

# Creating Sidelobe-Free Range Zone Around Detected Radar Target

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**Abstract**—To achieve good range response pulse radars transmit modulated or coded waveforms with good aperiodic autocorrelation function (ACF) and use a matched filter (MF) receiver. Range sidelobe (SL) can be considerably lowered by using a mismatched-filter (MMF), designed to minimize the integrated or peak SL. Low SL prevent masking weak neighboring targets and reduce the contribution from surrounding clutter. This paper suggests a different approach to SL reduction. It is based on designing a Modified MMF (MMMMF) in which the near SL of the cross-correlation function (CCF) are given higher weight in the minimization process and are therefore more drastically reduced. A combination of the MMMMF and a standard MF can create a SL-free response.

**Index Terms**—Radar, pulse compression, matched filter, mismatched filter, sidelobe reduction.

## I. INTRODUCTION

For good delay response, pulse radars utilize modulated or coded waveforms with good aperiodic autocorrelation function (ACF). Basic waveform examples [1,2] are Linear-FM, phase-coded (binary or polyphase), multicarrier, and more. Our paper deals with phase-coded waveforms. The interest in the (ACF) stems from the fact that a matched filter (MF) in the radar receiver produces a delay response identical to the ACF. For many years the search for binary waveforms with good delay response involved searching ACF with low peak sidelobe (SL) ratio (PSLR) [3] or low integrated SL ratio (ISLR), also known as the merit factor [4,5], and similarly for polyphase waveforms [6-8]. As the length  $N$  of the code sequence increases, such a search becomes more difficult [9].

With a penalty of small signal-to-noise ratio (SNR) loss, the ISLR (or PSLR) can be considerably improved by using a mismatched-filter (MMF) [10].

With MMF the delay response is determined by the cross-correlation function (CCF) between the signal and the MMF. Designing a MMF optimized with respect to ISLR, for a given sequence and filters length, is a straight forward calculation [11]. The ISLR improvement increases with the length  $P$  of the MMF sequence, but so do the SNR loss and the processor's complexity [12]. Typical MMF length is  $P=3N$ . The search complexity increases considerably [13-15] because in addition to looking for waveform/MMF combination with good ISLR, it is necessary to add a constraint on the acceptable SNR loss. Tolerated SNR loss is usually  $< 2$  dB.

This paper suggests a different approach to SL reduction. It is based on designing a MMF of length  $P$ , in which the near CCF SL are given higher weight and thus are being more drastically reduced. When  $P = 3N$ , a typical near-SL span extends as far as  $K \geq N+1$ . With these typical parameters the reduction of the near-SL can be very drastic, with only minor increase in the SNR loss.

In a parallel track the same received radar signal from a point target is processed by a matched filter. The resulted signal-induced output is the ACF, which extends as far as  $N$ . Beyond  $N$  the output of this track is caused by noise, with no contribution from the signal. The outputs from the two tracks are continuously compared and the smaller of the two is selected.

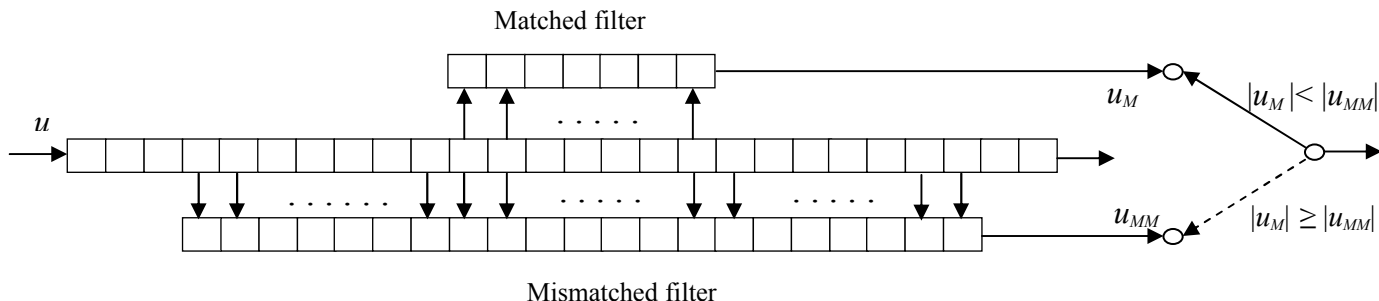


Fig. 1. Combining matched and mismatched filters.

