MORE SIGNALS AND TECHNIQUE
for
ACF SIDELOBE REDUCTION

Single pulse:

- Variable amplitude (amplitude weighting, phase/amplitude coding)
- Mismatch receiver

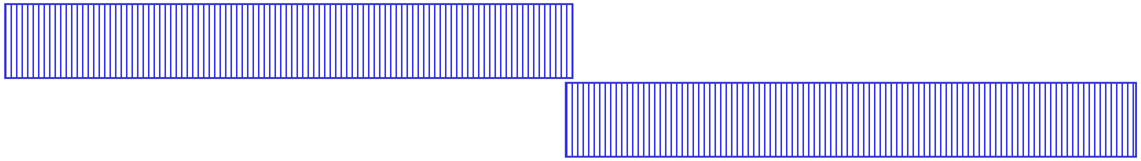
Pulse train:

- Complementary pulses
- Stepped-frequency pulses

Can we design a single pulse with ideal (zero sidelobes) a-periodic autocorrelation function ?

No !

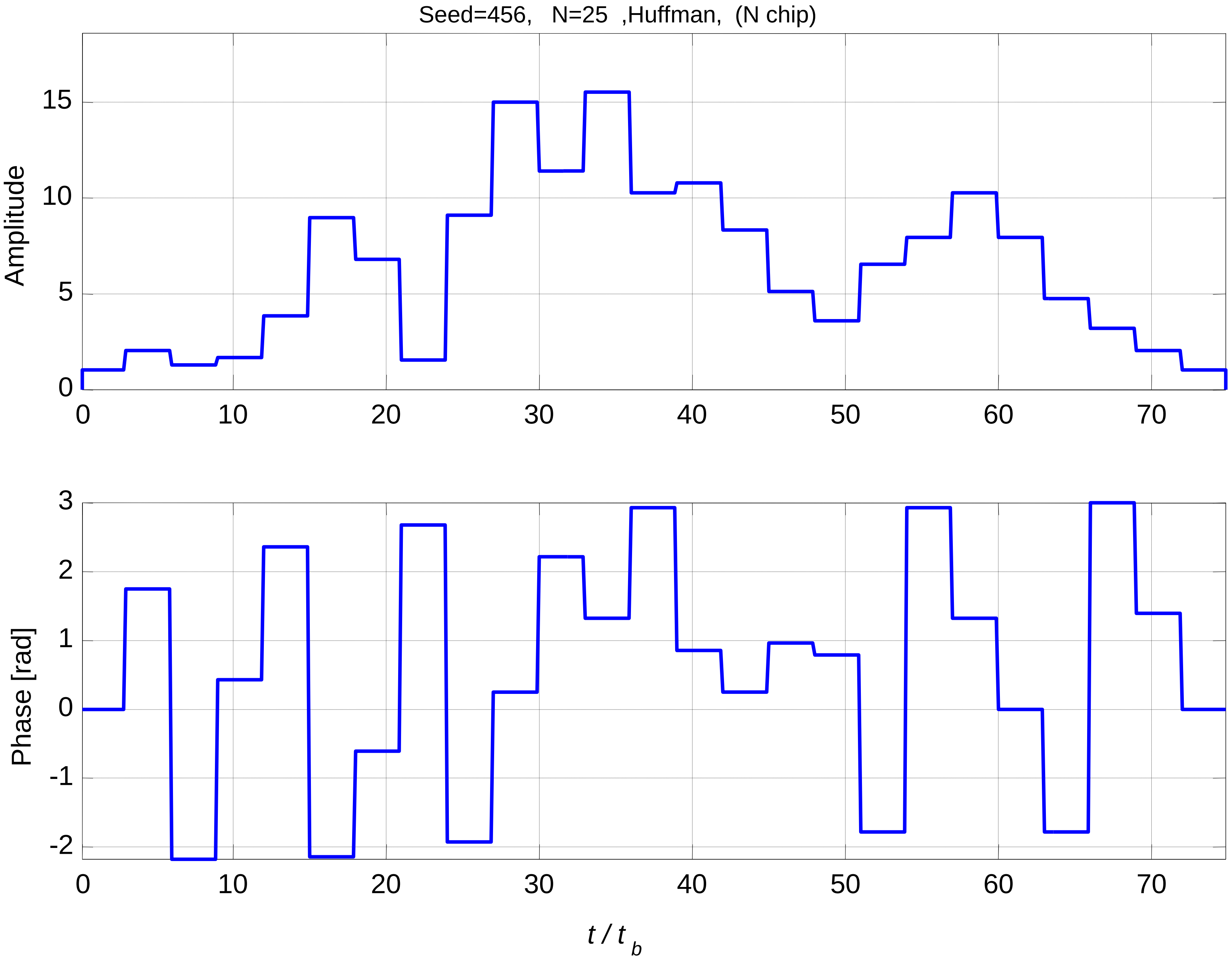
No matter how clever we code it, at the edges of the ACF the last bit multiplies the first bit, and nothing can cancel that product.



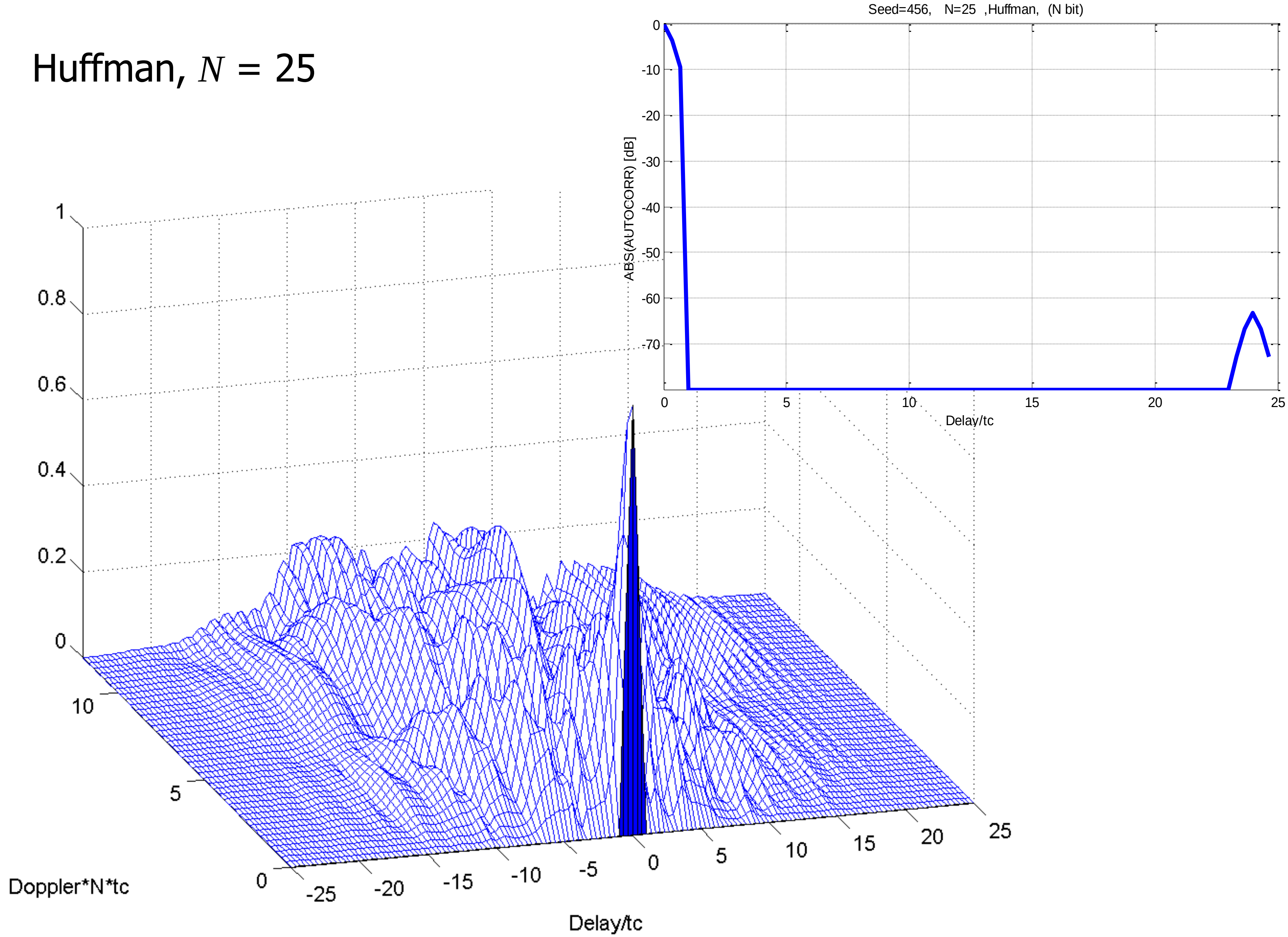
Huffman coding produces the closest to an ideal a-periodic autocorrelation.
The price: requires both phase and amplitude coding

Hufman A. D. "The generation of impulse-equivalent pulse trains", *IRE Trans. Information Theory*, Vol. IT-8, Sept 1962, pp. S10-S16.

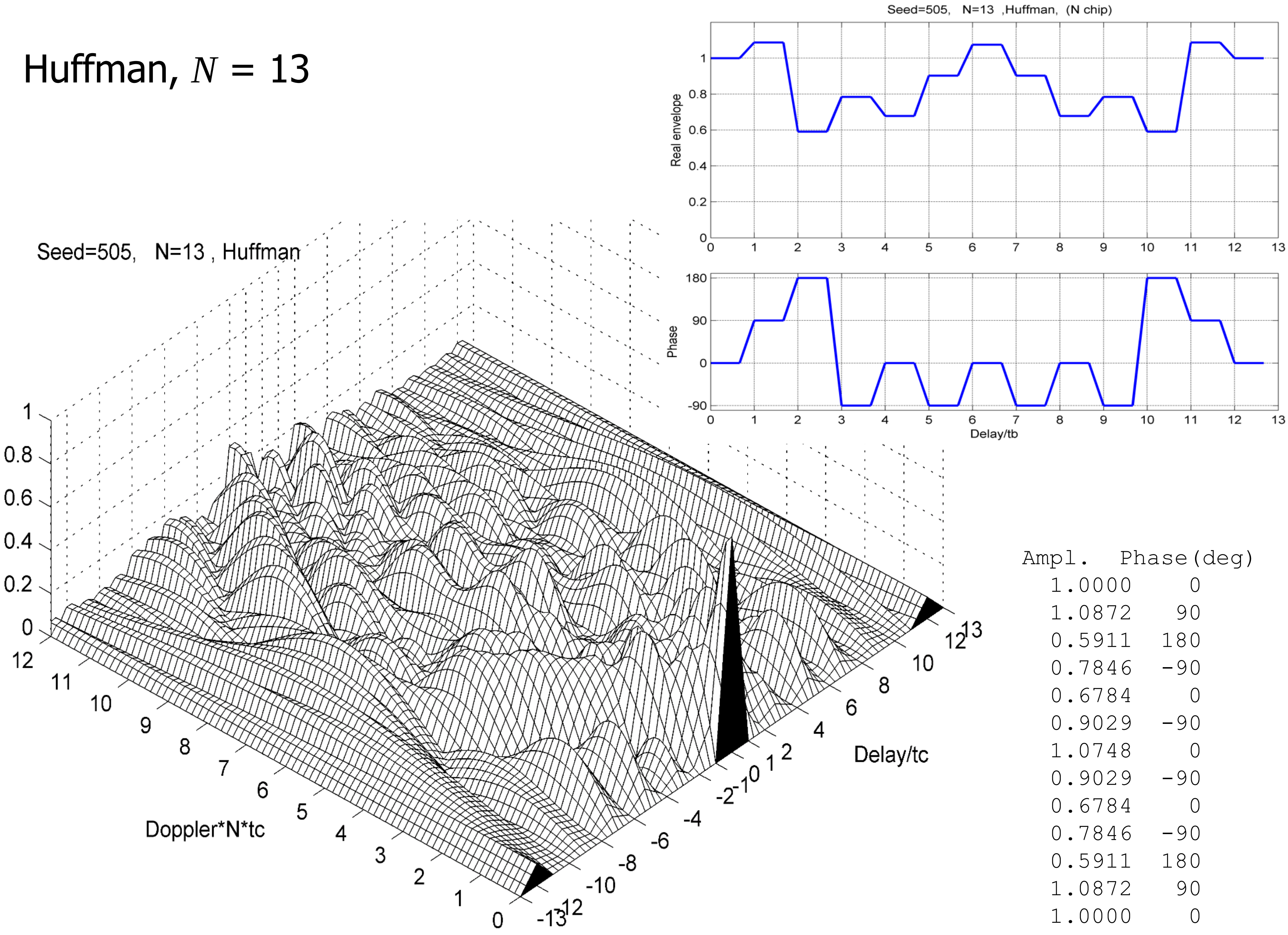
Ackroyd, M.H. "The design of Huffman Sequences", *IEEE Trans. Aerospace Electron. Syst.*, Vol 6, Nov. 1970, pp. 790-796.


Seed=456, N=25 ,Huffman, (N chip)
Amplitude
15
10
5
0
0
10
20
30
40
50
60
70
Phase [rad]
3
2
1
0
-1
-2
0
10
20
30
40
50
60
70
t / t b

Huffman, N = 25


Seed=456,   N=25   ,Huffman,   (N bit)
ABS(AUTOCORR) [dB]
0
-10
-20
-30
-40
-50
-60
-70
0
5
10
15
20
25
Delay/tc
1
0.8
0.6
0.4
0.2
0
10
5
0
Doppler*N*tc
-25
-20
-15
-10
-5
0
5
10
15
20
25
Delay/tc

Huffman, N = 13



Ampl.	Phase(deg)
1.0000	0
1.0872	90
0.5911	180
0.7846	-90
0.6784	0
0.9029	-90
1.0748	0
0.9029	-90
0.6784	0
0.7846	-90
0.5911	180
1.0872	90
1.0000	0

MISMATCHED FILTERS

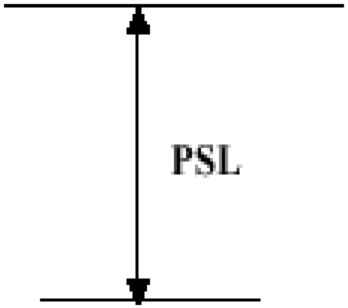
Range sidelobes is a major shortcoming of radar pulse compression

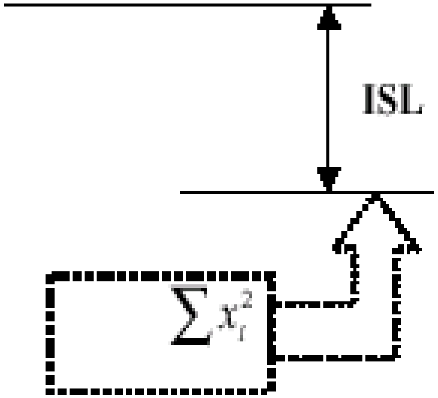
The task of achieving low range sidelobes follows two routes:

- Search for signals with good matched filter response (= ACF).
  The task is on the signal.
  The search is one dimensional (the signal dimension).
- Use mismatched filters and search for good signal/filter pairs.
  The task is shared by the signal and the filter.
  The search is two dimensional.

Figures of Merit

- The Peak Sidelobe Level (PSL) is defined as a ratio of the largest range sidelobe to the mainlobe peak.
- The Integrated Sidelobe Level (ISL) is defined as a ratio of the integrated range sidelobes over the entire matched filter response (excluding the mainlobe) to the mainlobe peak.





$$S=\left\{s_1=\pm1,s_2,\ldots,s_N\right\}$$

Bipolar signal of length N

$$C_k\left(S\right)=\sum_{i=1}^{N-k} s_i\, s_{i+k}\quad ,\quad k=0,\,1,\,2,\,\ldots,\;N-1$$

A-periodic autocorrelation (positive delays)

$$E\left(S\right)=\sum_{k=1}^{N-1}C_k^2\left(S\right)$$

The energy in the autocorrelation sidelobes (positive delays)

$$F=\frac{N^2}{2E(S)}$$

Merit factor

$$H=\left\{h_1,h_2,\ldots,h_P\right\}\;,\;\;N\le P$$

Filter of length P

normalization requires that

$$HH^{\mathrm{T}}=SS^{\mathrm{T}}$$

$$R_k\left(H,S\right)$$

Cross-correlation

Loss

$$L=\frac{R_0^2}{C_0^2\left(S\right)}=\frac{R_0^2}{N^2}$$

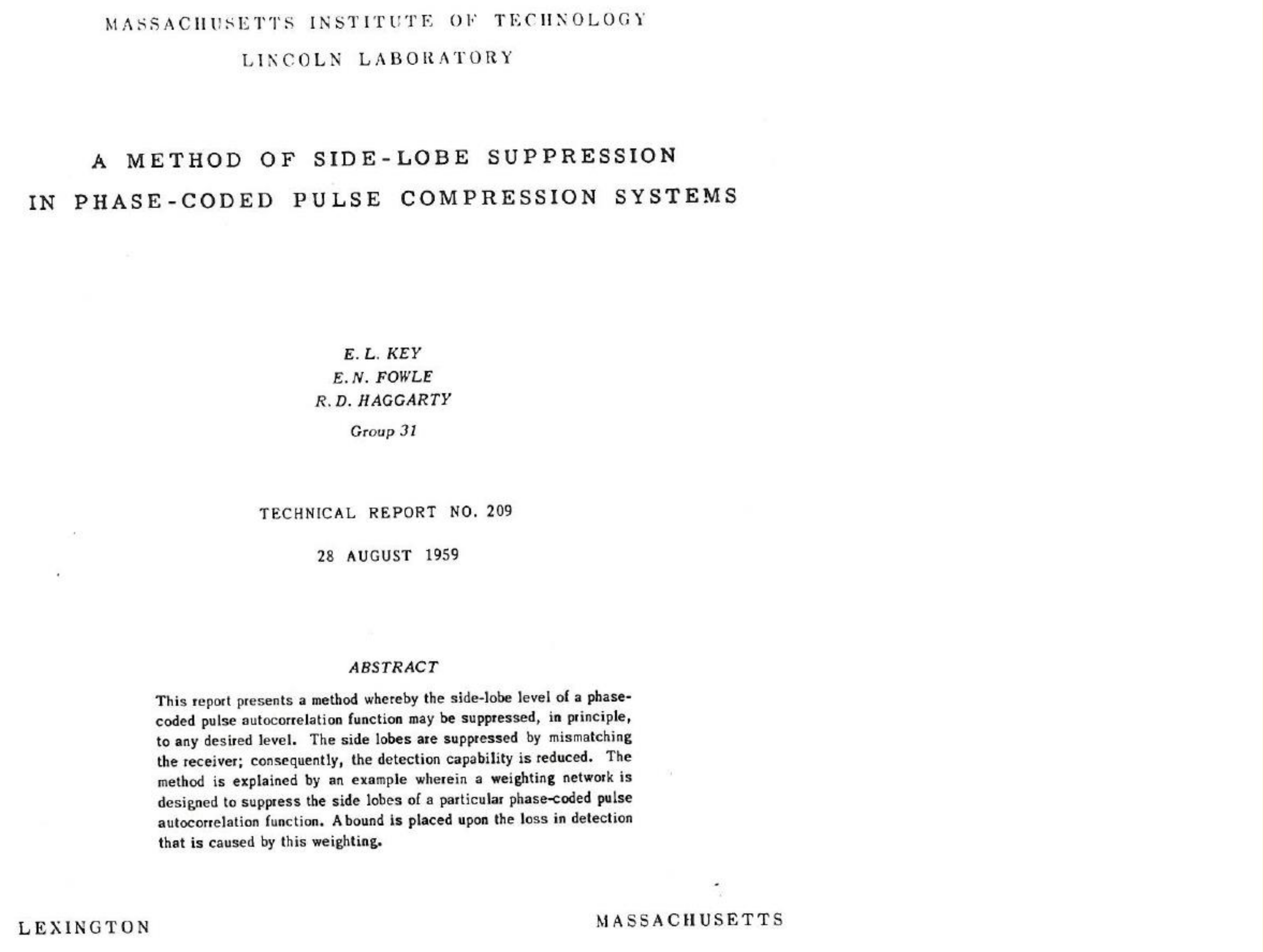
$$ISLR=\frac{1}{R_0^2}\sum_{k\neq0}R_k^2$$

$$ISLR=1/F$$

$$PSLR=\frac{1}{R_0^2}\left(\max_{k\neq0}\left|R_k\right|\right)^2$$

$$ISLR_2=\frac{1}{C_0^2\left(S\right)}\sum_{k\neq0}R_k^2=\frac{1}{N^2}\sum_{k\neq0}R_k^2=ISLR\cdot L$$

$$PSLR_2=\frac{1}{C_0^2\left(S\right)}\left(\max_{k\neq0}\left|R_k\right|\right)^2=\frac{1}{N^2}\left(\max_{k\neq0}\left|R_k\right|\right)^2=PSLR\cdot L$$



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

A METHOD OF SIDE-LOBE SUPPRESSION IN PHASE-CODED PULSE COMPRESSION SYSTEMS

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Group 31

TECHNICAL REPORT NO. 209

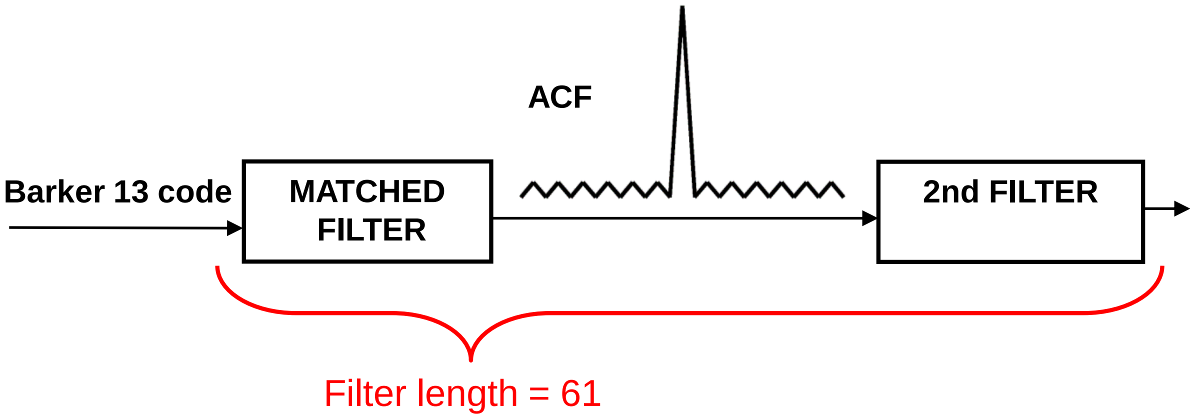
28 AUGUST 1959

ABSTRACT

This report presents a method whereby the side-lobe level of a phase-coded pulse autocorrelation function may be suppressed, in principle, to any desired level. The side lobes are suppressed by mismatching the receiver; consequently, the detection capability is reduced. The method is explained by an example wherein a weighting network is designed to suppress the side lobes of a particular phase-coded pulse autocorrelation function. A bound is placed upon the loss in detection that is caused by this weighting.

LEXINGTON MASSACHUSETTS

Key Fowle Haggarty approach



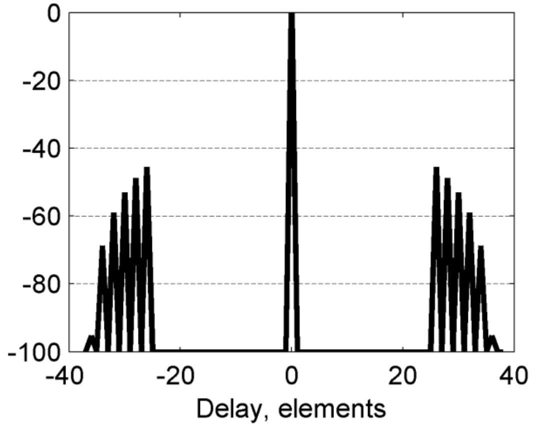
ACF

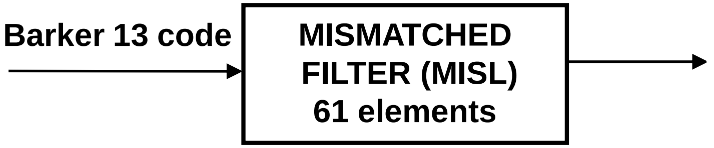
Barker 13 code

MATCHED FILTER

2nd FILTER

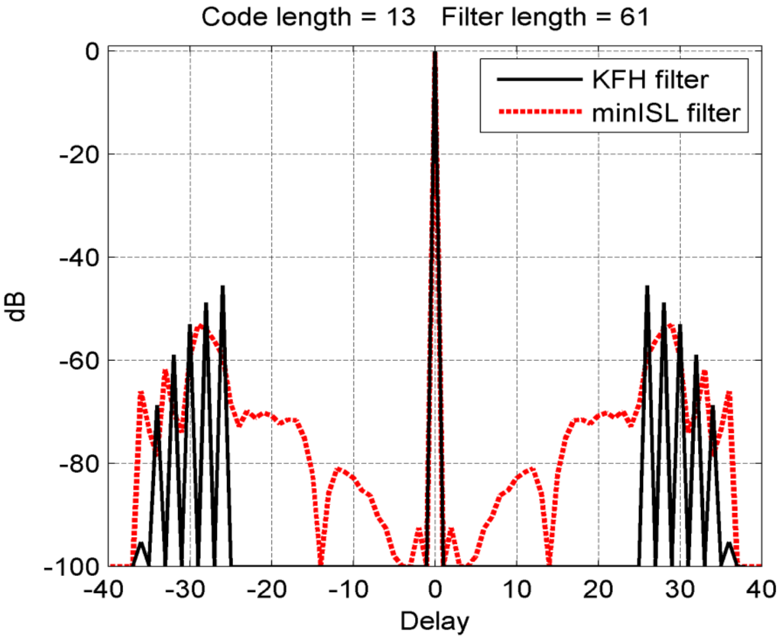
Filter length = 61



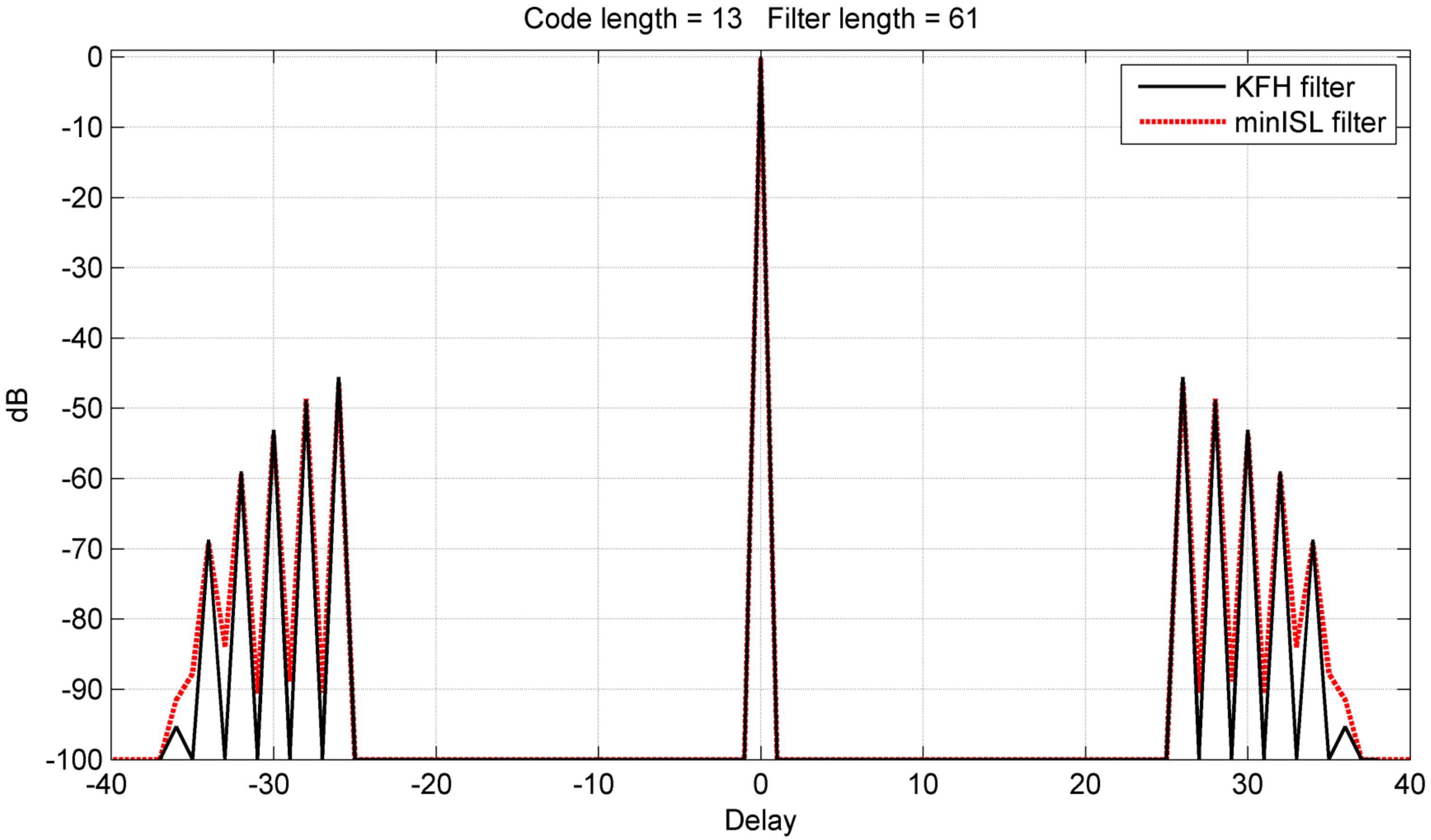


Barker 13 code

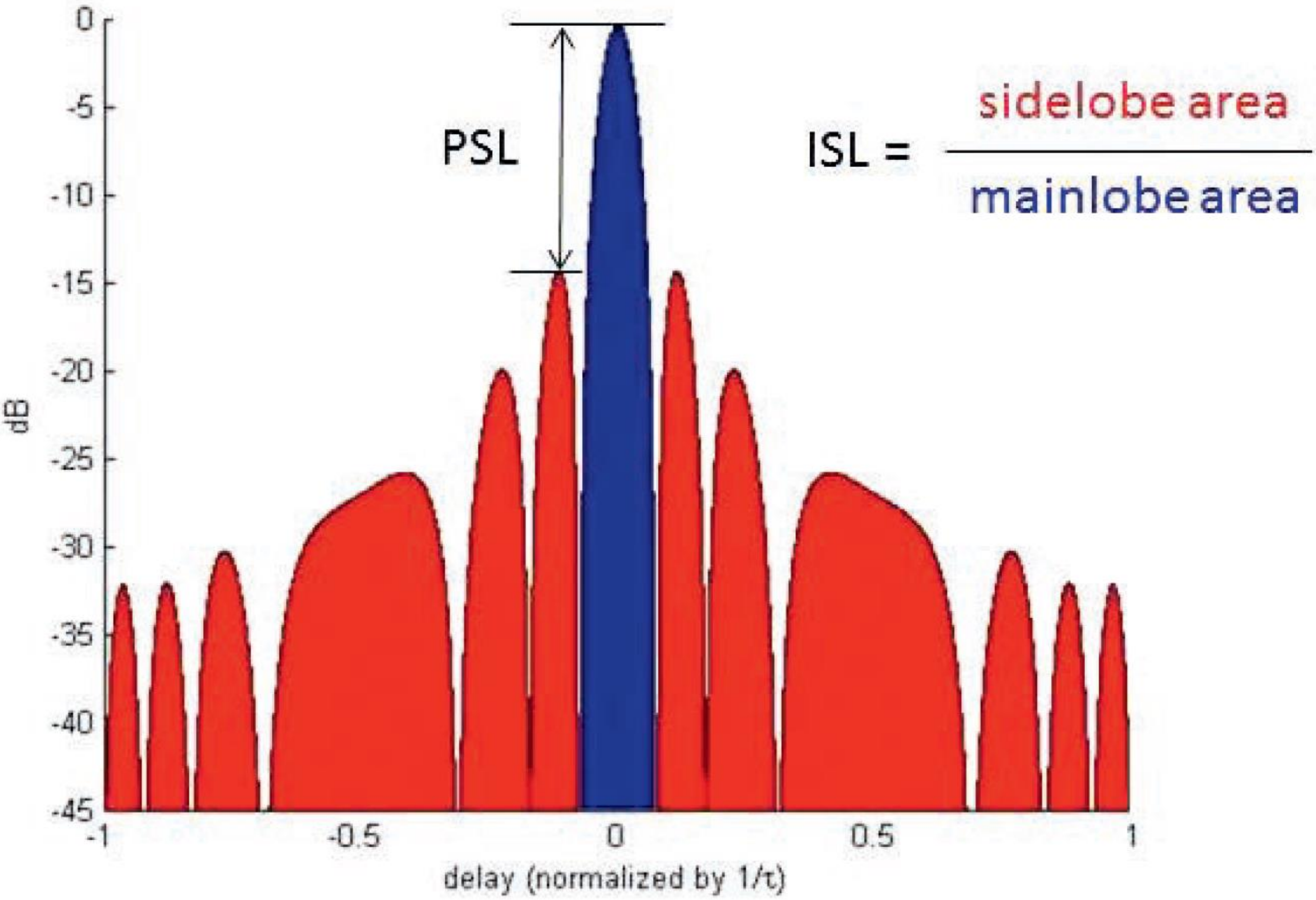
MISMATCHED FILTER (MISL) 61 elements



Barker 13 code → **MISMATCHED FILTER (Controlled SL weights)**
61 elements →



Peak sidelobe (PSL) Vs. Integrated sidelobes (ISL)



Optimal integrated sidelobe (ISL) reduction by mismatched filter

For a given coded signal of length N find a mismatched filter of length P ($\geq N$), that will yield the lowest possible ISL.

The signal s will be extended to have the same length P as the length of the filter h by zero-padding the original signal.

$$R_k(S) = \sum_{i=1}^{P-k} s_i \, h_{i+k} \quad , \quad k = 0, 1, 2, \ldots, \; P-1$$

$$ISL = \frac{1}{R_0^2} \sum_{k \neq 0} R_k^2$$

UNMATCHED LONG FILTERS FOR MINIMUM INTEGRATED SIDELOBES

Based on K.R. Griep *et. al.* “Poly-phase codes and optimal filters for multiple user ranging”, *IEEE Trans. on AES*, vol.31,(2), Apr.1995, pp. 752-767

code = 1 1 1 -1 1 Filter length = 7

Code length = N, Filter length = P

$$\mathbf{c}' = \left[c_0 \; c_1 \; \ldots\ldots\ldots \quad c_{N-1} \right]$$

Nearly-symmetrical zero padding of **C** to reach length P

$$\mathbf{x}' = \left[0 \;\; 0 \;\; c_0 \; c_1 \; \ldots\ldots\ldots \quad c_{N-1} \;\; 0 \;\; 0 \right]$$

$$\mathbf{x} = \begin{matrix} 0 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 0 \end{matrix}$$

$\mathbf{\Psi}$ is a Px $(2P\text{-}1)$ Hankel matrix of $\mathbf{X}$

$$\Psi = \begin{bmatrix} 0 & 0 & \ldots\ldots & x_{P-2} & x_{P-1} \\ 0 & 0 & \ldots\ldots & x_{P-1} & 0 \\ 0 & x_0 & \ldots\ldots & 0 & 0 \\ x_0 & x_1 & \ldots\ldots & 0 & 0 \end{bmatrix}$$

$$\Psi' = \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & -1 \\
0 & 0 & 1 & 1 & 1 & -1 & 1 \\
0 & 1 & 1 & 1 & -1 & 1 & 0 \\
1 & 1 & 1 & -1 & 1 & 0 & 0 \\
1 & 1 & -1 & 1 & 0 & 0 & 0 \\
1 & -1 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}$$

Filter

$$\mathbf{h}' = \left[h_0 \; h_1 \; \ldots\ldots\ldots \quad h_{P-1} \right]$$

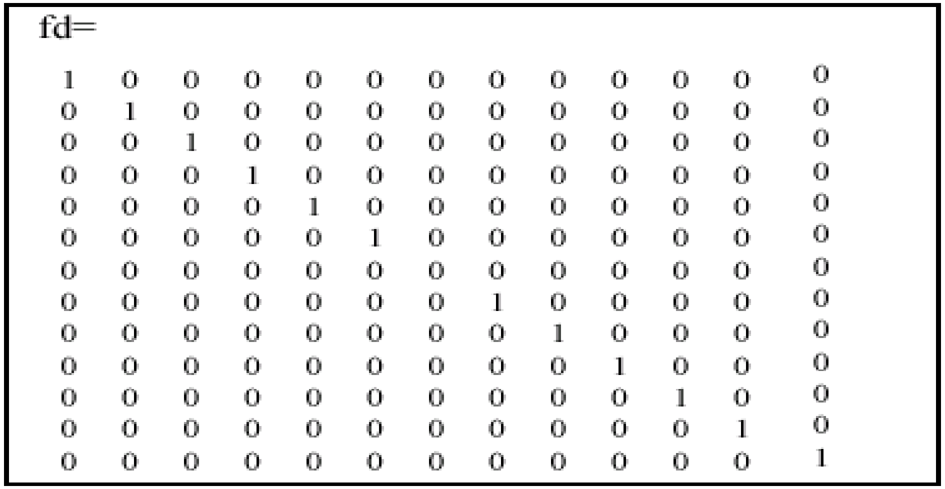
Response

$$y_m = \sum_{n=0}^{P-1} x_n h^*_{n-m} \quad , \quad m = -(P-1), \; ..... \; , (P-1)$$

$\mathbf{y} = \mathbf{h}'\Psi$, ( )' implies conjugate transpose

F is a (2*P*-1)x(2*P*-1) identity matrix in which the (p,p) element is zero, e.g., for *P*=3

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



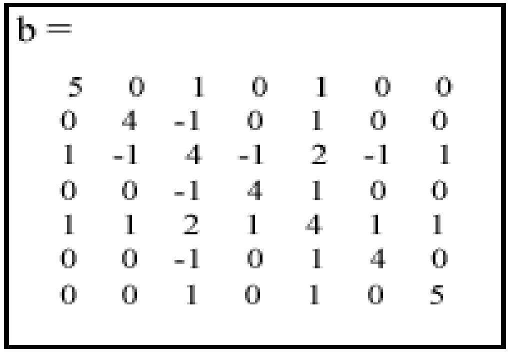
```
fd=

  1  0  0  0  0  0  0  0  0  0  0  0  0
  0  1  0  0  0  0  0  0  0  0  0  0  0
  0  0  1  0  0  0  0  0  0  0  0  0  0
  0  0  0  1  0  0  0  0  0  0  0  0  0
  0  0  0  0  1  0  0  0  0  0  0  0  0
  0  0  0  0  0  1  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  1  0  0  0  0  0
  0  0  0  0  0  0  0  0  1  0  0  0  0
  0  0  0  0  0  0  0  0  0  1  0  0  0
  0  0  0  0  0  0  0  0  0  0  1  0  0
  0  0  0  0  0  0  0  0  0  0  0  1  0
  0  0  0  0  0  0  0  0  0  0  0  0  1
```

E , the total sidelobe energy is given by

$$\mathbf{E} = \mathbf{y}\,\mathbf{F}\,\mathbf{y}' = (\mathbf{h}'\Psi)\mathbf{F}(\mathbf{h}'\Psi)' = \mathbf{h}'(\Psi\mathbf{F}\Psi')\,\mathbf{h} = \mathbf{h}'\,\mathbf{B}\,\mathbf{h}$$

The filter **h** which minimizes the sidelobe total energy **E**, is given by

$$\mathbf{h}_0 = \mathbf{B}^{-1}\mathbf{x}$$



```
b =

   5   0   1   0   1   0   0
   0   4  -1   0   1   0   0
   1  -1   4  -1   2  -1   1
   0   0  -1   4   1   0   0
   1   1   2   1   4   1   1
   0   0  -1   0   1   4   0
   0   0   1   0   1   0   5
```

Normalization of **h** can follow so that the energy of the filter **h'h** will be equal to the energy of a matched filter **x'x**

$$\mathbf{h} = \mathbf{h}_0\sqrt{\frac{\mathbf{x}'\mathbf{x}}{\mathbf{h}_0'\mathbf{h}_0}}$$

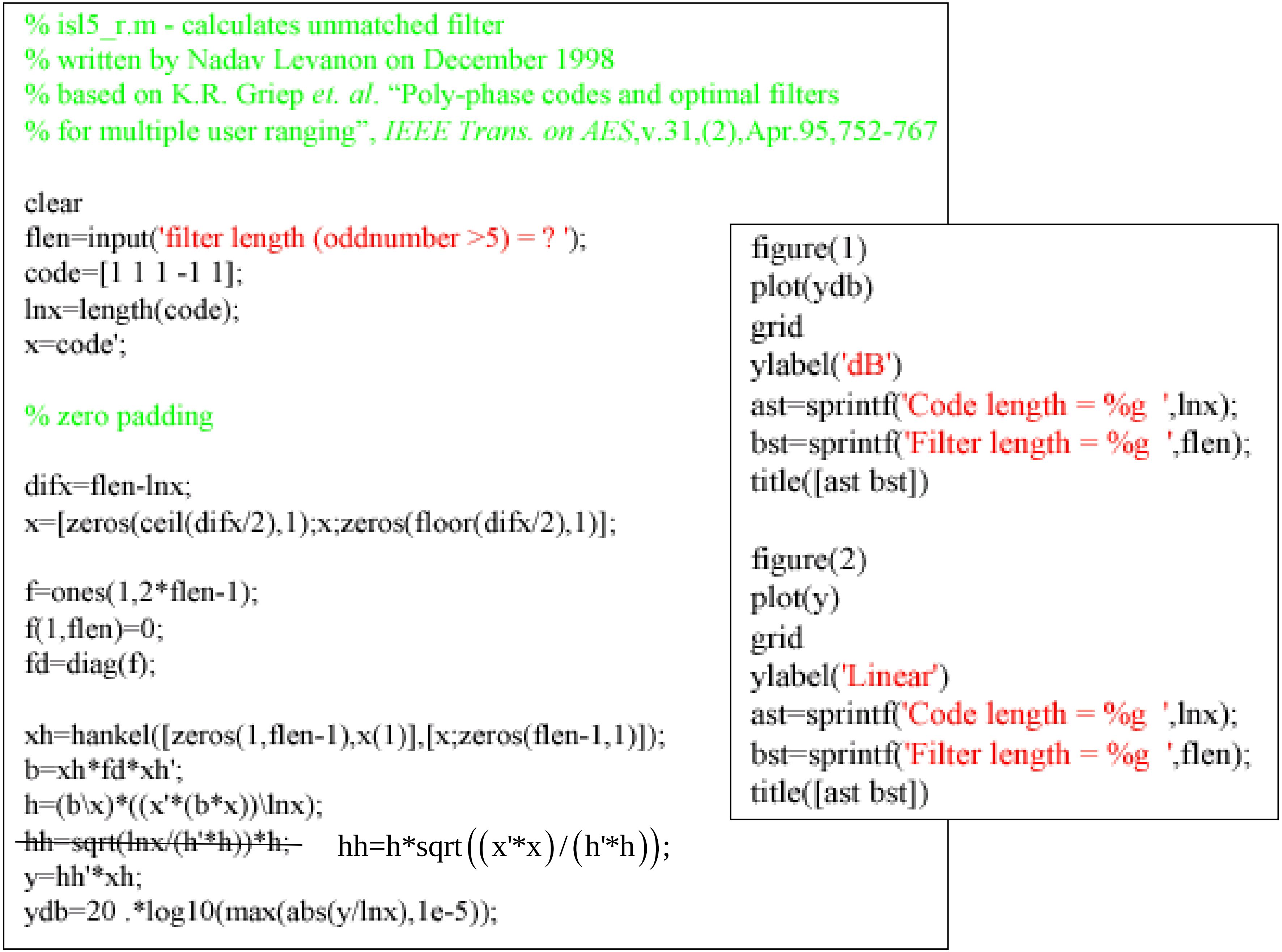
$h_0=$
0.0000
2.5000
4.3750
2.5000
-4.3750
2.5000
0

$h=$
0.0000
0.7402
1.2954
0.7402
-1.2954
0.7402
0

$y' =$
0
0
0.7402
-0.5552
0.1851
0.0000
4.8115
-0.0000
0.1851
0.5552
0.7402
0.0000
0

