Performance Difference Between Differently-Coded, Same-Length, Binary Complementary Pairs

Nadav Levanon, and Itzik Cohen

Tel Aviv University, Tel Aviv, 6997801, Israel

***Abstract* — Radar pulse waveforms, binary-coded according to Golay's complementary pairs, exhibit two valuable properties: (a) Pulse-compression to the duration of one code element, (b) Zero autocorrelation (ACF) sidelobes for the entire pulse duration. Number theorists are interested in enumerating as many different code sequences as possible. However, radar engineers may ponder the value of a large selection of different sequences (at a given length) beyond the benefit of enhanced waveform diversity. The paper considers two sequence-dependent properties: (a) The height of the recurrent ACF peaks around delay equal to odd number of PRIs, (b) The delay-Doppler response, namely, the ambiguity function.**

***Index Terms* — Radar waveforms, pulse compression, delay sidelobes, complementary pairs, ambiguity function.**

I. Introduction

A Golay [1] pair is a set of two binary sequences whose autocorrelation functions (ACF) sum to yield zero sidelobes. When used as radar waveforms, they are referred to as complementary pairs. Complex elements can also be used [2]. Devising construction algorithms of binary complementary pairs, and performing exhaustive searches, are research topics in number theory [3-5].

As a radar pulse waveform, the main advantage of complementary pairs is the zero level of the ACF near-sidelobes, which is a rare property of pulse compression waveforms. Unfortunately, it comes with an inherent poor recurrent sidelobes, around delays equal to the pulse repetition interval (PRI). At that delay the received first-pulse coincides with the second-pulse of the stored reference pair. These properties are demonstrated using a complementary pair of length 26. The phases of the pair are:

$$\phi_1 = \pi[00011000101101010110010000]$$
$$\phi_2 = \pi[00001001101000001011100111]$$

The a-periodic autocorrelation functions (ACF) of each coded pulse are shown in subplots (a) and (b) of Fig. 1. Note the equal magnitudes but opposite polarities at each delay, except at the origin. That fact is responsible for the sidelobes cancellation when the sidelobes of the two correlations are added. Such addition happens when a train of repeated complementary pulse pairs $\{s_1\ s_2\ s_1\ s_2\ s_1\ s_2\ s_1\ s_2\ .....\}$ is cross-correlated with at least one reference pair. The resulted periodic correlation, with a reference containing one complementary pulse pair $\{s_1\ s_2\}$, is shown in subplot (c) of Fig. 1. Selected duty cycle of d = 0.2 resulted in a PRI five times longer than the pulse duration, namely

$$T_r = t_p / d = L t_b / d = 26 t_b / 0.2 = 130 t_b \quad (1)$$

where t_b is the duration of a code element (bit), L is the code length, t_p is the pulse duration and T_r is the PRI.

The main property of a complementary pair is demonstrated in Fig. 1(c) by the zero near-sidelobes at $1 \le |\tau/t_b| \le L = 26$. When the delay equals the PRI, signal and reference pulses overlap again but now the overlapping pulses are not matched. Signal pulse #1 aligns with reference pulse #2 and signal pulse #2 overlaps reference pulse #1. This results in the recurrent delay lobes at the delay spans

$$\left(T_r - t_p\right)/t_b = (130-26) < |\tau/t_b| < (130+26) = \left(T_r + t_p\right)/t_b \quad (2)$$

Note from Fig. 1(c) that for this complementary pair the peak sidelobe ratio of the recurrent delay lobes is $20\log_{10}(8/52) = -16.25\,\text{dB}$. When Doppler, ν, resolution is desired, the pulse-pair is repeated to create a coherent pulse train. Coherently processing a train of P pulses (namely P/2 pairs) will result in Doppler resolution of $\Delta\nu = \left(P T_r\right)^{-1}$.