This two-track processing approach depends heavily on the ability to design a good MMF. In phase-coded waveforms it is easy to design a MMF as long as the complex envelope of a code element (bit) is represented by a rectangle with fix amplitude and phase. Such a waveform has the advantage of constant amplitude but occupies wide bandwidth with slow decaying spectral SL. There are several approaches for band-limiting phase-coded signals [16-18], but designing a good MMF for the modified signal may become difficult or impossible. To cover that issue the new processing concept is demonstrated also with a very efficient band-limited signal, for which a good MMF can be designed.

## II. MISMATCHED FILTER DESIGN FOR LOW NEAR-SL

The suggested new processor (Fig. 1) performs parallel processing of the streaming received signal samples  $u$  by both MF and MMF. With each new complex signal sample that enters the processor, the MF and MMF produce their new two respective output samples  $u_M$  and  $u_{MM}$ . The complex value, whose absolute value is the smaller, is chosen as the processor's complex output sample. There may be more than one sample per code sequence element.

Without noise the one-sided MF response SL extend as far as the signal sequence length  $N$ . We would therefore like to design the MMF with very low CCF SL up to the  $N$ 'th delay element, or slightly beyond, so that those reduced SL will be chosen instead of the higher MF SL.

Our MMF length is  $P$  ( $\approx 3N$ ). Its design (see Appendix) follows the approach in [11, 2 (sec. 6.6)], in which the ISLR is minimized. The MMF design assumes one sample per sequence element. The cross-correlation sequence extends over

$$n = -(P-1), \dots, -2, -1, 0, 1, 2, \dots, (P-1) \quad (1)$$

Normally, the minimization operation gives a uniform weight (=1) to all the cross-correlation elements except the center element (the mainlobe), which gets the weight value 0 because it is not supposed to be minimized

$$w(n) = \begin{cases} 0, & n = 0 \\ 1, & \text{elsewhere} \end{cases} \quad (2)$$

In order to further reduce the near SL, the new weighting is not uniform anymore:

$$w(n) = \begin{cases} 0, & n = 0 \\ M, & n = \pm 1, \pm 2, \dots, \pm K \\ 1, & \text{elsewhere} \end{cases} \quad (3)$$

where  $K \geq N+1$  and  $M \gg 1$  (e.g.,  $M = 200$ ). Increasing  $K$  much beyond  $N$  would not allow meaningful SL reduction. An example of conventional MMF response (uniform weighting) and a low near-SL MMF response is presented in Fig. 2. The signal is a 48 element MPSL binary  $\{\pm 1\}$  signal, whose  $\{0,1\}$  representation is:  $\{1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\}$ .