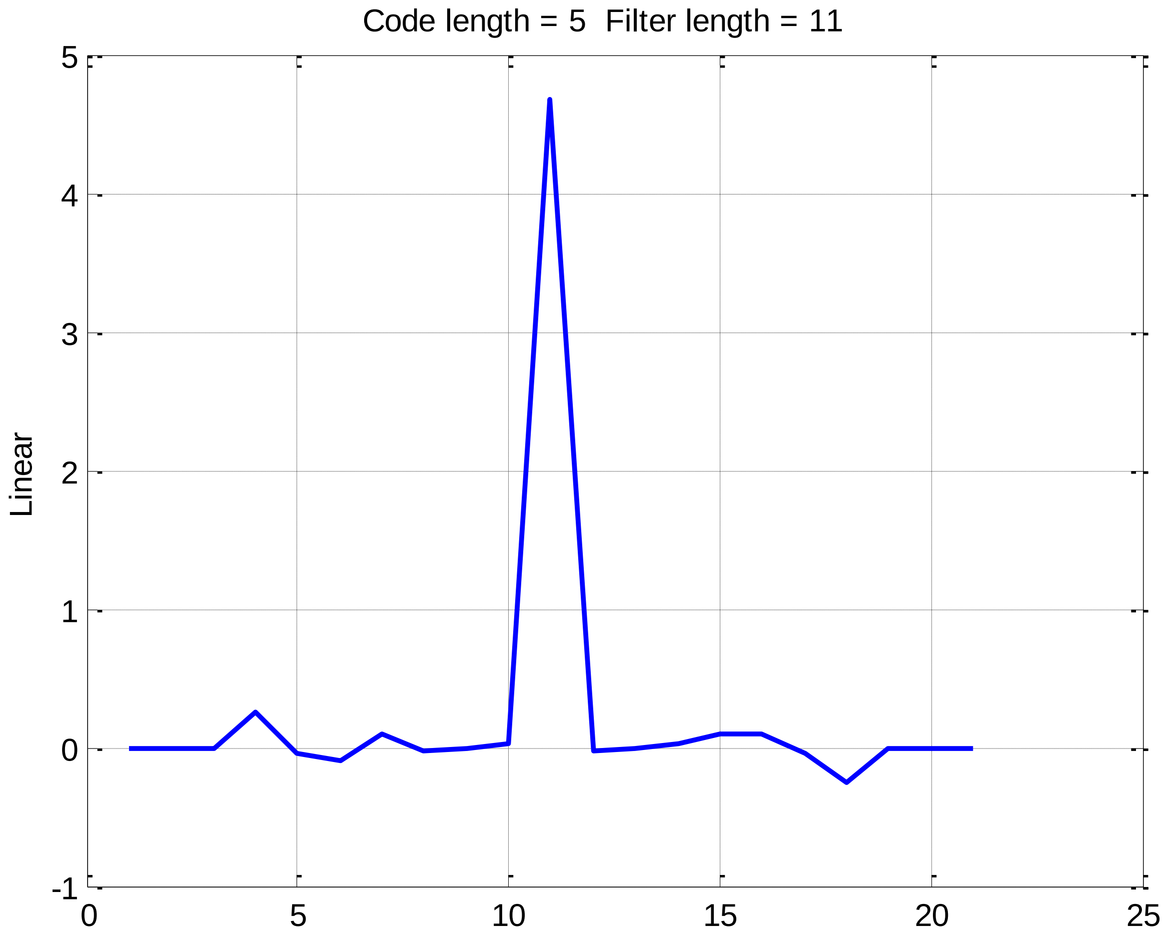
Normalized filter

5 in matched filter



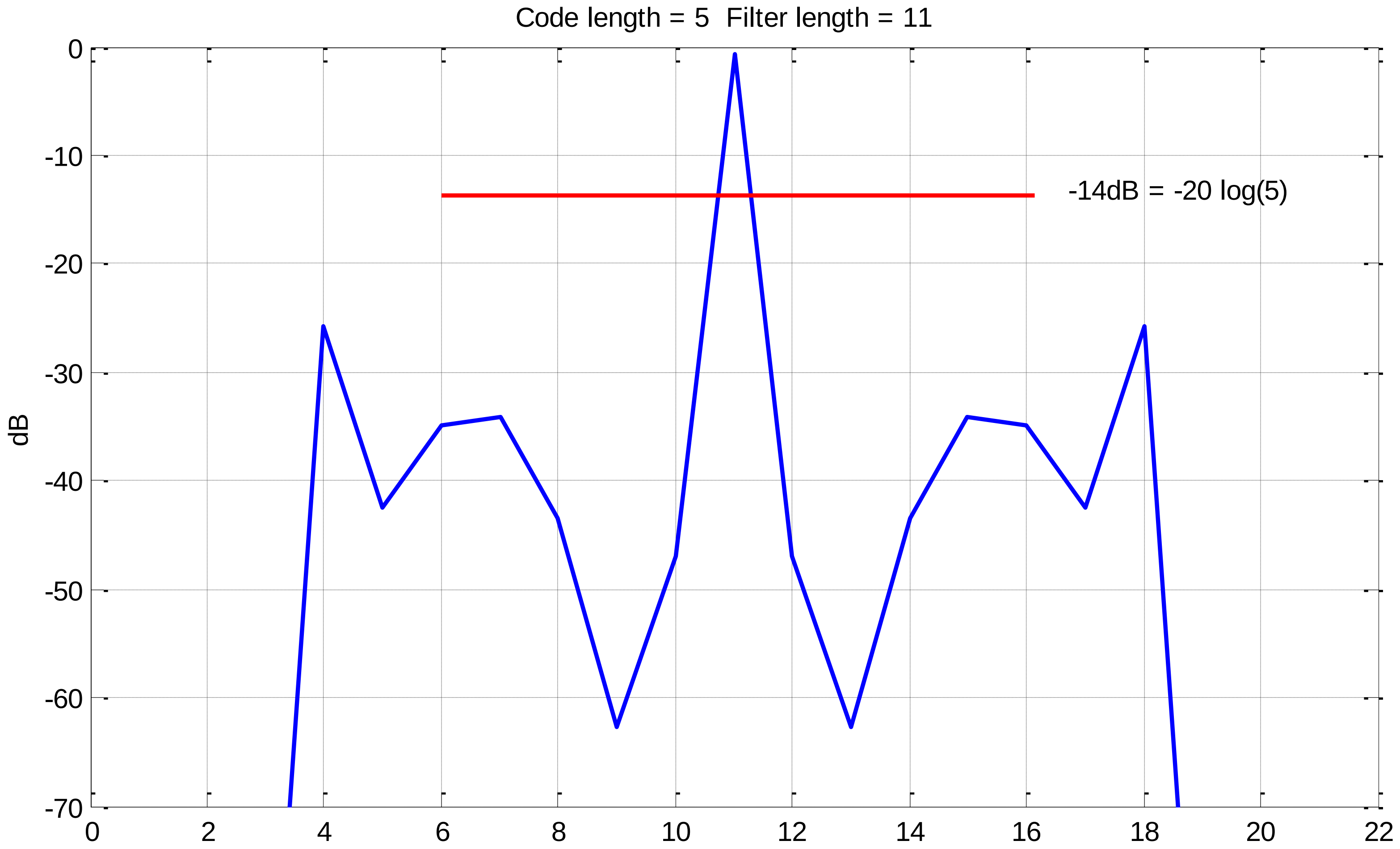
```
% isl5_r.m - calculates unmatched filter
% written by Nadav Levanon on December 1998
% based on K.R. Griep et. al. "Poly-phase codes and optimal filters
% for multiple user ranging", IEEE Trans. on AES,v.31,(2),Apr.95,752-767

clear
flen=input('filter length (oddnumber >5) = ? ');
code=[1 1 1 -1 1];
lnx=length(code);
x=code';

% zero padding

difx=flen-lnx;
x=[zeros(ceil(difx/2),1);x;zeros(floor(difx/2),1)];

f=ones(1,2*flen-1);
f(1,flen)=0;
fd=diag(f);

xh=hankel([zeros(1,flen-1),x(1)],[x;zeros(flen-1,1)]);
b=xh*fd*xh';
h=(b\x)*((x'*(b*x))\lnx);
~~hh=sqrt(lnx/(h'*h))*h;~~   hh=h*sqrt((x'*x)/(h'*h));
y=hh'*xh;
ydb=20 .*log10(max(abs(y/lnx),1e-5));
```

```
figure(1)
plot(ydb)
grid
ylabel('dB')
ast=sprintf('Code length = %g  ',lnx);
bst=sprintf('Filter length = %g  ',flen);
title([ast bst])

figure(2)
plot(y)
grid
ylabel('Linear')
ast=sprintf('Code length = %g  ',lnx);
bst=sprintf('Filter length = %g  ',flen);
title([ast bst])
```



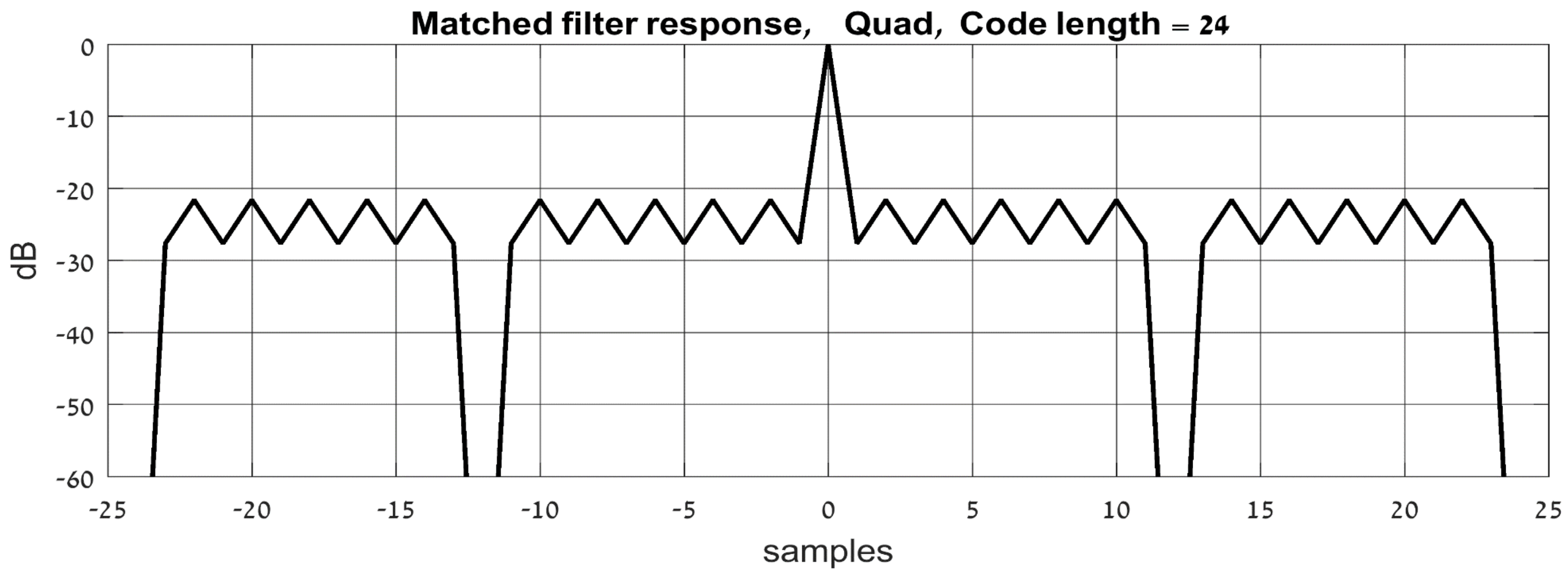
-0.2588
-0.2967
0.0518
0.7046
1.2421
0.7787
-1.2421
0.7046
-0.0518
-0.2967
0.2588

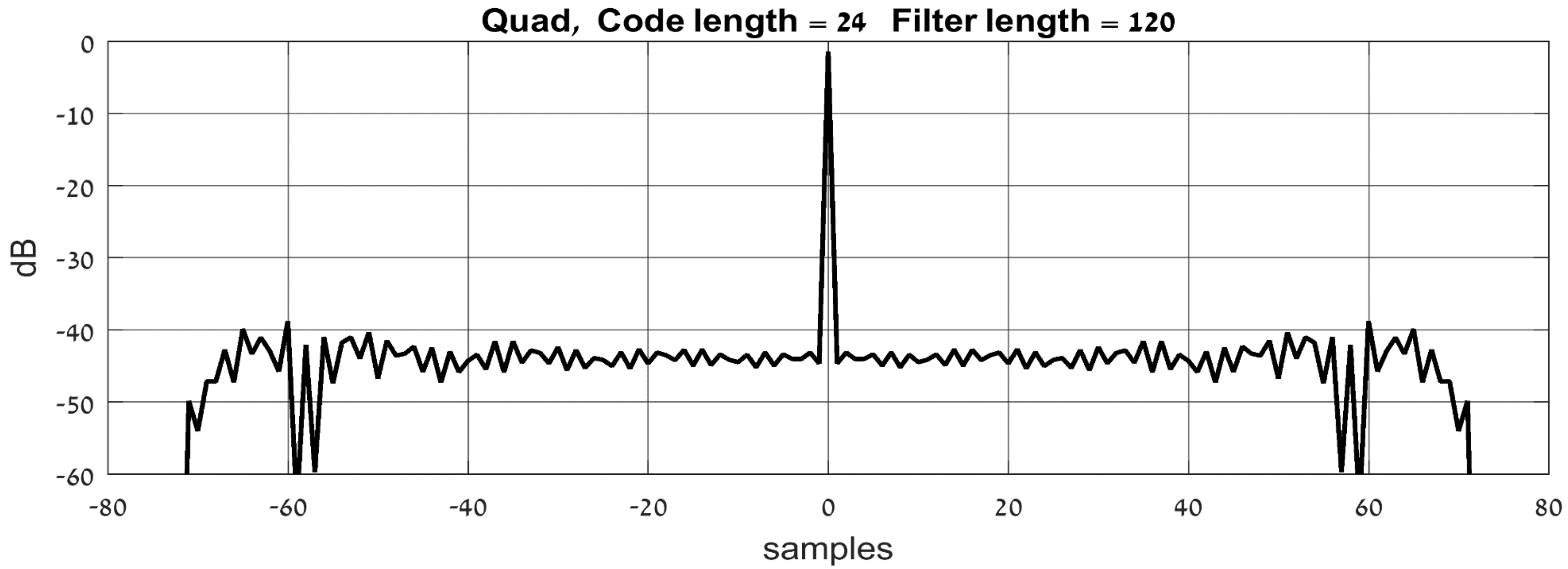
MM Filter

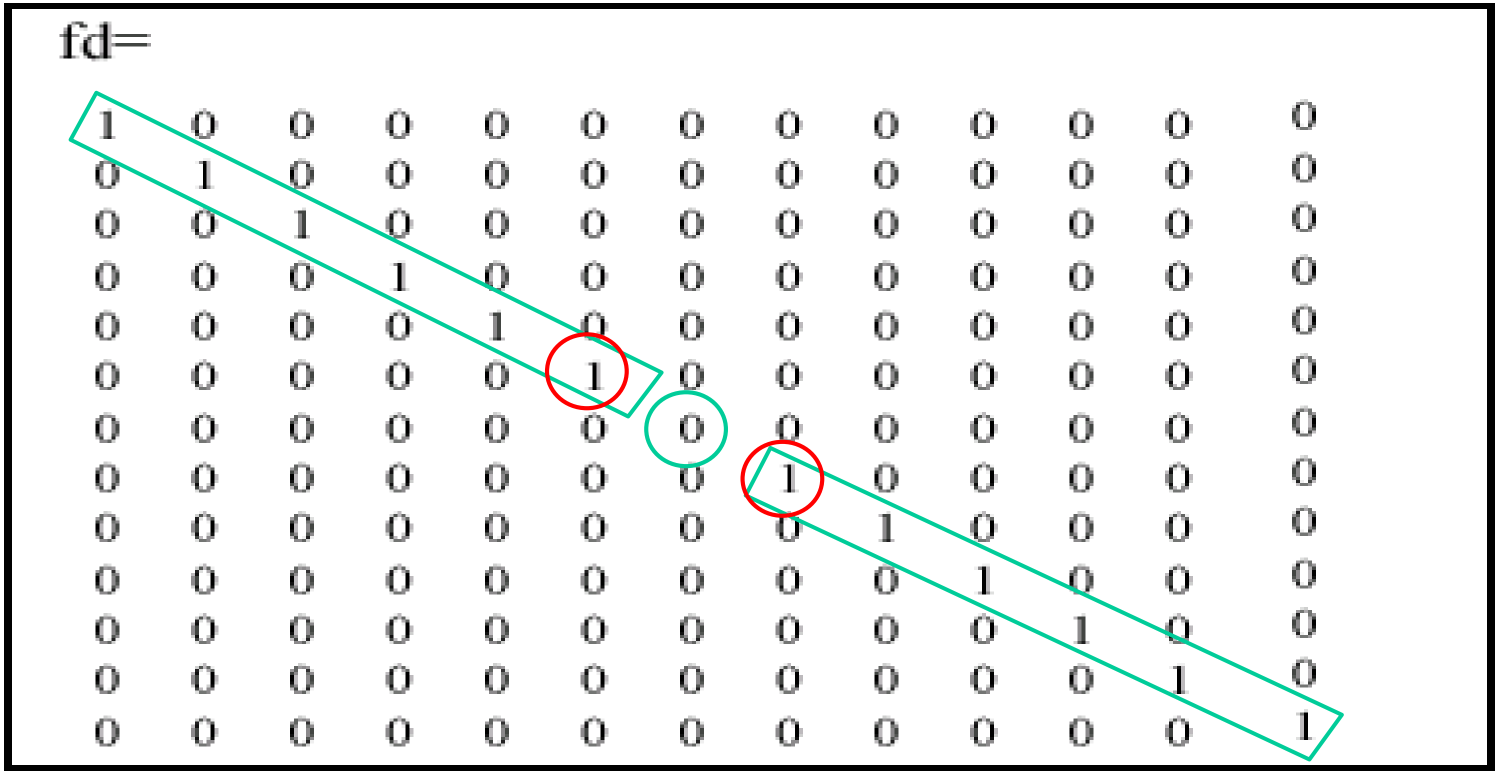

Code length = 5 Filter length = 11
dB
0
-10
-20
-30
-40
-50
-60
-70
0
2
4
6
8
10
12
14
16
18
20
22
-14dB = -20 log(5)

```
% isl_24_quad_short.m - mismatched filter for min ISL pulse,
% 24 elements quad phase sequences
% written by Nadav Levanon on 9 Jan 2017
clear
mm=24;
code=[1 1 1i 1 -1 -1i -1i 1 -1i -1 1i ...
    -1i -1i 1i -1 -1i 1 -1i -1i -1 1 1i 1 1];
dst='  Quad,   ';
flen=5*mm;
m=1:mm;
lnx=length(code);
x=code.'; % ←
% zero padding
difx=flen-lnx;
x=[zeros(ceil(difx/2),1);x;zeros(floor(difx/2),1)];
f=ones(1,2*flen-1);
f(1,flen)=0;
fd=diag(f);
xh=hankel([zeros(1,flen-1),x(1)],[x;zeros(flen-1,1)]);
b=xh*fd*xh';
h=(b\x)*((x'*(b*x))\lnx);
% hh=sqrt(lnx/(h'*h))*h;
hh=h*sqrt((x'*x)/(h'*h));
y=hh'*xh;
ydb=20 .*log10(max(abs(y/lnx),1e-5));
% plot(abs(y))
xc=xcorr(code);
xc_abs=abs(xc)/max(abs(xc));
isl_sig=1/(abs(xc(lnx))).^2 * 2*sum((abs(xc(1:lnx-1))).^2 )
delay_of_peak=find(y==max(y));
isl_filter=1/max(abs(y)).^2 * (sum((abs(y(1:delay_of_peak-1))).^2 ) ...
    +sum((abs(y(delay_of_peak+1:length(y)))).^2 ))
ydb_s=ydb(mm:9*mm);
xyscale=-mm*4:mm*4;
```

```
figure(1)
subplot(2,1,2)
plot(xyscale,ydb_s,'k','linewidth',1.5)
Grid on
ylabel('dB')
xlabel('samples')
ast=sprintf('Code length = %g   ',lnx);
bst=sprintf('Filter length = %g  ',flen);
title([dst ast bst ])
xc_out_db=20*log10(max(xc_abs,1e-5));
xc_scale=-mm*2:mm*2;
xc_out_db_2=[-100*ones(1,mm+1) xc_out_db...
    -100*ones(1,mm+1)];
ylim([-60,0])
subplot(2,1,1)
plot(xc_scale , xc_out_db_2, 'k','linewidth',1.5)
grid on
title([' Matched filter response,  '  dst ast])
xlabel('samples')
ylabel('dB')
ylim([-60,0])
```


Matched filter response, Quad, Code length = 24
dB
0
-10
-20
-30
-40
-50
-60
-25
-20
-15
-10
-5
0
5
10
15
20
25
samples

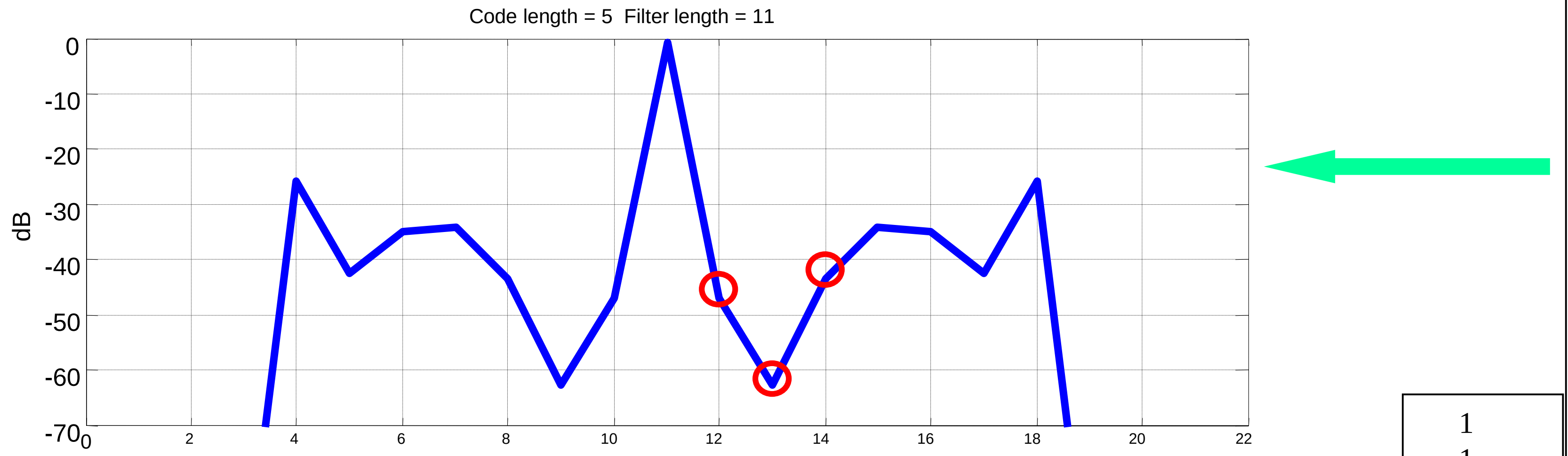

Quad, Code length = 24 Filter length = 120
dB
0
-10
-20
-30
-40
-50
-60
-80
-60
-40
-20
0
20
40
60
80
samples



```
fd=

  1  0  0  0  0  0  0  0  0  0  0  0   0
  0  1  0  0  0  0  0  0  0  0  0  0   0
  0  0  1  0  0  0  0  0  0  0  0  0   0
  0  0  0  1  0  0  0  0  0  0  0  0   0
  0  0  0  0  1  0  0  0  0  0  0  0   0
  0  0  0  0  0  1  0  0  0  0  0  0   0
  0  0  0  0  0  0  0  0  0  0  0  0   0
  0  0  0  0  0  0  0  1  0  0  0  0   0
  0  0  0  0  0  0  0  0  1  0  0  0   0
  0  0  0  0  0  0  0  0  0  1  0  0   0
  0  0  0  0  0  0  0  0  0  0  1  0   0
  0  0  0  0  0  0  0  0  0  0  0  1   0
  0  0  0  0  0  0  0  0  0  0  0  0   1
```

Increasing values on the diagonal will tend to lower the corresponding sidelobes;

E.g., increasing the two “ones” near the mainlobe’s “zero” will lower the near-sidelobes.



1
1
1
1
1
1
1
1
1
1
0
1
1
1
1
1
1
1
1
1
1

1
1
1
1
1
1
1
20
20
20
0
20
20
20
1
1
1
1
1
1
1

$-20\log_{10}(13) = -22.28\text{dB}$

Mismatched filter for Barker 13

`fmincon.m`

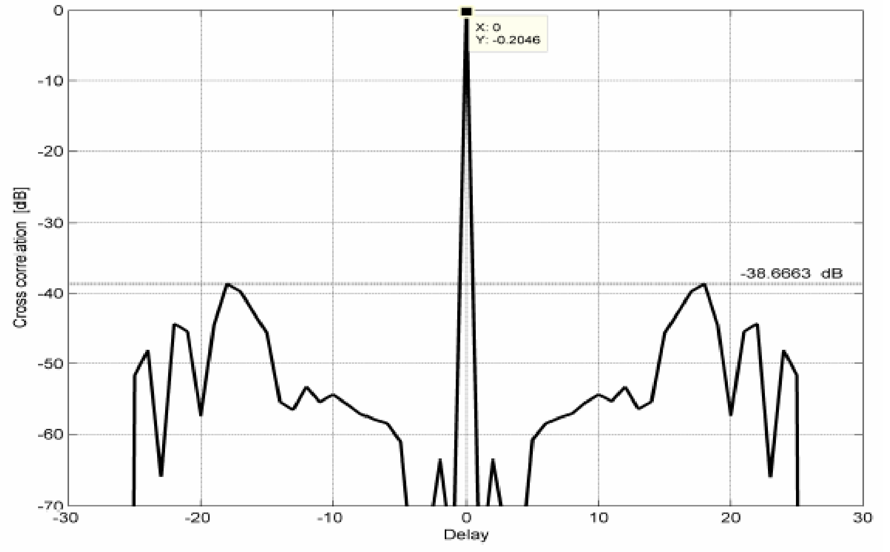


Fig. 1 *Output of mismatched min ISL filter with M = 39, for a Barker 13 signal*

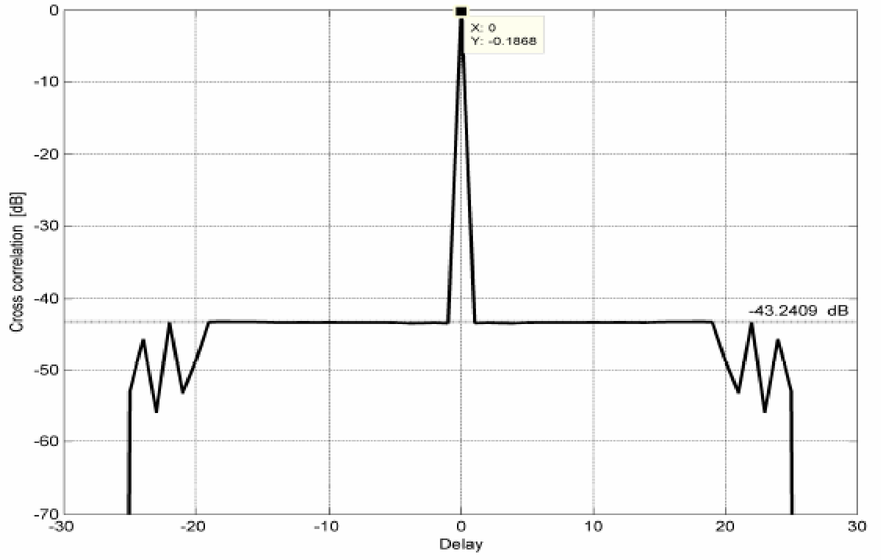
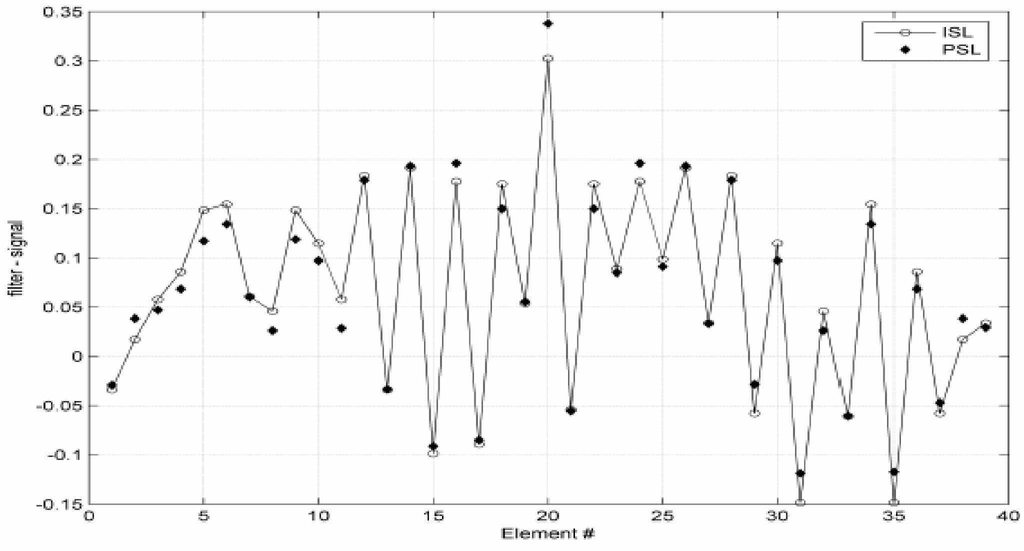


Fig. 2 *Output of mismatched min PSL filter with M = 39, for a Barker 13 signal*

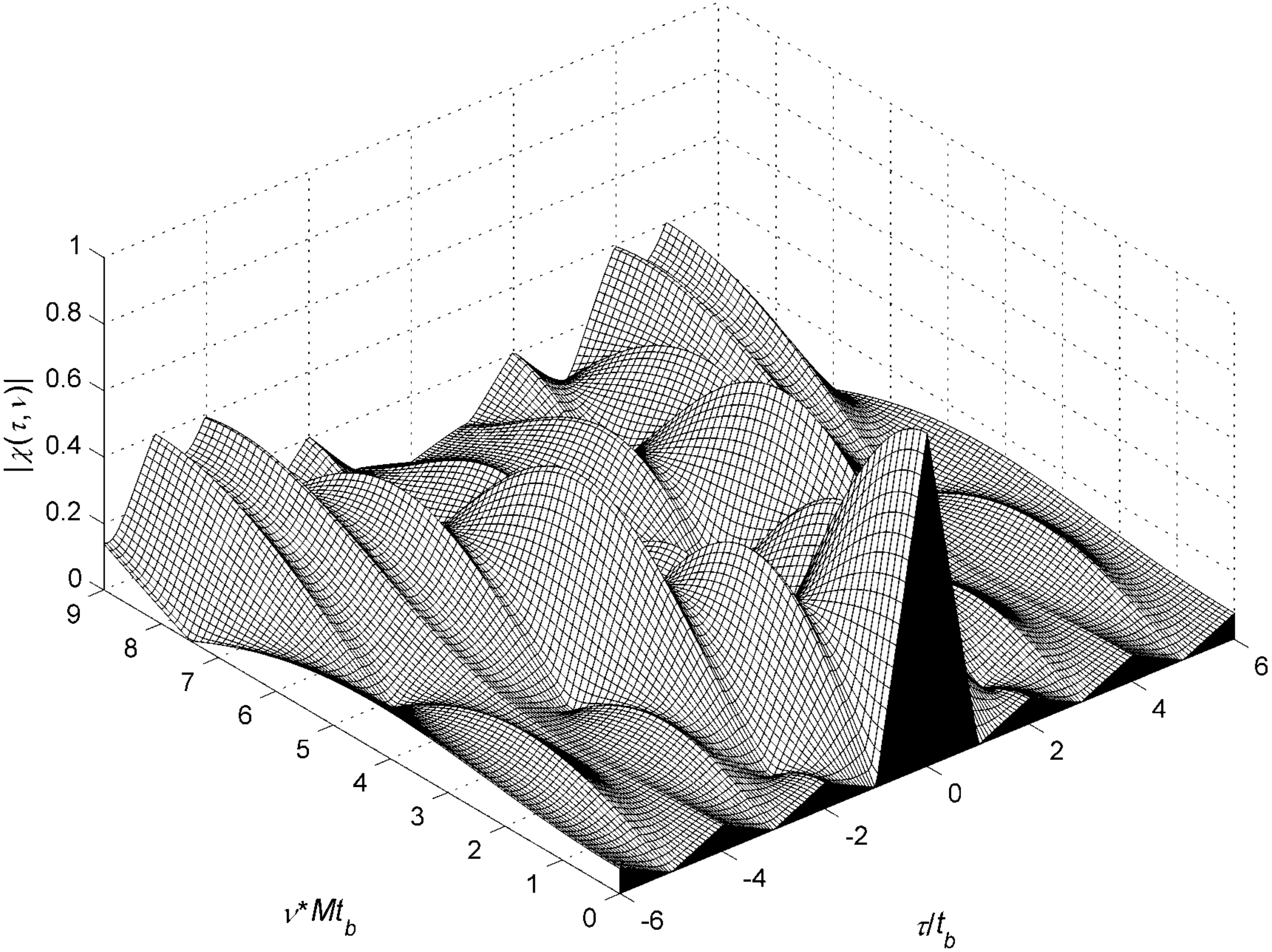


Deviation of mismatched filter elements from Barker 13 signal values (The signal occupies elements 14 to 26)

N. Levanon : "Cross-correlation of long binary signals with longer mismatched filters", *IEE Proc. - Radar, Sonar and Navigation*, 152 (6), 372-382, 2005

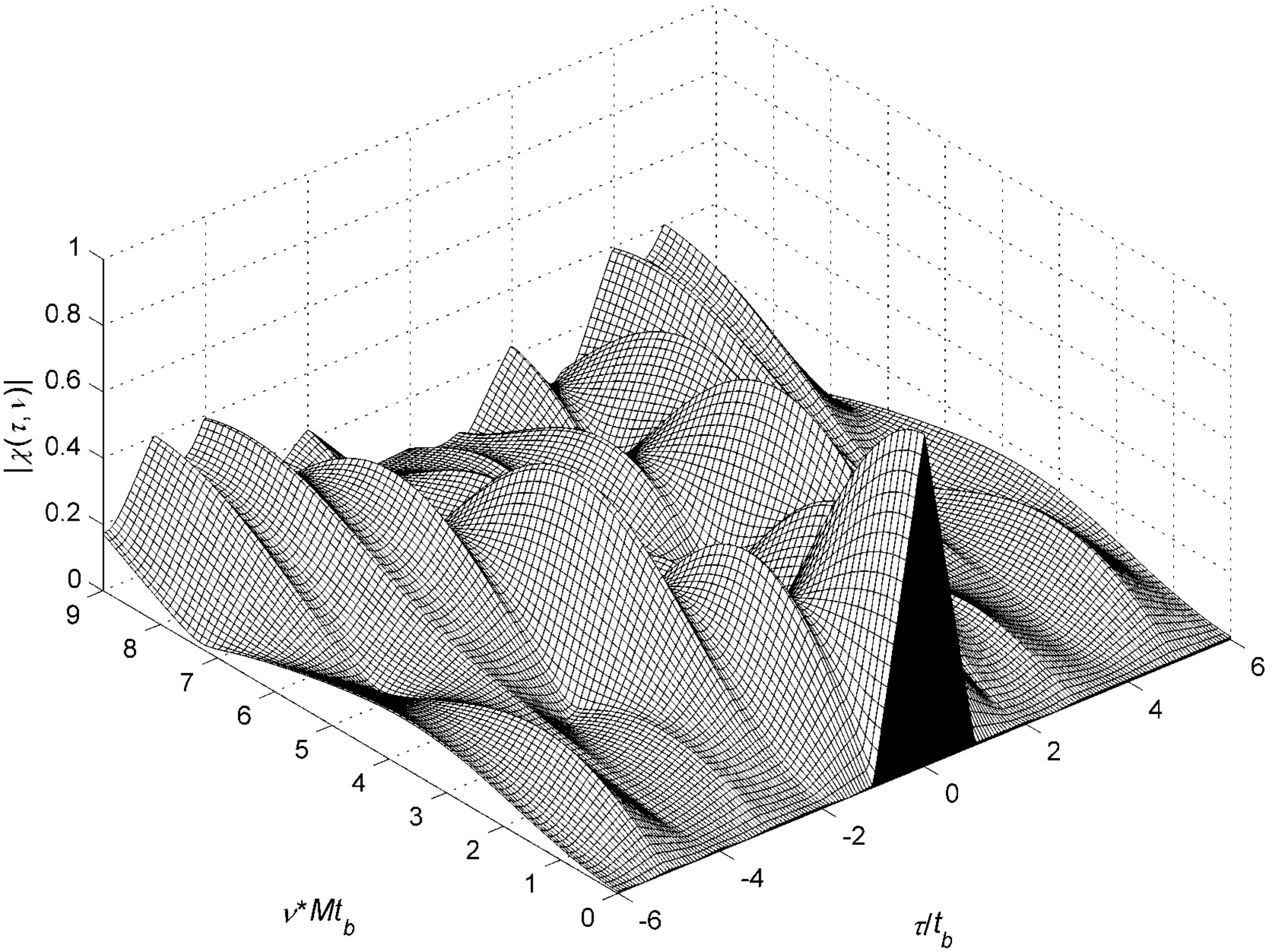
Matched filter for Barker 13

Signal	Reference
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
-1	-1
-1	-1
-1	-1
-1	-1
-1	-1
1	1
1	1
-1	-1
-1	-1
1	1
-1	-1
1	1
-1	-1
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0



Mismatched (min PSL) filter for Barker 13

Signal	Reference
0	-0.0293
0	0.0383
0	0.0471
0	0.0684
0	0.1171
0	0.1343
0	0.0603
0	0.0262
0	0.1188
0	0.0972
0	0.0285
0	0.1789
0	-0.0330
-1	-0.8068
-1	-1.0913
-1	-0.8040
-1	-1.0850
-1	-0.8501
1	1.0554
1	1.3379
-1	-1.0554
-1	-0.8501
1	1.0850
-1	-0.8041
1	1.0914
-1	-0.8069
0	0.0331
0	0.1789
0	-0.0285
0	0.0971
0	-0.1188
0	0.0262
0	-0.0603
0	0.1343
0	-0.1171
0	0.0684
0	-0.0471
0	0.0383
0	0.0292



Radar Principles - extended

Nadav Levanon, Tel-Aviv University

$$t_p = 0.65\,\mu\text{sec} = \text{pulse width}$$

$$t_b = \frac{t_p}{13} = 0.05\,\mu\text{sec} \Rightarrow \Delta R = 7.5\text{m}$$

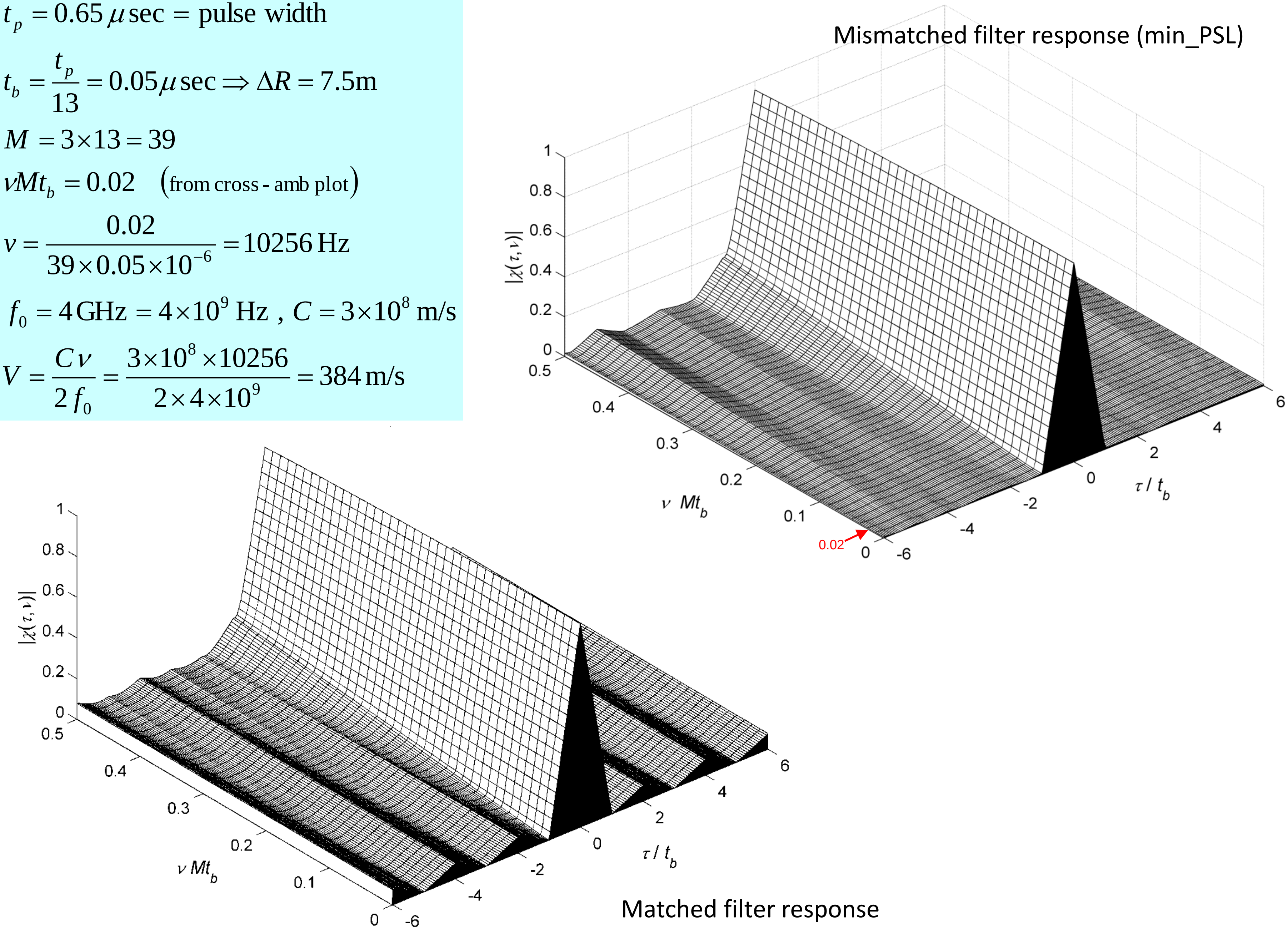
$$M = 3\times 13 = 39$$

$$\nu M t_b = 0.02 \quad (\text{from cross - amb plot})$$

$$\nu = \frac{0.02}{39\times 0.05\times 10^{-6}} = 10256\text{ Hz}$$

$$f_0 = 4\,\text{GHz} = 4\times 10^9\text{ Hz}\ ,\ C = 3\times 10^8\text{ m/s}$$

$$V = \frac{C\,\nu}{2\,f_0} = \frac{3\times 10^8\times 10256}{2\times 4\times 10^9} = 384\,\text{m/s}$$

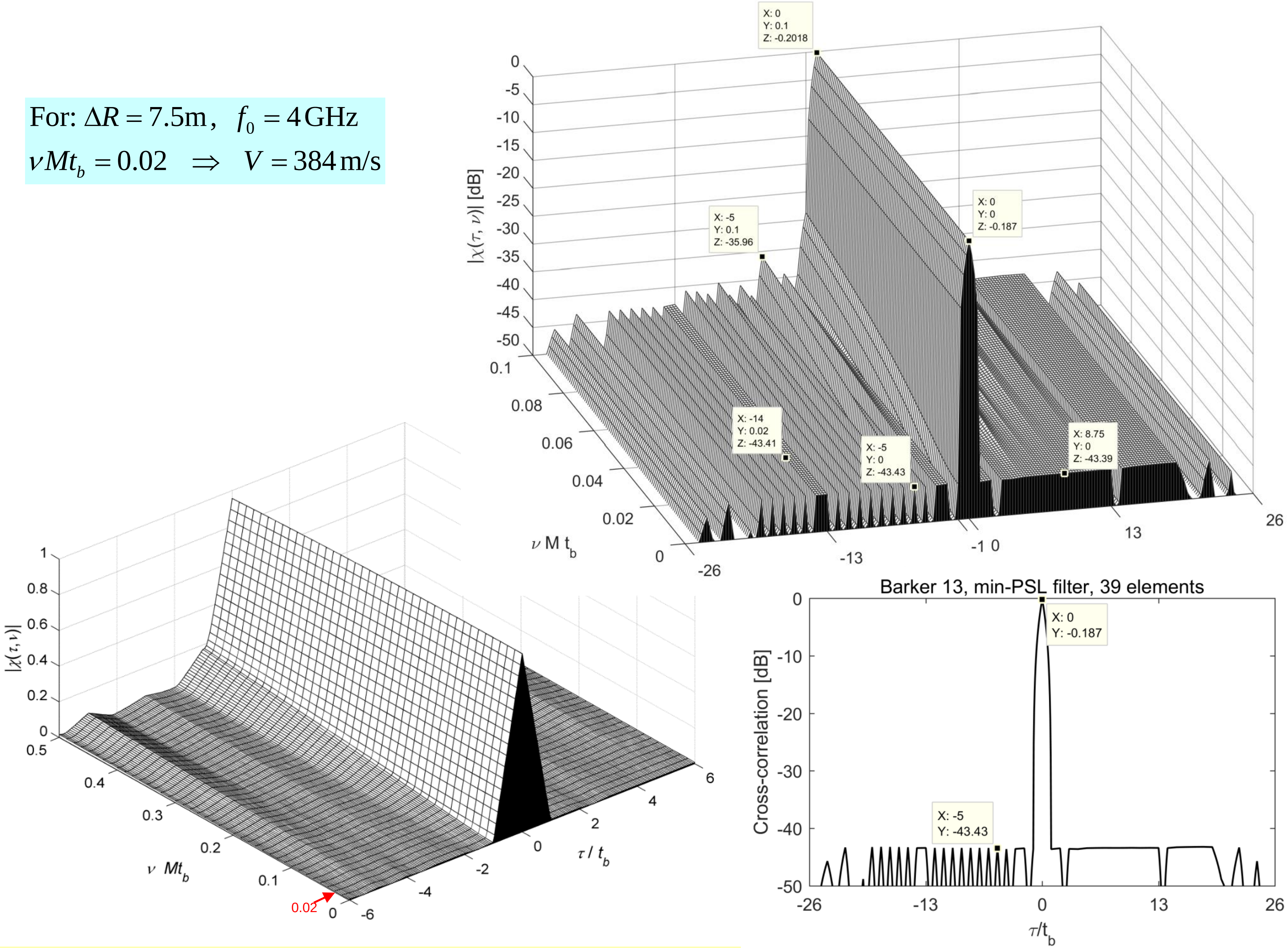


Mismatched filter response (min_PSL)