II. Performance differences

The contribution of our paper relates to the previous two paragraphs. We show that different complementary pairs produce different correlation recurrent delay sidelobes, with peak values that can be considerably different. Using the periodic ambiguity function [6], we will also show that complementary pairs, exhibiting low recurrent correlation sidelobes, produce more favorable delay-Doppler response.

"Best" and "Worst" complementary pairs were obtained from 109 pairs of length 40, listed in Table 3 of [4]. The best/worst pair yields the lowest/highest recurrent sidelobe peak. Their hexadecimal forms are

$$\texttt{best} = \begin{cases}\texttt{F11255BB8F}\\ \texttt{F1DDA5B770}\end{cases} \quad \texttt{worst} = \begin{cases}\texttt{C91F505239}\\ \texttt{C91F5FADC6}\end{cases}$$

The corresponding delay-Doppler responses appear in Figs. 2 and 3, respectively. Note that 16 complementary pairs (P=32 periods), with PRI $T_r = 90 t_b$, were processed coherently. Note also that the reference signal was inter-pulse Hamming weighted (Fig. 4) in order to lower the Doppler sidelobes of the delay-Doppler response.

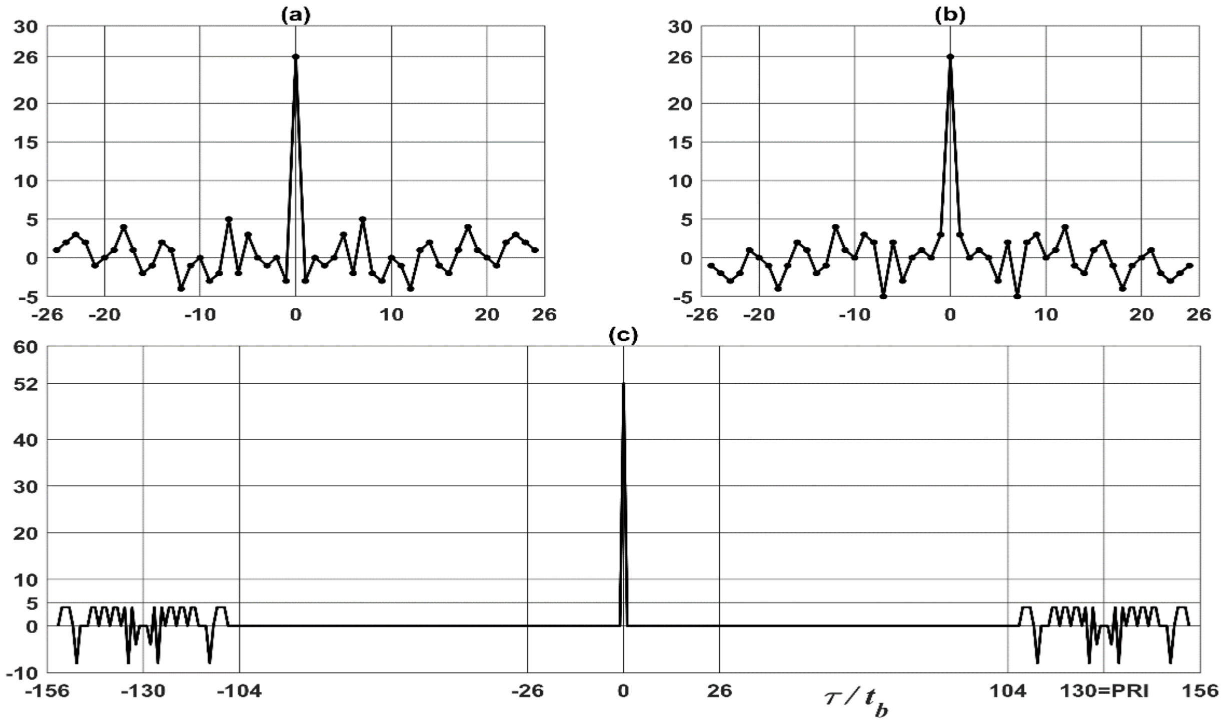

Fig. 1. (a) ACF of 1st coded pulse; (b) ACF of 2nd coded pulse; (c) Periodic cross-correlation between a train of repeated pulse pairs and a reference containing one pair. The separation between pulses is 78 bits.