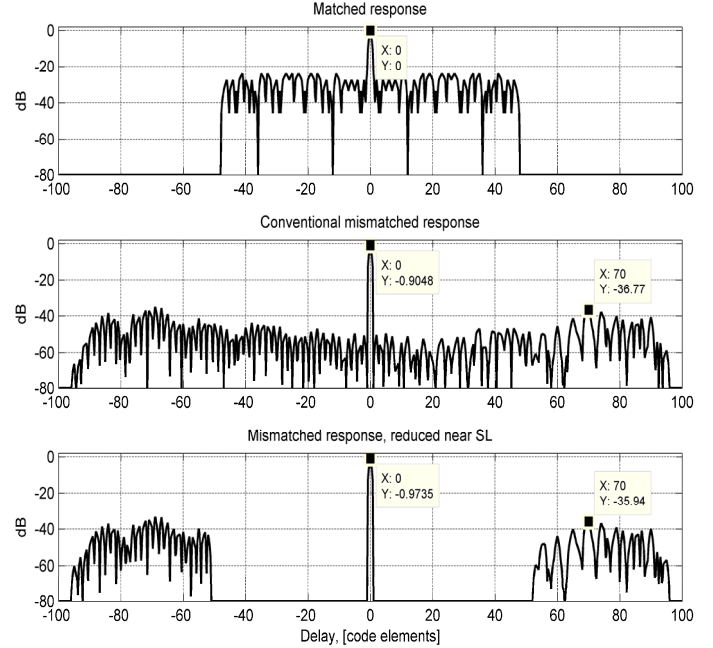


Fig. 2. MPSL 48 responses: MF (top), min-ISLR MMF (middle), Low near-SL MMF (bottom)

The top sub-plot of Fig. 2 shows the MF response. There is no SNR loss and the peak SL is  $-24.08\text{dB}$  ( $=20\log_{10}(3/48)$ ). The middle sub-plot shows that conventional min-ISLR MMF exhibits SNR loss of  $0.905\text{dB}$ , and an improved peak SL of  $-36.8\text{dB}$ . The bottom sub-plot shows that the Modified MMF (MMMMF) version achieved near-SL level below  $-80\text{dB}$ , with negligible increases in SNR loss and in peak SL, compared to the conventional MMF.

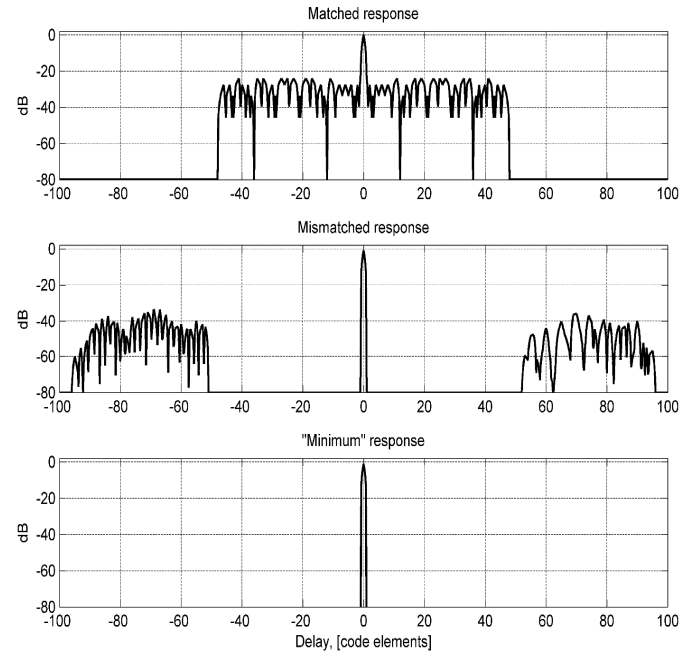


Fig. 3. MPSL-48 responses: MF (top), Low near-SL MMMF (middle), Minimum (bottom).

### III. OUTPUTTING THE MINIMUM OF THE MF AND MMMF

The last step of the pulse compression processor (Fig. 1) is to choose one of the two complex output samples (MF and MMMF). The chosen sample is the one with the smaller absolute value. The two outputs and the selected minimum, in a noise-free case, are plotted in Fig. 3. For the noise-free case, the bottom subplot of Fig. 3 demonstrates SL-free output. The non-linear “minimum” operation is not new to radar, it can be found reducing SL in SAR images [19].