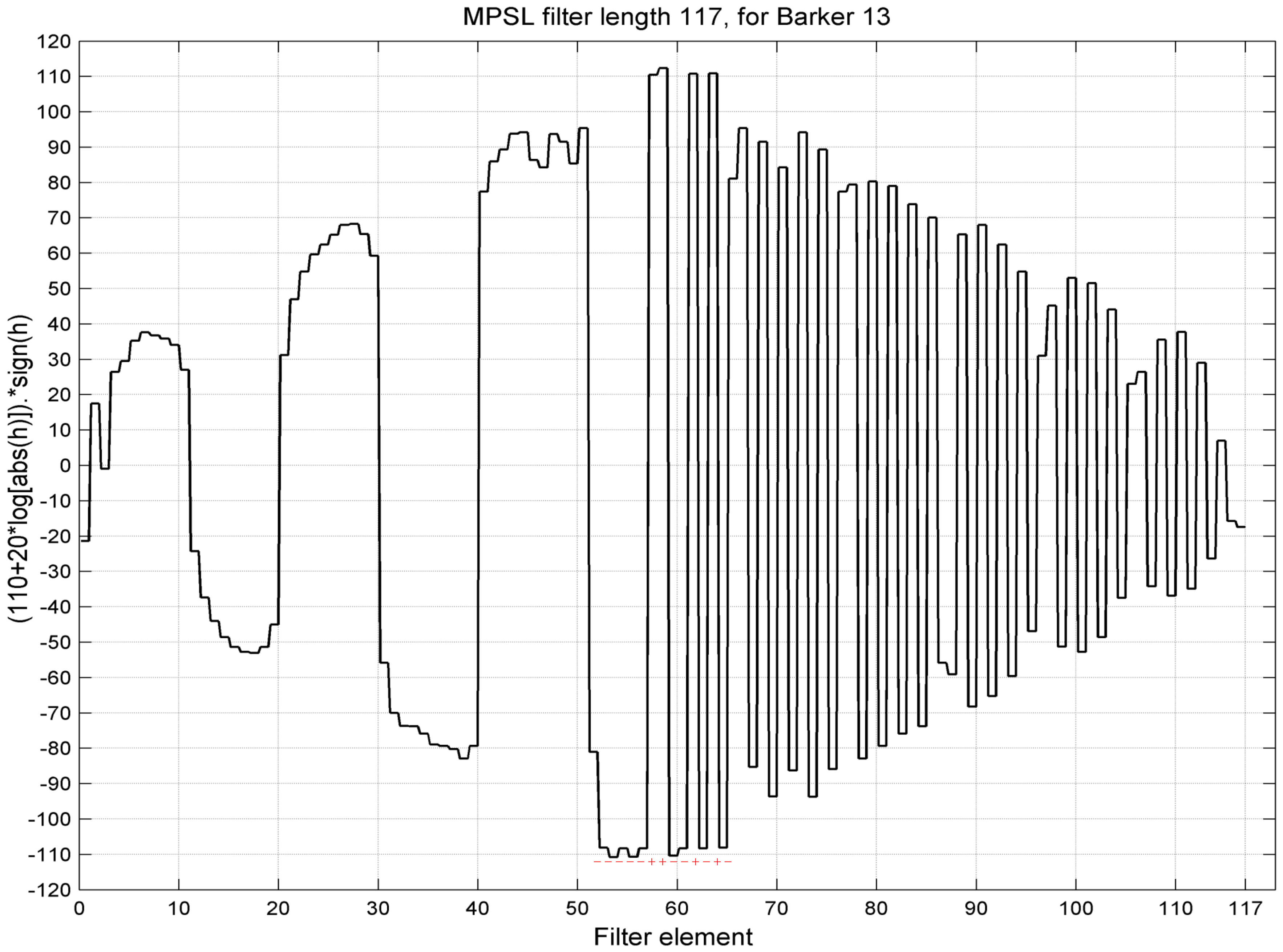
Matched filter response

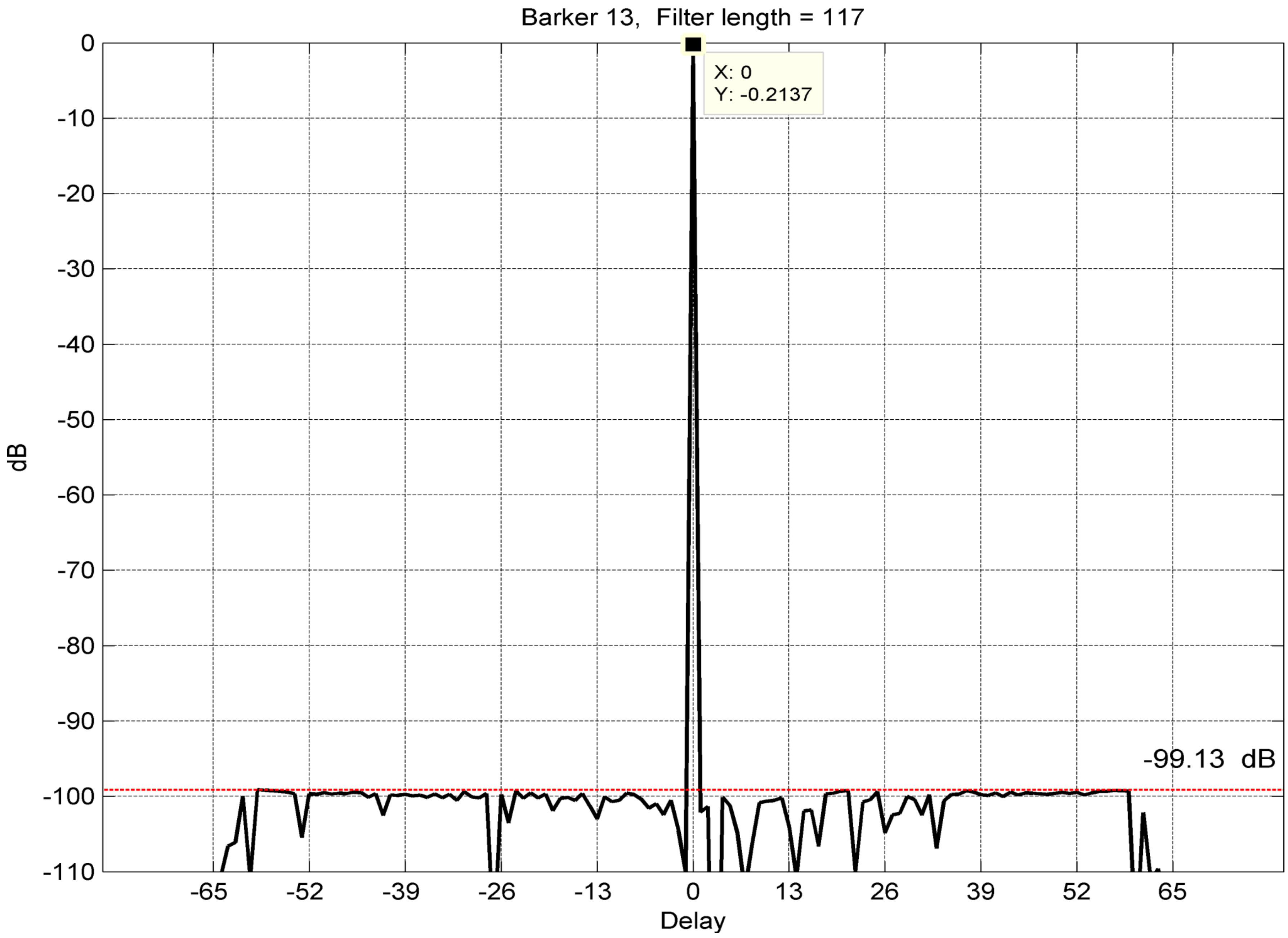
For: $\Delta R = 7.5\mathrm{m}, \quad f_0 = 4\,\mathrm{GHz}$

$\nu M t_b = 0.02 \quad \Rightarrow \quad V = 384\,\mathrm{m/s}$



MPSL filter length 117, for Barker 13




Barker 13, Filter length = 117
X: 0
Y: -0.2137
-99.13 dB
dB
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-65
-52
-39
-26
-13
0
13
26
39
52
65
Delay

In LFM, frequency weighting on-receive will:

Reduce range sidelobes.

The penalties:

Wider mainlobe

SNR loss

In chirp-like phase-coded signals (e.g., P4) with mismatched filter on receive, setting zero weight to the two nearest sidelobes will:

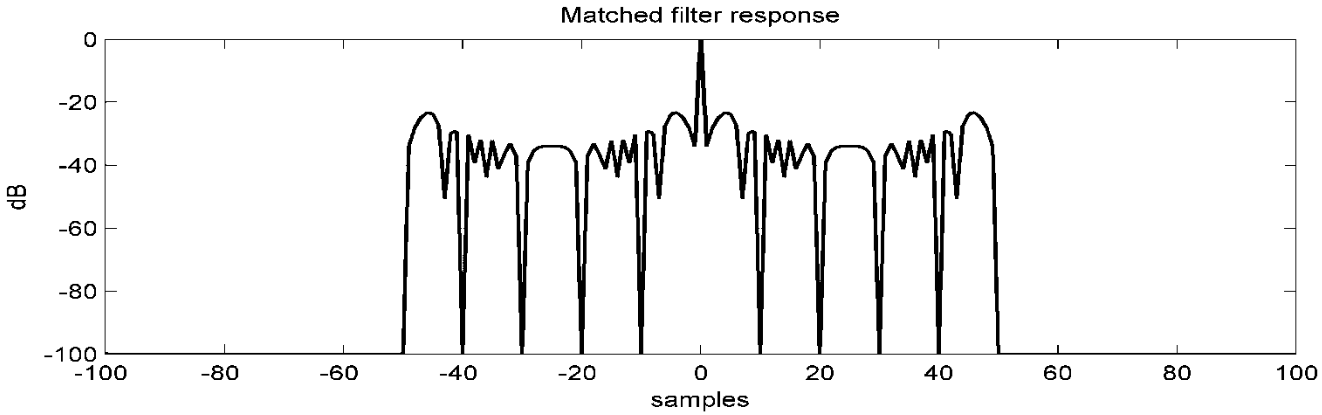
Reduce range sidelobes

Widen mainlobe

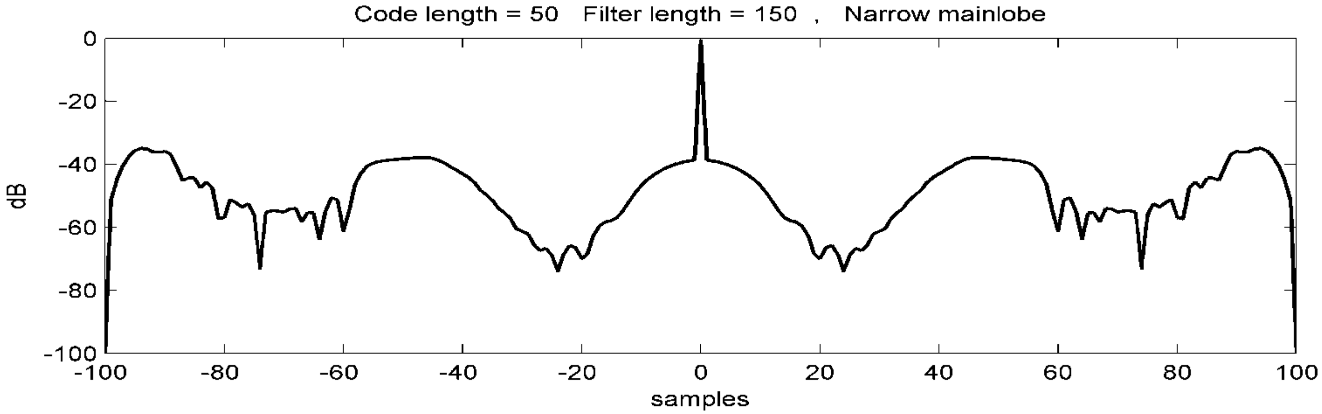
Increase SNR loss

$$\begin{matrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{matrix}$$

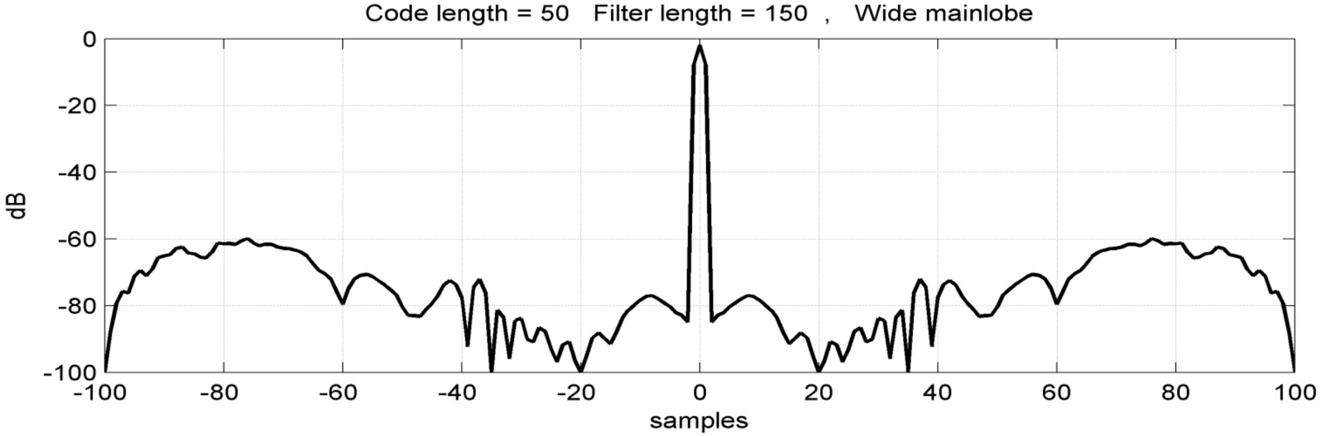
P4, N = 50
Filter length = 150


Matched filter response

PSL=-34.87 dB
SNR loss=0.47 dB


Code length = 50 Filter length = 150 , Narrow mainlobe

PSL=-59.93 dB
SNR loss=1.87 dB


Code length = 50 Filter length = 150 , Wide mainlobe

Mismatched filter for good* binary signal of length 169

+ + + − − − − − + + + − − − + + + + + − − − + + − − − + − − − + + + − + + + − − − + + + − + − + + − − −
+ + + + + + + − + − − − − − + − + + + − + + − + − + + + + + + − + + + − + − + − − − − − + + + − + + +
+ − + − + + + + − + − + − + + − + + − − − − − + − − + − − + − − − + − − + − − − + − − + + − + + − + − +
+ − + + − + + − + − + + − +

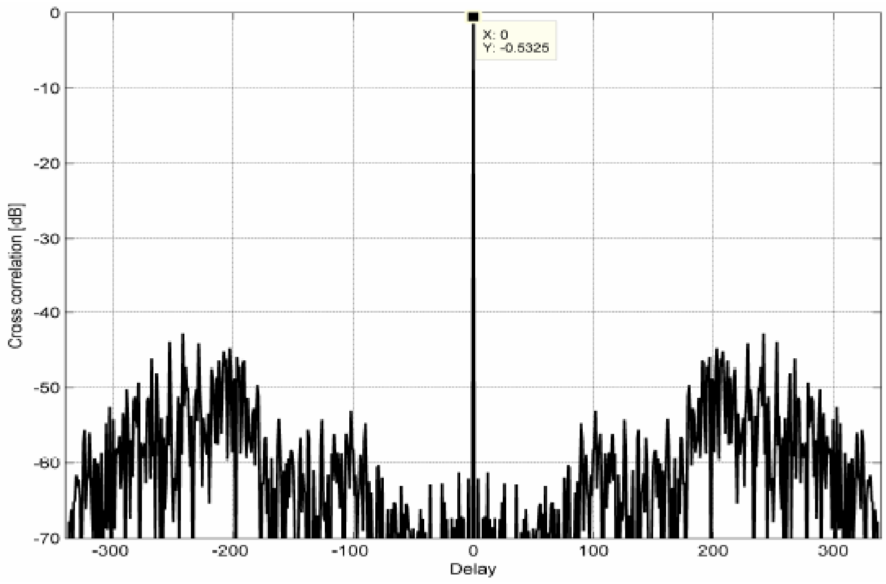


Fig. 6 *Output of mismatched min ISL filter with M = 507, to a low ISL signal of length 169*

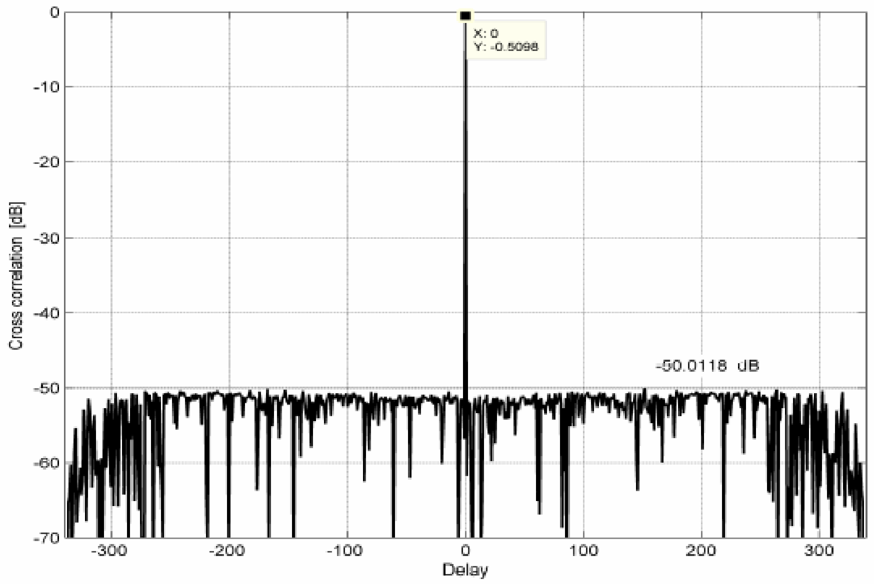


Fig. 9 *Output of mismatched min PSL filter with M = 507, to a low ISL signal of length 169*

good* = ACF with low ISL

$$R_k(S) = \sum_{i=1}^{P-k} s_i \, h_{i+k} \ , \quad k = 0, 1, 2, \ldots, \ P-1$$

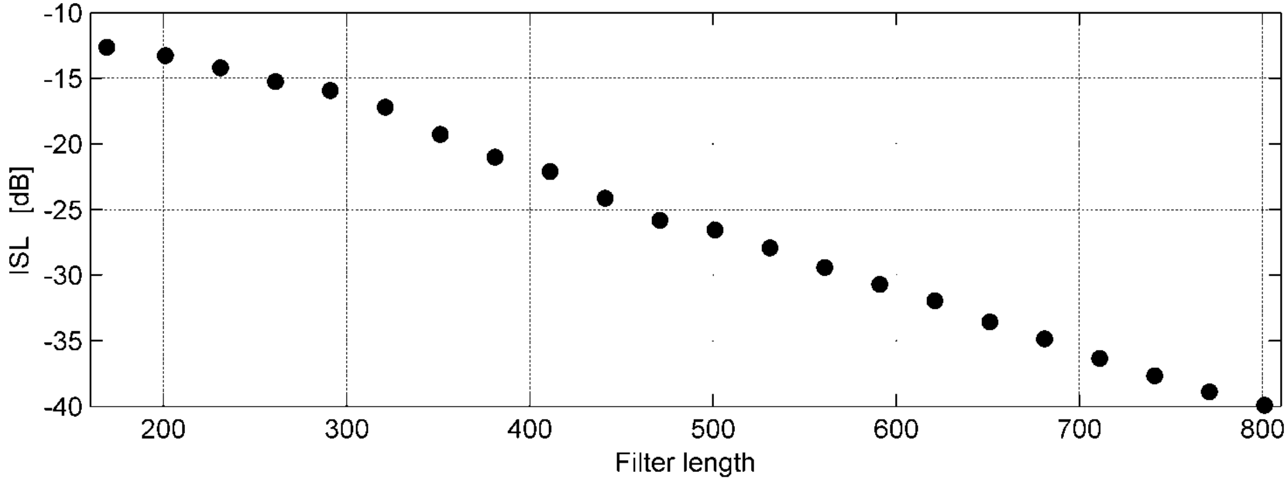
Good binary signal of length 169

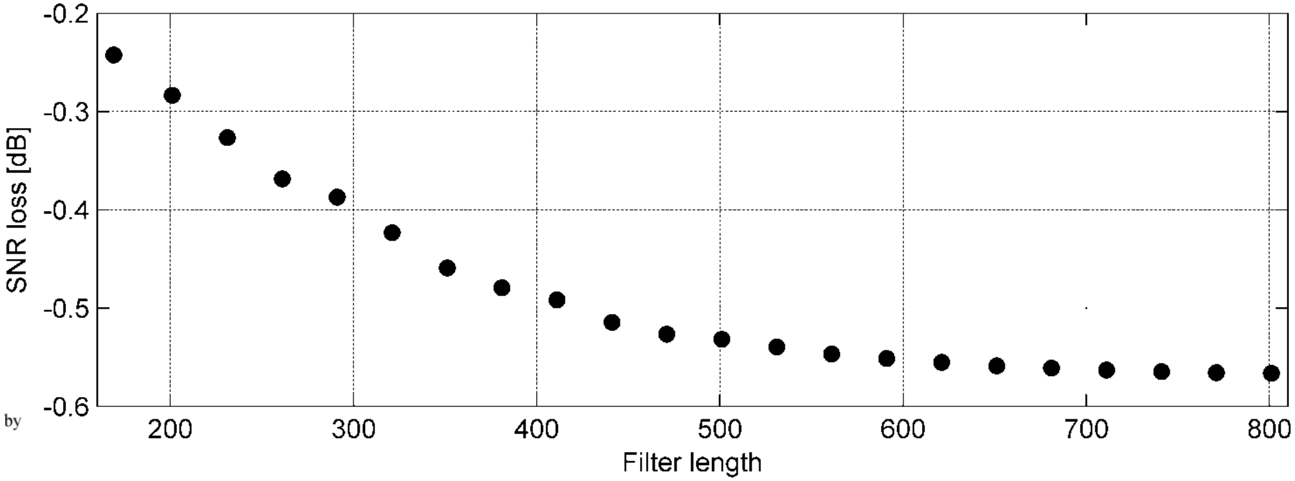
The signal s and the filter h are of the same length P due to zero-padding of the signal.

+++-----+++---+++++---++---+---+++-+++---+++-+-++---
+++++++-+-----+-+++-++-+-++++++-+++-+-+-----+++-+++
+-+-++++-+-+-++-++-----+--+--+---+--+---+--++-++-+-+
+-++-++-+-++-+

$$ISL = \frac{1}{R_0^2} \sum_{k \neq 0} R_k^2$$

ISL and SNR loss vs. filter length





E , the total sidelobe energy is given by

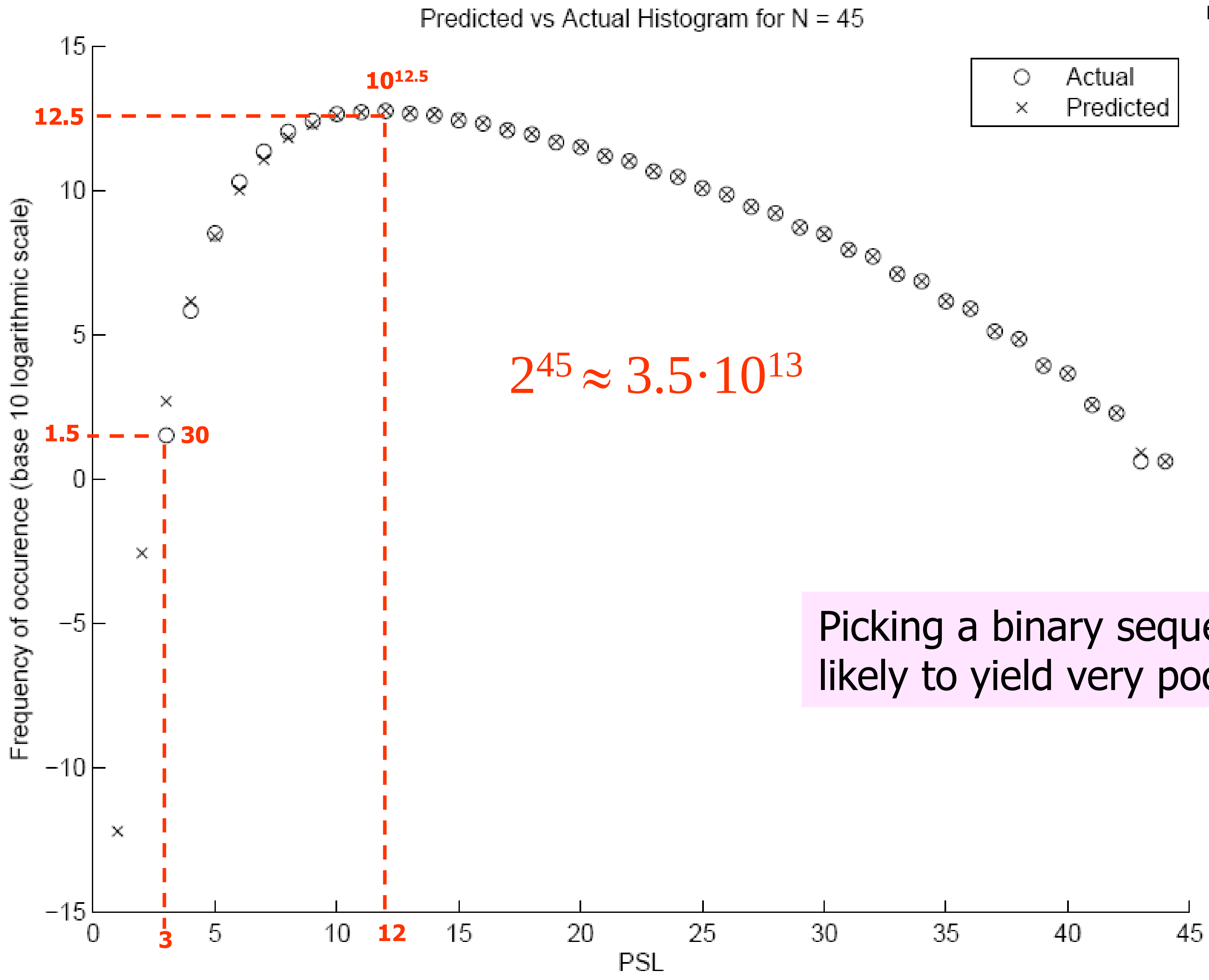
$$\mathbf{E} = \mathbf{y}\,\mathbf{F}\,\mathbf{y}' = (\mathbf{h}'\Psi)\mathbf{F}(\mathbf{h}'\Psi)' = \mathbf{h}'(\Psi\mathbf{F}\Psi')\,\mathbf{h} = \mathbf{h}'\,\mathbf{B}\,\mathbf{h}$$

The filter **h** which minimizes the sidelobe total energy **E**, is given by

$$\mathbf{h}_0 = \mathbf{B}^{-1}\mathbf{x}$$

From Ferrara, Kupferschmid and Coxson

Int'l waveform diversity and design conf., Pisa, 2007

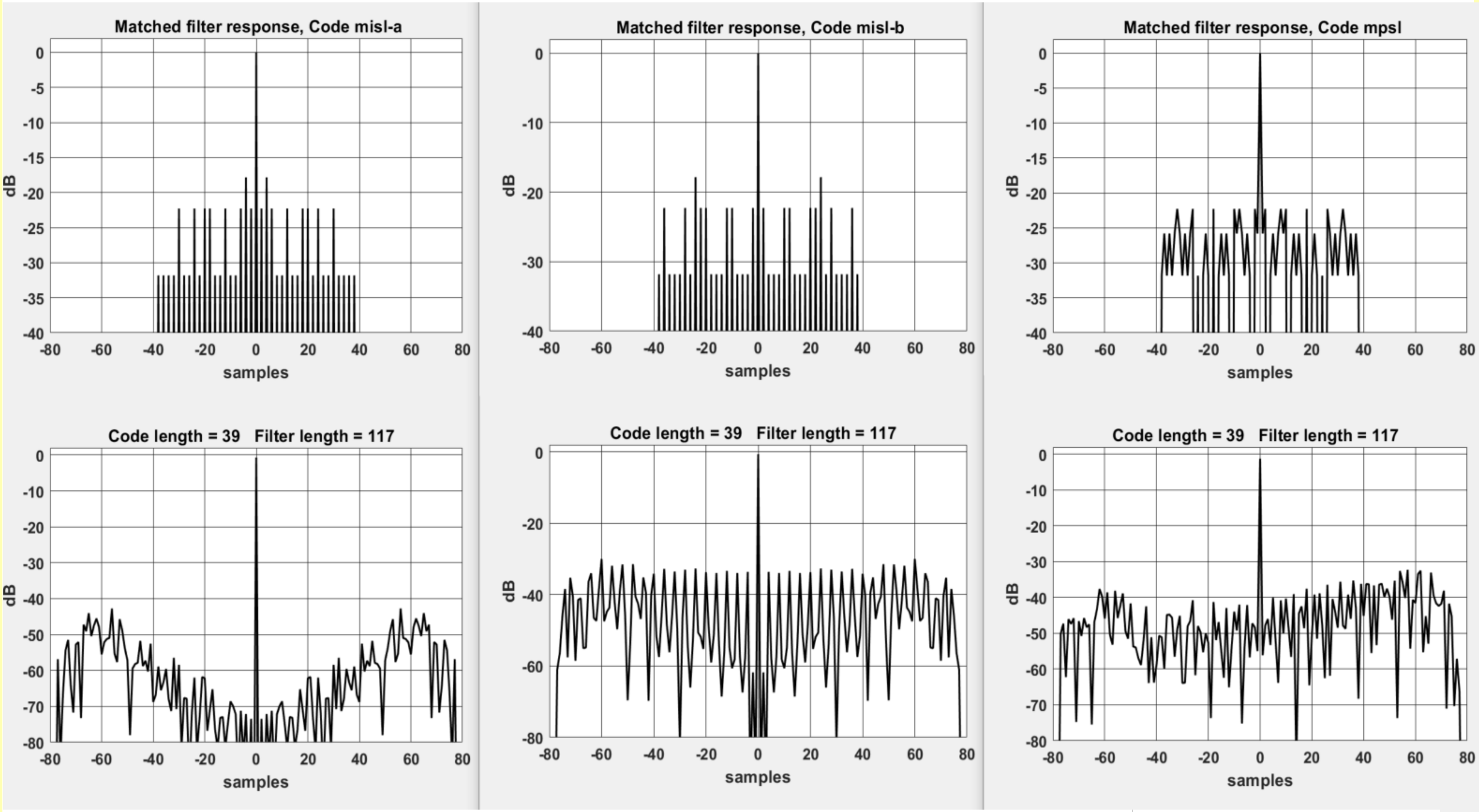


$$2^{45} \approx 3.5 \cdot 10^{13}$$

Picking a binary sequence randomly is likely to yield very poor PSLR

Histogram generation and prediction results for $N = 45$.

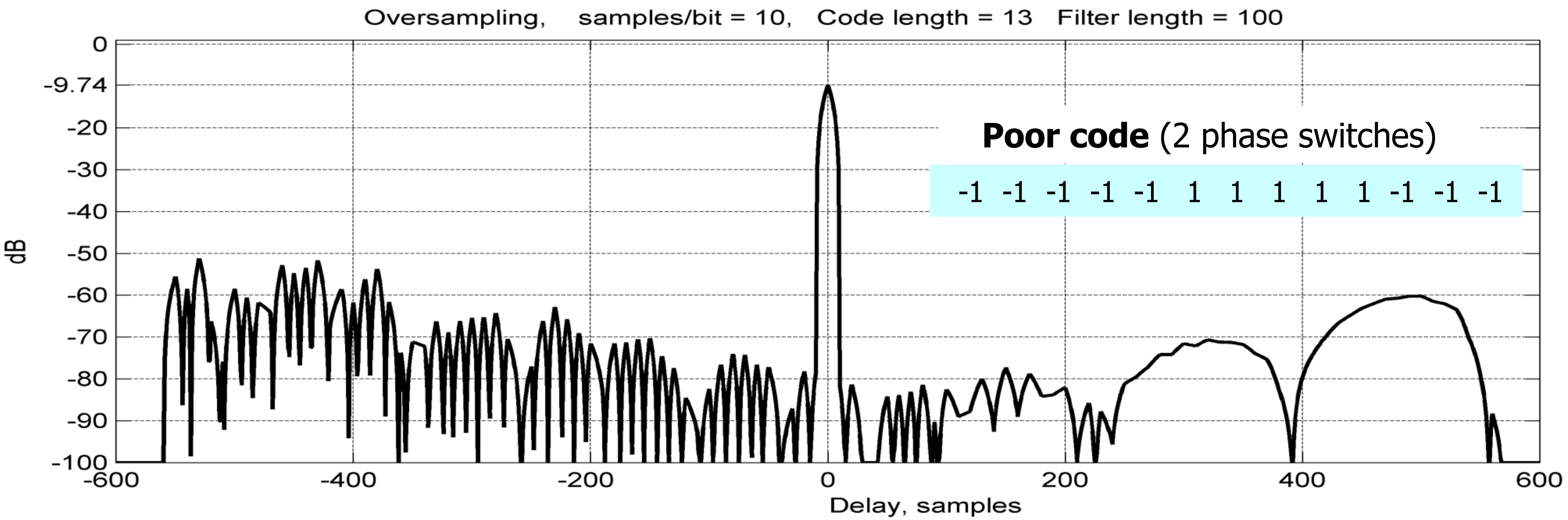
Comparing min-ISL MMF response of the “best” 39 element binary codes: two MISLs and one MPSL



```
misl_39_a=-1+2*[1 1 1 1 1 1 1 1 0 0 1 0 0 1 0 1 1 0 1 1 0 0 0 1 1 1 1 0 0 0 1 1 0 1 0 1 0 1 0];
misl_39_b=-1+2*[1 1 0 0 0 1 1 0 0 0 0 1 0 1 1 1 1 1 1 1 0 1 0 1 0 1 1 1 1 0 1 0 0 1 1 0 1 1 0];
mpsl_39  =-1+2*[0 0 1 0 0 1 1 0 0 1 1 0 1 0 1 0 0 0 0 1 0 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 0 0];
```

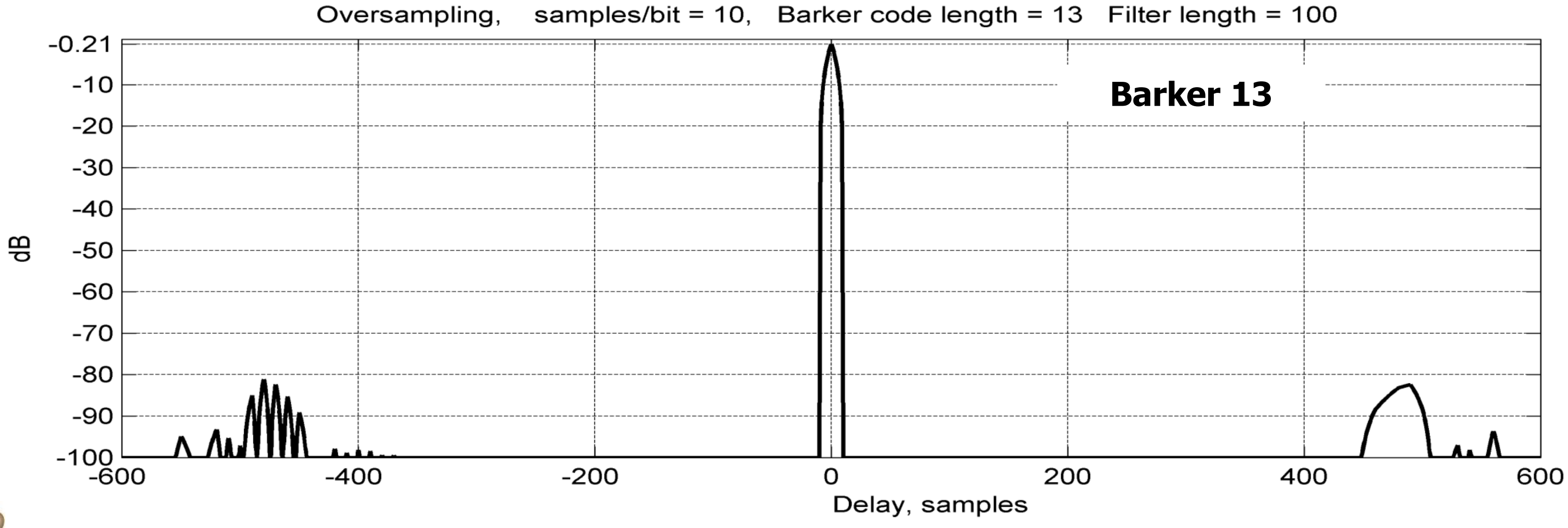
It is hard to predict which code will suit mismatched filtering

Comparing MISL mismatched outputs (good and poor codes)



Poor code (2 phase switches)

-1 -1 -1 -1 -1 1 1 1 1 1 -1 -1 -1



Barker 13

Signal/Mismatched filter - conclusions

- The signal should have a relatively good autocorrelation function
    - Good ACF ➔ low sidelobes, especially low far sidelobes
- A mismatched filter that is longer than the signal can then reduce the sidelobes considerably, without introducing large SNR loss.
- The ISL continues to drop as the filter length increases.
- The SNR loss levels off as the filter length increases.
- A mismatched filter does not necessarily degrade the Doppler tolerance.

Good initial code + longer mismatched filter ➔ good delay response

Good initial codes are hard to find!
Random selection has no chance of producing good codes!

Coherent train of non-identical pulses

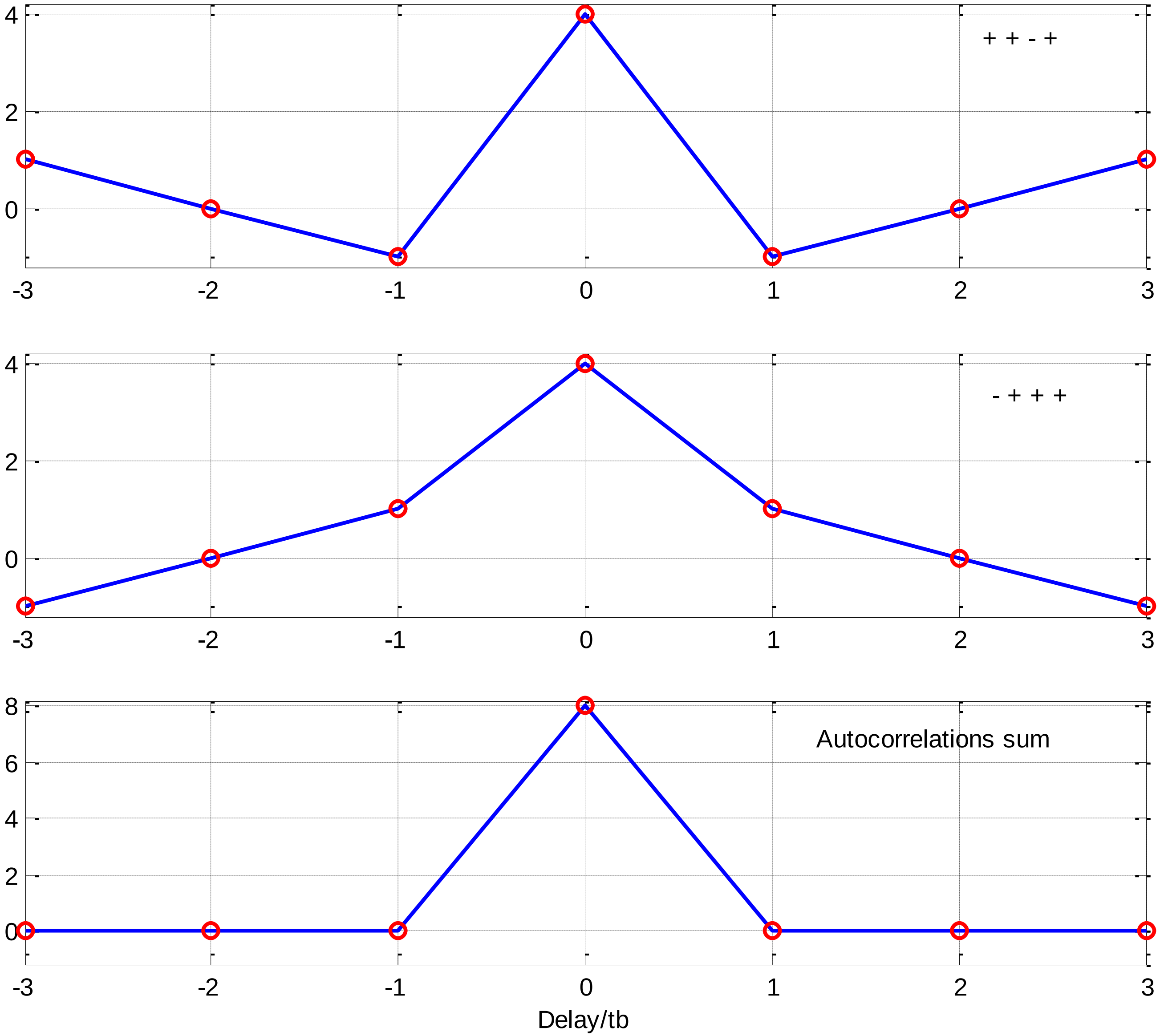
- Complementary pulses - reduce ACF sidelobes

