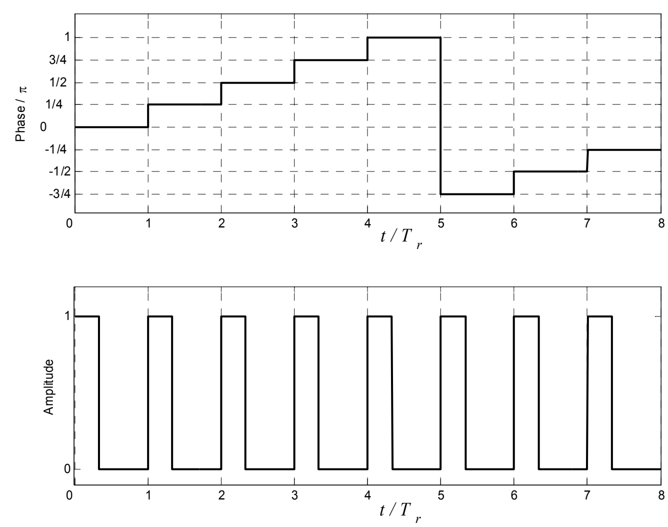

Fig. 9. Inter-pulse phase compensation for a reference signal matched to $\nu = 1/(NT_r)$

VI. MATCHED FILTERS FOR HIGHER DOPPLER SHIFTS

The zero-delay cut of the ambiguity function is identical to the zero-delay cut of the AF of any signal with the same real amplitude (no matter what other frequency or phase modulation is used, including none). As is often done in a coherent train of identical pulses, filters matched to higher Doppler shifts can be implemented by performing DFT on the N pulse-compression outputs. This is equivalent to adding inter-pulse phase steps to the reference signal. Fig. 9 shows the inter-pulse phase steps that should be added to the reference signal in order for it to match a return signal with Doppler shift of $\nu = 1/(NT_r)$. Fig. 10 shows the delay-Doppler response of a correlation receiver for the orthogonal train of modified Costas pulses, which uses a reference signal with the added inter-pulse phase steps. The peak has moved in Doppler to $\nu = 1/(NT_r)$. The new

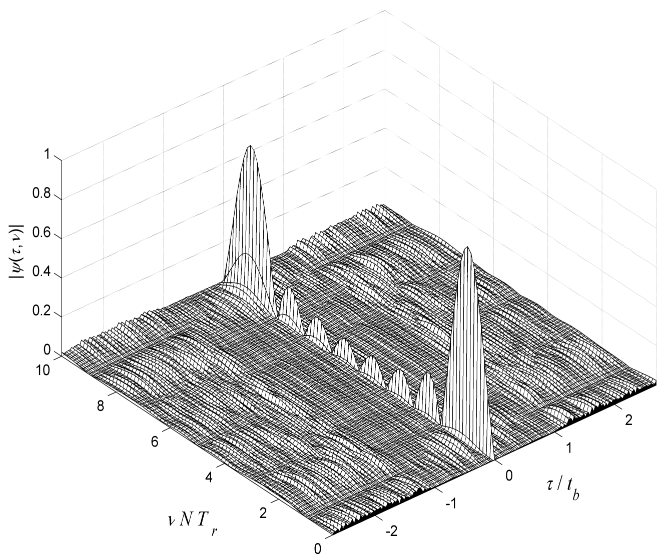

Fig. 10. Delay-Doppler response with a reference signal matched to $\nu = 1/(NT_r)$

delay-Doppler response looks like a copy of the ambiguity function, shifted in Doppler by $1/(NT_r)$.

The Doppler axis in Fig. 10 extends as far as the end of the first Doppler recurrent lobe, whose center is spaced $1/T_r$ from the center of the main Doppler lobe. Because the zero-delay cut of the AF is independent of any phase and/or frequency modulation, it soars above the otherwise low sidelobes pedestal, achieved thanks to the frequency and phase modulation. The only mean to reduce Doppler sidelobes on the zero-delay cut is to add amplitude modulation. One possibility is to introduce inter-pulse amplitude steps according to one of the many well known windows (Hann, Hamming, etc.). However, inter-pulse amplitude weighting will violate the orthogonality between the different pulses in the train. Without orthogonality the property of zero ACF sidelobes beyond the first bit (= slice), will be lost.

Fig. 10 showes some attenuation of the first Doppler recurrent lobe relative to main Doppler lobe. If the ratio T_r/T were considerably larger than 3, that attenuation would have been much smaller. Since LFM pulses exhibit better Doppler tolerance than the modified Costas pulses, in their delay-Doppler response the Doppler recurrent lobe will be less attenuated, but slightly shifted in delay.

VII. Conclusions

Our example of 8 modified Costas pulses, of 8 elements each, yielded pulse compression of 320. Conventional 8 element Costas pulses would have yielded pulse compression of 64. The added orthogonal phase-coded pulse diversity limited the non-zero ACF sidelobes to only 1/8 of the pulse duration, with the peak of those sidelobes at approximately -28 dB. The diversity attenuated the recurrent lobes to a level of approximately -20 dB. We also showed that despite the pulse diversity the signal lends iteslf to simple Doppler processing, commonly used in more simple coherent trains of identical pulses. A similar orthogonal train of LFM pulses (with the same bandwidth) exhibited higher near-sidelobes.

In addition to the improved ambiguity function, the new signal offers advanges in the contex of coexistance with other radars, and reduced probability of intercept. LFM pulses have only two basic permutations - positive and negative frequency slope. Eight element Costas array can be produced in 444 different permutations (16 element array in 21104 permutations). Furthermore, the 8 different pulses (different due to the overlay) can be arranged in 8! = 40320 differrent orders. There can also be several different binary overlays, each one providing its *N*! different orders. Thus, a signal with a given set of parameters can still be produced in milions different permutations. Each permutation will be detected properly only by its own matched filter. The cross ambiguity between different permutations of the same general signal (using the same subcarrier frequencies) is likely to exhibit a low pedestal shape.

References

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