### IV. SPECTRALLY EFFICIENT SIGNAL AND MMF

When the code element (bit) is a rectangle, it is represented by a single complex number and designing a MMF for it is trivial, but its spectrum utilization is inefficient, with slow decaying spectral SL. This prompts using band limited modifications. Unfortunately some band limited modifications, e.g., Quadriphase coding [16] does not lend themselves to effective MMF design. One representation that is suitable for MMF design is Gaussian Weighted Sinc (GWS) [17, 18]. An example of a bit representation appears in Fig. 4. The single bit representation stretches over 4 bits, creating overlap, and amplitude variations. The phase of the 4 bit representation is that of the original bit. The amplitude is multiplied by the original amplitude (relevant to the MMF). The resulted variable amplitude of the transmitted signal (Fig. 5) requires a linear power amplifier. A noise-free response is shown in Fig. 6. Note that the mainlobe peak is  $-1.2\text{dB}$ , compared to  $-0.97\text{dB}$ , obtained with a rectangular bit representation (Fig. 3).

### V. DOPPLER TOLERANCE

Phase-coded pulses are not as Doppler tolerant as Linear Frequency Modulated (LFM) pulses. The least Doppler tolerant are the SL, whose reduction is the main subject of this work. Hence, the SL tolerance with Doppler needs to be studied.

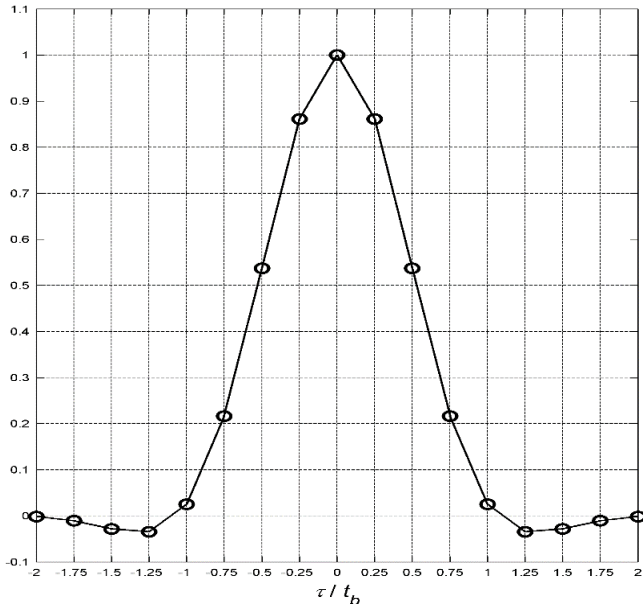


Fig. 4. Gaussian-weighted-sinc with 4 samples-per-bit.

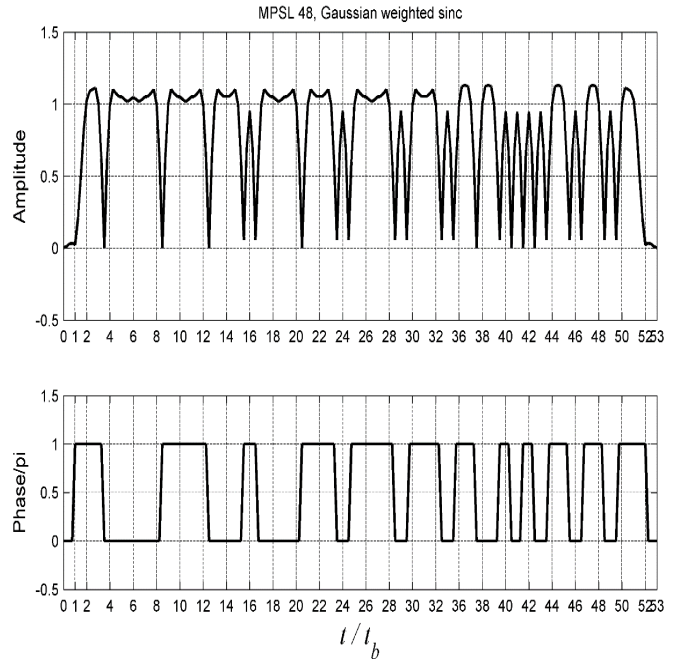


Fig. 5. MPSL-48 waveform, Gaussian-weighted-sinc bit.