- Step-frequency pulses - extend time-bandwidth product

- Diverse pulses - reduce recurrent ACF lobes

Complementary pulses

- Two or more coded coherent pulses
- With delay spacing larger than the pulse duration
- The sum of their individual autocorrelation sidelobes equals zero for all non-zero delays



comp_dem

Complex valued code

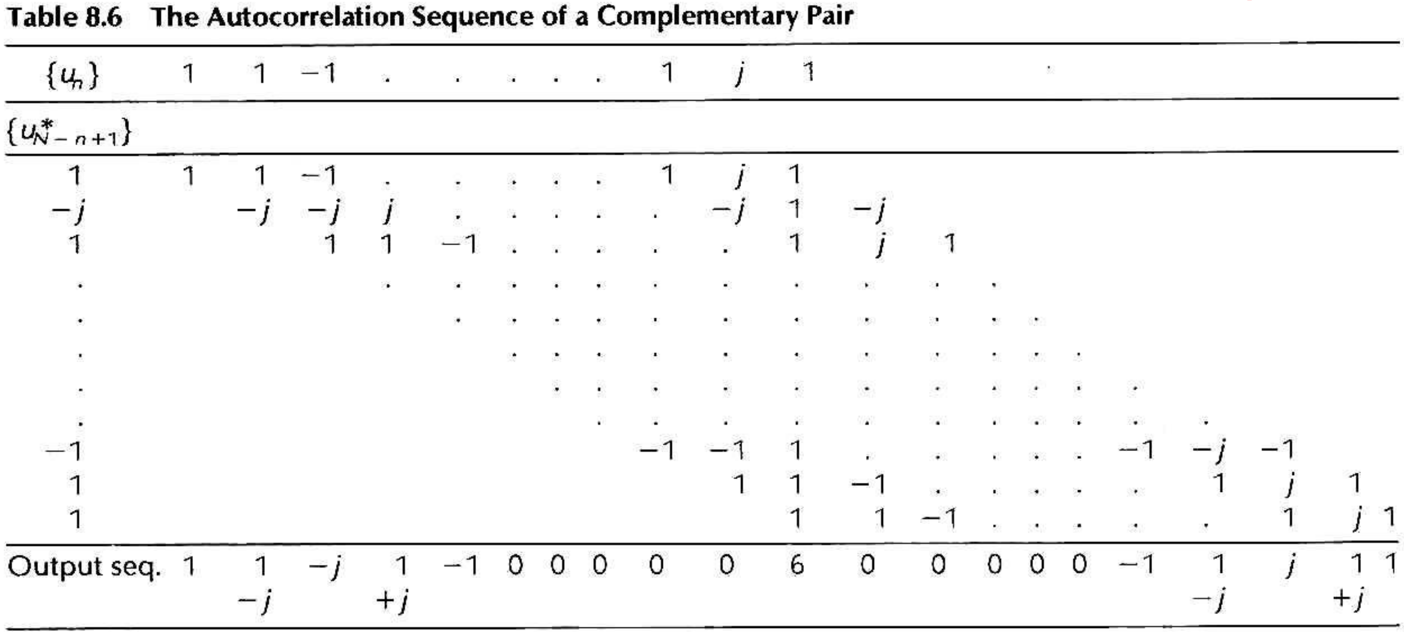
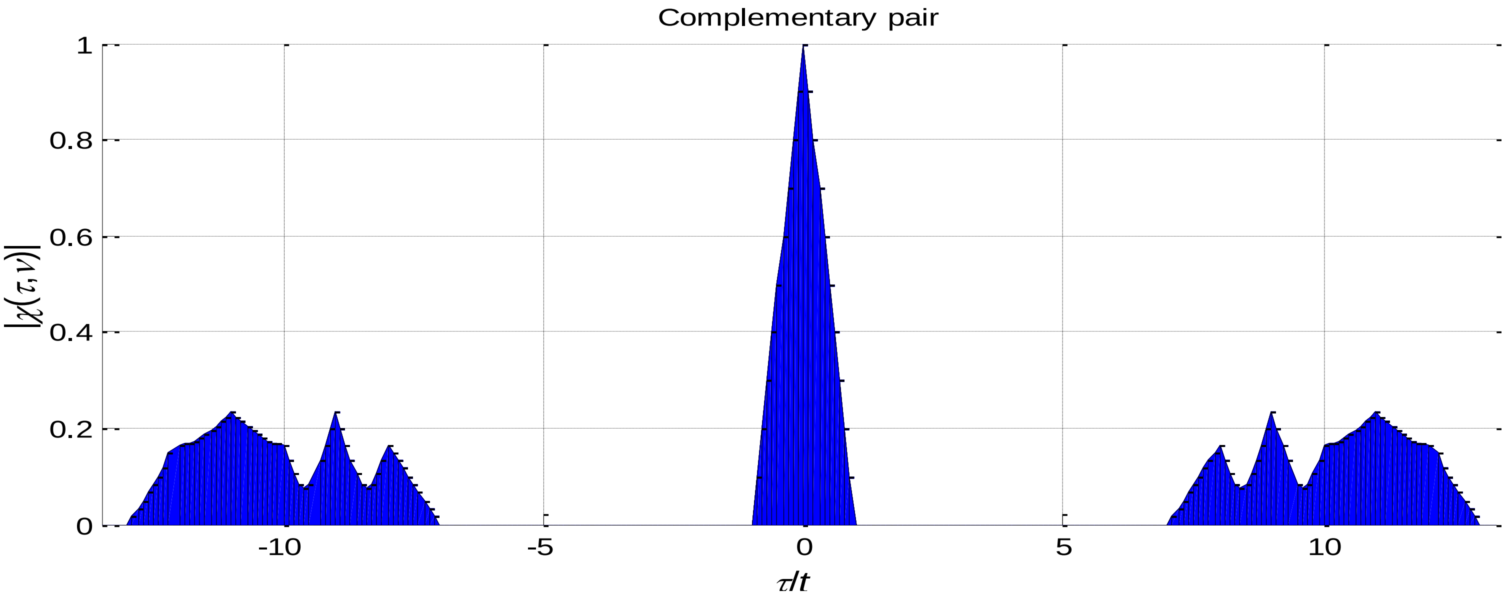
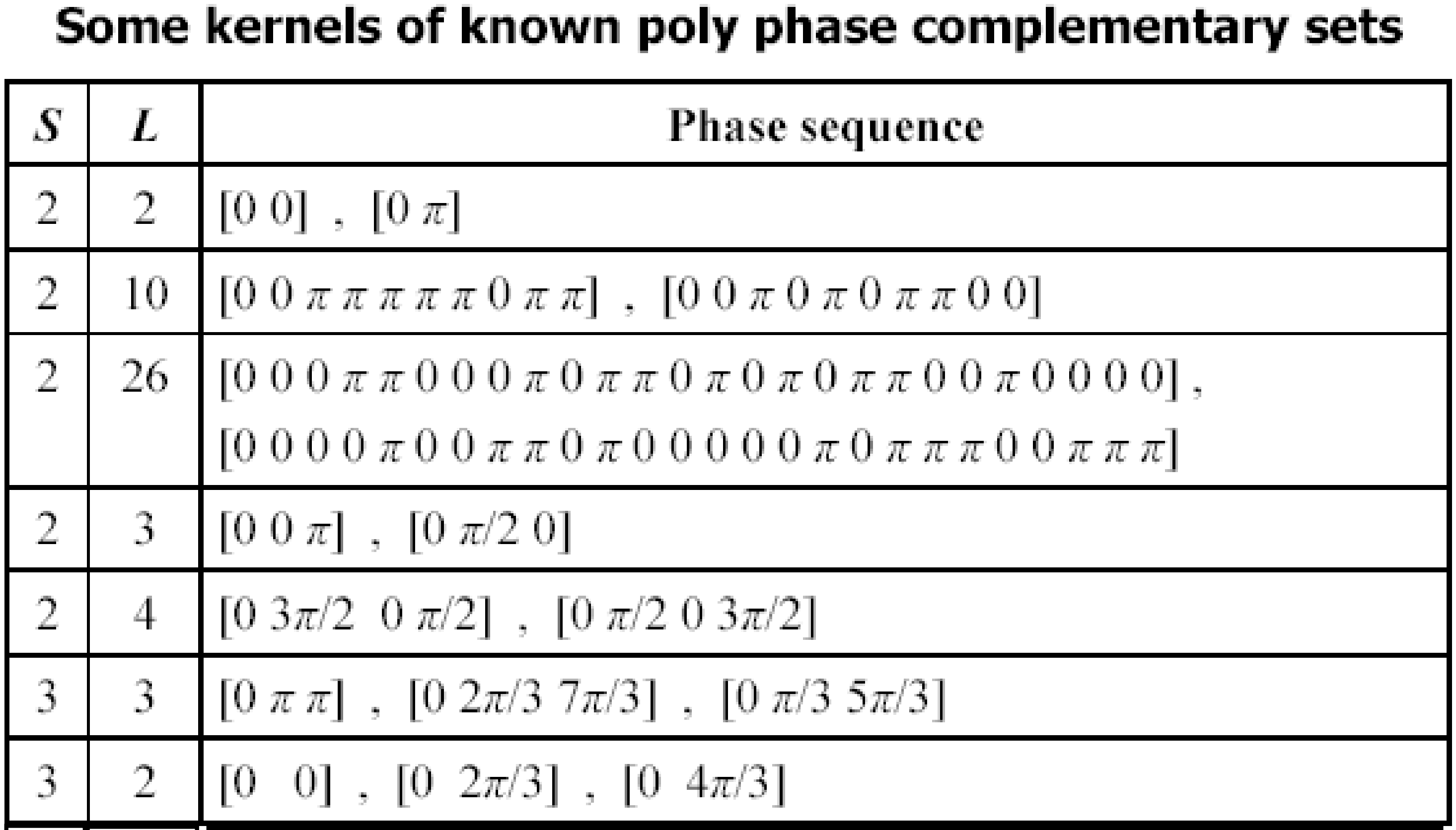


Table 8.6 The Autocorrelation Sequence of a Complementary Pair

$\{u_n\}$	1	1	−1	.	.	.	.	.	1	j	1										
$\{u^*_{N-n+1}\}$																					
1	1	1	−1	.	.	.	.	.	1	j	1										
$-j$		$-j$	$-j$	j	.	.	.	.	.	$-j$	1	$-j$									
1			1	1	−1	.	.	.	.	.	1	j	1								
.				.	.	.	.	.	.	.	.	.	.	.							
.					.	.	.	.	.	.	.	.	.	.	.						
.						.	.	.	.	.	.	.	.	.	.	.					
.							.	.	.	.	.	.	.	.	.	.	.				
.								.	.	.	.	.	.	.	.	.	.	.			
−1									−1	−1	1	.	.	.	.	.	−1	$-j$	−1		
1										1	1	−1	.	.	.	.	.	1	j	1	
1											1	1	−1	.	.	.	.	.	1	j	1
Output seq.	1	1 $-j$	$-j$	1 $+j$	−1	0	0	0	0	0	6	0	0	0	0	0	−1	1 $-j$	j	1 $+j$	1





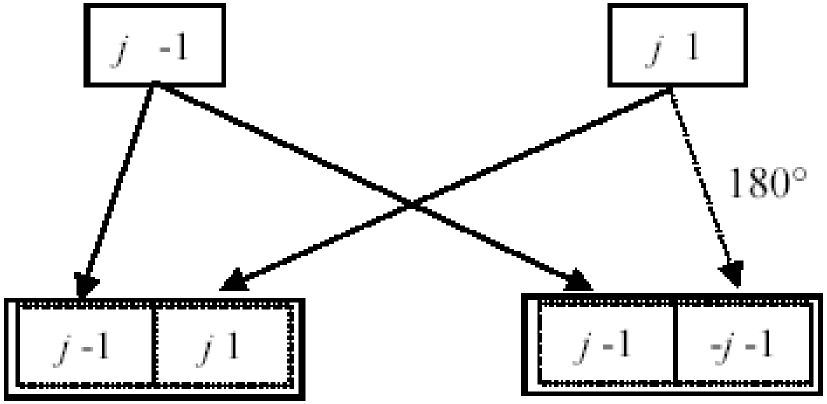
Some kernels of known poly phase complementary sets

S	*L*	Phase sequence
2	2	$[0\ 0]$, $[0\ \pi]$
2	10	$[0\ 0\ \pi\ \pi\ \pi\ \pi\ \pi\ 0\ \pi\ \pi]$, $[0\ 0\ \pi\ 0\ \pi\ 0\ \pi\ \pi\ 0\ 0]$
2	26	$[0\ 0\ 0\ \pi\ \pi\ 0\ 0\ 0\ \pi\ 0\ \pi\ \pi\ 0\ \pi\ 0\ \pi\ 0\ \pi\ \pi\ 0\ 0\ \pi\ 0\ 0\ 0\ 0]$, $[0\ 0\ 0\ 0\ \pi\ 0\ 0\ \pi\ \pi\ 0\ \pi\ 0\ 0\ 0\ 0\ 0\ \pi\ 0\ \pi\ \pi\ \pi\ 0\ 0\ \pi\ \pi\ \pi]$
2	3	$[0\ 0\ \pi]$, $[0\ \pi/2\ 0]$
2	4	$[0\ 3\pi/2\ \ 0\ \pi/2]$, $[0\ \pi/2\ 0\ 3\pi/2]$
3	3	$[0\ \pi\ \pi]$, $[0\ 2\pi/3\ 7\pi/3]$, $[0\ \pi/3\ 5\pi/3]$
3	2	$[0\ \ \ 0]$, $[0\ \ 2\pi/3]$, $[0\ \ 4\pi/3]$
2	5	$[\pi\ \ 0\ \ \pi\ \ \pi/2\ \ \ \pi/2]$, $[\pi/2\ \ \pi\ \ -\pi/2\ \ -\pi/2\ \ \pi]$

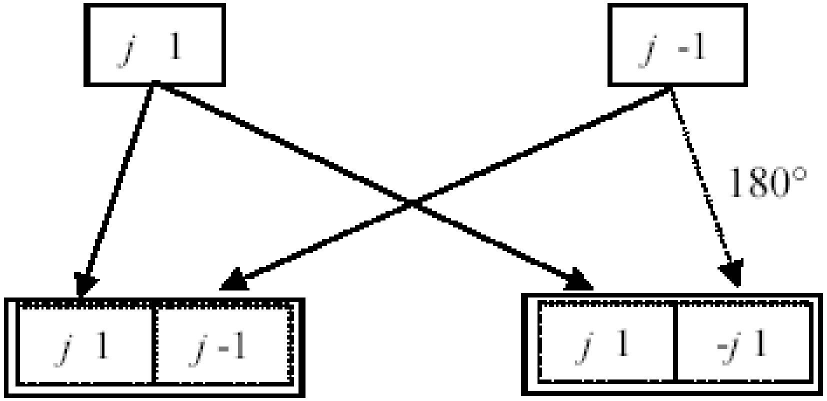
Complementary binary code pairs are known only for code length N of the form: $N = 2^a 10^b 26^c$, where $a, b,$ and c are non-negative integers. For $N \le 100$, only those CC pairs of length $N = 1,2,4,8,10,16,20,32,40,52,64,80,100$ were found.

Generating a complementary pair of 4-element sequences

using a complementary pair of 2-element sequences

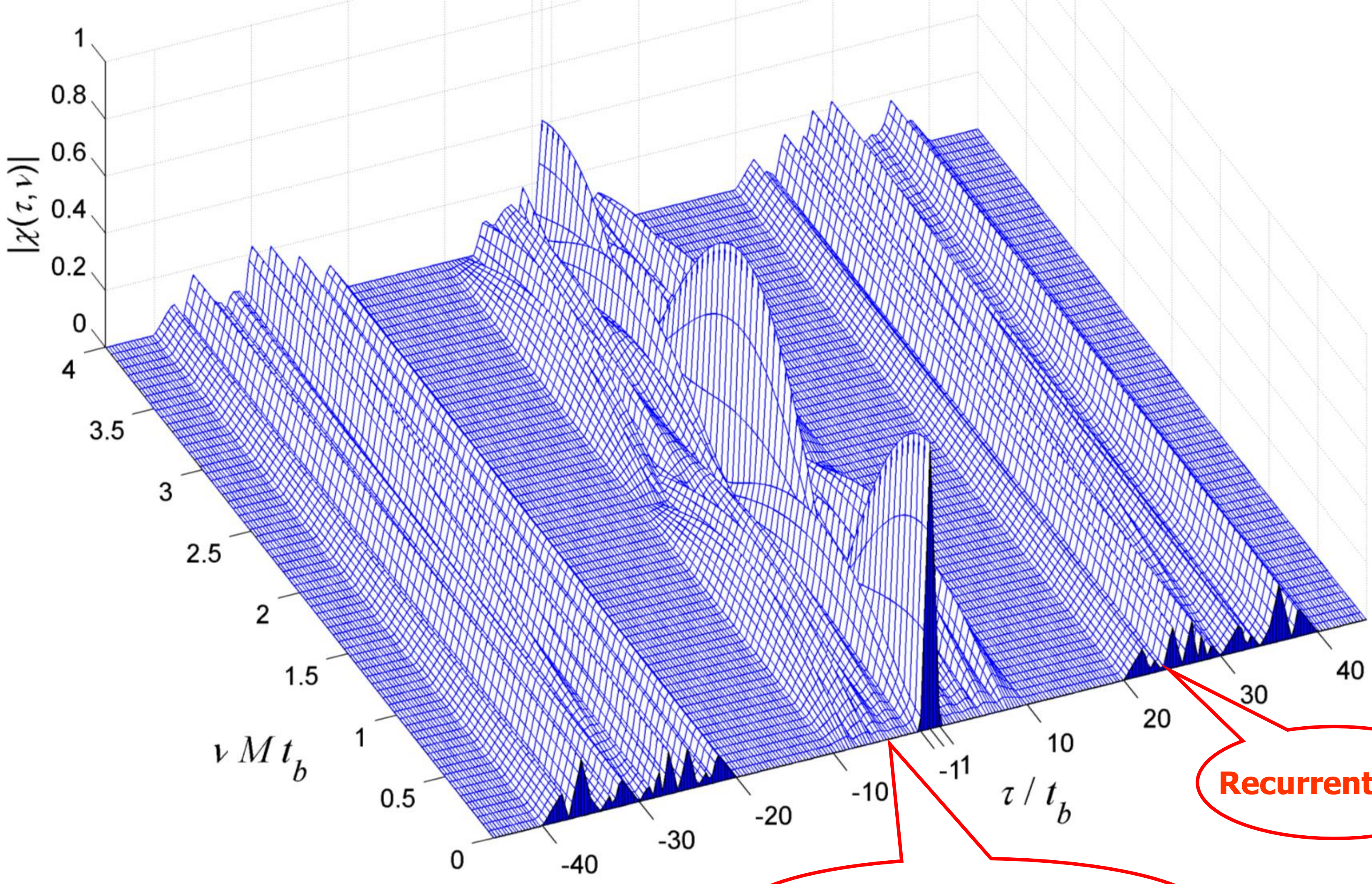


switch roles



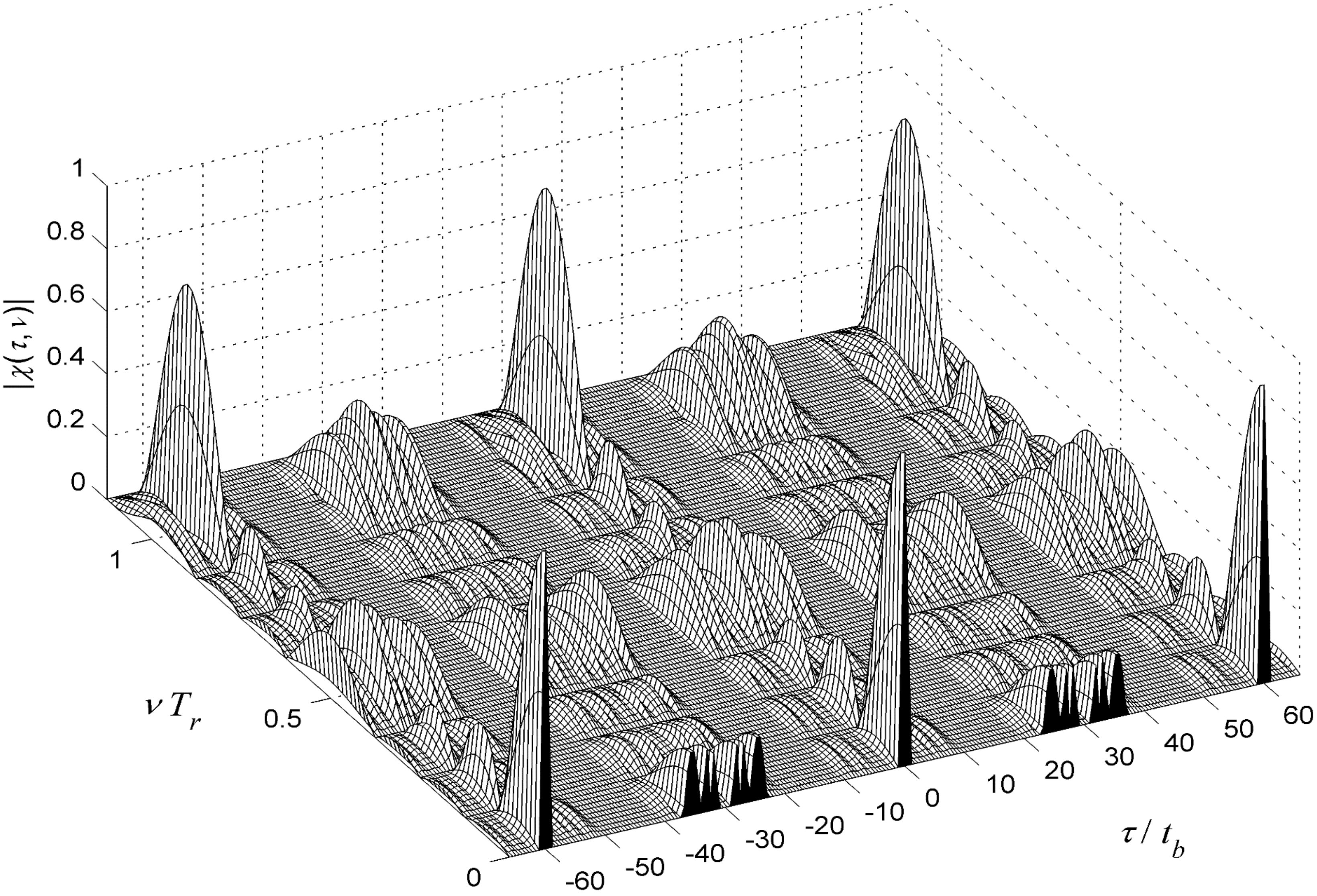
Complementary pair

$\pi[0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1]$ $\pi[0\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0]$

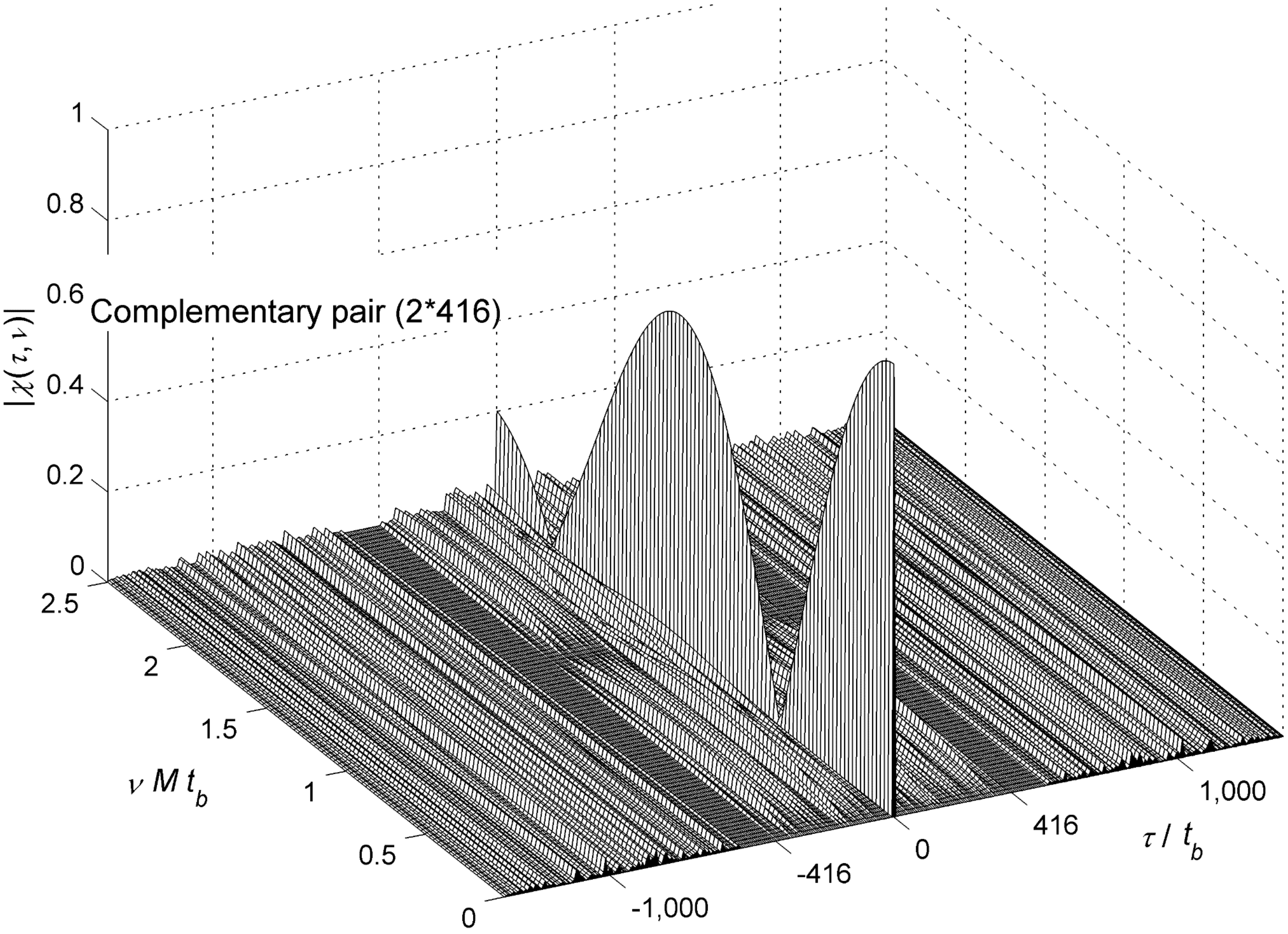


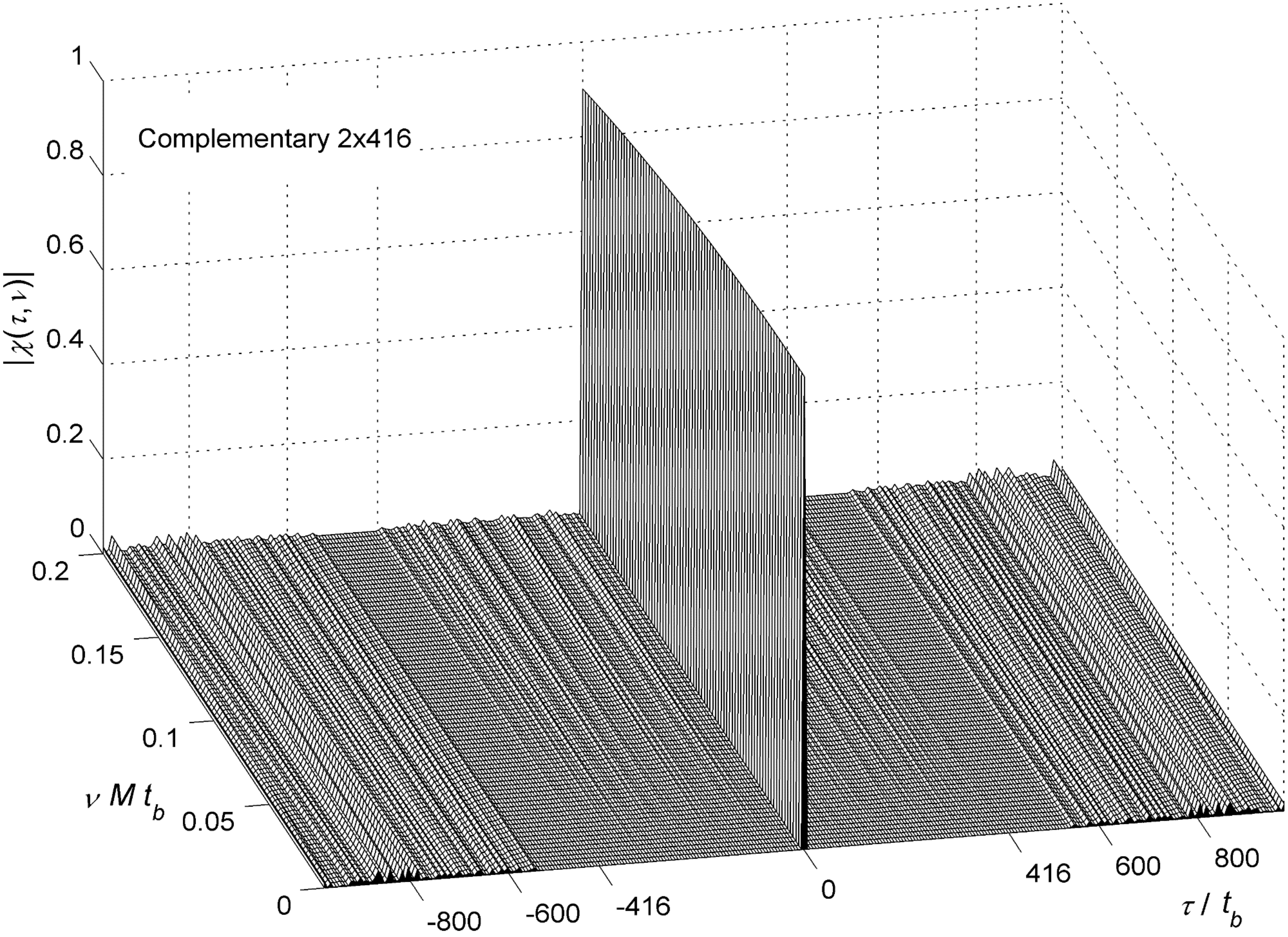
Recurrent lobe

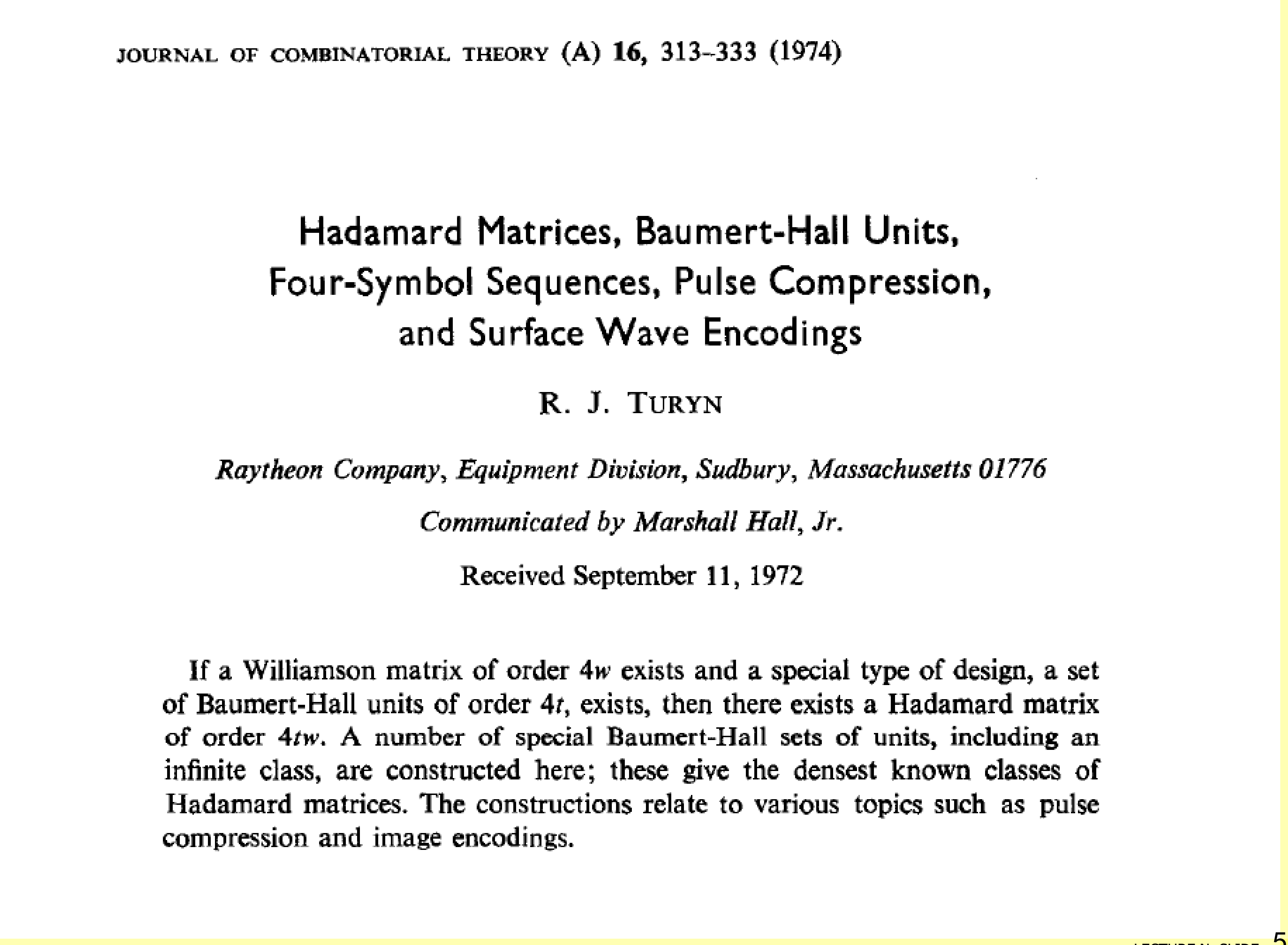
Cancelled near-sidelobes



PAF of a train of 4 complementary pairs (t_p=10t_b, T_r=30t_b)


Complementary pair (2*416)
$|\chi(\tau,\nu)|$
1
0.8
0.6
0.4
0.2
0
2.5
2
1.5
1
0.5
0
$\nu M t_b$
-1,000
-416
0
416
1,000
τ / t_b


Complementary 2x416
$|\chi(\tau,\nu)|$
1
0.8
0.6
0.4
0.2
0
0.2
0.15
0.1
$\nu M t_b$
0.05
0
-800
-600
-416
0
416
600
800
τ / t_b



JOURNAL OF COMBINATORIAL THEORY (A) **16**, 313–333 (1974)

Hadamard Matrices, Baumert-Hall Units, Four-Symbol Sequences, Pulse Compression, and Surface Wave Encodings

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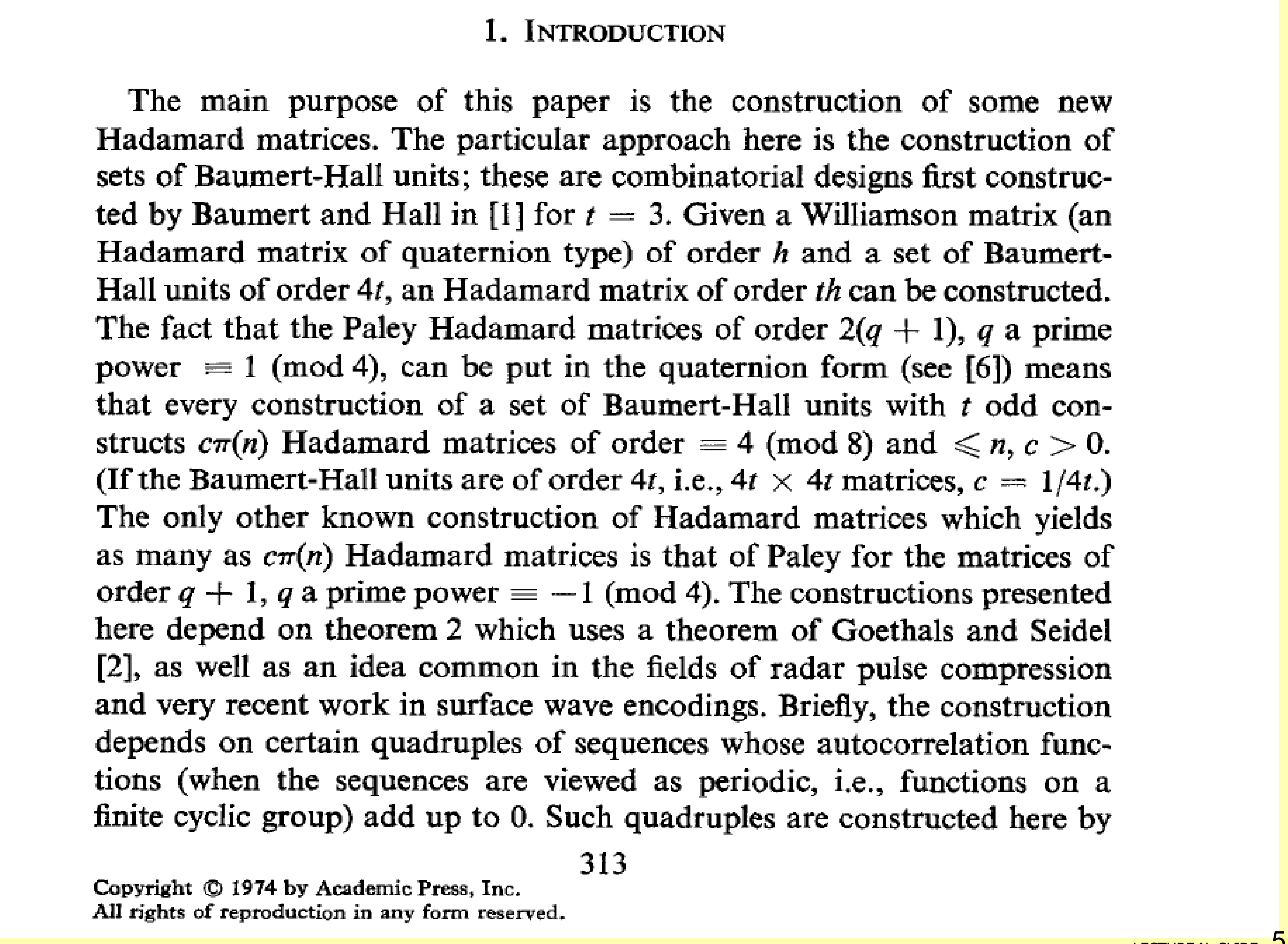
Communicated by Marshall Hall, Jr.

Received September 11, 1972

If a Williamson matrix of order $4w$ exists and a special type of design, a set of Baumert-Hall units of order $4t$, exists, then there exists a Hadamard matrix of order $4tw$. A number of special Baumert-Hall sets of units, including an infinite class, are constructed here; these give the densest known classes of Hadamard matrices. The constructions relate to various topics such as pulse compression and image encodings.

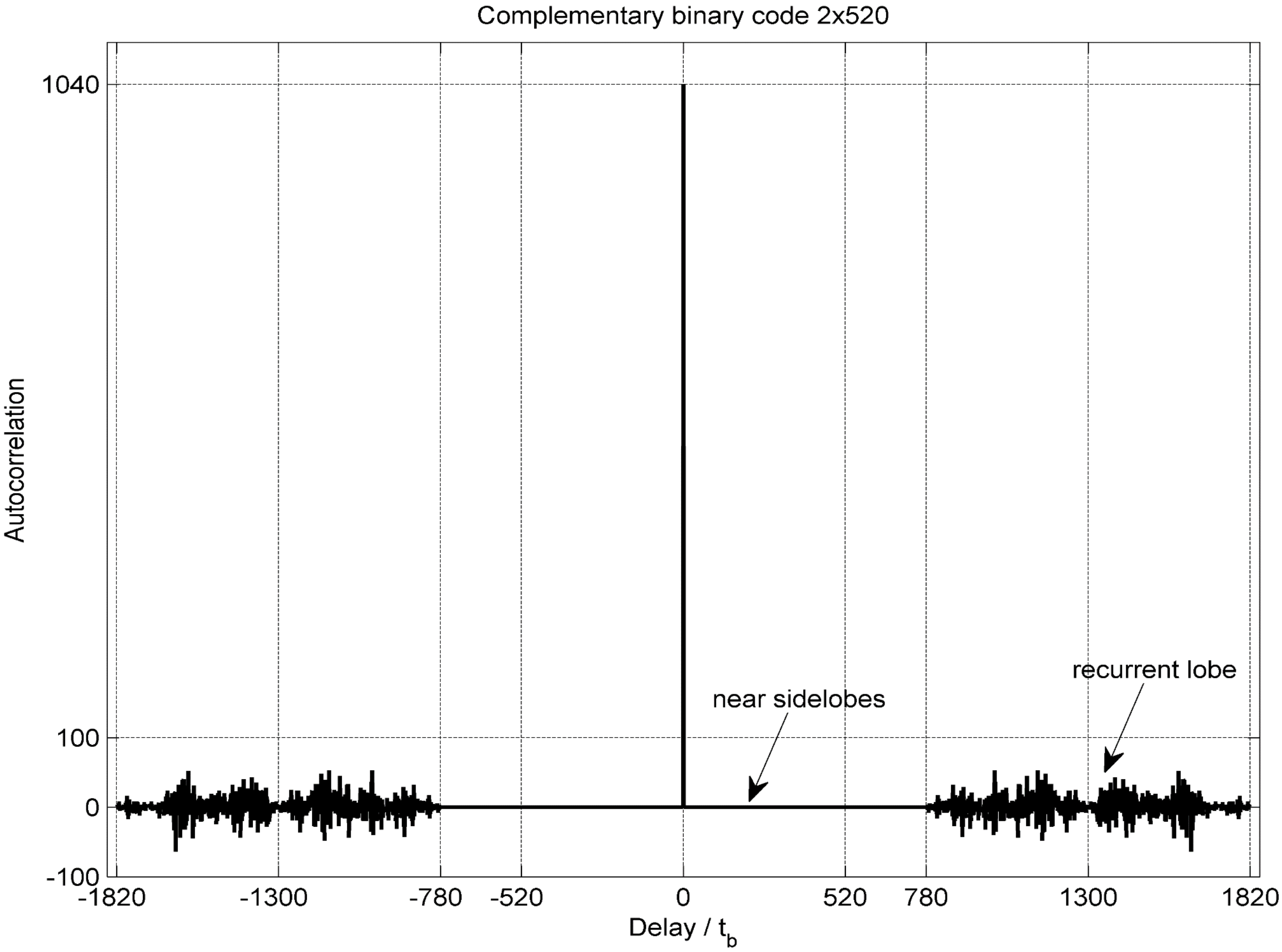


1. INTRODUCTION

The main purpose of this paper is the construction of some new Hadamard matrices. The particular approach here is the construction of sets of Baumert-Hall units; these are combinatorial designs first constructed by Baumert and Hall in [1] for $t = 3$. Given a Williamson matrix (an Hadamard matrix of quaternion type) of order h and a set of Baumert-Hall units of order $4t$, an Hadamard matrix of order th can be constructed. The fact that the Paley Hadamard matrices of order $2(q + 1)$, q a prime power $\equiv 1 \pmod 4$, can be put in the quaternion form (see [6]) means that every construction of a set of Baumert-Hall units with t odd constructs $c\pi(n)$ Hadamard matrices of order $\equiv 4 \pmod 8$ and $\leqslant n$, $c > 0$. (If the Baumert-Hall units are of order $4t$, i.e., $4t \times 4t$ matrices, $c = 1/4t$.) The only other known construction of Hadamard matrices which yields as many as $c\pi(n)$ Hadamard matrices is that of Paley for the matrices of order $q + 1$, q a prime power $\equiv -1 \pmod 4$. The constructions presented here depend on theorem 2 which uses a theorem of Goethals and Seidel [2], as well as an idea common in the fields of radar pulse compression and very recent work in surface wave encodings. Briefly, the construction depends on certain quadruples of sequences whose autocorrelation functions (when the sequences are viewed as periodic, i.e., functions on a finite cyclic group) add up to 0. Such quadruples are constructed here by

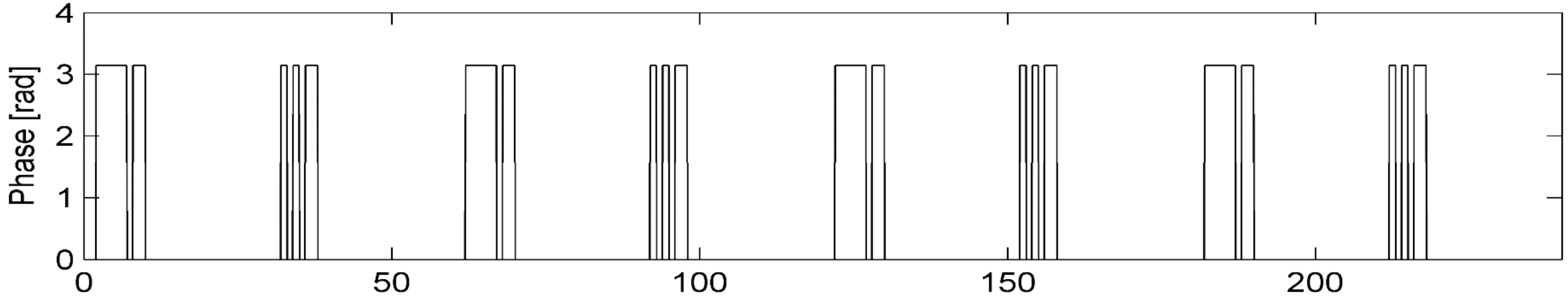
Constructing a pair of binary complementary sequences, with sequence length 2MN, from two pairs of binary complementary sequences with sequences lengths of M and N.