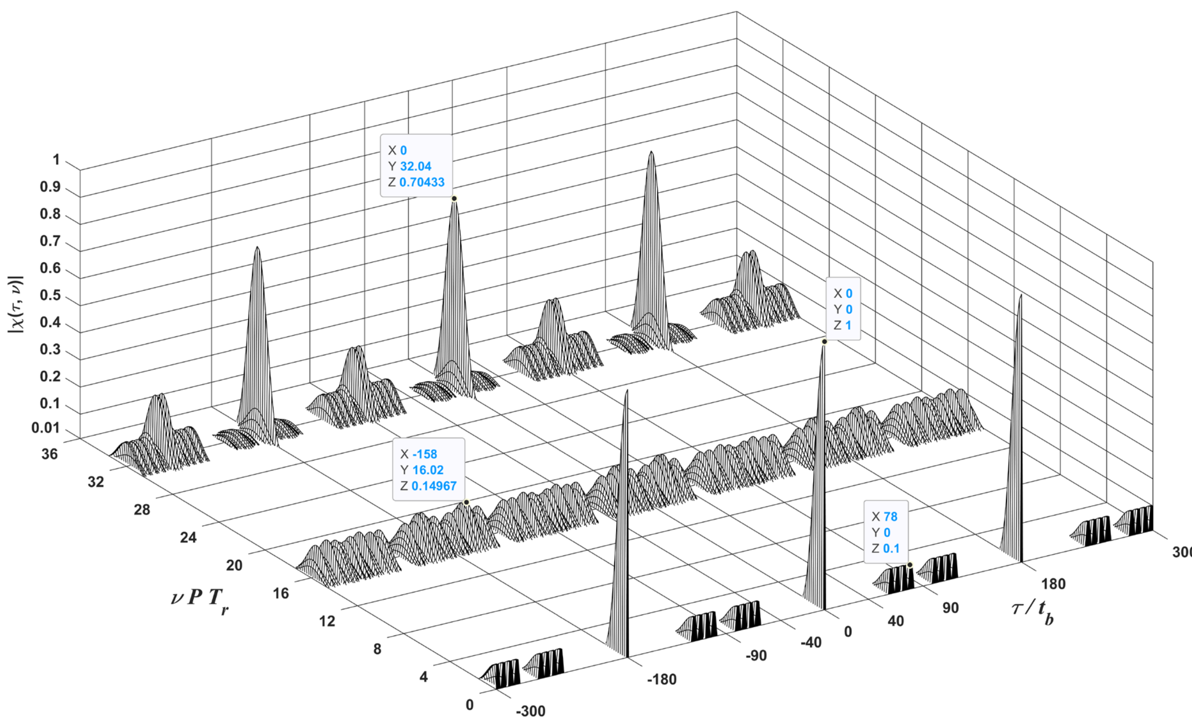

Fig. 2. Periodic delay-Doppler response of the "best" complementary pair of length 40

(The source of "ridge" at $\nu PT_r = 16$ is attributed to the fact that the true period of the signal is $2T_r$.)

A simple MATLAB script to obtain the normalized periodic recurrent sidelobes at zero Doppler is:

```
abs(xcorr(s1,s2)+xcorr(s2,s1))/(max(abs(xcorr(s1)+xcorr(s2))))
```