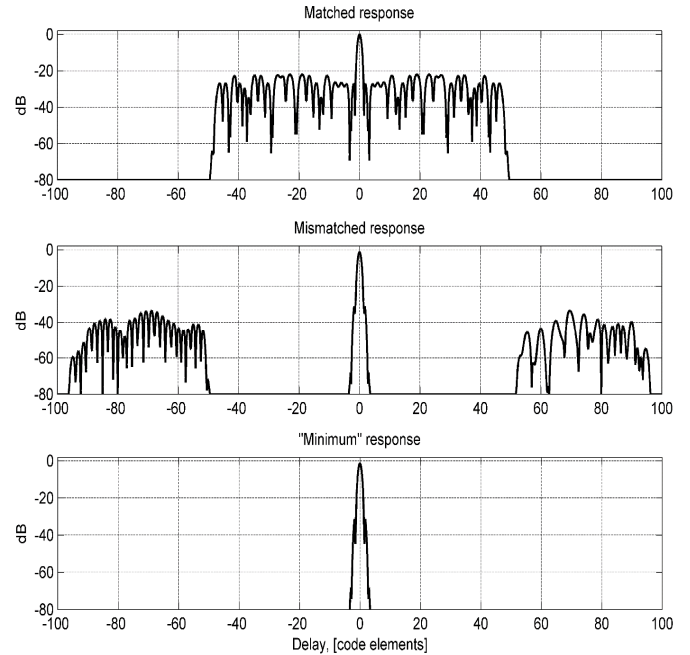


Fig. 6. MPSL-48, Gaussian-weighted-sinc: MF response (top), “Low near-SL” MMF response (middle), “Minimum” output response (bottom).

The effect of Doppler shift is shown by a plot the periodic cross-ambiguity function between the transmitted signal and the MMMF. Doppler tolerance of a pulse signal needs to be examined in relation to the pulse train parameters. The most relevant parameter is the pulse repetition frequency (PRF). The unambiguous Doppler span is  $-PRF/2 \leq f_D \leq PRF/2$ . For radar that does not operate with Doppler ambiguity we need to examine the near-SL

behavior up to  $f_D = PRF/2$ . We will examine a practical pulse-train signal in which the pulse is a GWS MPSL 48, the duty-cycle is 0.1. The reference is a train of 16 MMMFs. To lower the Doppler SL the reference train will include inter-pulse Hamming weighting.

Fig. 7 displays the central part (delay wise) of the resulted periodic cross-ambiguity function. The vertical scale is in dB, starting at a floor level of  $-50\text{dB}$ . On the zero-Doppler axis we see the expected cross-correlation response of the MMF. The near SL (at  $-82\text{dB}$ ) are below the  $-50\text{dB}$  floor. The recurrent normalized-Doppler peak at  $\nu T_r = 1$ , or  $\nu 16 T_r = 16$  reaches

almost the same height as the peak at the origin. ( $\nu$  is the Doppler shift and  $T_r$  is the pulse repetition interval.) However, the near-SL at that Doppler increased from below  $-80\text{dB}$  to about  $-37\text{dB}$ . On the zero-delay axis we see the  $-41\text{dB}$  SL expected with Hamming inter-pulse weighting. Recall, however, that the unambiguous positive Doppler span extends up to half that Doppler shift. The delay-Doppler response of a processor matched to half the PRF, namely to  $\nu 16 T_r = 8$ , is shown in Fig. 8.

In Fig. 8 the near-SL reach a level of  $-42\text{dB}$  (around sample  $-68$ ). Recall that outside samples  $\pm 200$  the SL will be removed thanks to the “minimum” operation with the response of the MF. We can conclude that for the specified pulse train the signal/MMF combination is sufficiently Doppler tolerant. The delay-Doppler response in Fig. 8 was obtained with the reference signal described in Fig. 9. In the top subplot note the Hamming weighted pulse amplitudes. In the bottom subplot note how the Doppler compensation is performed step-wise, between pulses, but not intra-pulse. This is why we are concerned with the Doppler tolerance of the pulse signal. Finally note that the delay scale is in samples, and there are 4 samples per bit.