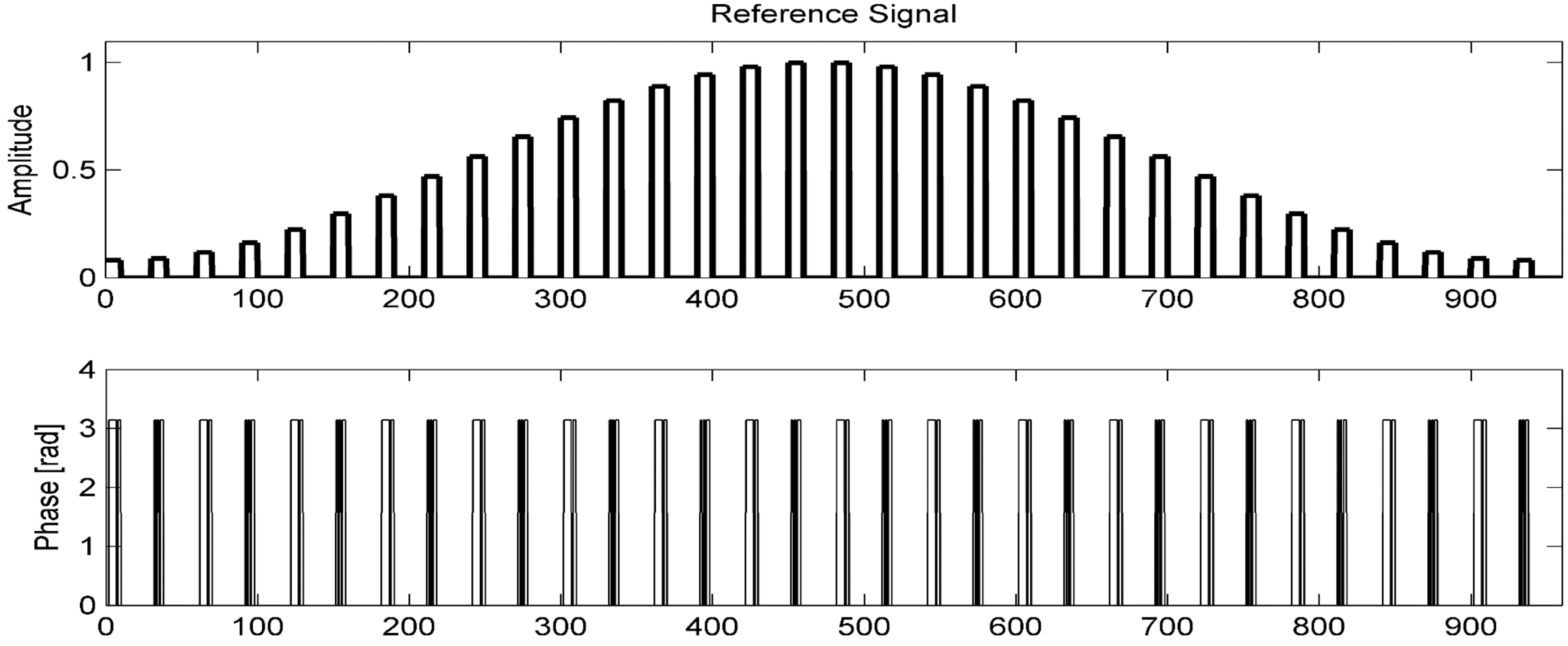
```
a= -1+2*[0 0 0 1 1 0 0 0 1 0 1 1 0 1 0 1 0 1 1 0 0 1 0 0 0 0];
b= -1+2*[0 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 1 0 1 1 1 0 0 1 1 1];
c= -1+2*[1 1 0 1 0 1 0 0 1 1];
d= -1+2*[1 1 0 1 1 1 1 1 0 0];

ac=kron(c,a); % Kronecker product
bd=kron(d,b);
sig_left=[ac bd]; % concutanation

df=fliplr(d); % flipping left-right
ncf=fliplr(-c); % negation and flipping
adf=kron(df,a); % Kronecker product
bncf=kron(ncf,b);
sig_right=[adf bncf]; % concutanation

mac=length(a)*length(c);
sr=round(mac*3);
space2=zeros(1,sr);

ss_all=[sig_left  space2 sig_right];
ss_all_cor=xcorr(ss_all);

peak_pos=0.5*(length(ss_all_cor)+1);
xscale=0:length(ss_all_cor)-1;
xscale=xscale-peak_pos +1;

xtick_locations=[-(2*length(sig_left)+sr)  -(length(sig_left)+sr) -sr  -length(sig_left) ...
  0 length(sig_left) sr (length(sig_left)+sr) (2*length(sig_left)+sr)] ;

figure(1), clf
plot(xscale, ss_all_cor,'k', 'linewidth',1.5)
xlabel(' Delay / t_b ')
ylabel(' Autocorrelation ')
title(' Complementary binary code 2x520 ')
axis([xtick_locations(1)-30  xtick_locations(end)+30 -100  1100])
set( gca, 'YTick',[-100 0 100 1040], 'YGrid','on', 'XTick',[xtick_locations], 'XGrid','on')
```


Complementary binary code 2x520
Autocorrelation
1040
100
0
-100
-1820
-1300
-780
-520
0
520
780
1300
1820
Delay / t_b
near sidelobes
recurrent lobe

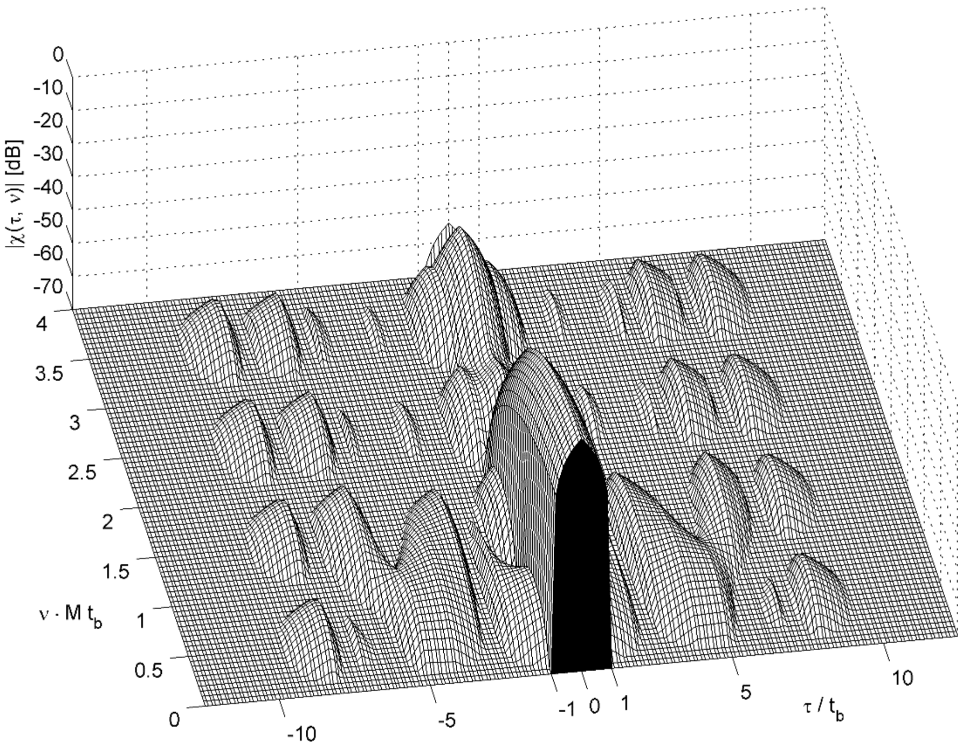
Reducing Doppler sidelobes by transmitting a train of complementary pairs with amplitude-weighted reference

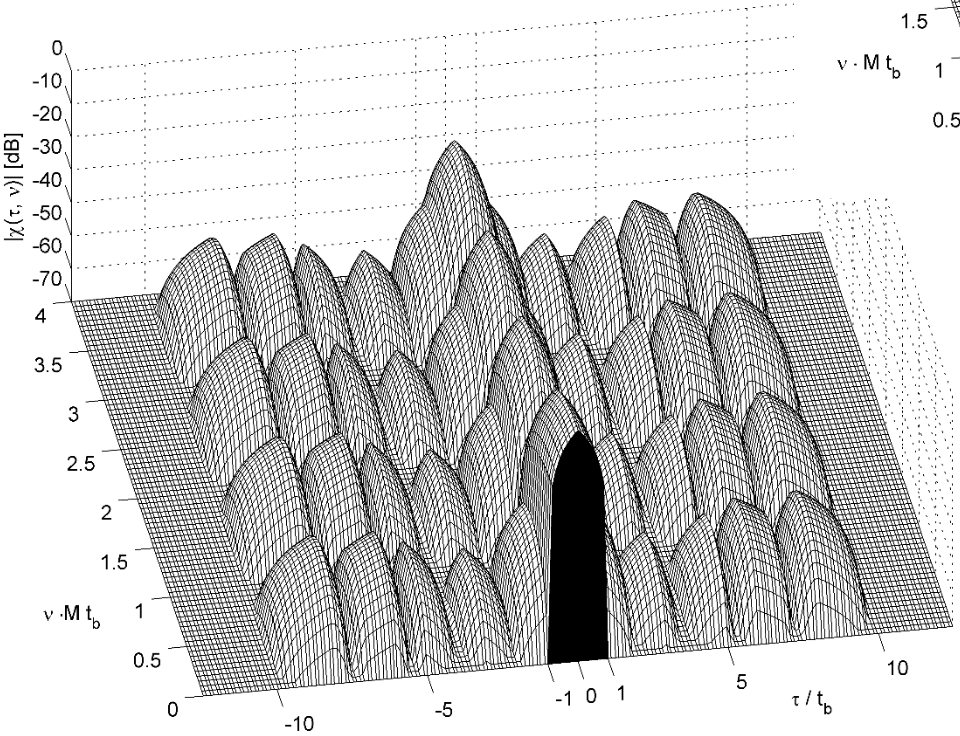
```
x0=[1 1 b b b b b 1 b b];   x1=[1 1 b 1 b 1 b b 1 1];   b=-1
```





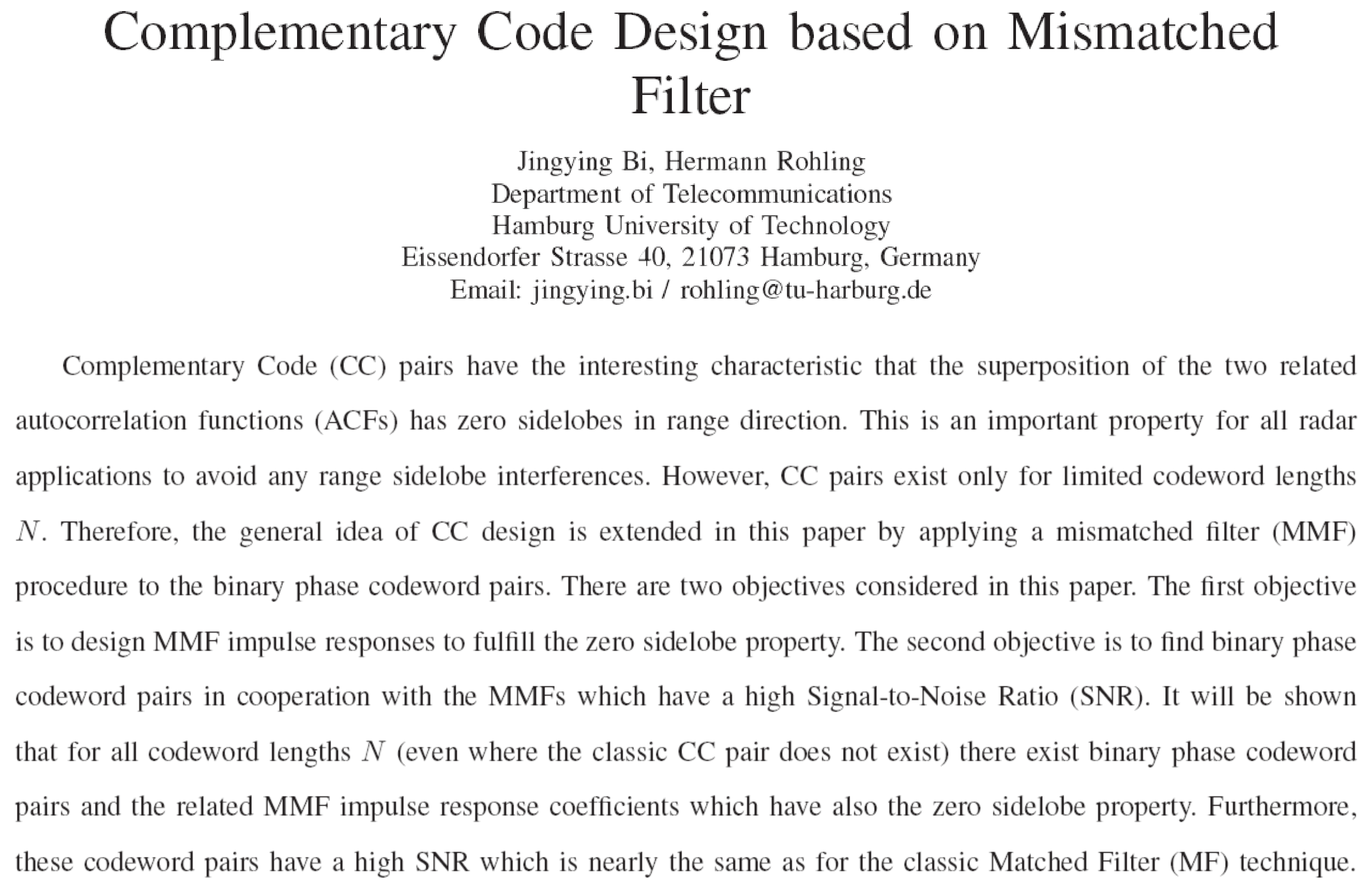
Amplitude-weighted (Hamming) reference train





Fixed-amplitude reference train

IEEE Trans. on Aerospace and Electronic Systems, Vol. 48, No. 2, April 2012, pp. 1793-1797.



Complementary Code Design based on Mismatched Filter

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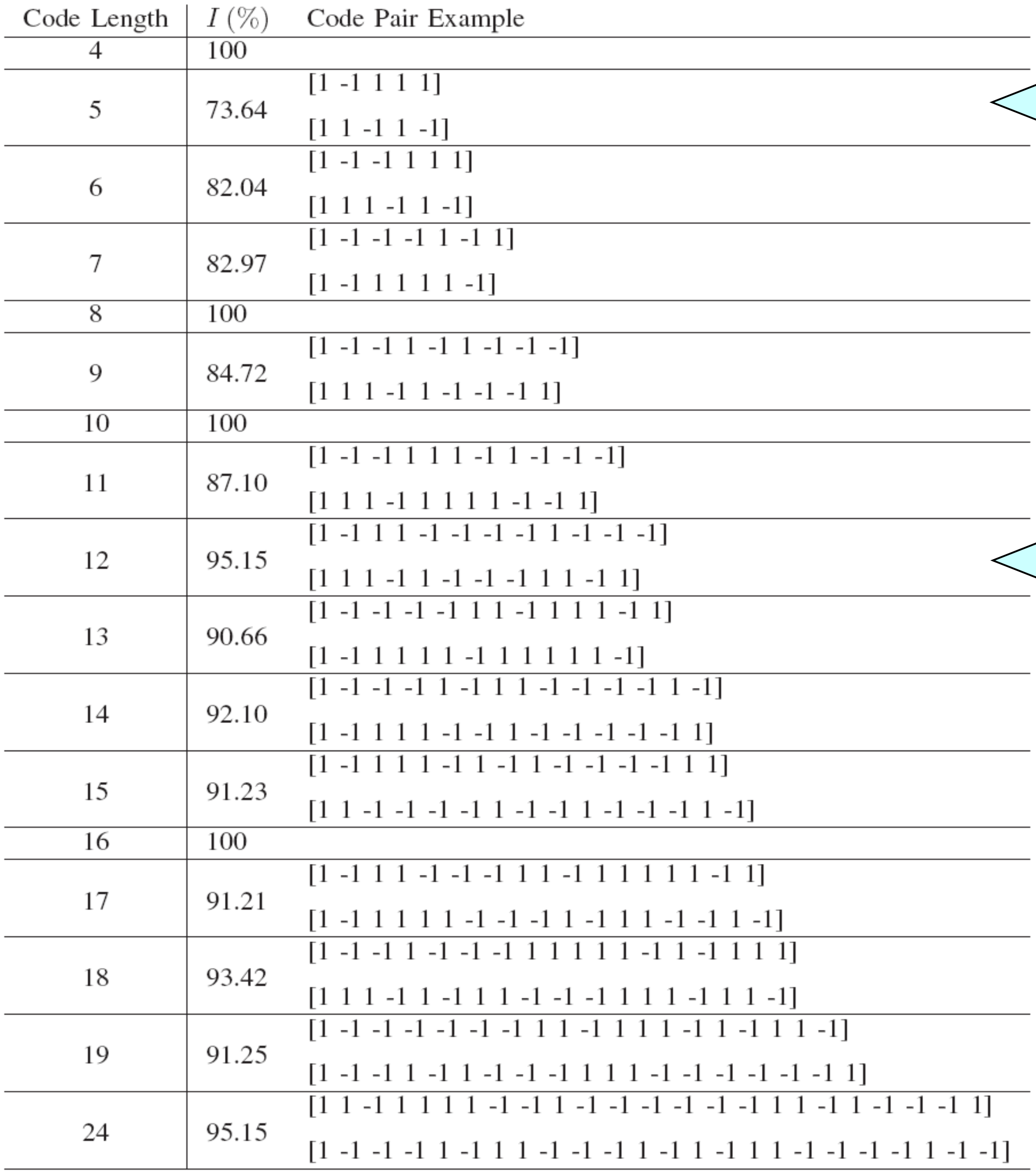
Complementary Code (CC) pairs have the interesting characteristic that the superposition of the two related autocorrelation functions (ACFs) has zero sidelobes in range direction. This is an important property for all radar applications to avoid any range sidelobe interferences. However, CC pairs exist only for limited codeword lengths N. Therefore, the general idea of CC design is extended in this paper by applying a mismatched filter (MMF) procedure to the binary phase codeword pairs. There are two objectives considered in this paper. The first objective is to design MMF impulse responses to fulfill the zero sidelobe property. The second objective is to find binary phase codeword pairs in cooperation with the MMFs which have a high Signal-to-Noise Ratio (SNR). It will be shown that for all codeword lengths N (even where the classic CC pair does not exist) there exist binary phase codeword pairs and the related MMF impulse response coefficients which have also the zero sidelobe property. Furthermore, these codeword pairs have a high SNR which is nearly the same as for the classic Matched Filter (MF) technique.



Code Length	I (%)	Code Pair Example
4	100	
5	73.64	[1 -1 1 1 1] [1 1 -1 1 -1]
6	82.04	[1 -1 -1 1 1 1] [1 1 1 -1 1 -1]
7	82.97	[1 -1 -1 -1 1 -1 1] [1 -1 1 1 1 1 -1]
8	100	
9	84.72	[1 -1 -1 1 -1 1 -1 -1 -1] [1 1 1 -1 1 -1 -1 -1 1]
10	100	
11	87.10	[1 -1 -1 1 1 1 -1 1 -1 -1 -1] [1 1 1 -1 1 1 1 1 -1 -1 1]
12	95.15	[1 -1 1 1 -1 -1 -1 -1 1 -1 -1 -1] [1 1 1 -1 1 -1 -1 -1 1 1 -1 1]
13	90.66	[1 -1 -1 -1 -1 1 1 -1 1 1 1 -1 1] [1 -1 1 1 1 1 -1 1 1 1 1 1 -1]
14	92.10	[1 -1 -1 -1 1 -1 1 1 -1 -1 -1 -1 1 -1] [1 -1 1 1 1 -1 -1 1 -1 -1 -1 -1 -1 1]
15	91.23	[1 -1 1 1 1 -1 1 -1 1 -1 -1 -1 -1 1 1] [1 1 -1 -1 -1 -1 1 -1 -1 1 -1 -1 -1 1 -1]
16	100	
17	91.21	[1 -1 1 1 -1 -1 -1 1 1 -1 1 1 1 1 1 -1 1] [1 -1 1 1 1 1 -1 -1 -1 1 -1 1 1 -1 -1 1 -1]
18	93.42	[1 -1 -1 1 -1 -1 -1 1 1 1 1 1 -1 1 -1 1 1 1] [1 1 1 -1 1 -1 1 1 -1 -1 -1 1 1 1 -1 1 1 -1]
19	91.25	[1 -1 -1 -1 -1 -1 -1 1 1 -1 1 1 1 -1 1 -1 1 1 -1] [1 -1 -1 1 -1 1 -1 -1 -1 1 1 1 -1 -1 -1 -1 -1 -1 1]
24	95.15	[1 1 -1 1 1 1 1 -1 -1 1 -1 -1 -1 -1 -1 -1 1 1 -1 1 -1 -1 -1 1] [1 -1 -1 -1 1 -1 1 1 -1 -1 -1 1 -1 1 -1 1 1 -1 -1 -1 -1 1 -1 -1]

TABLE I
COMPLEMENTARY CODE PAIRS FOR MMF DESIGN WITH MAXIMUM EFFICIENCY I.

```
    1.0000    1.1111
   -1.0000   -1.6667
    1.0000    0.5556
    1.0000    2.2222
    1.0000    1.1111

    1.0000    1.1111
    1.0000    0.5556
   -1.0000   -0.5556
    1.0000         0
   -1.0000   -1.1111
```

```
    1.0000    0.9646
   -1.0000   -0.9479
    1.0000    1.2228
    1.0000    0.7731
   -1.0000   -1.1384
   -1.0000   -1.2745
   -1.0000   -0.9795
   -1.0000   -1.4256
    1.0000    1.3757
   -1.0000   -0.7538
   -1.0000   -1.0200
   -1.0000   -1.0608

    1.0000    0.9646
    1.0000    0.9813
    1.0000    1.2561
   -1.0000   -0.6065
    1.0000    0.8908
   -1.0000   -0.6588
   -1.0000   -0.6673
   -1.0000   -0.7618
    1.0000    1.2782
    1.0000    0.8355
   -1.0000   -1.1016
    1.0000    1.0608
```

```
% Rohling_complementary_mismatched.m -  design based on Rohling 2010 paper
% Calculates two mis-matched filter for effective complementary binary pair
% Written by Nadav Levanon on 8 December 2010
clear
long=input(' Code length  =  5 (=1) ,   12 (=2) , 14 (=3),  14 (=4),   24 (=5)    =    ?   ');
if long==1
code1=-1+2*[1 0 1 1 1 ];
code2=-1+2*[1 1 0 1 0 ];
elseif long==2
code1=-1+2*[ 1 0 1 1 0 0 0 0 1 0 0 0];
code2=-1+2*[ 1 1 1 0 1 0 0 0 1 1 0 1];
elseif long==3
code1=-1+2*[ 1 0 0 1 1 1 0 1 1 1 1 1 0 1];
code2=-1+2*[ 1 0 1 1 1 1 0 0 0 0 0 1 1 0];
elseif long==4
code1=-1+2*[ 1 0 0 0 1 0 1 1 0 0 0 0 1 0];
code2=-1+2*[ 1 0 1 1 1 0 0 1 0 0 0 0 0 1];
else
code1=-1+2*[ 1 1 0 1 1 1 1 0 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 1  ];
code2=-1+2*[ 1 0 0 0 1 0 1 1 0 0 0 1 0 1 0 1 1 0 0 0 0 1 0 0  ];
end
flen=length(code1);
x1=code1';
x2=code2';
xx=[x1;x2];
dd=zeros(1,2*flen-1);
dd(1,flen)=2*flen;
dd=dd.';
xh1=hankel([zeros(1,flen-1),x1(1)],[x1;zeros(flen-1,1)]);
xh1=flipud(xh1.');
xh2=hankel([zeros(1,flen-1),x2(1)],[x2;zeros(flen-1,1)]);
xh2=flipud(xh2.');
ss=[xh1,xh2];
bb=ss\dd;
ss_short=ss(:,1:(2*flen-1));
zz_short=[zeros(flen-1,1) ; -flipud(x2)];
aa_short=inv(ss_short)*zz_short;
aa=[aa_short;1];
ab=sum(aa.*bb);
aas=sum(aa.^2);
alfa= -ab/aas;
hh=alfa*aa+bb;
output=ss*hh
I=2*flen/sum(hh.^2)
snr_loss_db=-10*log10(I)
disp([xx, hh])
```

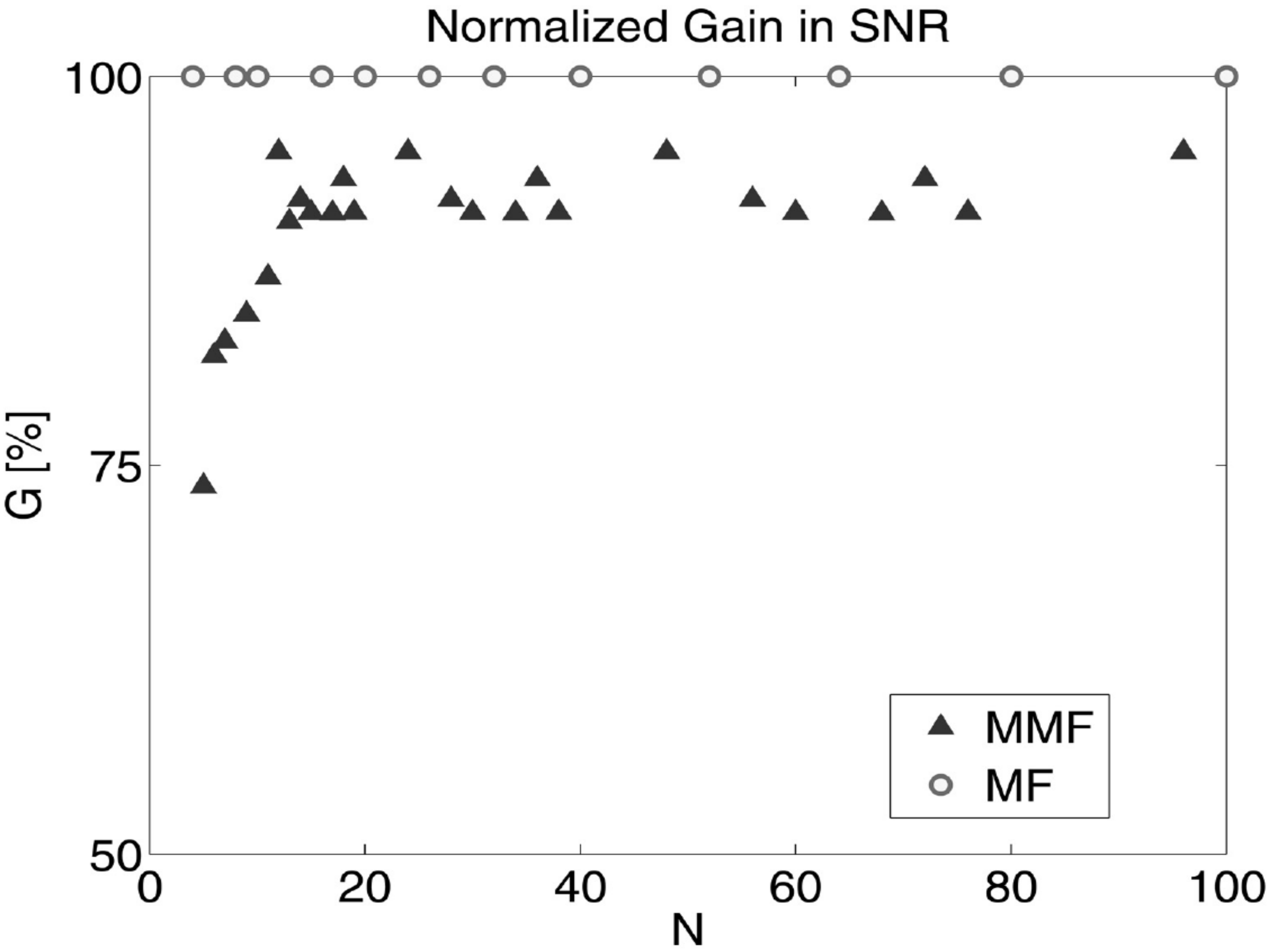


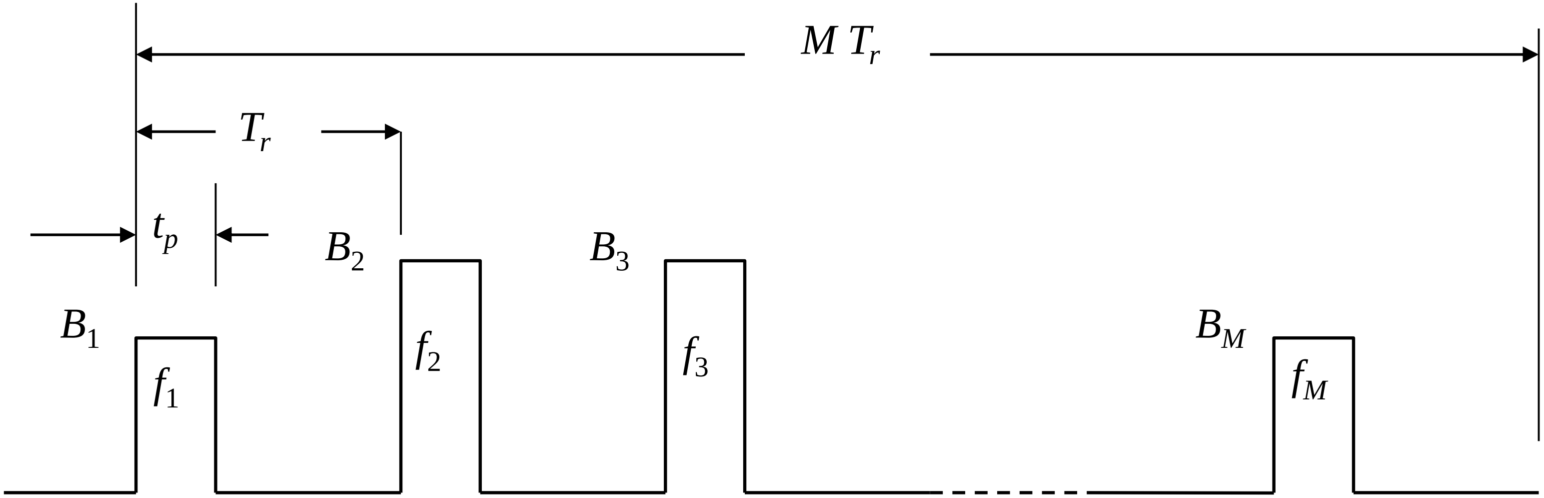
Fig. 4. Normalized gain in SNR over codeword length.

76		-1	-1	-1	1	1	1	1	-1
	1	1	1	-1	-1	-1	1	-1	1
	1	1	-1	-1	-1	1	-1	1	1
	1	-1	1	1	-1	1	1	1	-1
	1	-1	-1	1	-1	-1	-1	-1	1
	-1	-1	-1	1	1	1	-1	1	1
	1	1	-1	1	1	-1	1	1	1
	1	-1	1	1	-1	1	1	1	-1
	1	-1	-1	1	-1				

-1	1	-1	-1	1	-1	1	1	1	-1
	1	1	-1	1	1	1	1	-1	1
	1	-1	1	1	1	1	-1	1	1
	1	-1	-1	-1	1	-1	-1	-1	-1
	1	1	1	-1	1	-1	-1	-1	1
	-1	-1	1	-1	-1	-1	1	-1	1
	1	1	-1	-1	-1	1	-1	1	1
	1	-1	-1	-1	1	-1	-1	-1	-1
	1	1	1						
96		-1	-1	1	-1	1	1	1	-1
	1	1	-1	1	1	1	1	-1	-1
	-1	1	-1	1	1	1	-1	-1	-1
	1	-1	-1	-1	-1	1	1	1	-1
	1	1	1	1	-1	1	1	-1	1
	-1	-1	-1	1	1	1	-1	1	-1
	-1	-1	1	1	1	-1	1	-1	-1
	-1	1	-1	-1	1	-1	1	1	1
	-1	1	1	-1	1	1	1	1	-1
	1	1	-1	1	-1	-1	-1	1	1
	1	-1	1	1	1	1	-1		

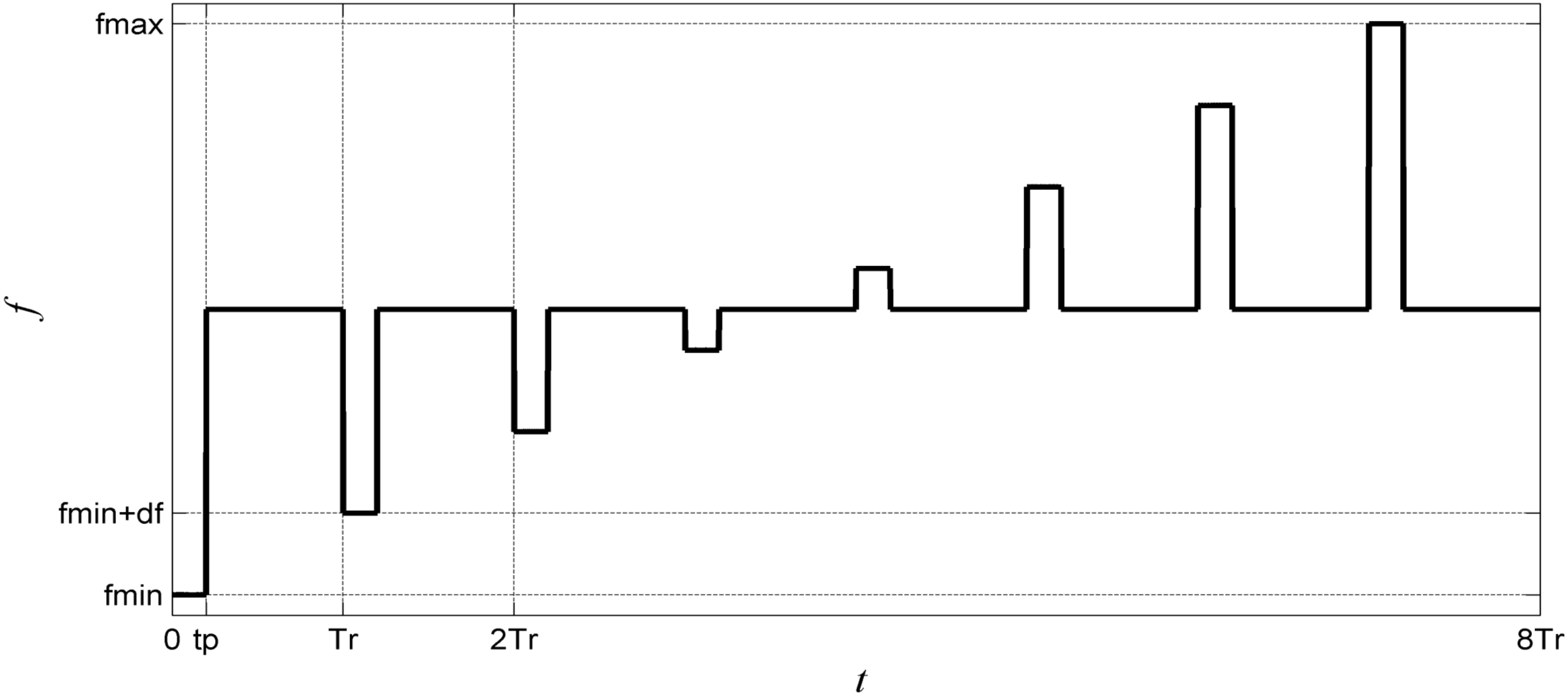
1	-1	-1	-1	-1	1	-1	-1	-1	1
	1	1	-1	1	-1	-1	1	-1	-1
	-1	-1	1	-1	-1	1	-1	-1	-1
	1	-1	1	1	-1	1	1	1	-1
	1	-1	-1	-1	1	1	1	1	-1
	1	1	-1	1	1	1	1	-1	1
	1	-1	1	1	1	1	-1	1	1
	1	-1	-1	-1	-1	1	-1	-1	-1
	1	1	1	-1	1	-1	-1	-1	1
	1	1	1	-1	1	1	-1	1	1
	1	-1	1	-1	-1				

Stepped-frequency pulse-train



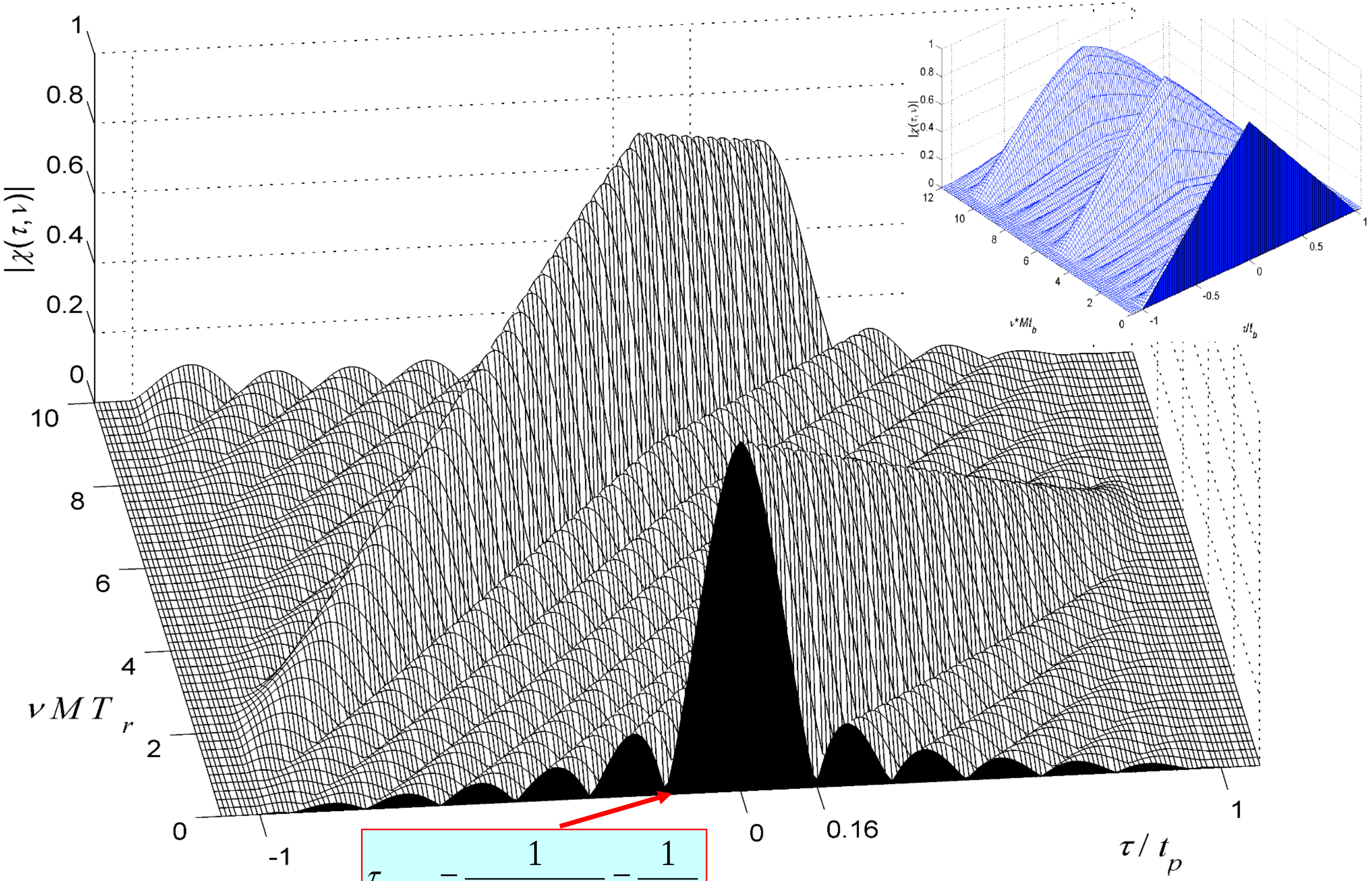
$$f_m - f_{m-1} = \Delta f$$

Stepped-frequency pulses

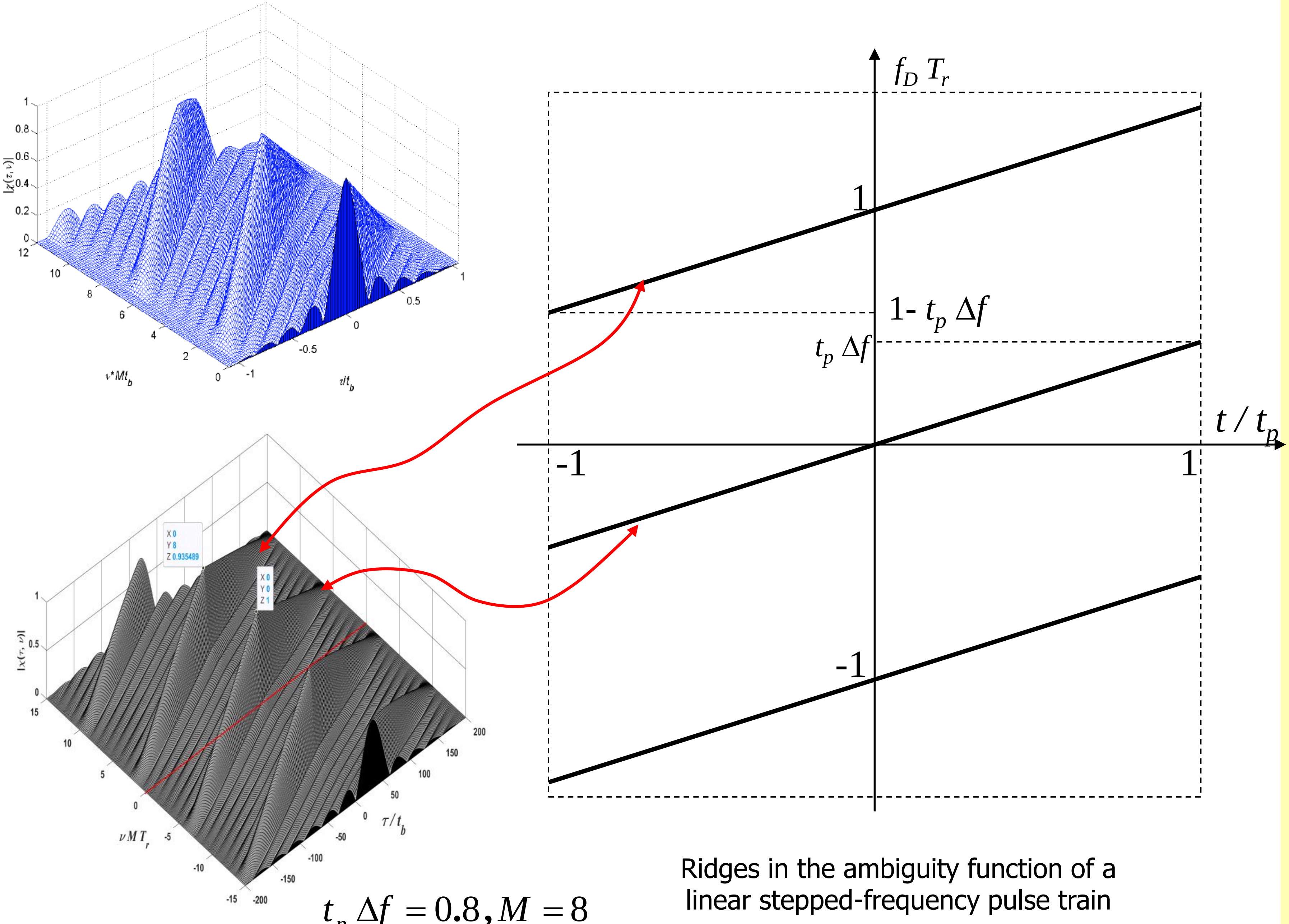


Frequency evolution of stepped-frequency pulse train

AF (zoom) of stepped-frequency pulse train with $t_p df = 0.8,\ M = 8,\ T_r / t_p = 5$



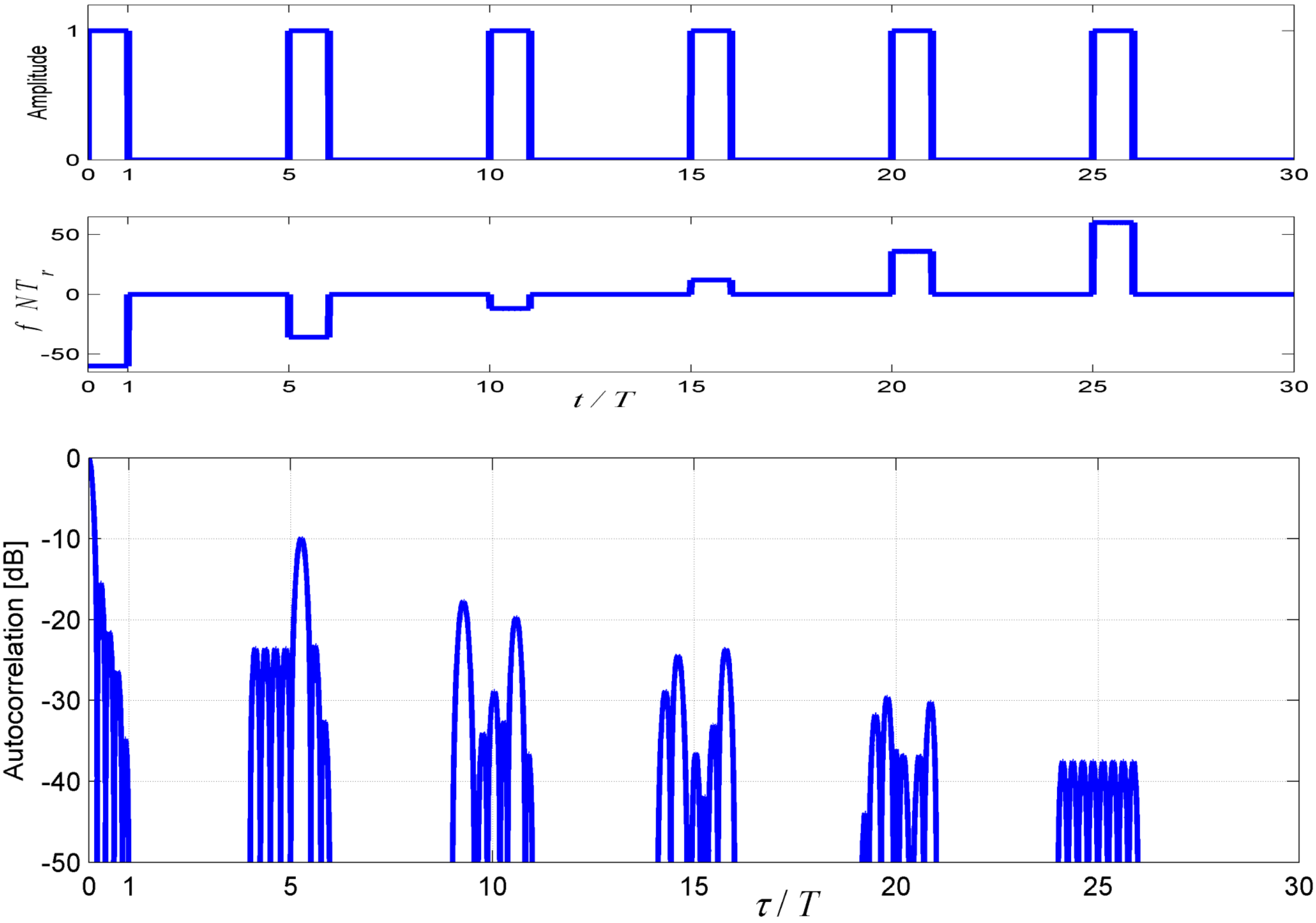
$$\tau_{\text{1st null}} = \frac{1}{\text{Bandwidth}} = \frac{1}{N\Delta f}$$