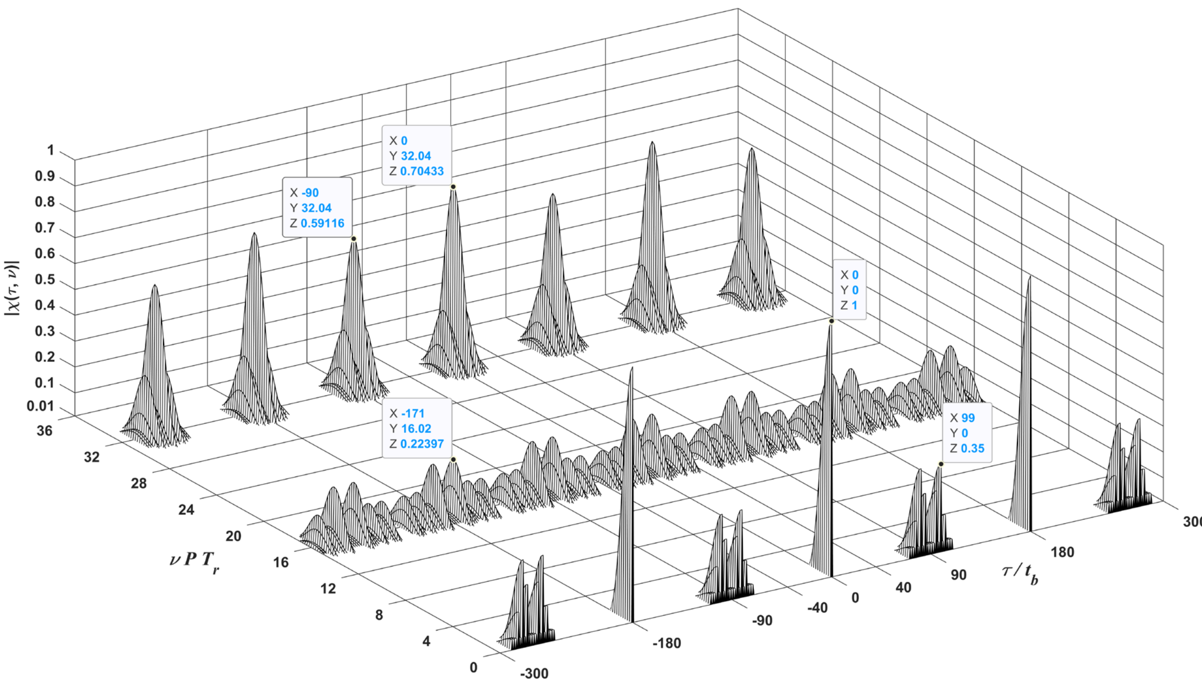

Fig. 3. Periodic delay-Doppler response of the “worst” complementary pair of length 40

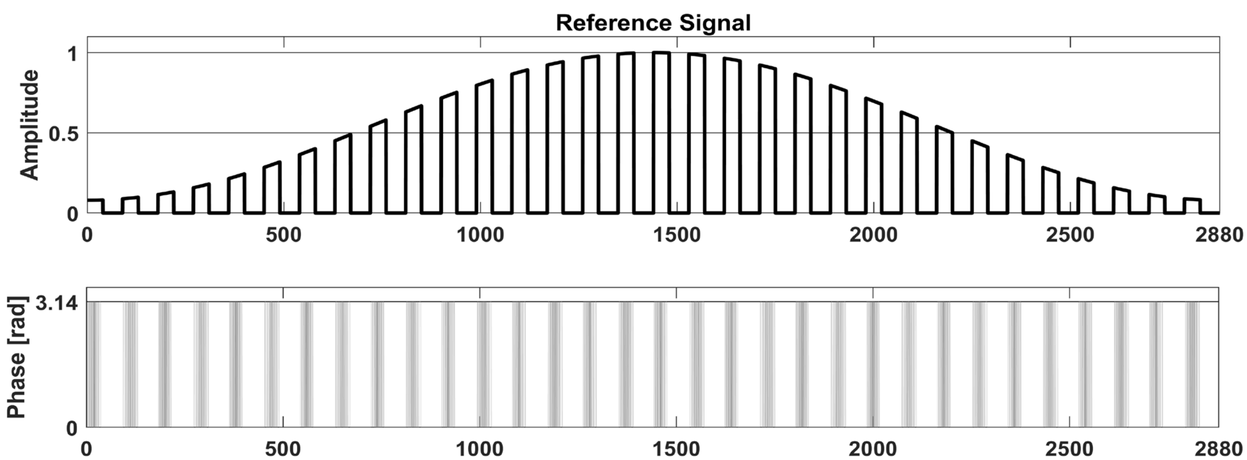

Fig. 4. Amplitude and phase evolution of the reference to the “best” 16 complementary pairs train