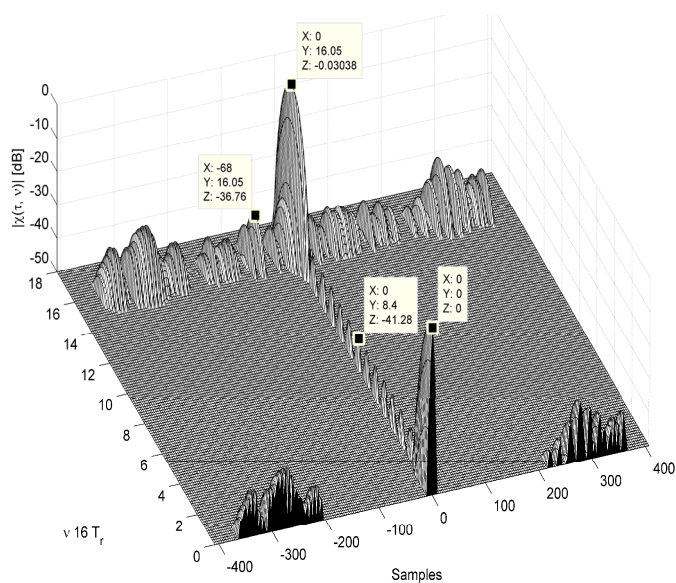


Fig. 7. Periodic cross-ambiguity function between a train of MPSL-48 GWS pulses and 16 “Low near-SL” MMMFs with Hamming inter-pulse weighting.

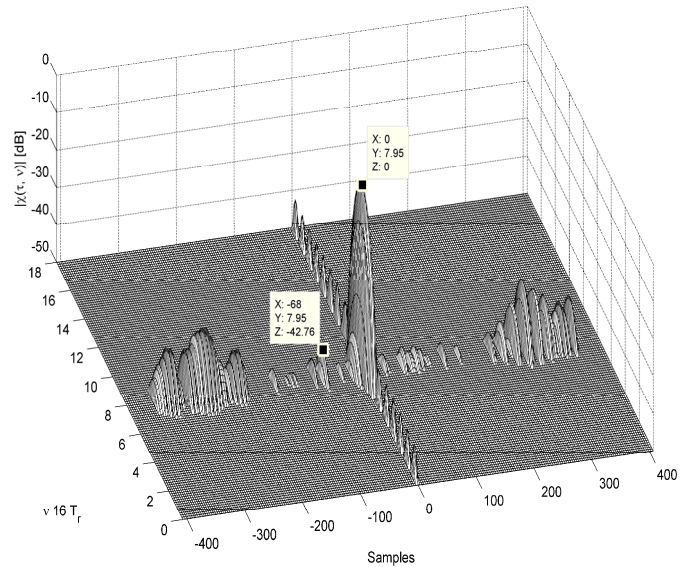


Fig. 8. Delay-Doppler response of 16 “Low near-SL” MMMFs when matched to Doppler =  $PRF/2$ .

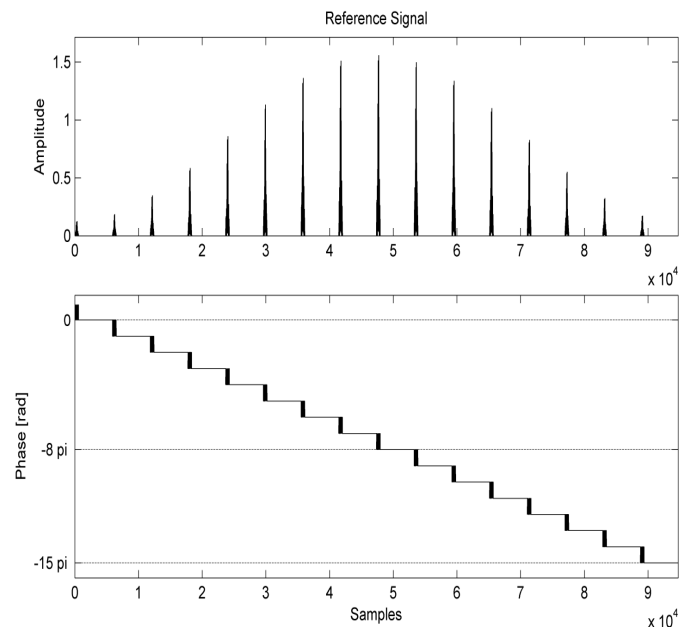


Fig. 9. The reference signal used to obtain the delay-Doppler response in Fig. 8.