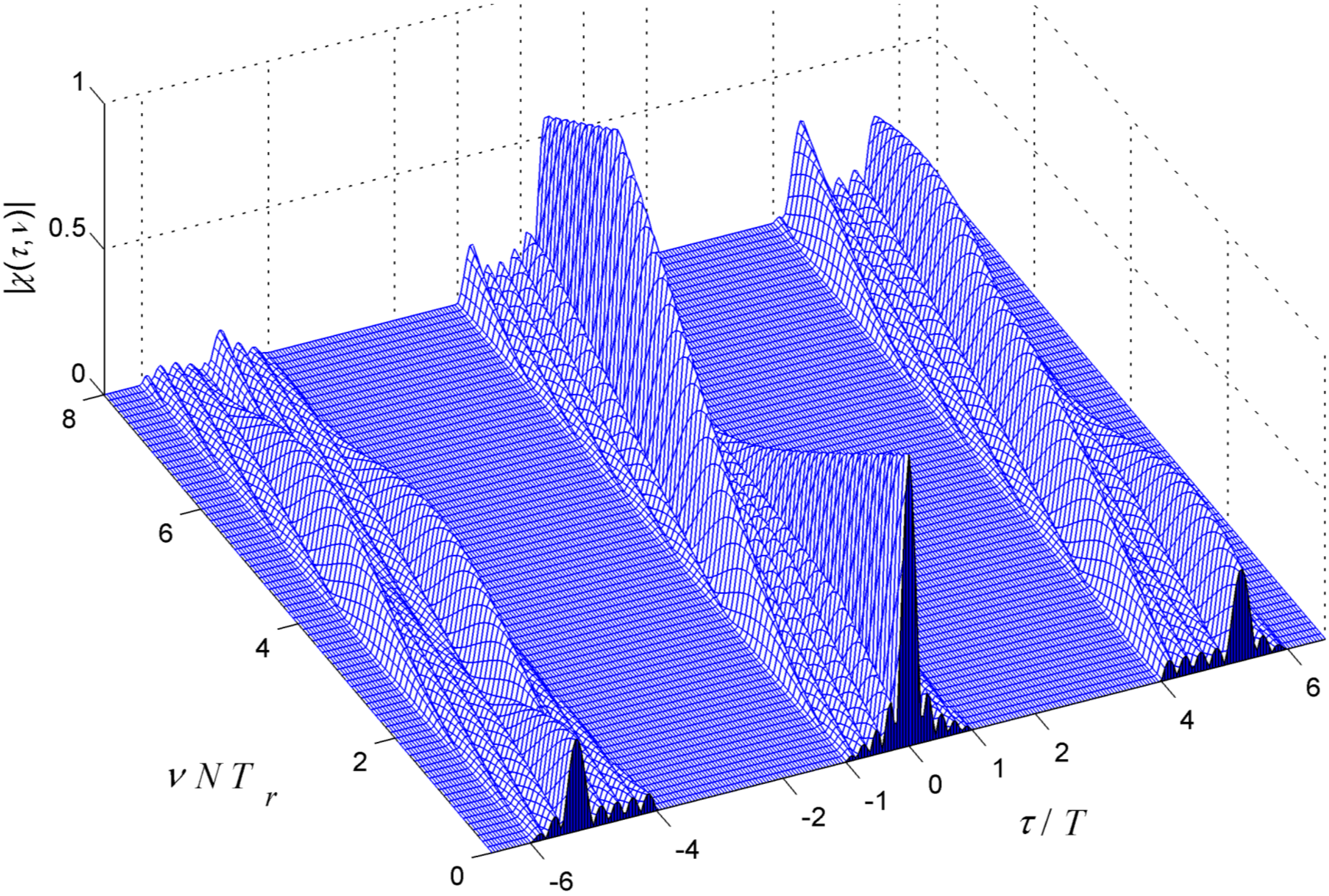
$$t_p\ \Delta f = 0.8, M = 8$$

Ridges in the ambiguity function of a linear stepped-frequency pulse train

Stepped frequency pulse train, 6 pulses, $T\Delta f = 0.8$, $T_r/T = 5$



Stepped frequency pulse train, 6 pulses, $T\Delta f = 0.8$, $T_r/T = 5$



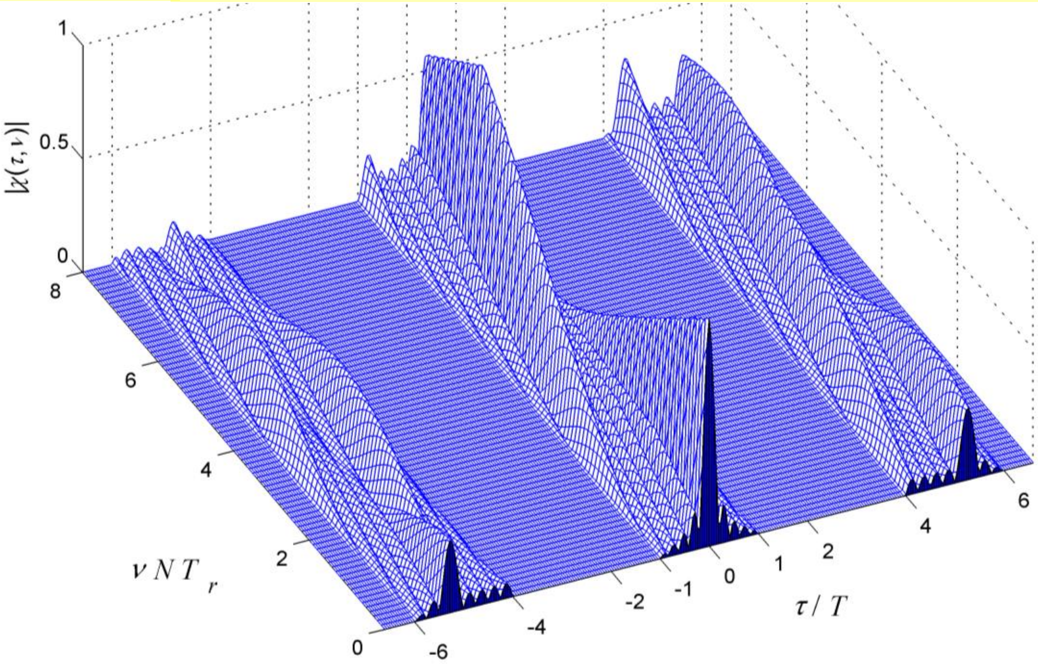
Stepped frequency pulse train, 6 pulses,
$T\Delta f = 0.8$, $T_r/T = 5$

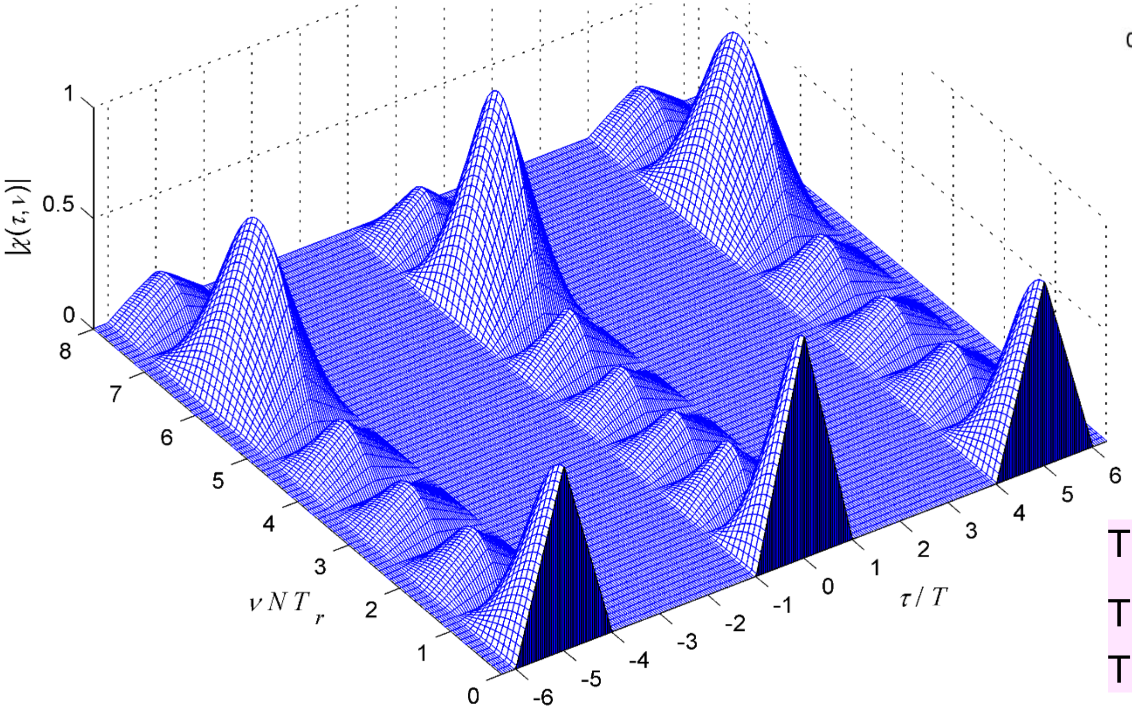
The 6 pulses are different.

The period is $6T_r$.

The ambiguous Doppler is at $\nu = 1/\ 6T_r$

(If the 6 pulses are repeated)



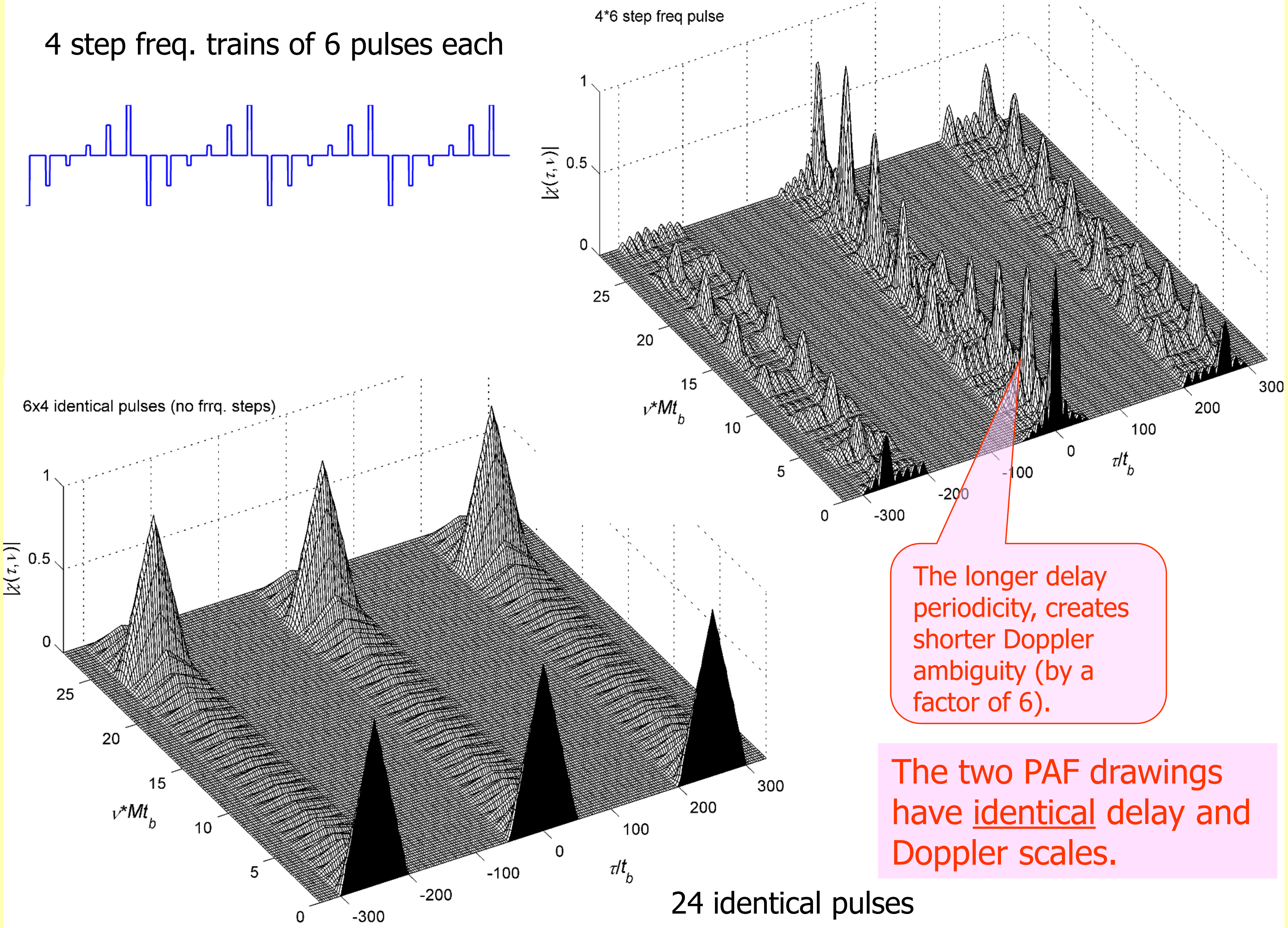


Pulse train, 6 pulses, $T_r/T = 5$

The 6 pulses are identical.

The period is T_r .

The ambiguous Doppler is at $\nu = 1/\ T_r$



4 step freq. trains of 6 pulses each

4*6 step freq pulse

6x4 identical pulses (no frq. steps)

The longer delay periodicity, creates shorter Doppler ambiguity (by a factor of 6).

24 identical pulses

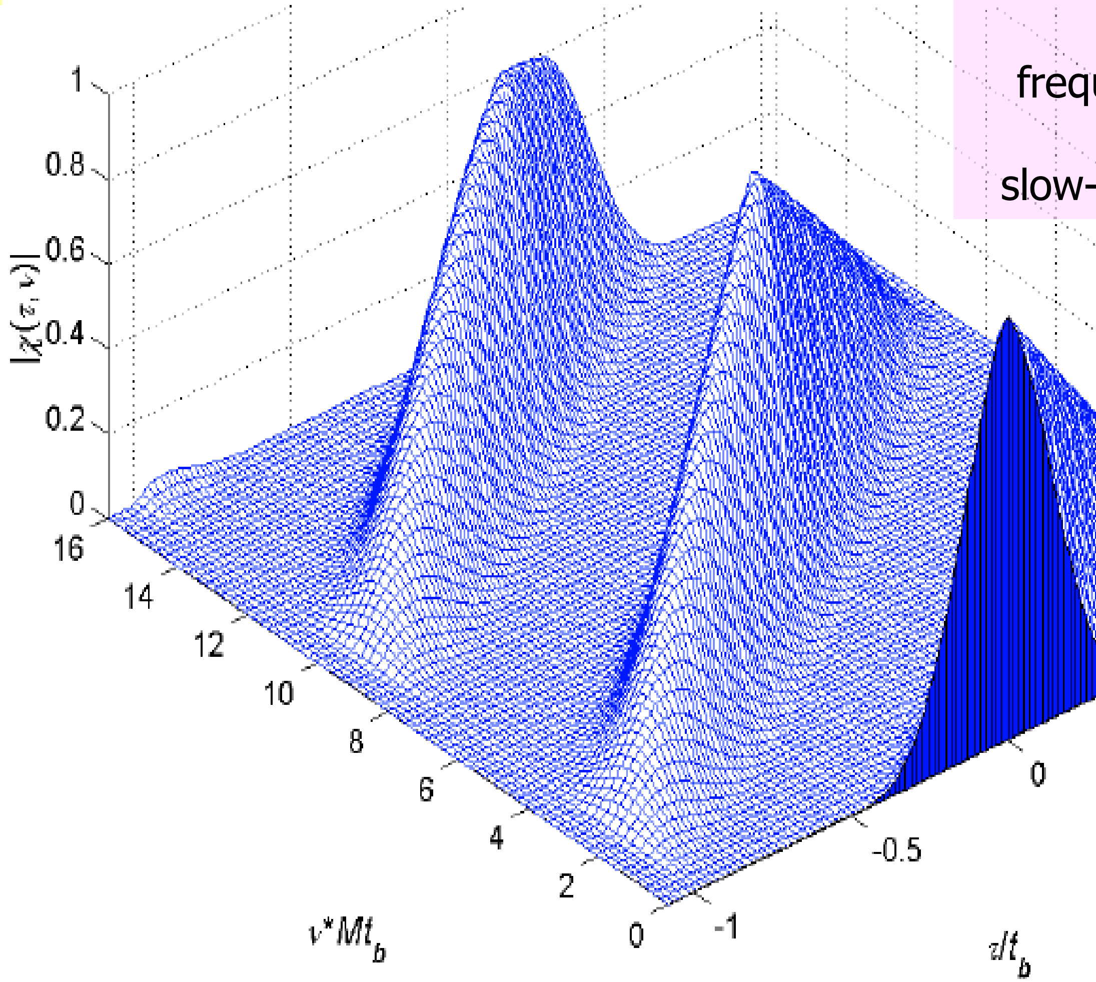
The two PAF drawings have <u>identical</u> delay and Doppler scales.

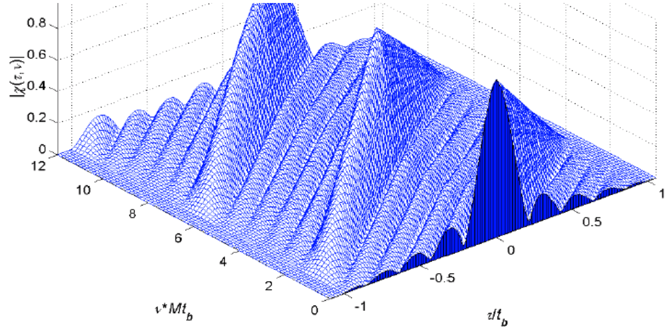
Weighting the pulses =

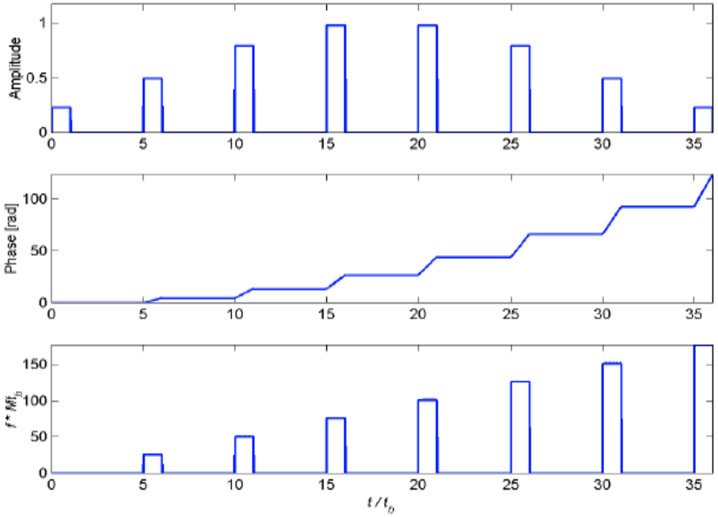
frequency weighting (lowers delay sidelobes)

and

slow-time weighting (lowers Doppler sidelobes)



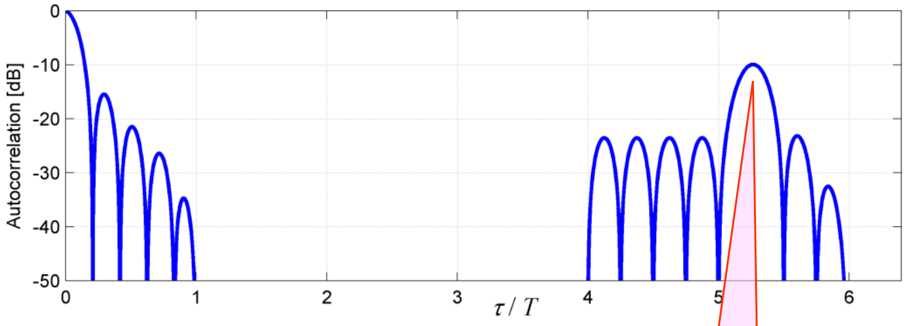




Delay resolution improves with bandwidth.
Bandwidth increases with N and/or Δf.

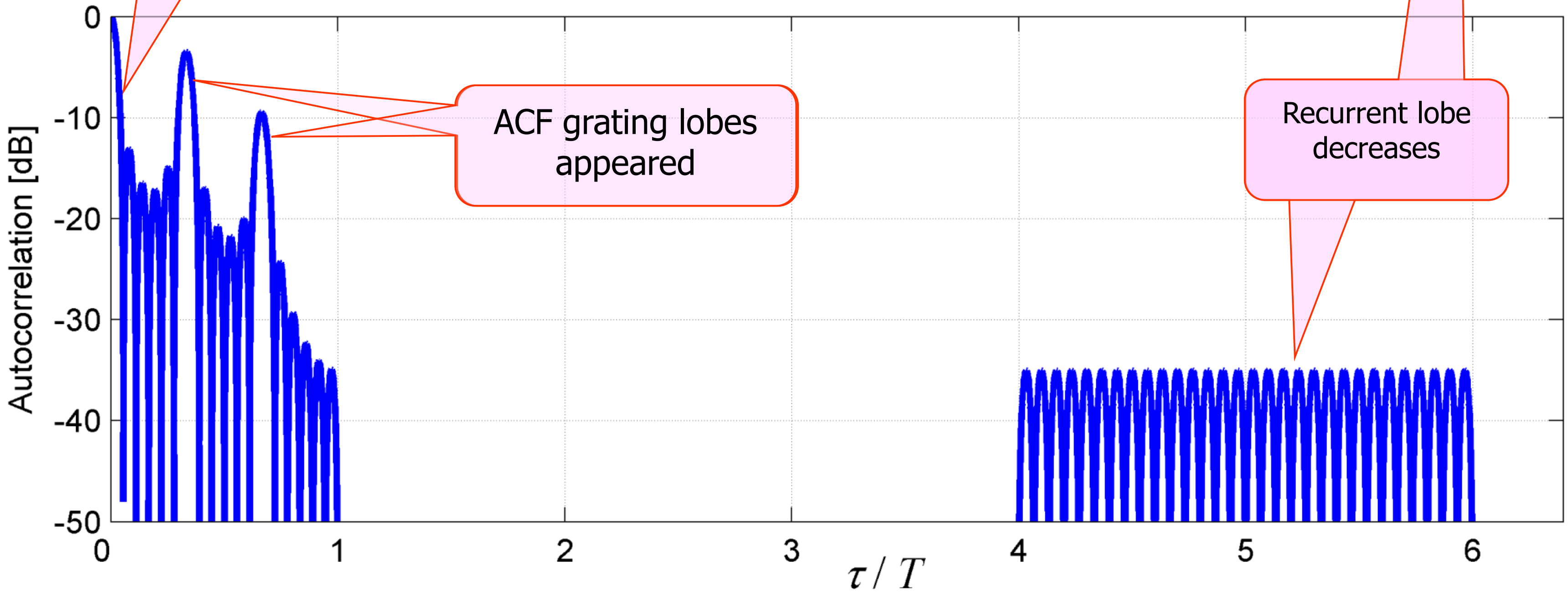
What happens when
$T\Delta f > 1$ **?**

$T\Delta f = 0.8$



ACF mainlobe width decreased

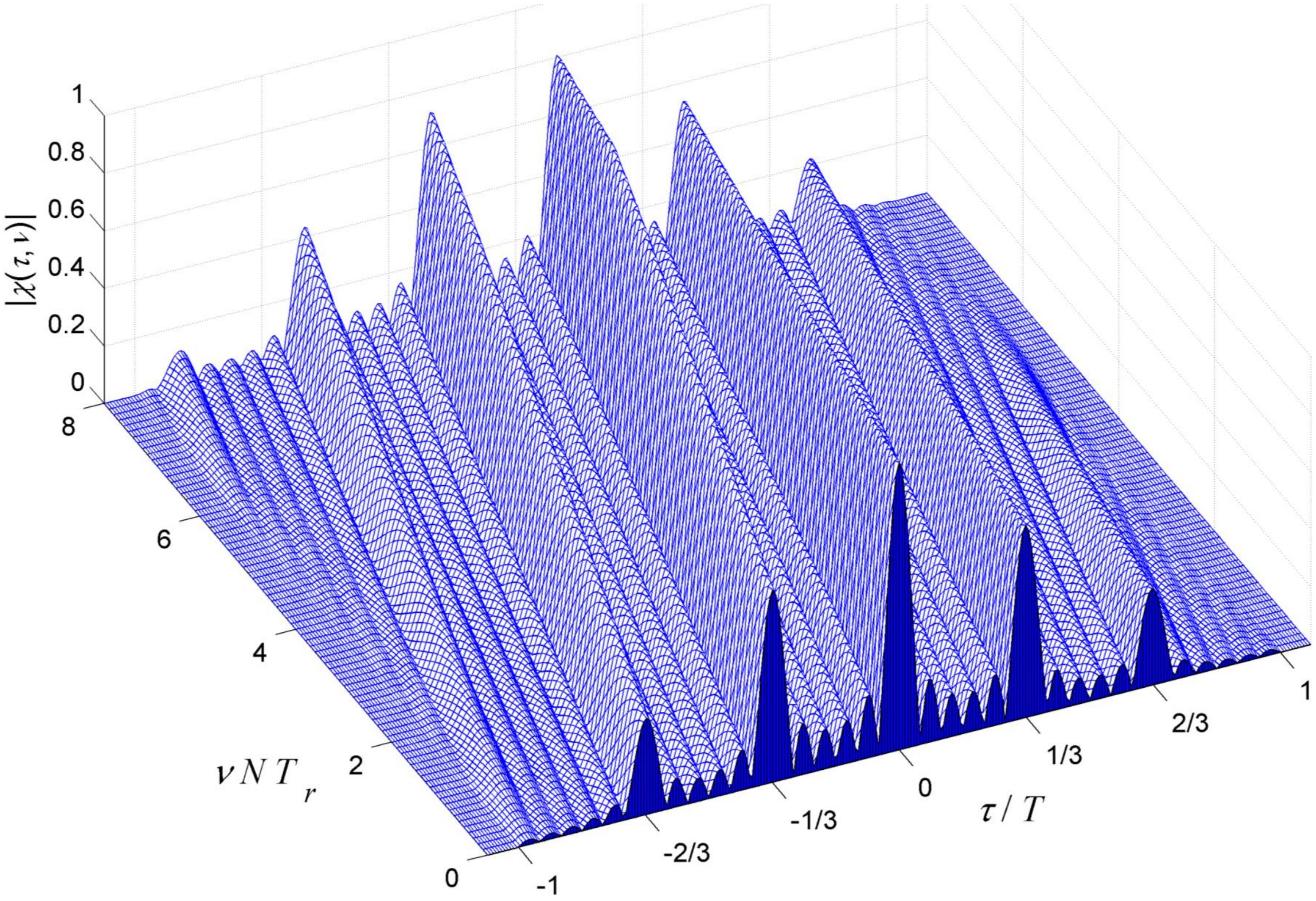
Stepped frequency pulse train, 6 pulses, $T\Delta f = 3$, $T_r/T = 5$



ACF grating lobes appeared

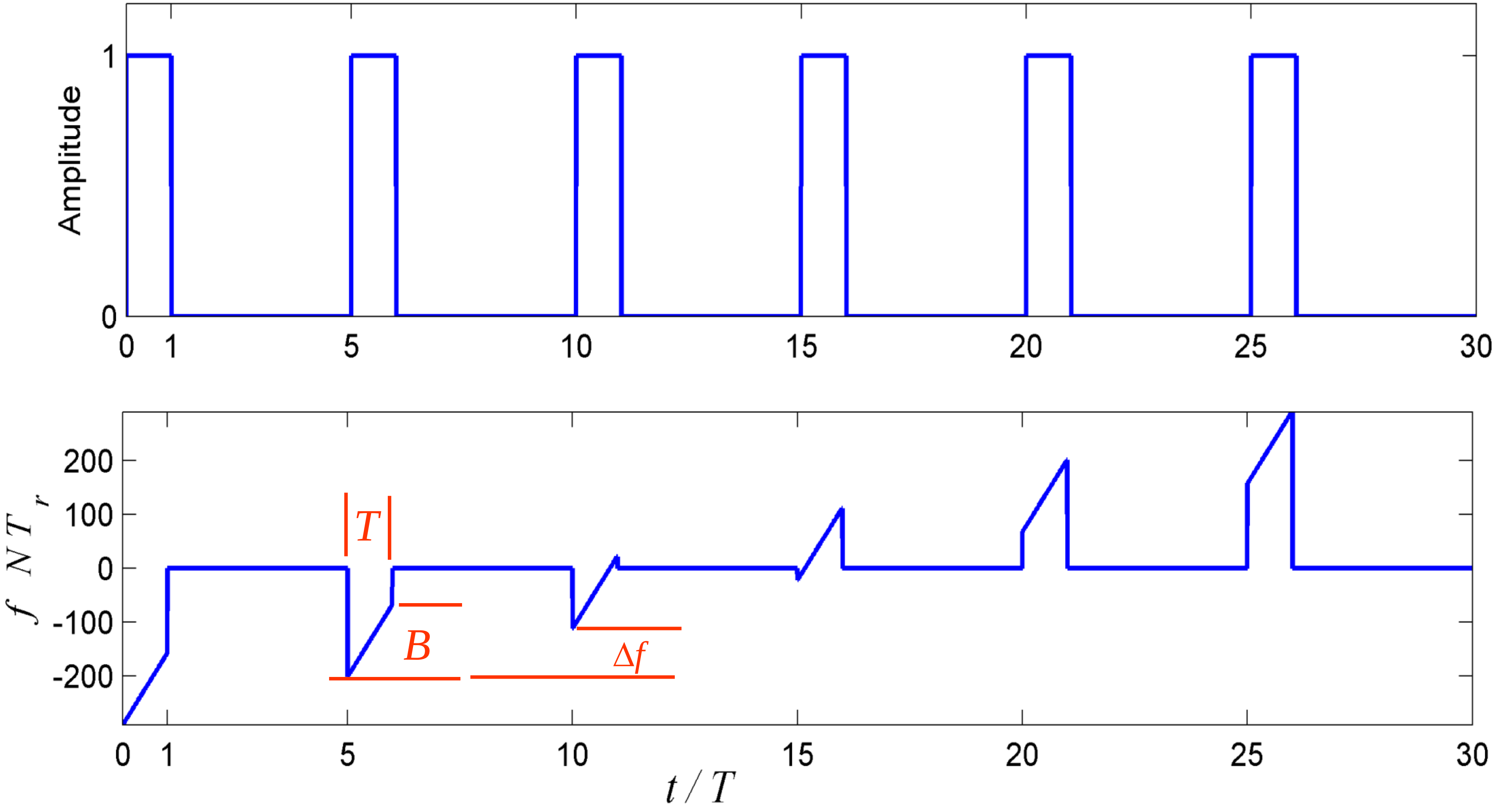
Recurrent lobe decreases

Stepped frequency pulse train, 6 pulses, $T\Delta f = 3$, $T_r/T = 5$

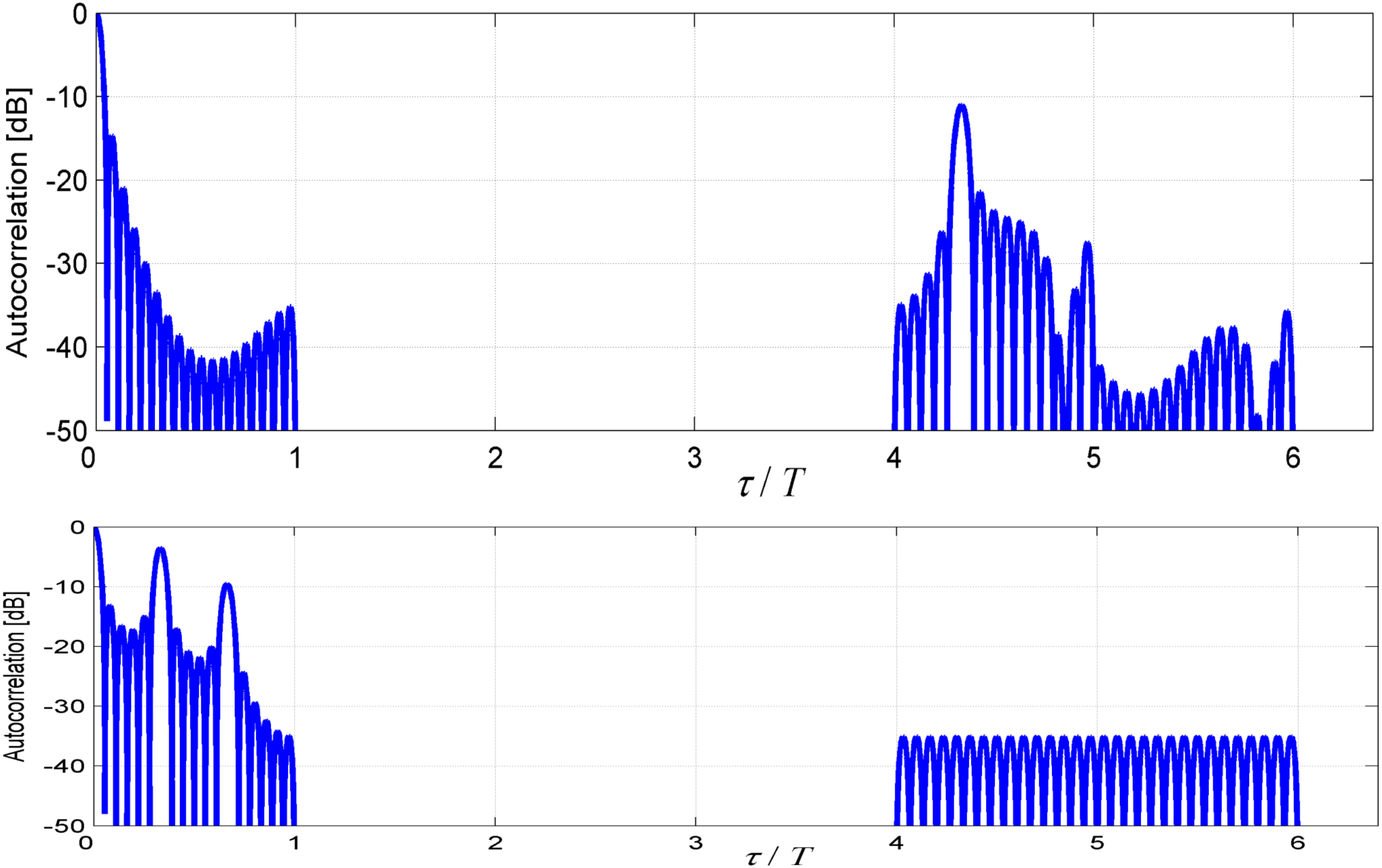


Nullifying the grating lobes by adding LFM

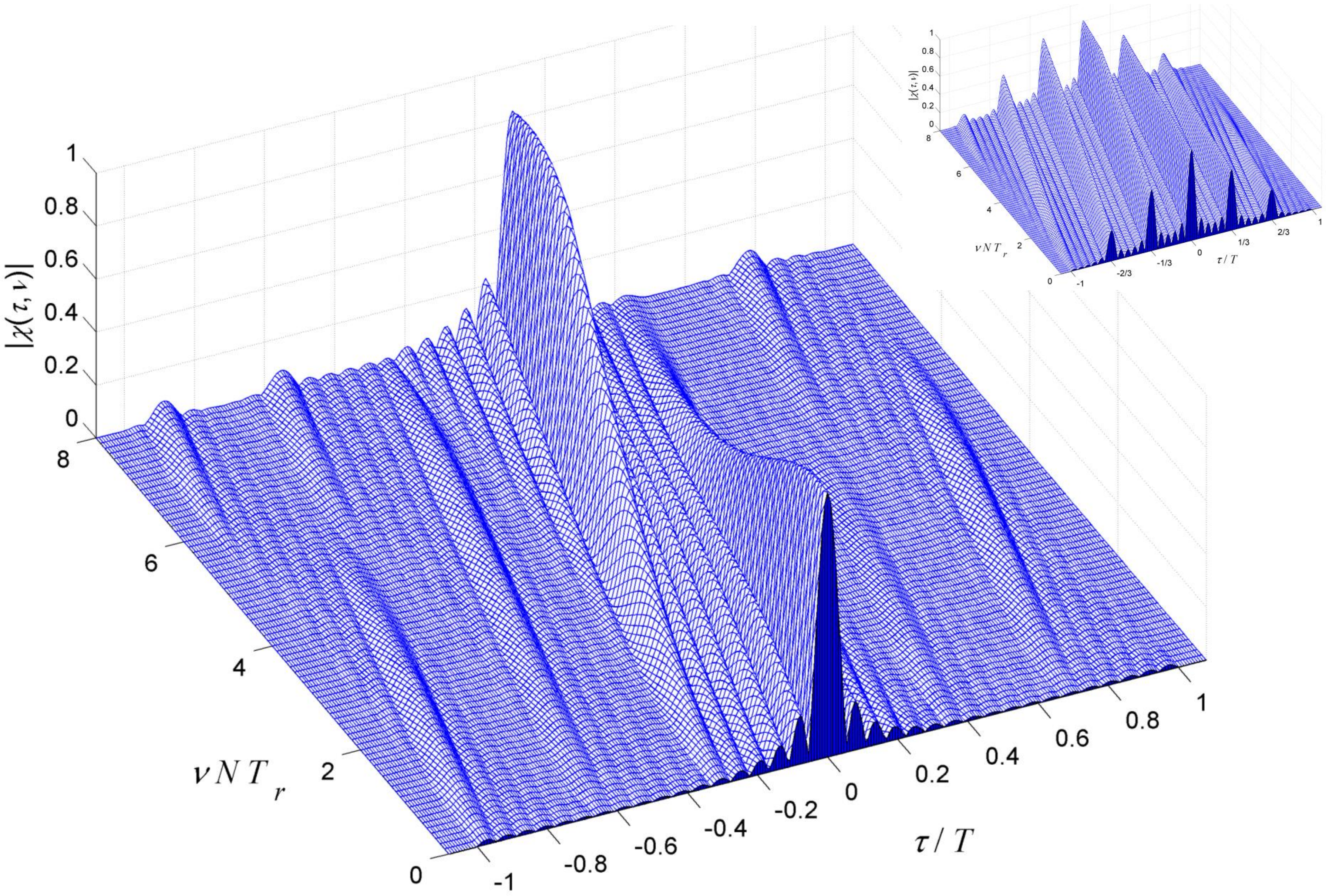
Stepped frequency pulse train, 6 LFM pulses, $T\Delta f = 3$, $TB = 4.5$, $T_r/T = 5$

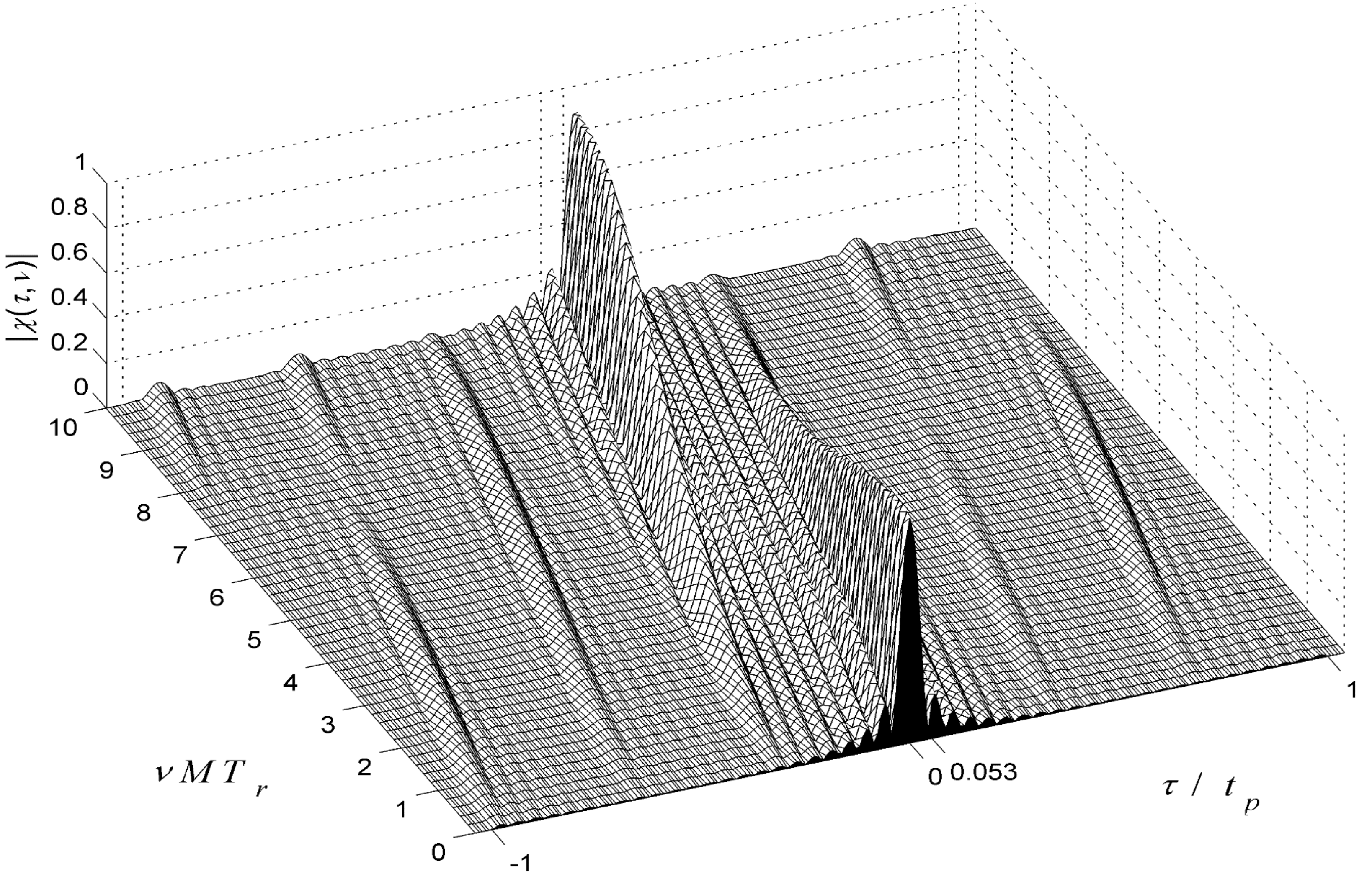


Stepped frequency pulse train, 6 LFM pulses, $T\Delta f = 3$, $TB = 4.5$, $T_r/T = 5$



Stepped frequency pulse train, 6 LFM pulses, $T\Delta f = 3$, $TB = 4.5$, $T_r/T = 5$

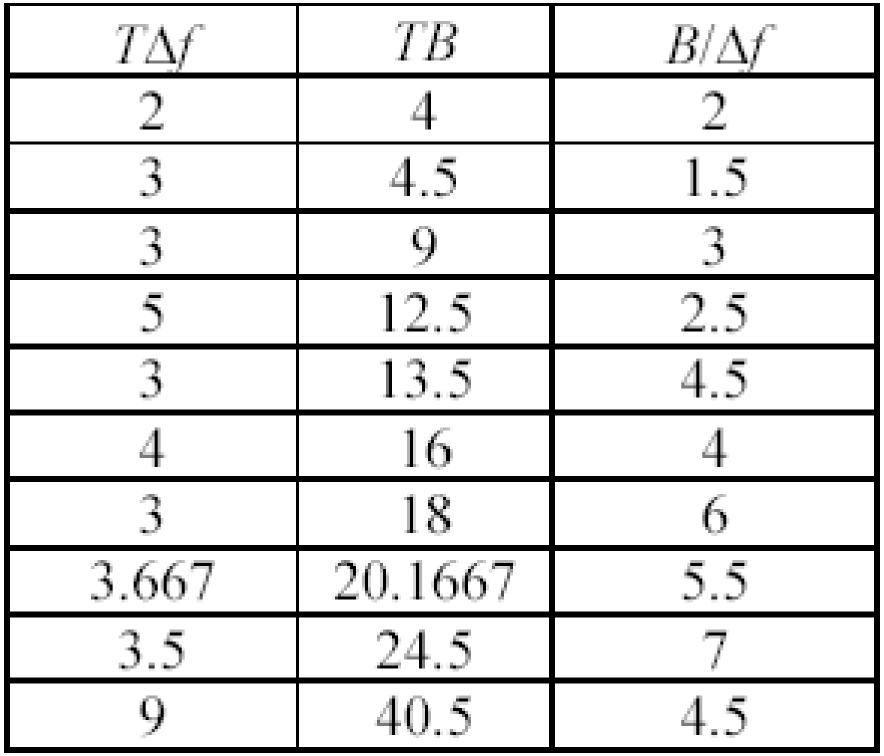




AF (zoom) of stepped-frequency train of LFM pulse with

$$t_p df = 3,\ t_p B = 4.5\ ,\ M = 8,\ T_r/t_p = 5$$

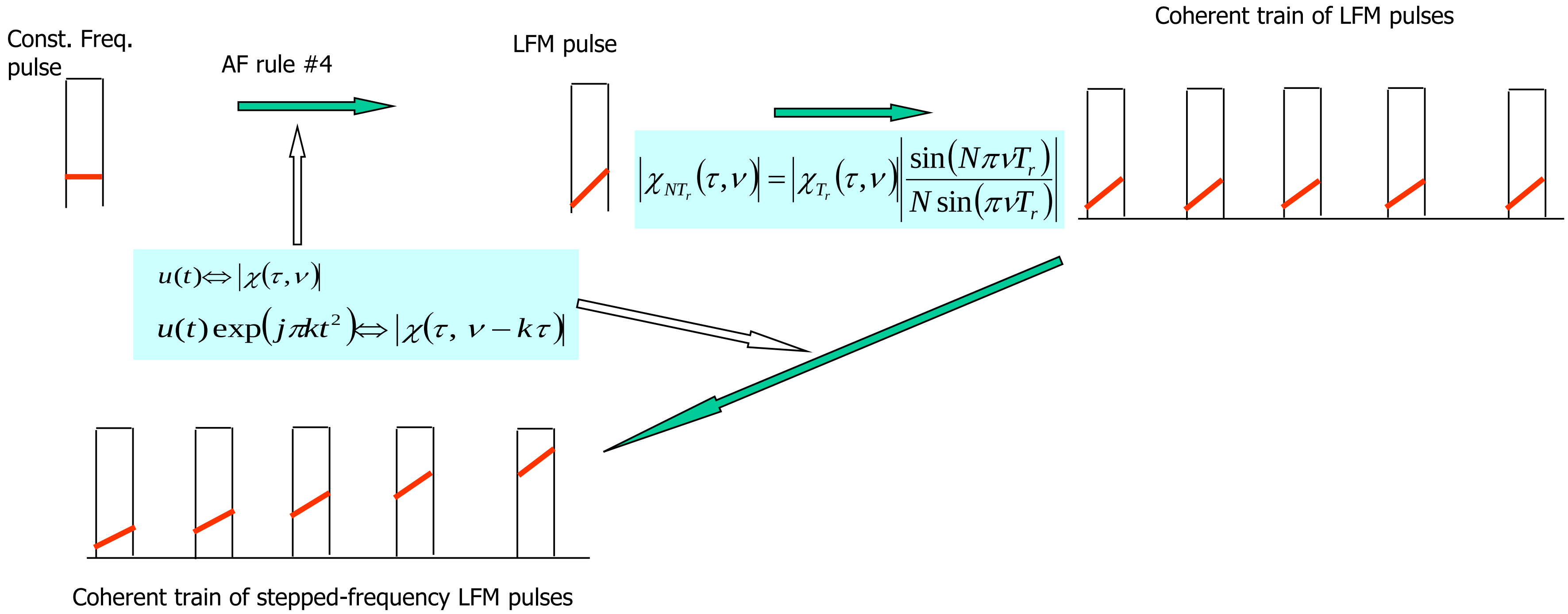
Some relationships that will nullify the grating lobes



$T\Delta f$	TB	$B/\Delta f$
2	4	2
3	4.5	1.5
3	9	3
5	12.5	2.5
3	13.5	4.5
4	16	4
3	18	6
3.667	20.1667	5.5
3.5	24.5	7
9	40.5	4.5

Levanon, N., and Mozeson, E. “Nullifying ACF grating lobes in stepped-frequency train of LFM pulses”, *IEEE Trans. on Aerospace and Electronic Systems*, 39, (2), Apr. 2003, pp. 694-703.