The zero-Doppler cut of Fig. 2 reveals much lower recurrent sidelobes peaks (= 0.1 = −20dB) than those of Fig. 3 (= 0.35 ≈ −9dB). Also higher in Fig. 3, are the corresponding peaks at $\nu PT_r = 32, \Rightarrow \nu = 1/T_r$.

III. Doppler tolerance

A well-known drawback of complementary pairs is their relatively poor Doppler tolerance, e.g., compared to a train of LFM pulses. Poor Doppler tolerance effects the delay-Doppler response when a receiver processor, matched to a given Doppler shift, compensates the inter-pulse Doppler shift, but not the intra-pulse shift (Fig. 5).

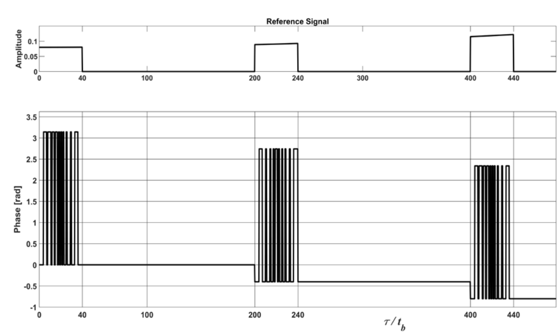

Fig. 5 Lack of intra-pulse Doppler compensation

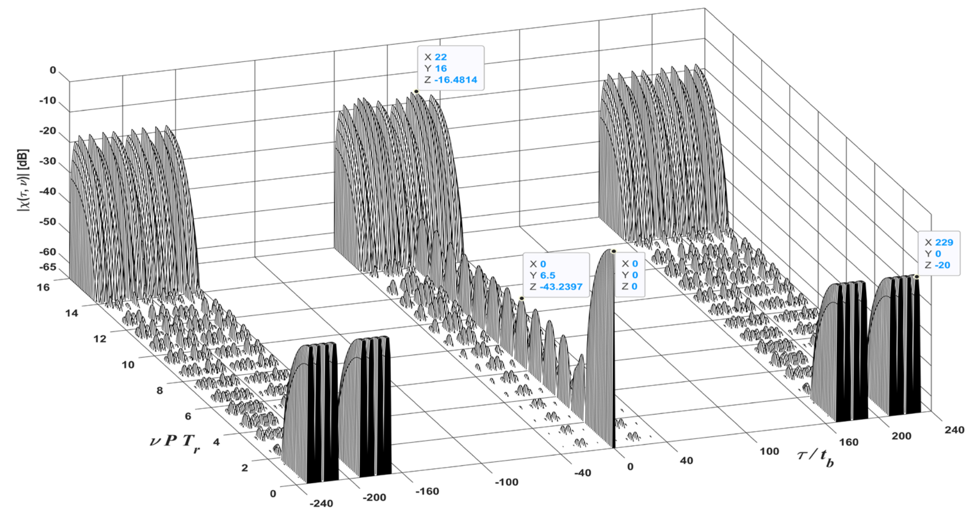

Fig. 6 Delay-Doppler response of a processor matched to zero Doppler (no intra-pulse compensation)