## VI. MULTIPLE CLOSE TARGETS

A critical test of the new approach for SL reduction is the performances when there are neighboring close targets. Here the conclusions are mixed and scene dependent. A noise-free example, with 3 targets, is presented in Fig. 10. The middle target is  $40\text{dB}$  weaker than the strongest target. The spacing between the more distant targets is 80 bit. In that scene the “minimum” output reveals all 3 targets. However, in such a scene the “minimum” output contains sections of the remote SL of the MMMFs. Depending on the threshold setting, those SL may be mistaken as weak targets, although in a practical scene they are likely to be below the noise level.

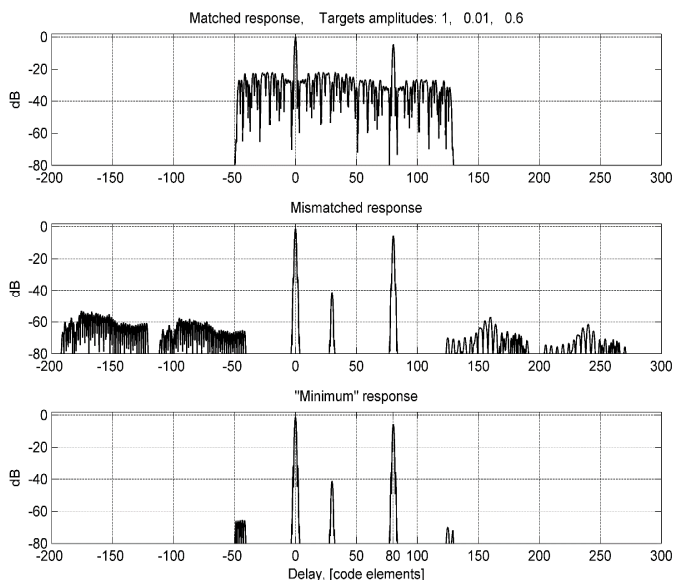


Fig. 10. Noise-free responses with 3 close targets.

## VII. CONCLUSIONS

A new approach for SL reduction was proposed and investigated. It is based on parallel pulse compression processing of a phase-coded waveform, using a MF in one track, and a specially designed MMMF in the second track. The minimum of the two tracks is selected for the output. The processor provides drastic reduction of delay SL. The reduction holds for practical Doppler shift, and its noise sensitivity is similar to that of a conventional MMF. In the presence of neighboring targets there may be cases of false weak targets. On the other hand miss-detections are less likely than when a conventional MMF is used.

## REFERENCES

- [1] C.E. Cook, and M. Bernfeld, Radar Signals: An Introduction to Theory and Application, New York, Academic Press, 1967.
- [2] N. Levanon, and E. Mozeson, Radar Signals, Hoboken, NJ, Wiley, 2004.
- [3] G. Coxson and J. Russo, "Efficient exhaustive search for optimal-peak-sidelobe binary codes," IEEE 2004 Radar Conf., Philadelphia, pp. 438-443. Also IEEE Trans. Aerospace and Electr. Sys., vol. 41, no. 1, pp. 302-308, January 2005.
- [4] P. Borwien, K.-K.S., Choi, and J. Jedweb, "Binary sequences with merit factor greater than 6.34," IEEE Trans. Inf. Theory, vol. 50, no. 12, pp. 3234-3249, 2004.
- [5] J.M. Baden, "Efficient optimization of the merit factor of long binary sequences," IEEE Trans. Inf. Theory, vol. 57, no. 12, pp. 8084-8094, 2011.
- [6] S. W. Golomb, and R. A. Scholtz, "Generalized Barker sequences," IEEE Trans. Inf. Theory, vol. IT-11, no. 4, pp. 533-537, April 1965.
- [7] T. Felhauer, "Design and analysis of new P(n,k) polyphase pulse compression codes," IEEE Trans. Aerospace and Elect. Sys., vol. 30, no. 3, pp. 865-874, July 1994.