Coherent train of stepped-frequency LFM pulses - theoretical derivation of the AF



$$|\chi_{NT_r}(\tau,\nu)| = |\chi_{T_r}(\tau,\nu)| \left|\frac{\sin(N\pi\nu T_r)}{N\sin(\pi\nu T_r)}\right|$$

$$u(t) \Leftrightarrow |\chi(\tau,\nu)|$$

$$u(t)\exp\left(j\pi k t^2\right) \Leftrightarrow |\chi(\tau,\ \nu - k\tau)|$$

Coherent train of stepped-frequency LFM pulses

$$\left|\chi(\tau,\nu)\right|_{|\tau|\le T} = \left|\left(1-\frac{|\tau|}{T}\right)\operatorname{sinc}\left[T\left(\nu + B\frac{\tau}{T}\right)\left(1-\frac{|\tau|}{T}\right)\right]\frac{\sin\left[N\pi\left(T_r\nu + \tau\,\Delta f\right)\right]}{N\sin\left[\pi\left(T_r\nu + \tau\,\Delta f\right)\right]}\right| \quad , \quad |\tau| \le T$$

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\left|\chi(\tau,\nu)\right|_{|\tau|\le T}=\left|\left(1-\frac{|\tau|}{T}\right)\mathrm{sinc}\left[T\left(\nu+B\frac{\tau}{T}\right)\left(1-\frac{|\tau|}{T}\right)\right]\frac{\sin\left[N\pi\left(T_r\nu+\tau\,\Delta f\right)\right]}{N\sin\left[\pi\left(T_r\nu+\tau\,\Delta f\right)\right]}\right|\;,\quad|\tau|\le T$$

AF ➔ |ACF| set $\nu=0$

$$\left|\chi(\tau,0)\right|_{|\tau|\le T}=\left|\left(1-\frac{|\tau|}{T}\right)\mathrm{sinc}\left[B\tau\left(1-\frac{|\tau|}{T}\right)\right]\frac{\sin\left(N\pi\tau\,\Delta f\right)}{N\sin\left(\pi\tau\,\Delta f\right)}\right|\;,\quad|\tau|\le T$$

$$\left|\chi\left(\frac{\tau}{T},0\right)\right|_{\left|\frac{\tau}{T}\right|\le 1}=\left|\left(1-\left|\frac{\tau}{T}\right|\right)\mathrm{sinc}\left[\boldsymbol{TB}\frac{\tau}{T}\left(1-\left|\frac{\tau}{T}\right|\right)\right]\frac{\sin\left(N\pi\frac{\tau}{T}\boldsymbol{T\Delta f}\right)}{N\sin\left(\pi\frac{\tau}{T}\boldsymbol{T\Delta f}\right)}\right|\;,\quad\left|\frac{\tau}{T}\right|\le 1$$

$$\left|\chi\left(\frac{\tau}{T},0\right)\right|_{\left|\frac{\tau}{T}\right|\le 1} = \underbrace{\left|\left(1-\left|\frac{\tau}{T}\right|\right)\mathrm{sinc}\left[\boldsymbol{TB}\frac{\tau}{T}\left(1-\left|\frac{\tau}{T}\right|\right)\right]\right|}_{} \underbrace{\left|\frac{\sin\left(N\pi\frac{\tau}{T}\boldsymbol{T\Delta f}\right)}{N\sin\left(\pi\frac{\tau}{T}\boldsymbol{T\Delta f}\right)}\right|}_{} , \quad \left|\frac{\tau}{T}\right|\le 1$$

Function of the LFM pulse
(contains nulls)

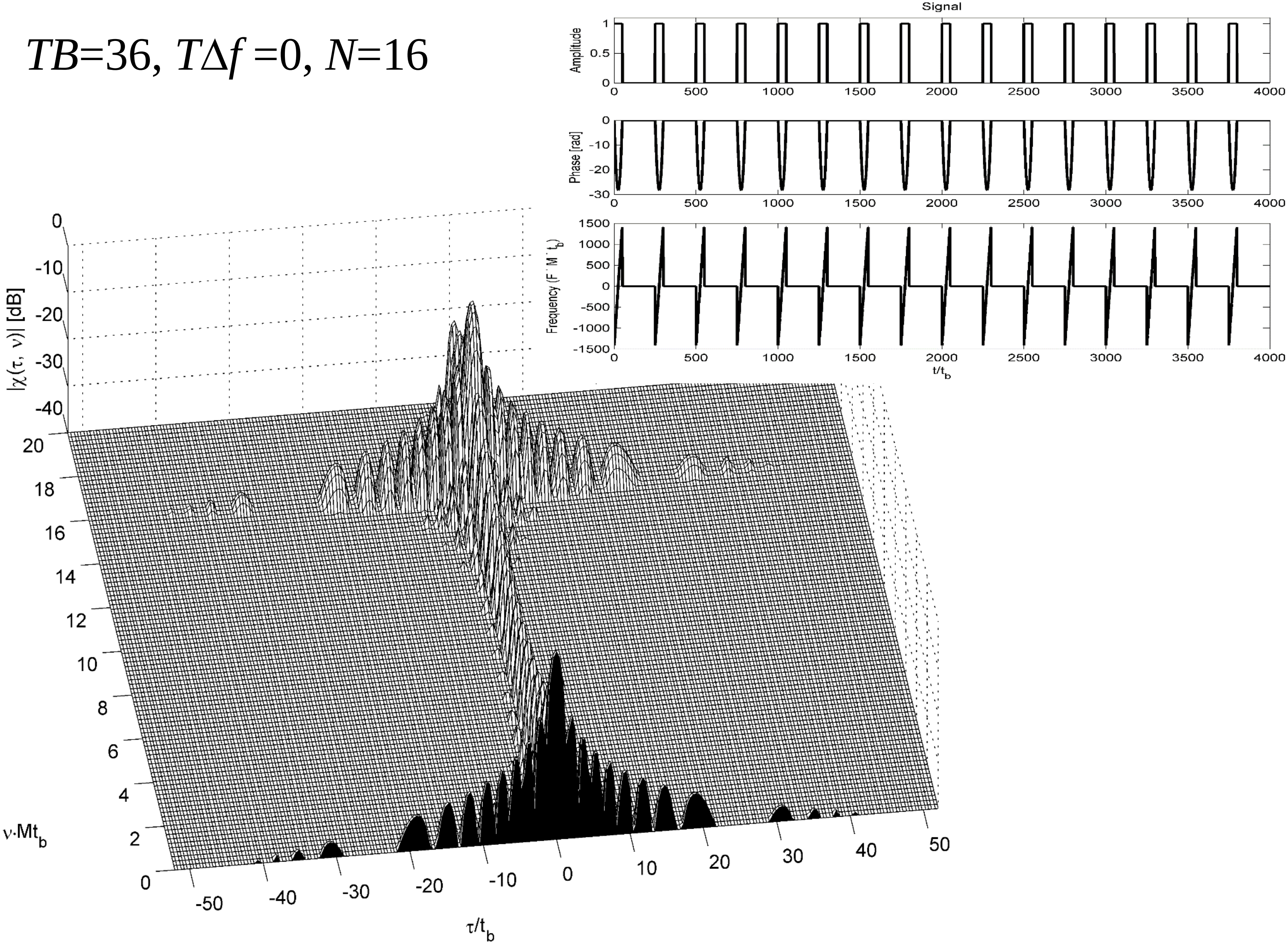
Function of N step-frequency pulses
(contains grating lobes)

Align the nulls with the grating lobes

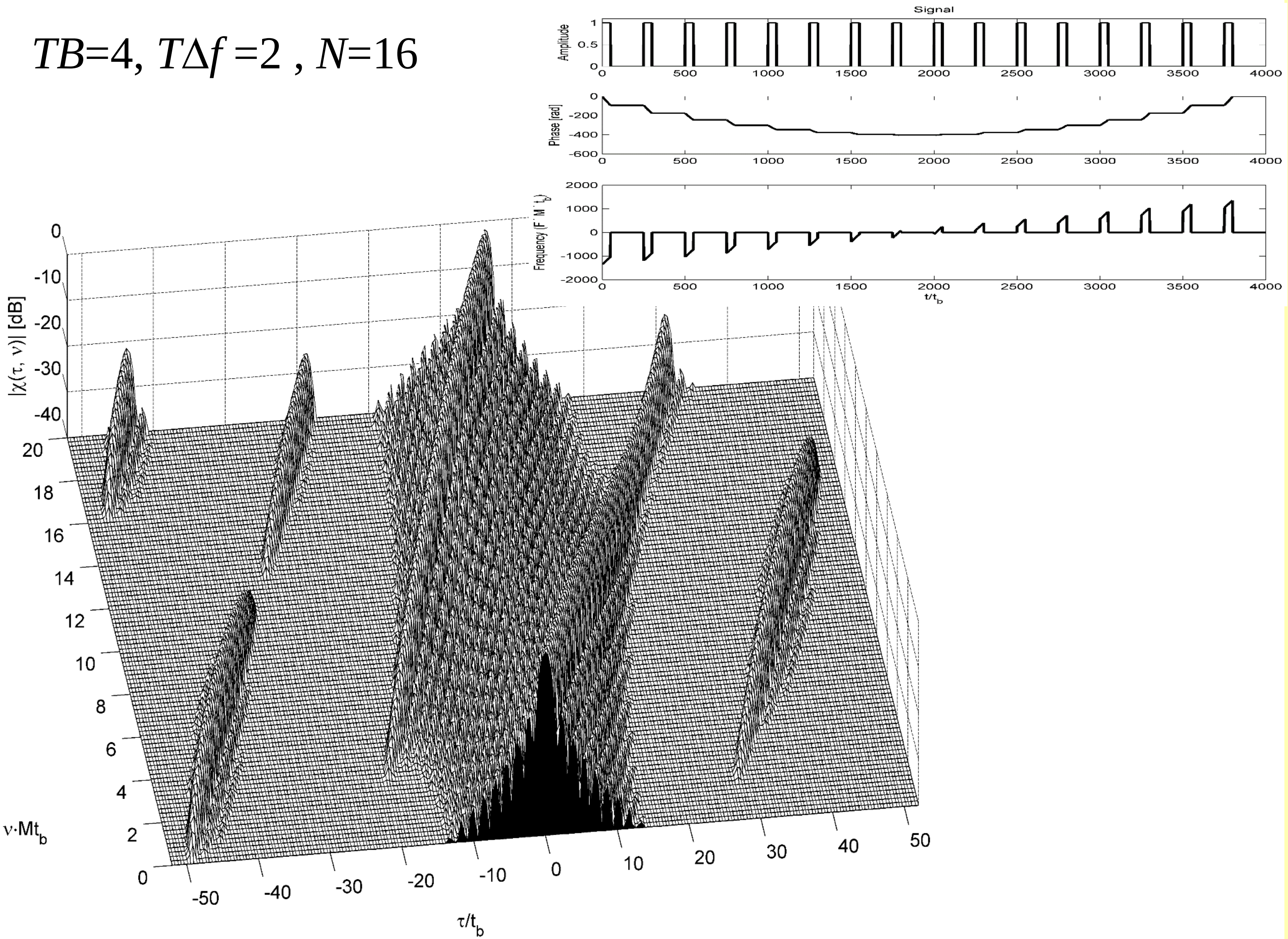
The ACF (over $|\tau|\le T$) is independent of $T_r{}^*$, of the order of steps, and of the polarity of the LFM slope.

*As long as $T_r > 2T$

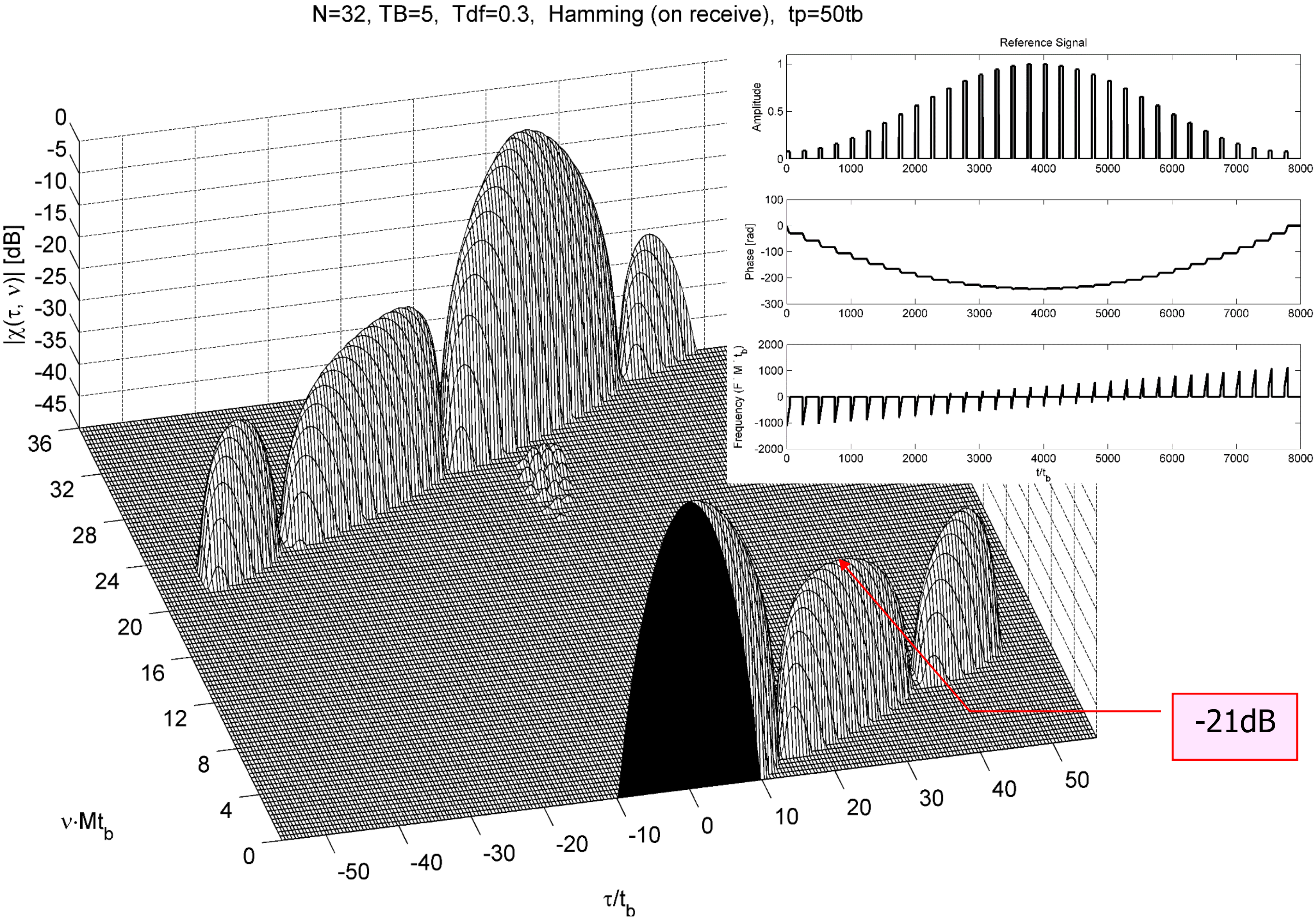
TB=36, $T\Delta f$ =0, N=16



TB=4, $T\Delta f$ =2 , N=16

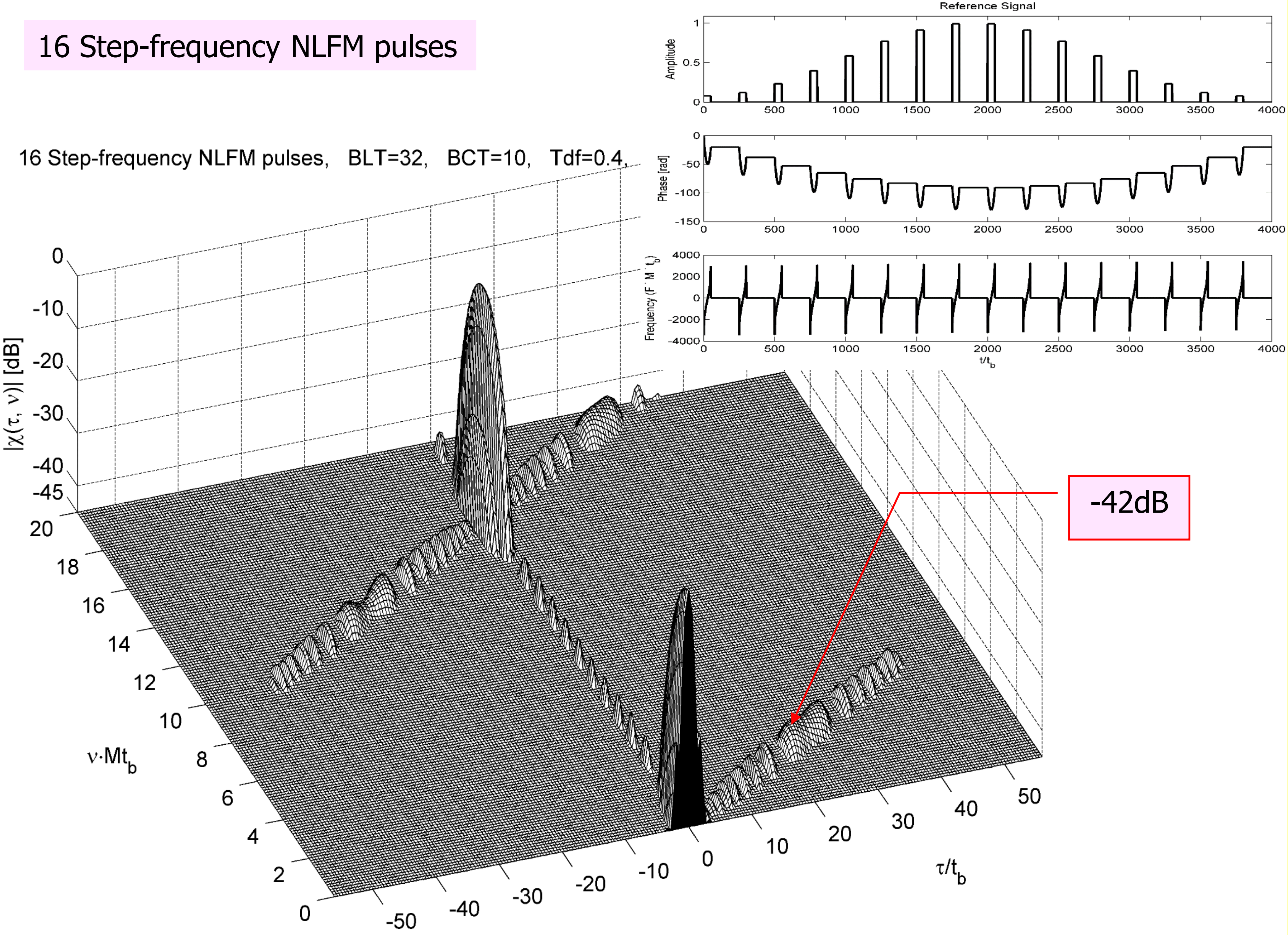


$TB=5,\ T\Delta f=0.3\ ,\ N=32$



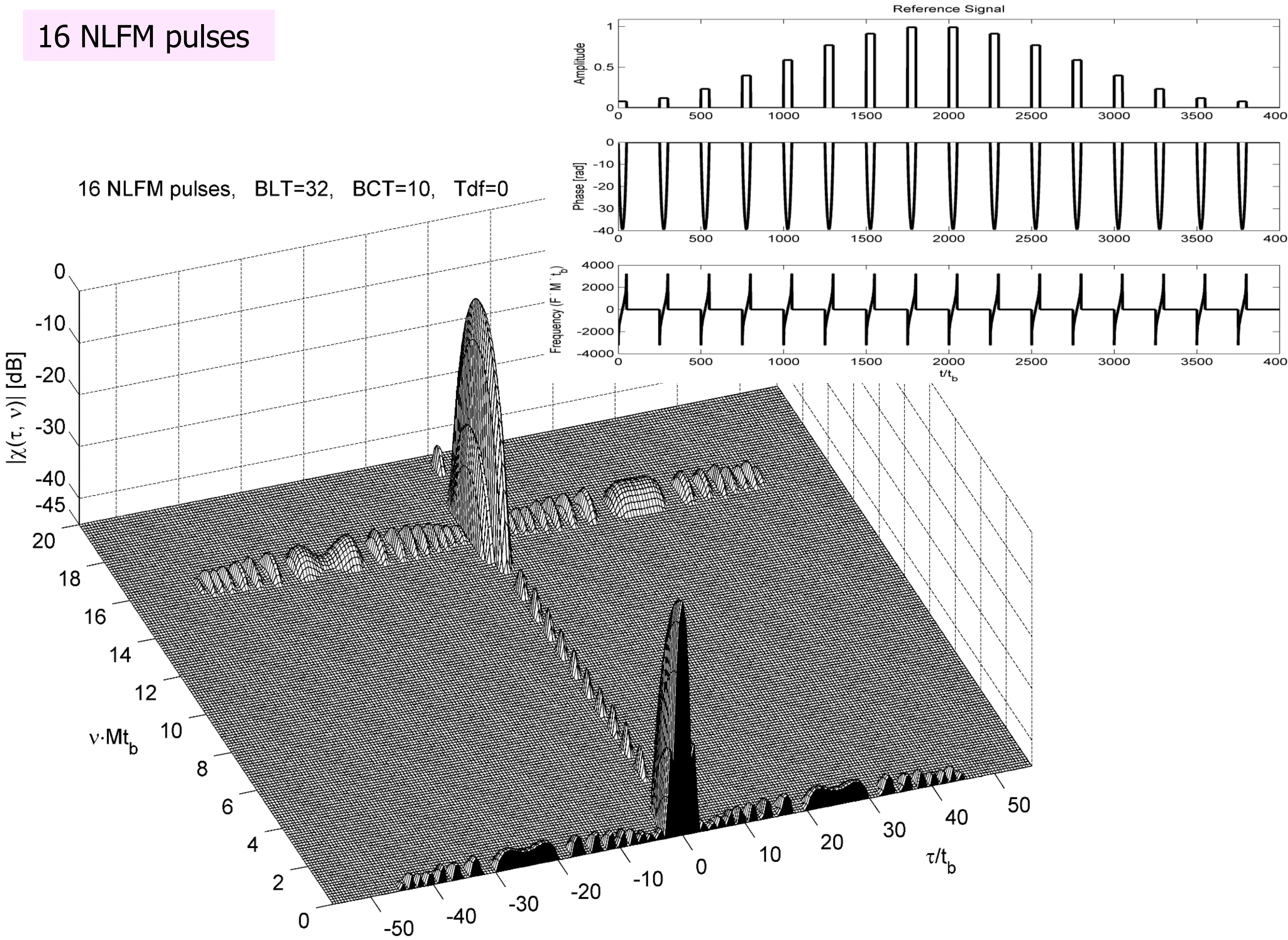
-21dB

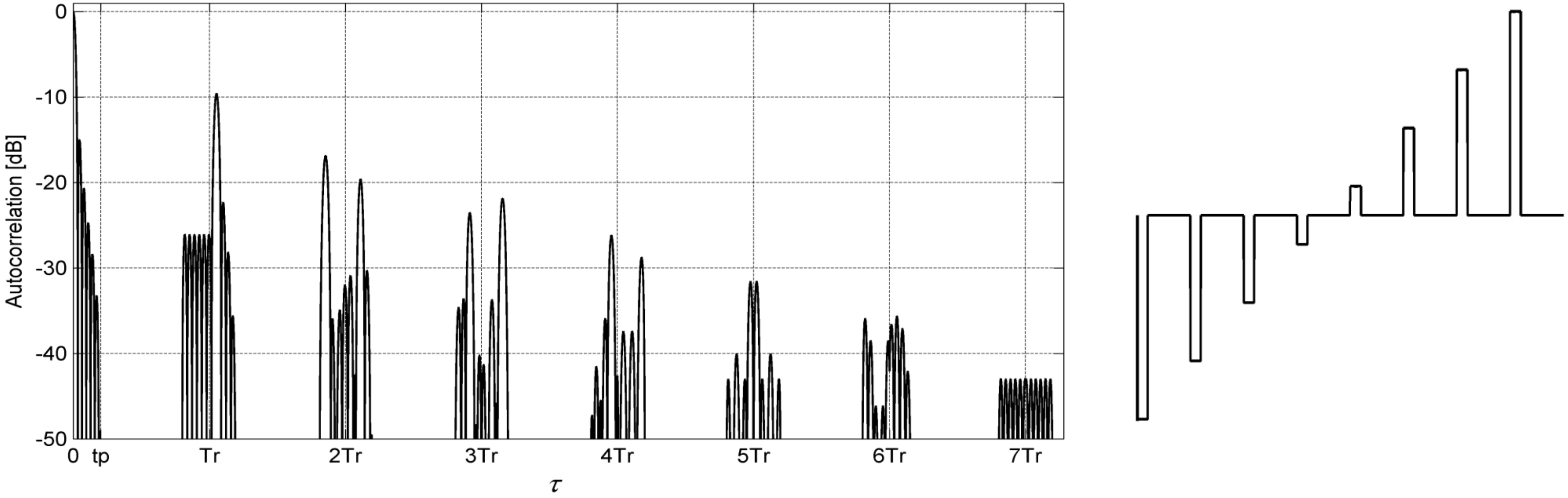
16 Step-frequency NLFM pulses



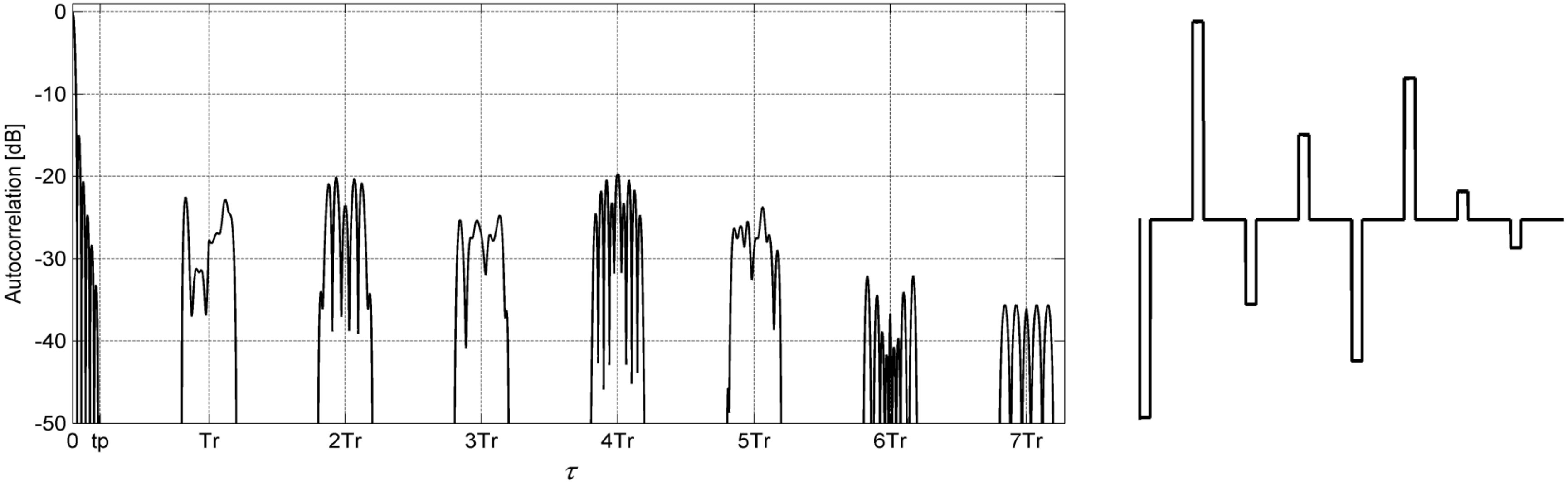
-42dB

16 NLFM pulses



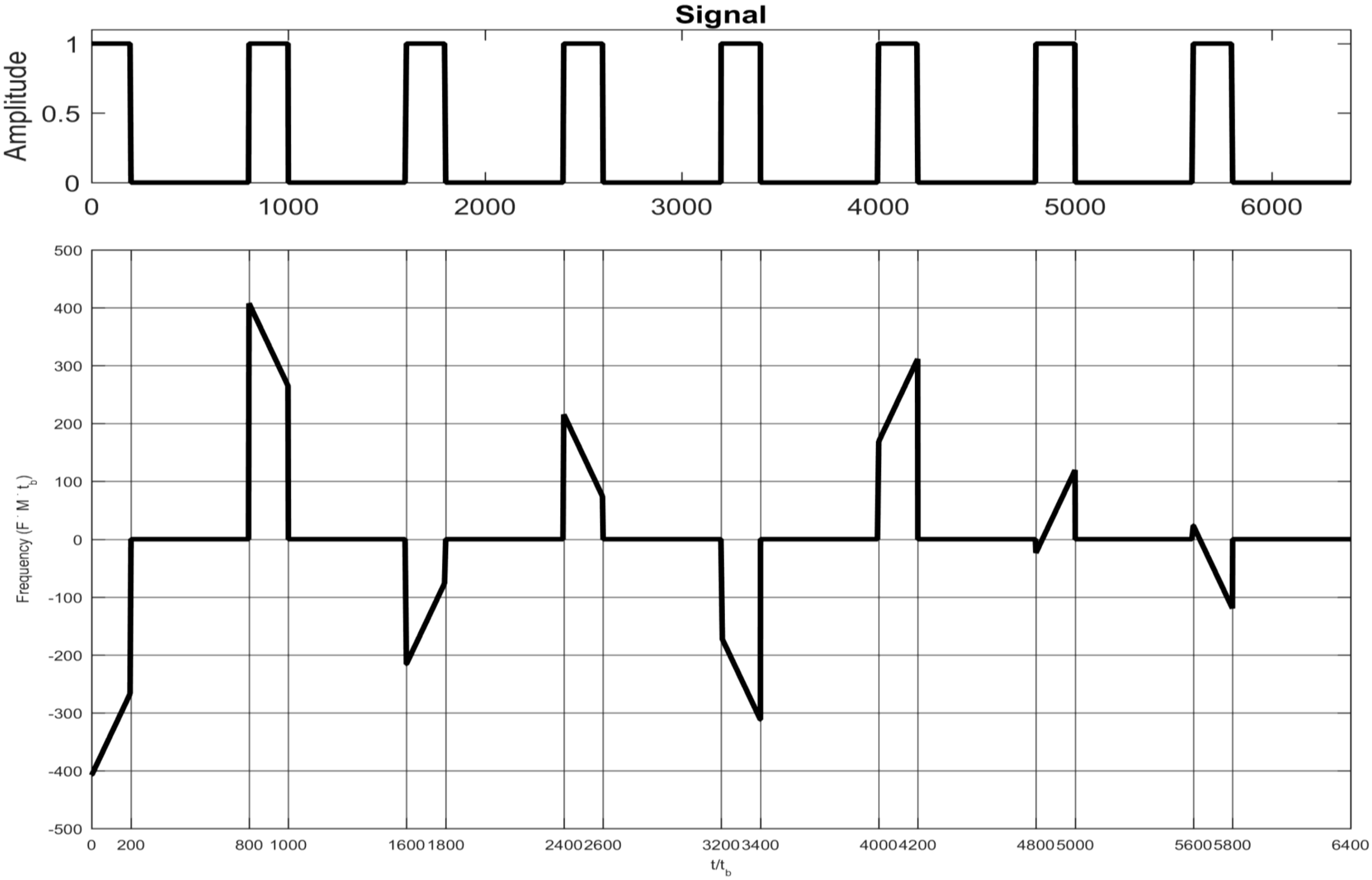


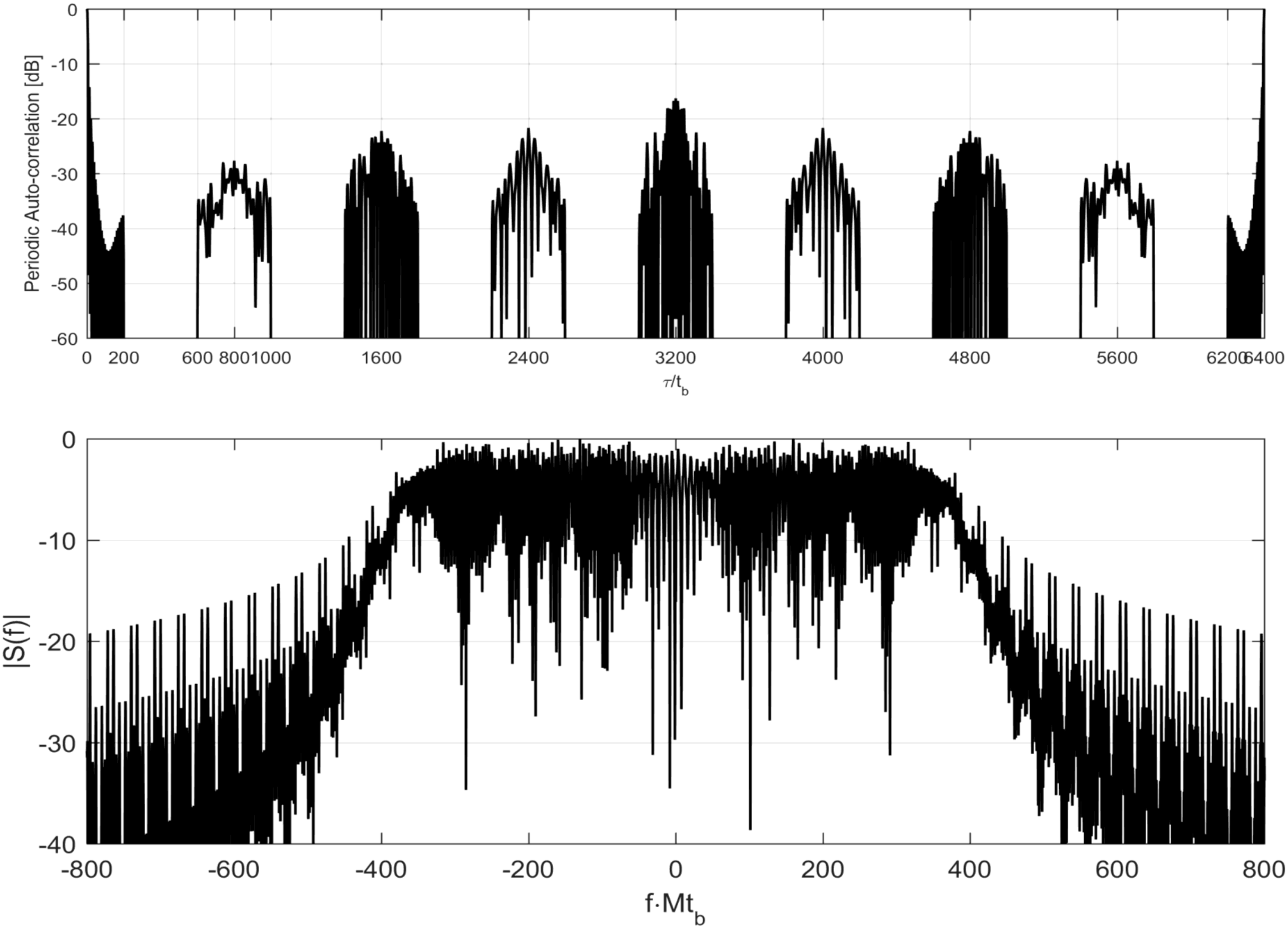
Recurrent ACF lobes (left) for a linearly-ordered stepped-frequency waveform (right)

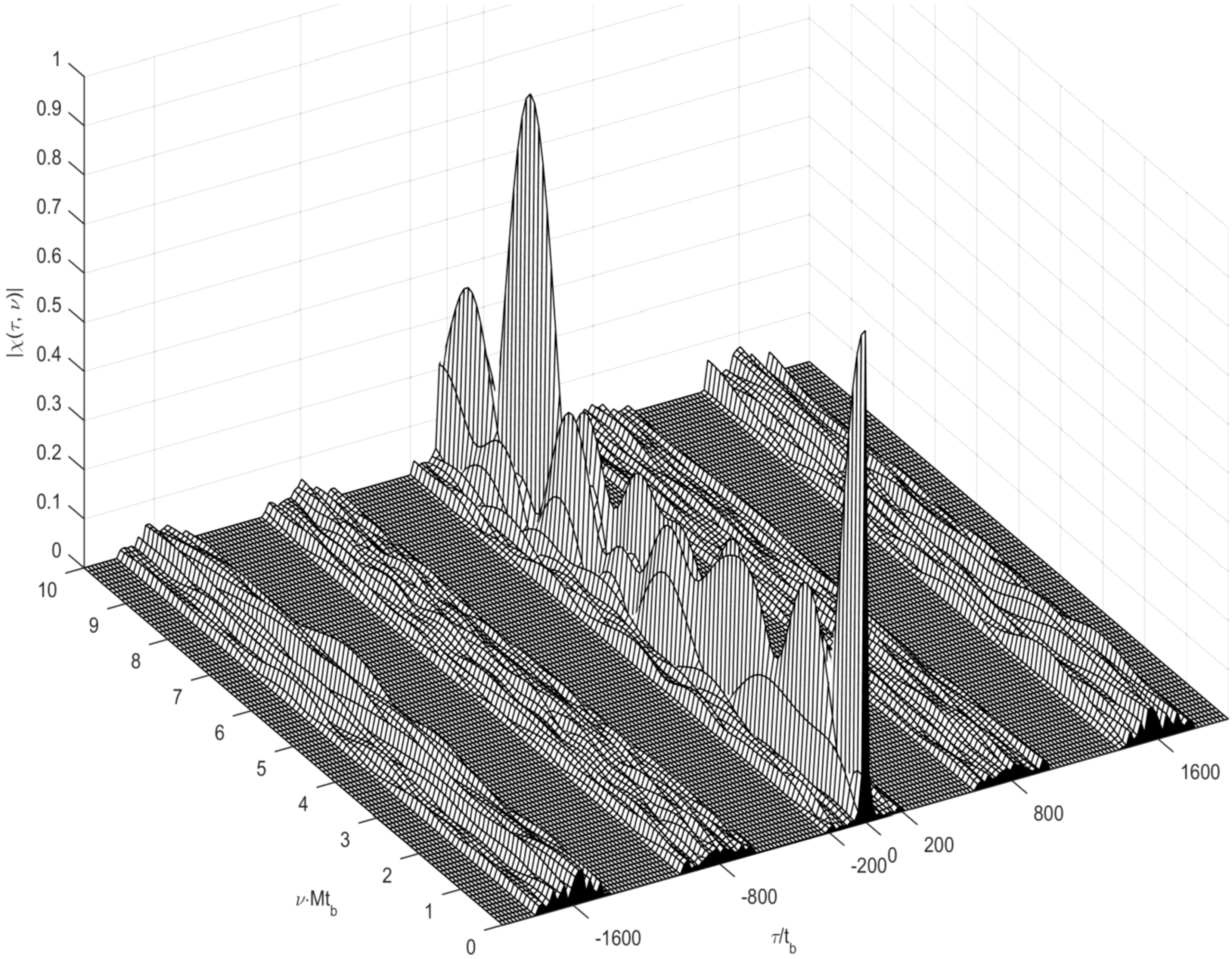


Recurrent ACF lobes (left) for a Costas-ordered stepped-frequency waveform (right)

$$t_p df = 3,\ t_p B = 4.5\ ,\ M = 8,\ T_r / t_p = 4$$



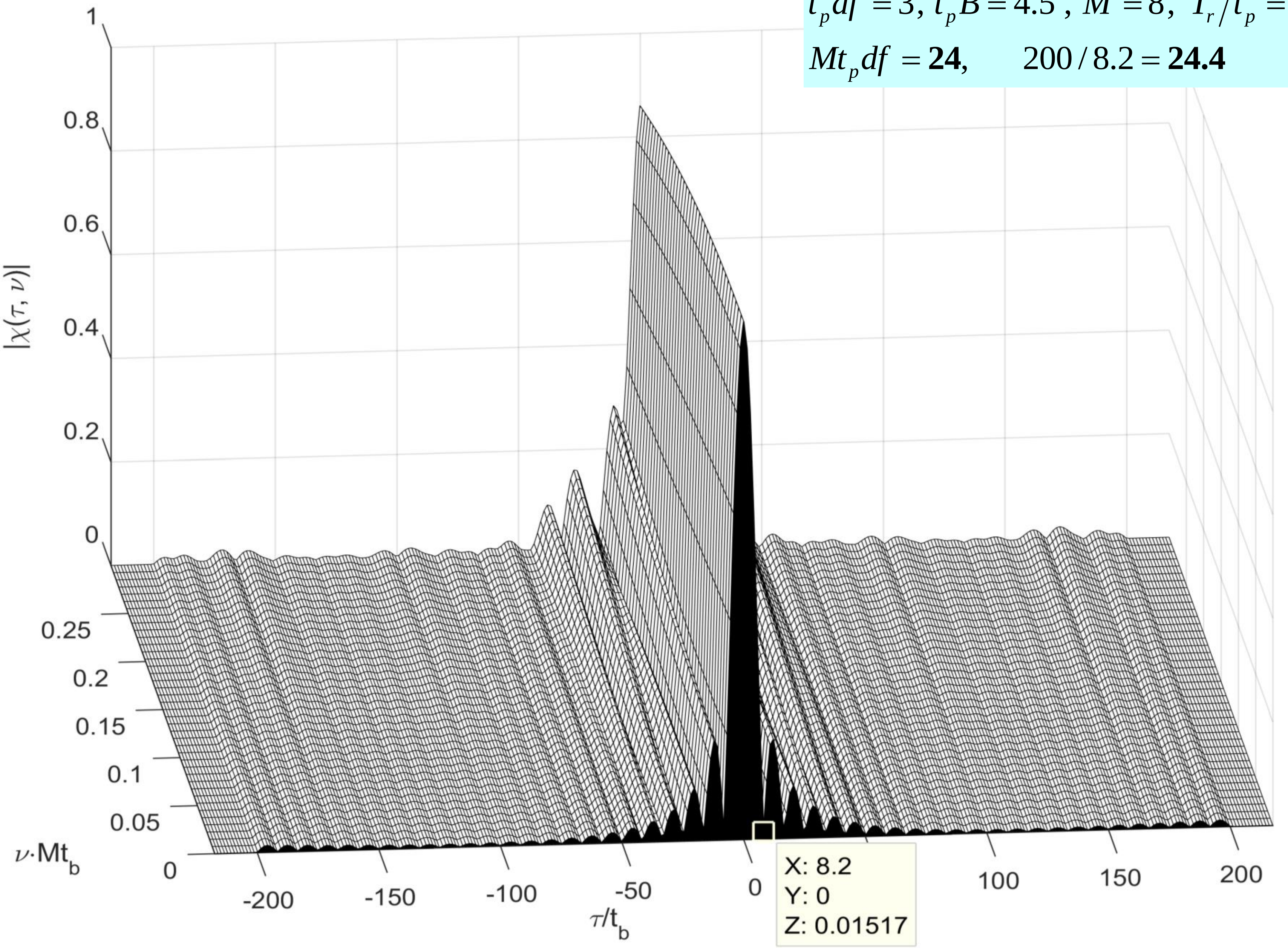

Periodic Auto-correlation [dB]
0
-10
-20
-30
-40
-50
-60
0
200
600
800
1000
1600
2400
3200
4000
4800
5600
6200
6400
τ/t_b
|S(f)|
0
-10
-20
-30
-40
-800
-600
-400
-200
0
200
400
600
800
$f \cdot Mt_b$



$|\chi(\tau,\nu)|$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
9
8
7
6
5
4
3
2
1
0
$\nu\cdot Mt_b$
-1600
-800
-200
0
200
800
1600
τ/t_b