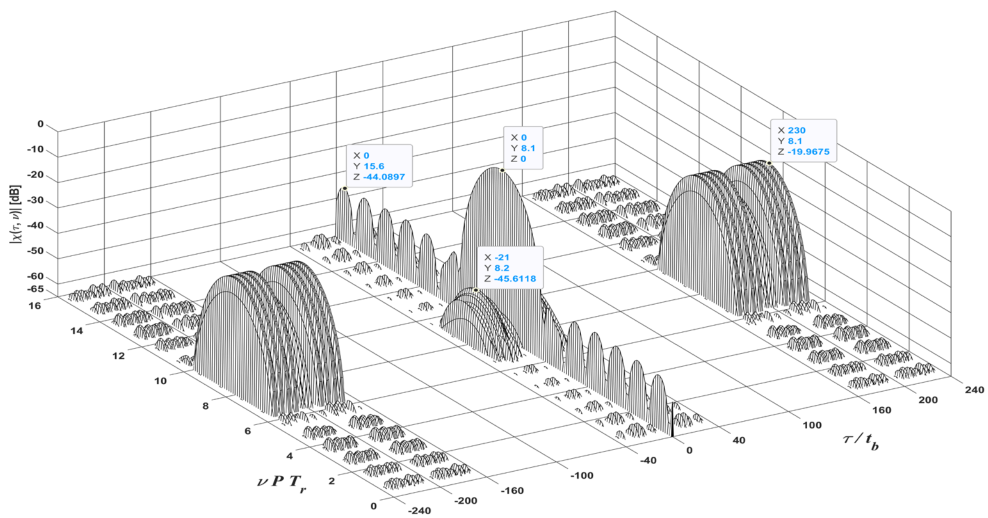

Fig. 7 Delay-Doppler response of a processor matched to $\nu = 8/(PT_r)$ (no intra-pulse compensation)

Figs. 6 and 7 display the corresponding delay-Doppler responses for the "best" complementary pair. A log scale, down to -65 dB, is used in order to see very low sidelobes. In Fig. 6, which displays the response of a processor matched to $\nu = 0$, lacking intra-pulse Doppler compensation has no effect. In Fig. 7, where the processor is matched to $\nu = 8/(PT_r)$, the missing intra-pulse Doppler compensation, causes a buildup of the near sidelobe to a level of -45dB. Fig. 8 repeats the response in Fig. 7, for the "worst" complementary pair. Note that the rise of the near sidelobes, because of missing intra-pulse Doppler compensation, grew from -45dB (Fig. 7) to -30dB (Fig. 8). Thus "worse", may apply to more than one aspect of the response, possibly thanks to different reasons. In our example, the larger sensitivity of the "worst" signal to lacking Doppler compensation may be due to its relatively high individual ACF sidelobes (Fig. 9).

IV. Conclusions

Selecting a complementary pair sequence, of a given length, out of the many available ones (at most lengths), should not be random, just for the sake of diversity. Considerable delay-Doppler response differences exist between different sequences, and they should be considered when making the selection. At some lengths, the number of different pairs is very large [3], and selection among *all* available pairs becomes impractical. There, the selection will be between "good" and "poor" pairs, among a relatively small group.