- [8] C. J. Nunn, and G. E. Coxson, "Polyphase pulse compression codes with optimal peak and integrated sidelobes," IEEE Trans. Aerospace and Elect. Sys., vol. 45, no.2, pp. 775-781, 2009.
- [9] M. Ferrara, M. Kupferschmid, and G. Coxson, "The peak sidelobe distribution of binary codes," Int'l Waveform Diversity and Design Conf., pp. 141-144, 2007.
- [10] E. L. Key, E. N. Fowle, and R. D. Haggarty, "A method for side-lobe suppression in phase-coded pulse compression systems," MIT, Lincoln Lab, Tech. Rep. No. 209, 28 Aug. 1959.
- [11] K. R. Griep, J. A. Ritcey and J. J. Burlingame, "Poly-phase codes and optimal filters for multiple user ranging," IEEE Trans. Aerospace and Elect. Sys., vol. 31, no. 2, pp. 752-767, April 1995.
- [12] N. Levanon, "Cross-correlation of long binary signals with longer mismatched filters," IEE Proc. - Radar, Sonar and Navigation, vol. 152, no. 6, pp. 372-382, 2005.
- [13] C. Nunn, "Constrained optimization applied to pulse compression codes, and filters," IEEE Int'l Radar Conf. 2005, pp 190-194.
- [14] C. Nunn and F.F. Kretschmer, "Performance of pulse compression code and filter pairs optimized for loss and integrated sidelobe level," IEEE Radar Conf., 2007, pp. 110-115.
- [15] P. Stoica, J. Li and M. Xue, "Transmit codes and receive filters for radar," IEEE Signal Processing Mag., vol. 94, November 2008.
- [16] J. W. Taylor and H.J. Blinichoff, "Quadriphase code: a radar pulse compression signal with unique characteristics," IEEE Trans. Aerospace and Elect. Sys., vol. 24, no. 2, pp. 156-170, March 1988.
- [17] R. Chen and B. Cantrell, "Highly bandlimited radar signals," IEEE Radar Conf. 2002, Long Beach CA, pp. 220-226, April 2002.
- [18] H. H. Faust, et al, "A spectrally clean transmitting system for solid-state phased-array radars," IEEE Radar Conf. 2004, pp. 140-144.
- [19] H.C. Stankwitz, R. J. Dallaire and J. R. Fienup, "Nonlinear Apodization for sidelobe control in SAR imagery," IEEE Trans. Aerospace and Elect. Sys., vol. 31, no. 1, pp. 267-279, Jan. 1995.

## APPENDIX

MATLAB script that calculates a minimum-ISL MMF

```
x=code'; % code is a binary {±1} row vector
flenc=input('Mismatched filter length = ? ');
low_w=input('1 sided width of the near SL=? ');
mm=200; % weight assigned to the reduced near SL
lenx=length(x);
difx=flenc-lenx;
x=[zeros(ceil(difx/2),1); x ;
zeros(floor(difx/2),1)];
w=ones(1,2*flenc-1);
w(1,flenc)=0;
if low_w>=1
    w(1,[flenc-low_w:flenc-1 flenc+1:flenc+low_w])=mm;
end
wd=diag(w);
xh=hankel([zeros(1,flenc-1),x(1)], [x ; zeros(flenc-1,1)]);
b=xh*wd*xh';
h=(b\X)*(X'(b*X)\lenx);
hh=sqrt(lenx/(h'*h))*h; % norm. mismatched filter
```