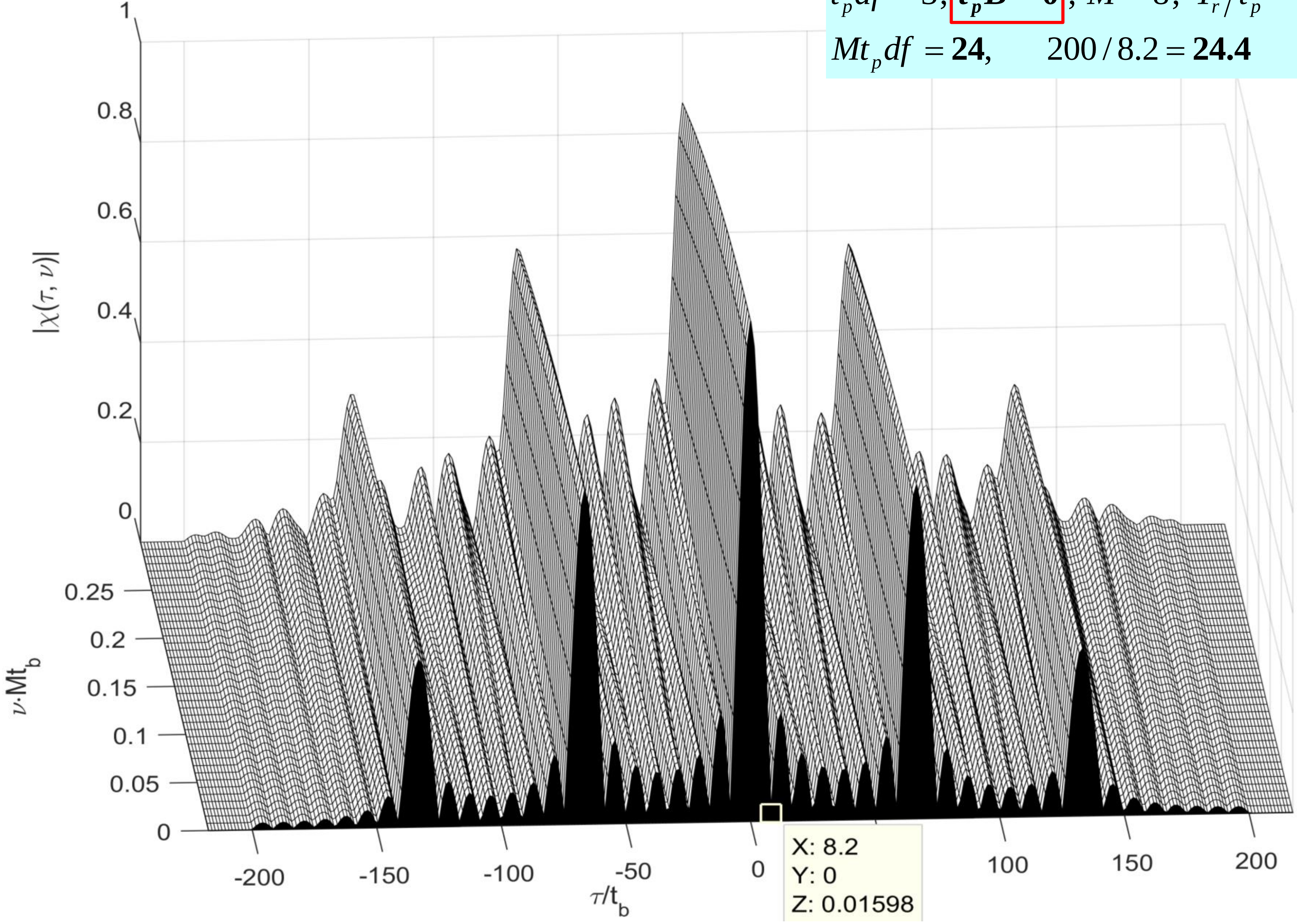
$t_p df = 3,\ t_p B = 4.5\ ,\ M = 8,\ T_r / t_p = 4$

$Mt_p df = \mathbf{24},\qquad 200/8.2 = \mathbf{24.4}$

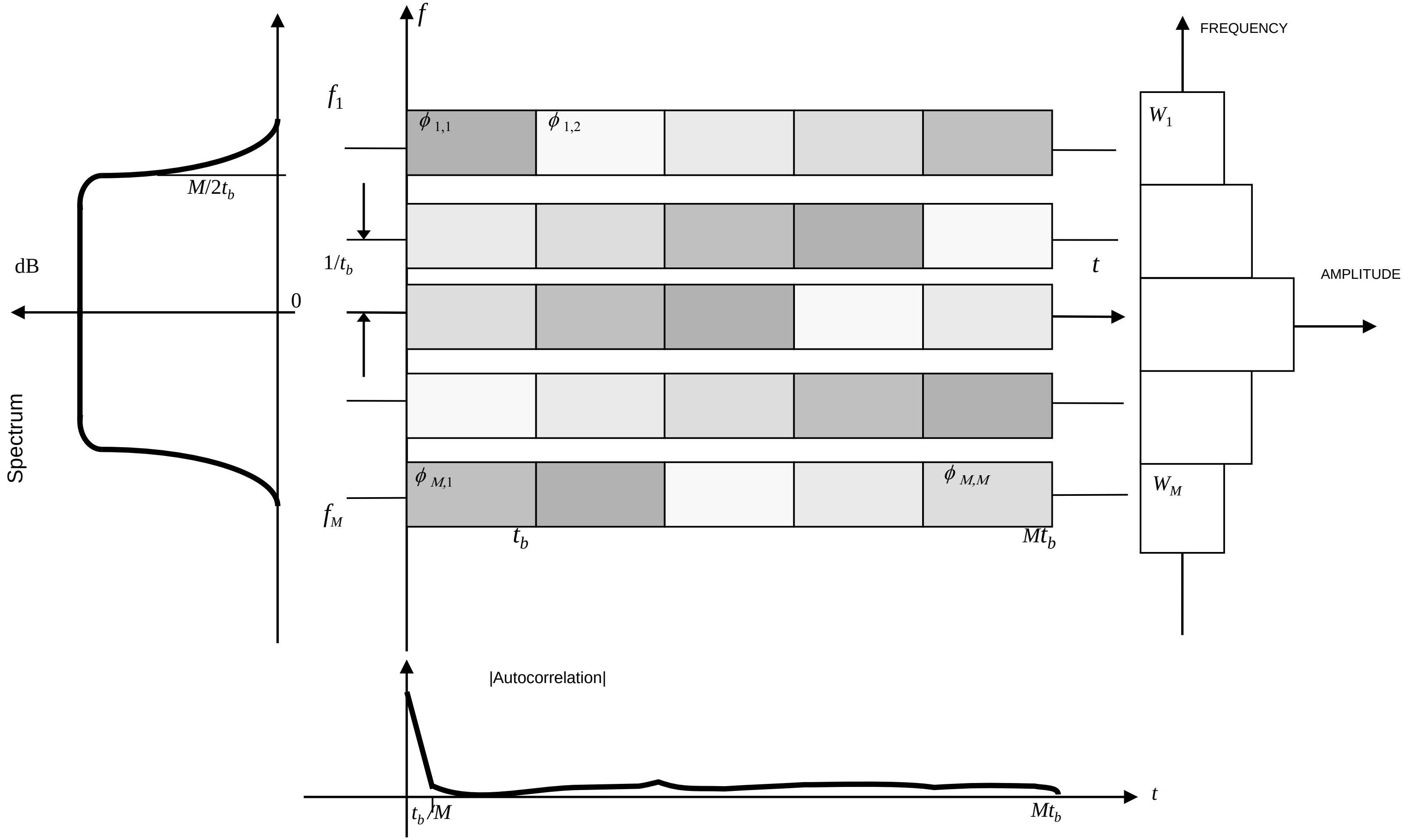


$t_p df = 3, \boxed{\mathbf{t_p B = 0}}, M = 8, T_r / t_p = 4$

$Mt_p df = \mathbf{24}, \quad 200/8.2 = \mathbf{24.4}$



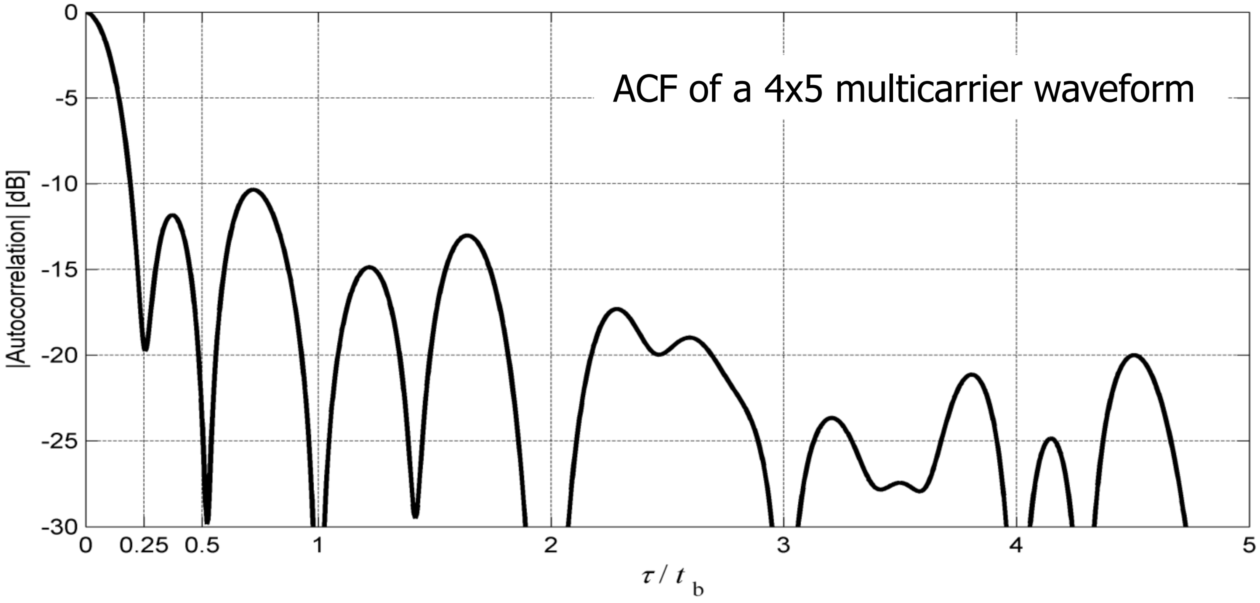
MULTICARRIER WAVEFORMS



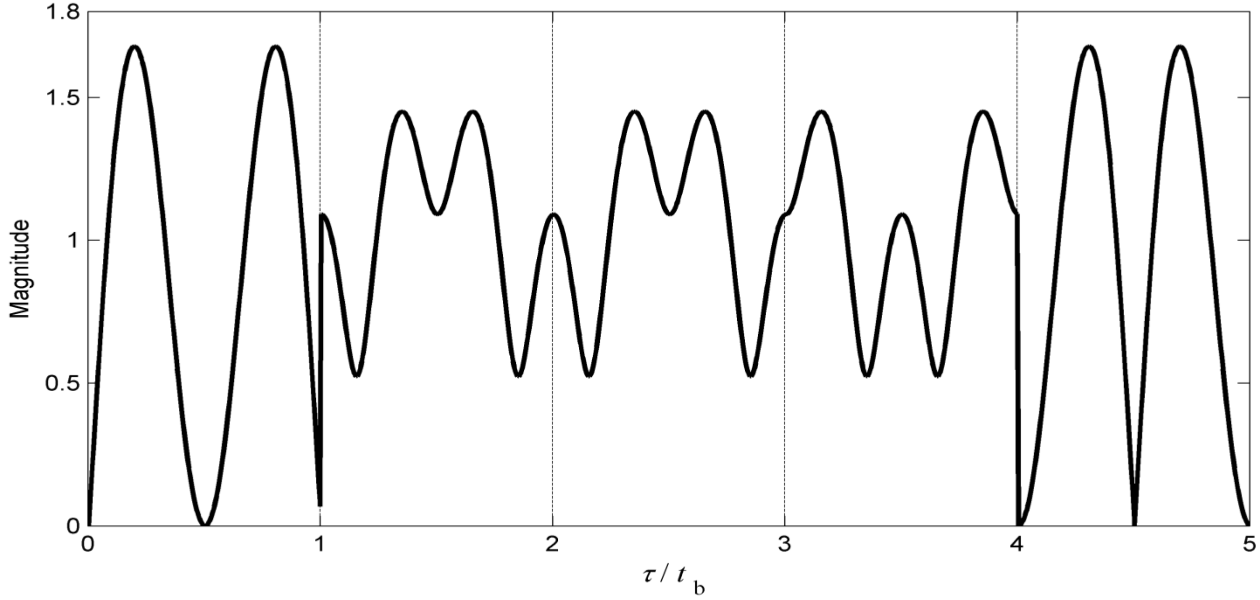
Structure of a multicarrier waveform

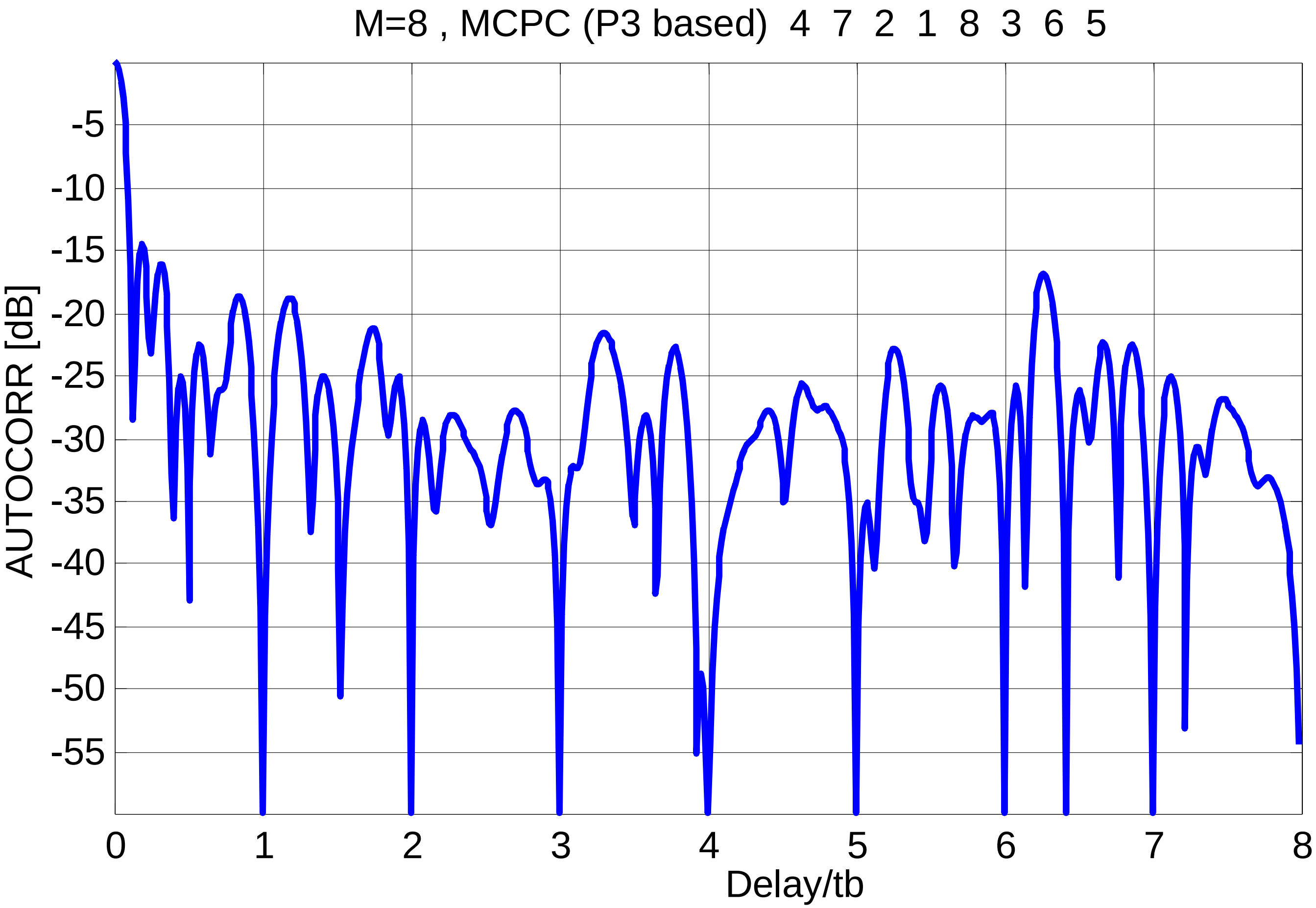
Phase/π

0	1	1	1	1
0	1	1	1	0
1	0	0	1	0
1	1	1	0	1

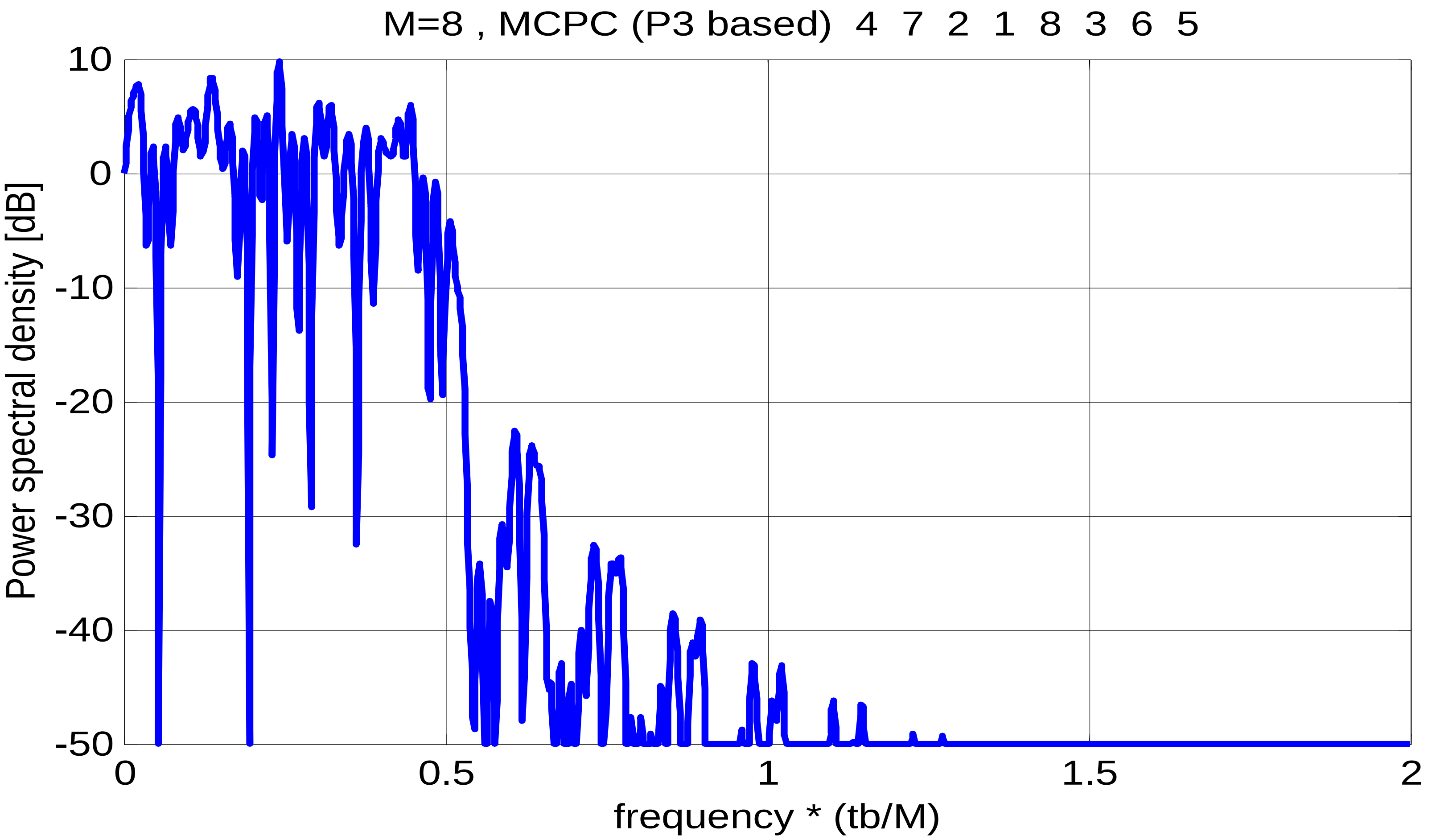

ACF of a 4x5 multicarrier waveform

Real envelope of a 4x5 multicarrier waveform

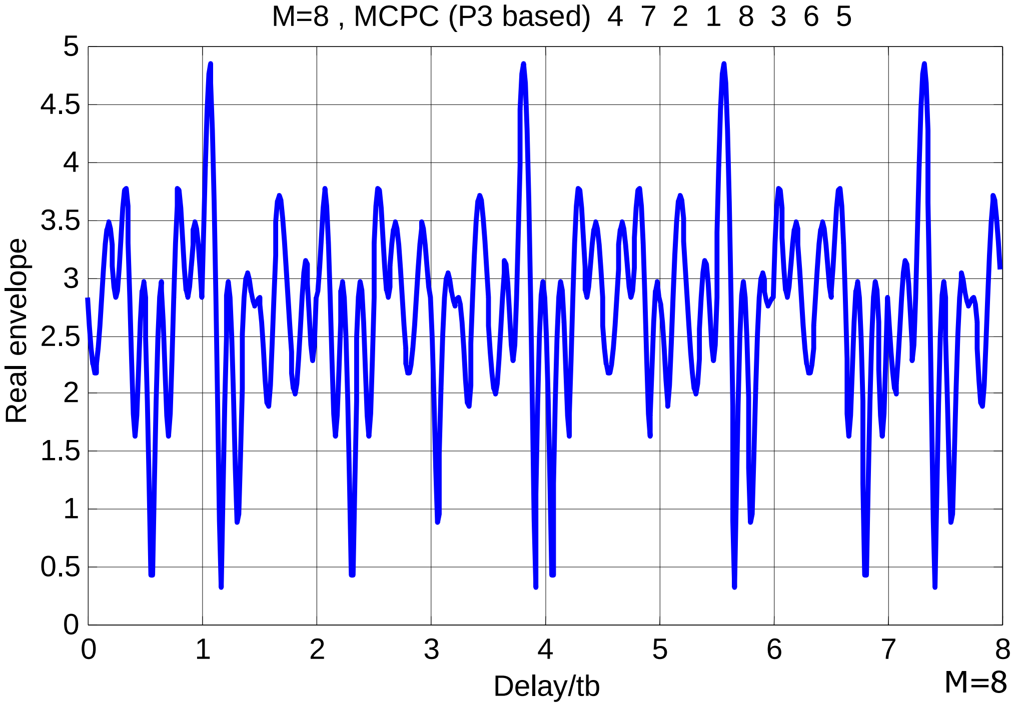




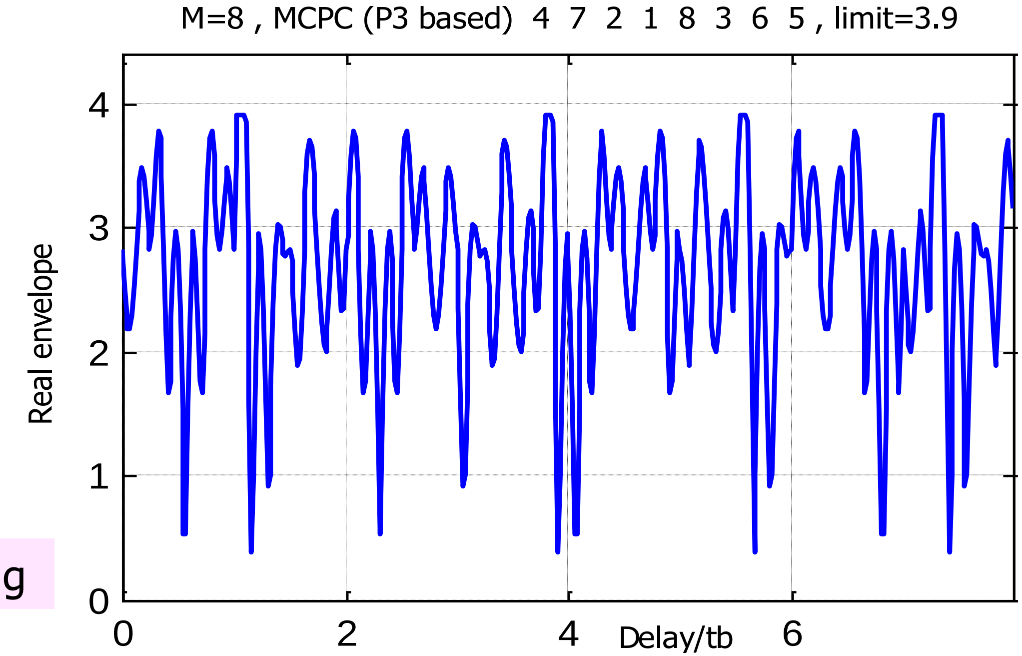
Autocorrelation of an 8x8 MCPC pulse based on a P3 sequence



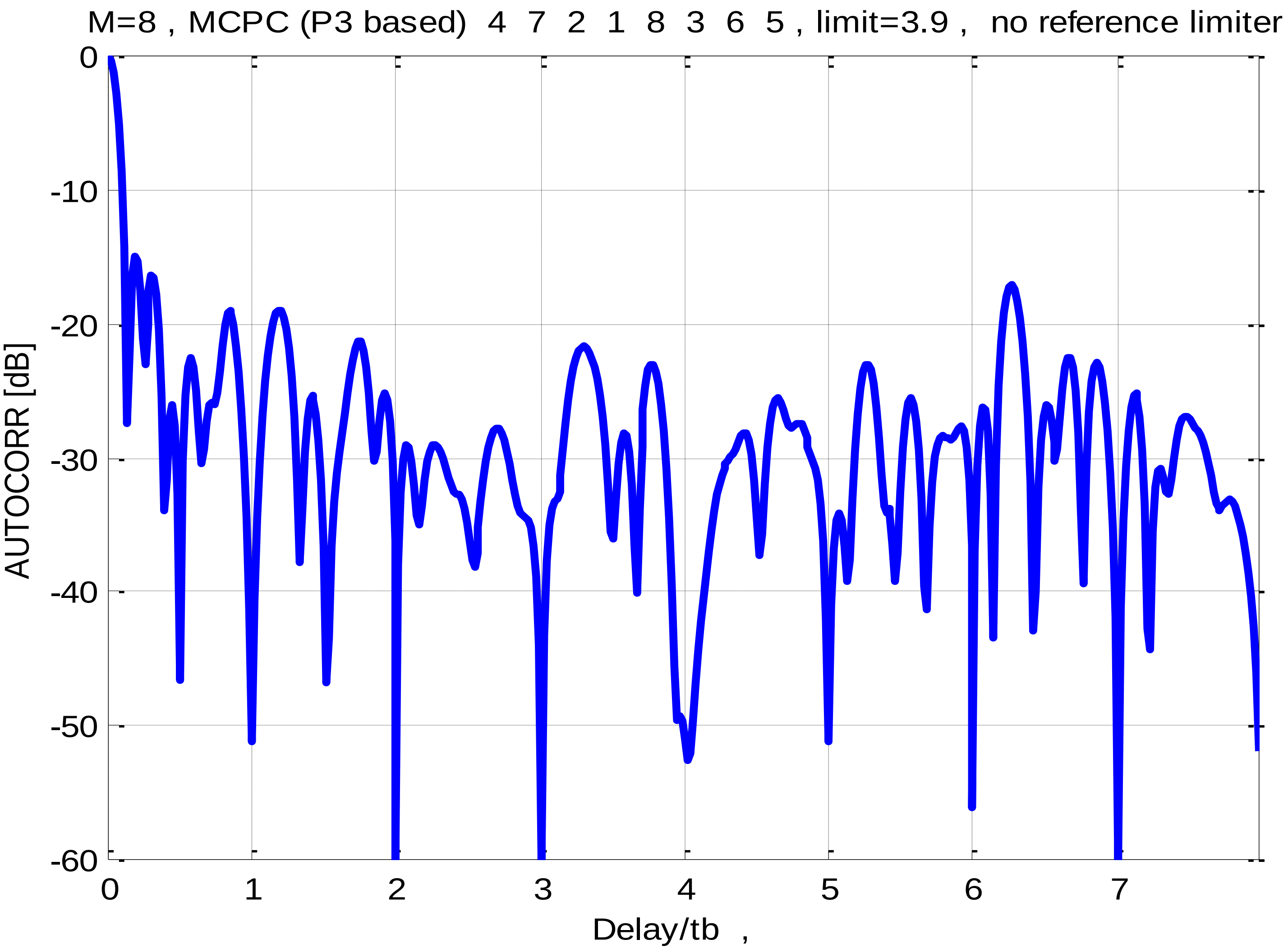
Power spectrum of the MCPC signal



Real envelope of the signal



Real envelope of the signal after limiting

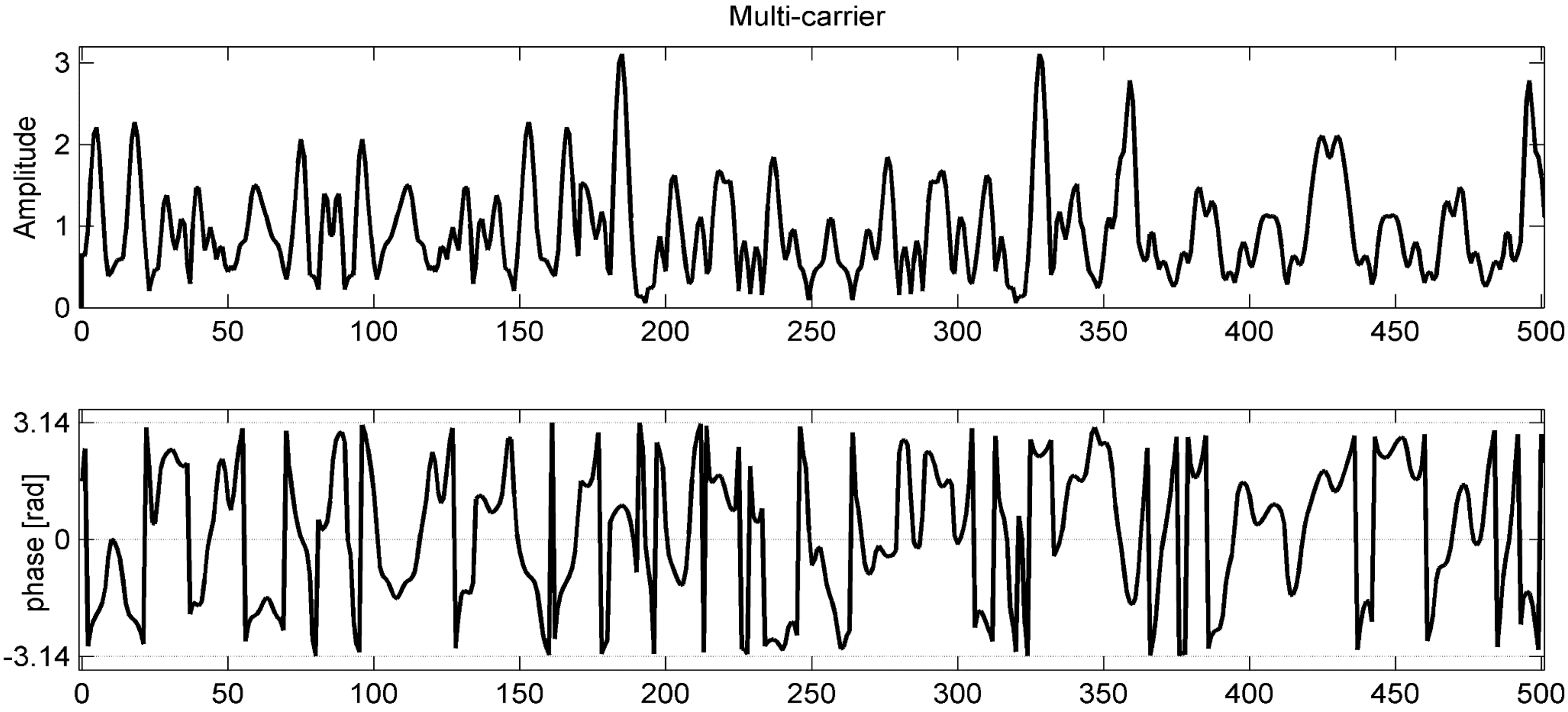


Crosscorrelation of the 8x8 MCPC pulse with hard limiting on transmit.

Long multi-carrier signal (57 carriers)

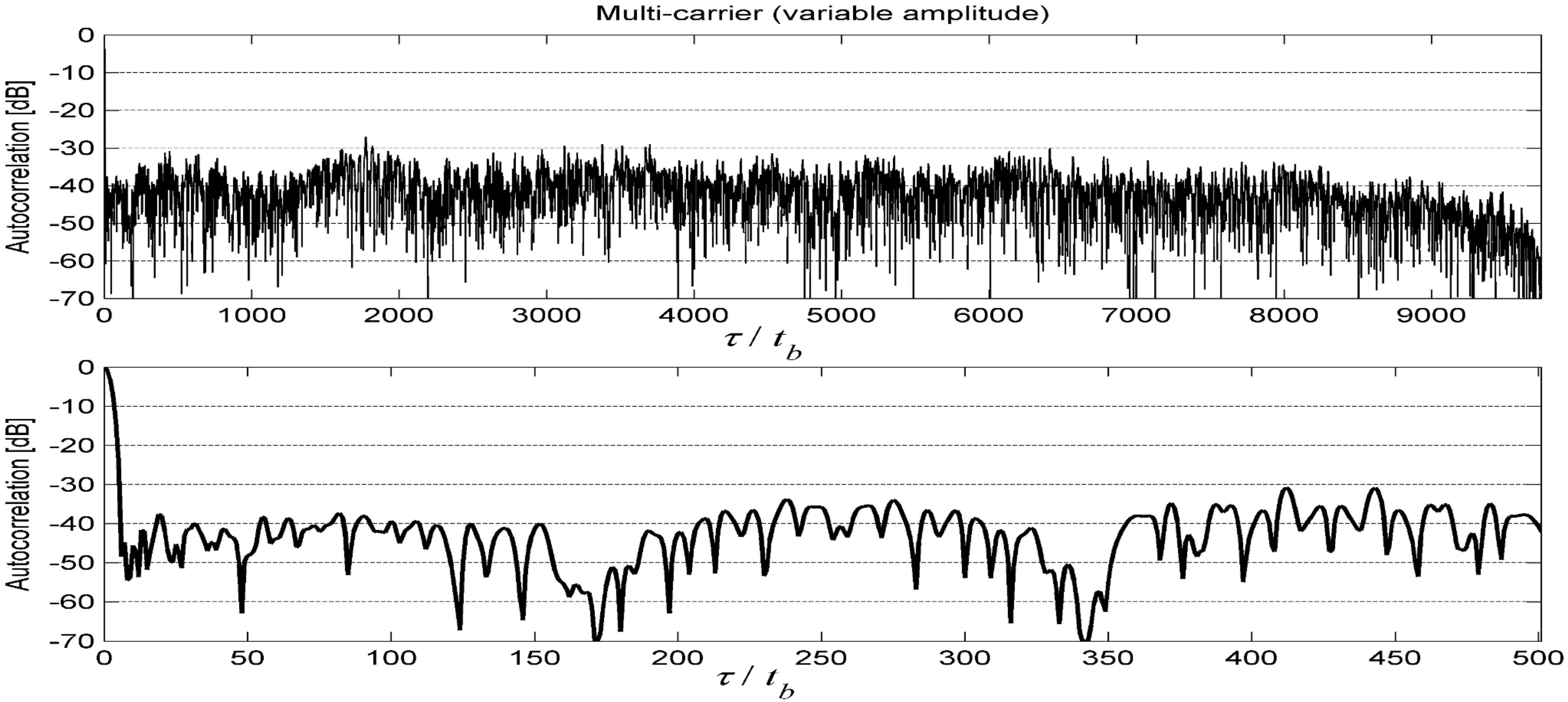
Modulated by all cyclic shifts of Ipatov 57 binary sequence
Carrier amplitudes are Hamming weighted

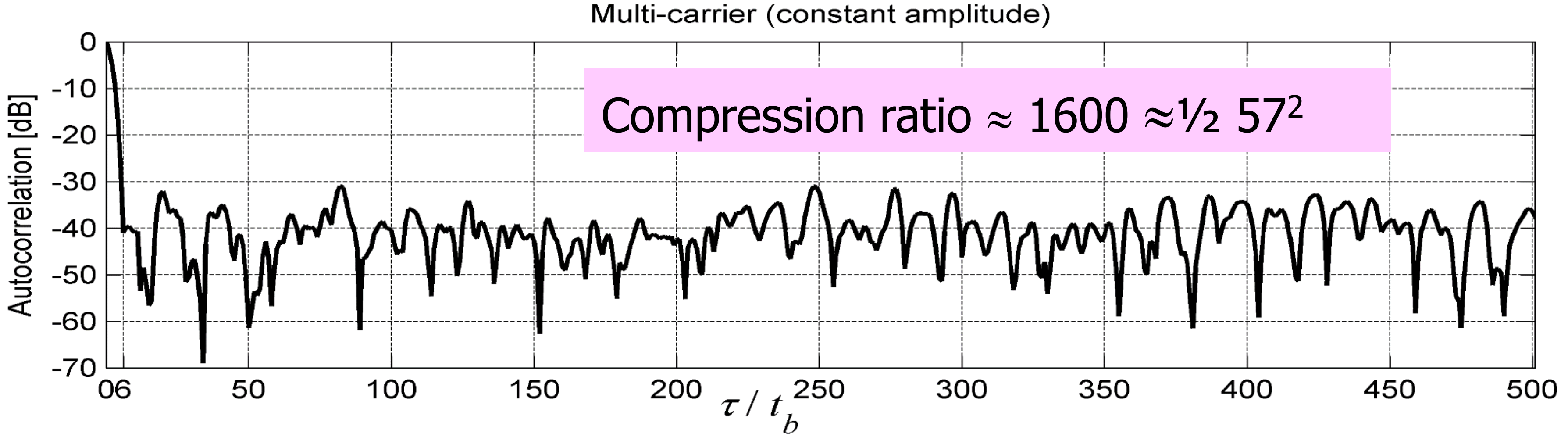
Plotted are 500 signal samples out of 9747 (=57·57·3)



Assigning the 57 cyclic shifts to the 57 carriers is random

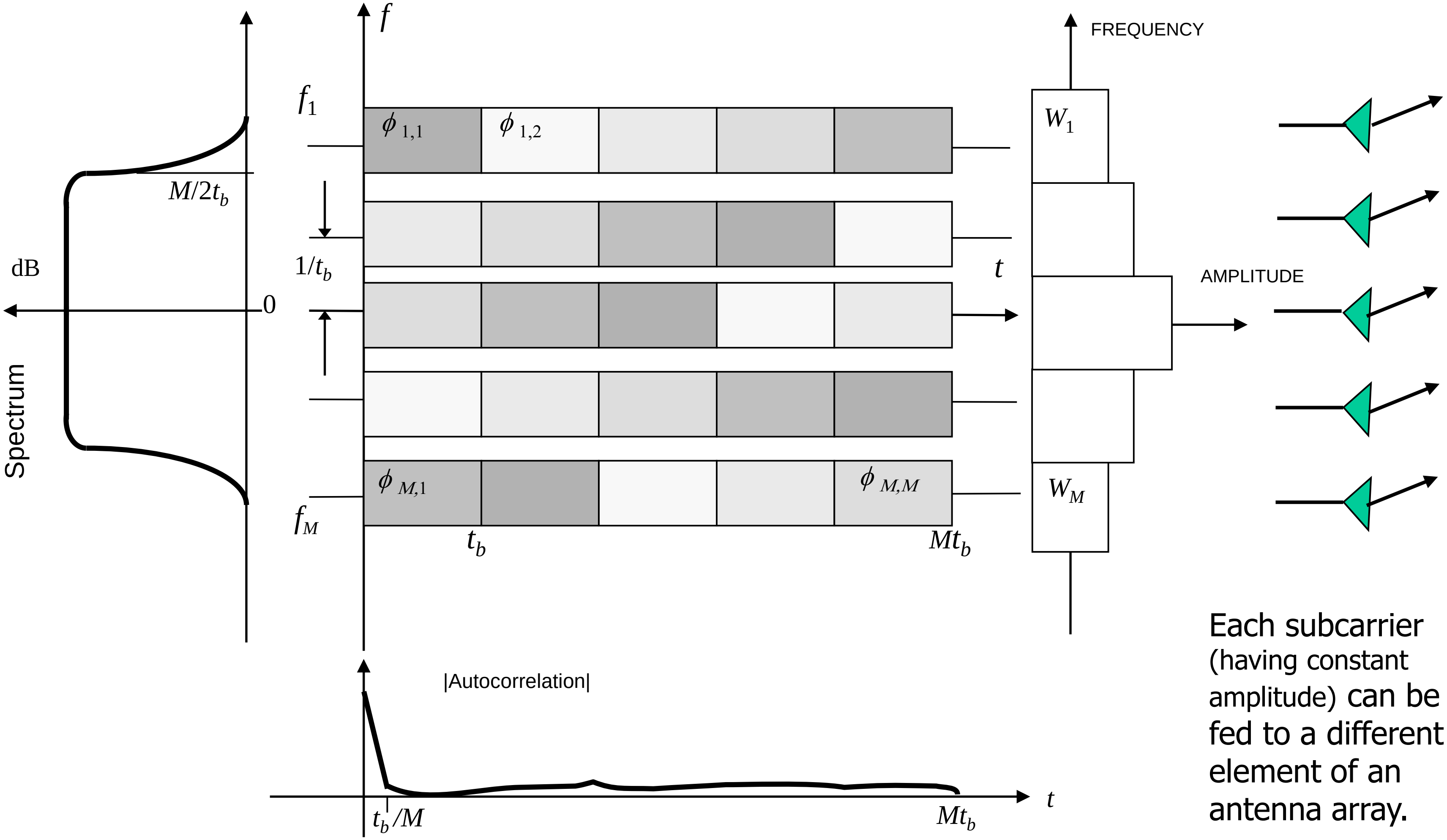
(57!=4·10^{76} different alternatives)





Compression ratio ≈ 1600 ≈½ 57^2

Multi-carrier waveforms



Each subcarrier (having constant amplitude) can be fed to a different element of an antenna array.

Structure of an MxM MCPC pulse

Chaotic signal

Sequence generator ➔

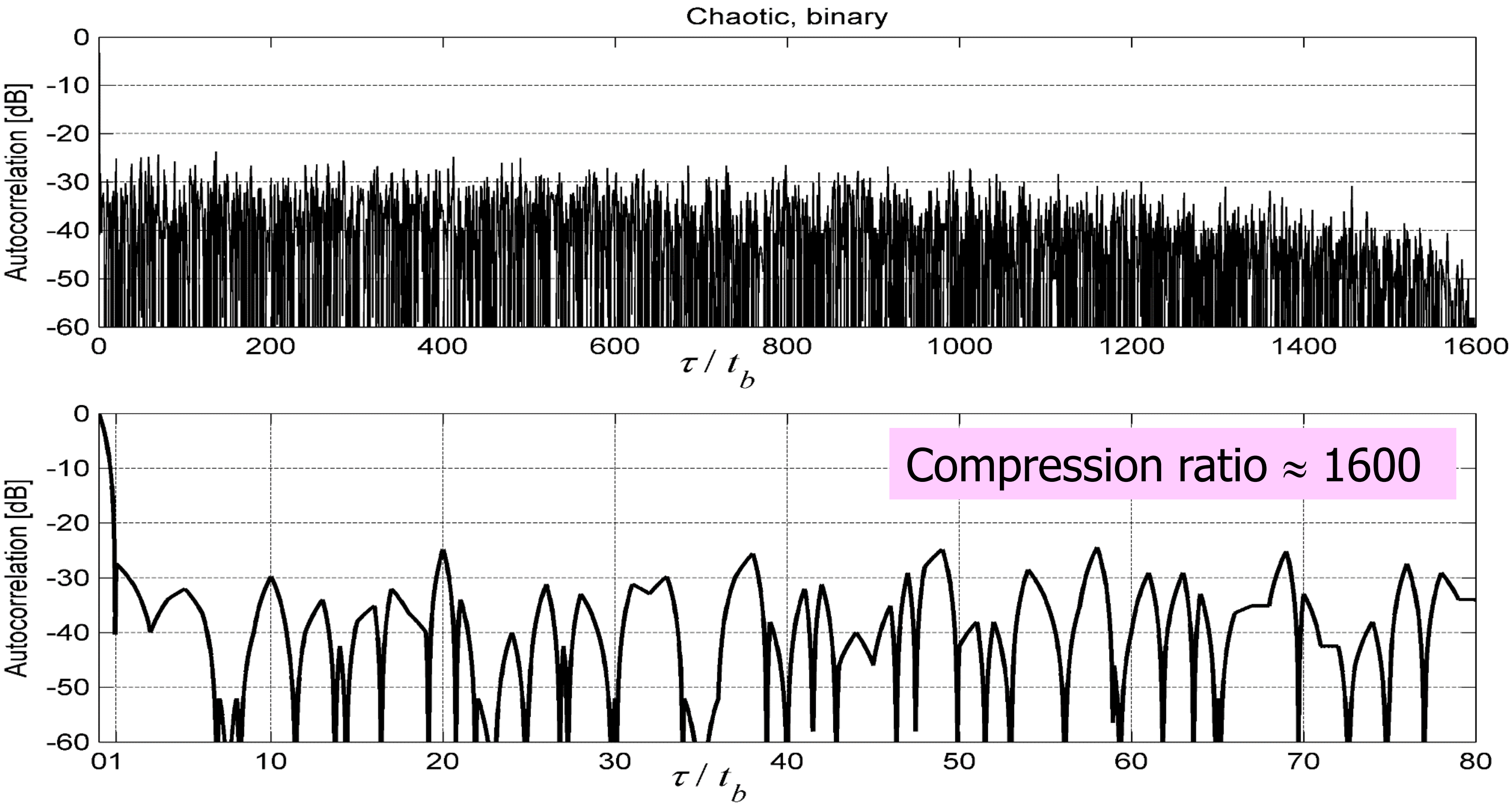
$$x_{n+1} = 4x_n\left(1 - x_n\right), \quad x_n \in [0,1]$$

- The sequence is strongly dependent on its initial value x_1 .
- The analog value of $x_n \in [0,1]$ can be easily converted to binary or m-valued phase coded sequence by setting boundaries. (Choosing 3 boundaries can create a 4-phase code, etc.)

Initial value

Analog to binary conversion

```
x=zeros(1,1600);
x(1)=0.1;
for q=2:1600
    x(q)=4*x(q-1)*(1-x(q-1));
end
u_amp=ones(1,1600);
u_phase=x>0.5;
```



Compression ratio ≈ 1600

The ACF of chaotic binary signals (of which there are many) is usually not as good as that of binary signals, exhaustively searched for low ISL (of which there are few).