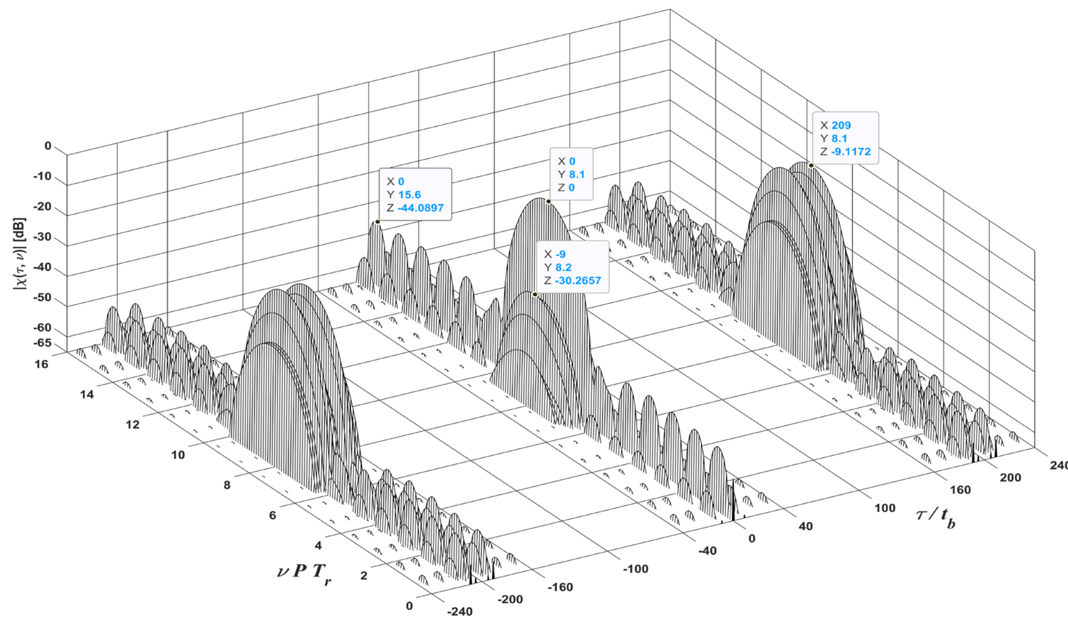

Fig. 8 Delay-Doppler response of a processor matched to $\nu = 8/(PT_r)$ (worst pair)

REFERENCES

[1] M. J. E. Golay, "Complementary Series," *IRE Trans. Inf. Theory*, vol. 7, no. 2, pp. 82-87, 1961.

[2] N. Levanon and E. Mozeson, *Radar Signals*, New York: J. Wiley & Sons, 2004, sec. 9.3.

[3] P. Borwein and R. Ferguson, "A complete description of Golay pairs for length up to 100", *Math. Comput.* vol. 73, pp. 967-985, 2004.

[4] D. Djokovic, "Equivalence classes and representatives of Golay sequences', *Discrete Math.*, vol. 189, pp. 79-93, 1998.

[5] G. E. Coxson, "Golay pairs having larger swap sets," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 57, no. 1, pp. 738-743, 2021.

[6] N. Levanon, "The periodic ambiguity function – its validity and value," *2010 IEEE Int'l Radar Conf.*, Washington, DC, USA, 10-14 May 2010, pp. 204-208.

[7] N. Levanon, I. Cohen, and P. Itkin, "Complementary pair radar waveforms – evaluating and mitigating some drawbacks," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 32, no. 3, March 2017, pp. 40-50.

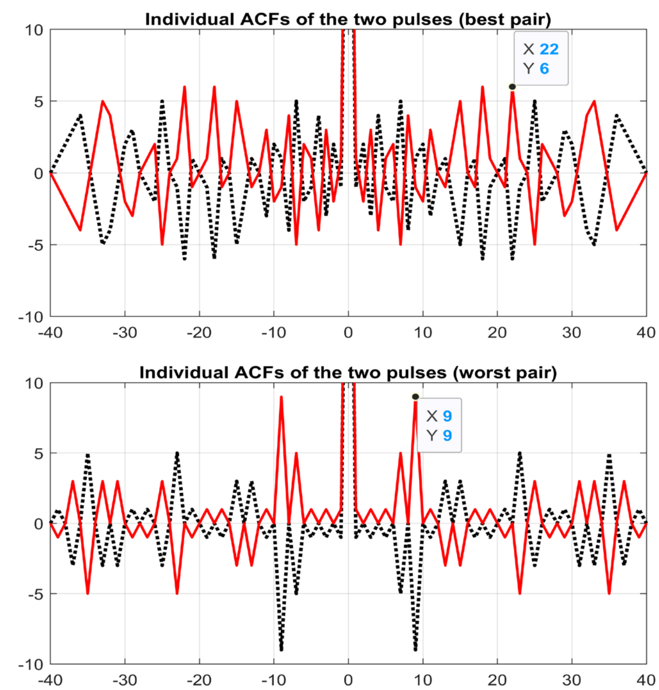

Fig. 9 Individual ACFs sidelobes of the "best" and "worst" pairs